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THE ADVENTURES OF SHERLOCK HOLMES

The 48% Solution

By Michael Carter

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An interesting debate on intellectual primacy was conducted through the pages of *Econometrica* in 1953. The French mathematician, Maurice Fréchet, proposed that Emile Borel should be recognised as the *initiator* of game theory. He submitted three notes by Borel originally published between 1921 and 1927. The notes (as translated by L. J. Savage) were published, along with a commentary by Fréchet and a dissenting note from von Neumann. The latter claimed that Fréchet's attribution lacked merit since Borel had been unable to prove the decisive **minimax theorem.**[1]

The minimax theorem established the **existence** of optimal strategies for every two person zero sum game. For many games, these optimal strategies are **mixed strategies**. That is optimal play requires the player to choose one of his/her pure strategies at random according to a particular probability distribution. This is done by the player conducting an appropriate random experiment at the commencement of the game.

The use of mixed strategies means that the actual outcome of a game is stochastic, depending upon the random experiments of the players. In order to cope with random payoffs, von Neumann and Morgenstern developed

[1]von Neumann reported the minimax theorem in 1926 and it was published in 1928 (von Neumann (1928)).

their theory of expected utility. This was necessary in order to provide a coherent theory of rational behaviour in a game.

Having read the exchange between Fréchet and von Neumann, I was intrigued to come across evidence that von Neumann and Morgenstern had themselves not fully absorbed the significance of this innovation at the time at which they wrote their celebrated book *The Theory of Games and Economic Behaviour*. The evidence occurs in a footnote to an example taken from *The Adventures of Sherlock Holmes*. Let von Neumann and Morgenstern describe the incident in their own words:

> Sherlock Holmes desires to proceed from London to Dover and hence to the Continent in order to escape from Professor Moriarty who pursues him. Having boarded the train he observes, as the train pulls out, the appearance of Professor Moriarty on the platform. Sherlock Holmes takes it for granted - and in this he is assumed to be fully justified - that his adversary, who has seen him, might secure a special train and overtake him. Sherlock Holmes is faced with the alternative of going to Dover or of leaving the train at Canterbury, the only intermediate station. His adversary - whose intelligence is assumed to be fully adequate to visualize these possibilities - has the same choice. Both opponents must choose the place of their detrainment in ignorance of the other's corresponding decision. If, as a result of these measures, they should find themselves, *in fine*, on the same platform, Sherlock Holmes may with certainty expect to be killed by Moriarty. If Sherlock Holmes reaches Dover unharmed he can make good his escape. (von Neumann & Morgenstern (1953) p177)

von Neumann and Morgenstern model this story as a game between the two celebrated antagonists, assigning payoffs to the four possible outcomes. These are represented in the following matrix:

		HOLMES	
		Dover	Canterbury
MORIARTY	Dover	100	0
	Canterbury	-50	100

The numbers represent the assigned payoffs to Moriarty. He is trying to maximise his payoff, which Holmes seeks to minimise.

The minimax theorem ensures that there is an optimal strategy for players. In this particular game there exists a unique pair of optimal mixed strategies {(0.6,0.4),(0.4,0.6)} which simultaneously maximise the expected payoff (minimise the expected loss) of the two players. Moriarty should carry on to Dover with probability 0.6 and get off at Canterbury with probability 0.4. Sherlock Holmes should carry on to Dover with probability 0.4. With these strategies, there is a 48% chance of Holmes being captured and killed, a 16% chance of his escaping to the Continent and a 36% chance of evading capture at Canterbury without making a complete getaway.

These are the expected returns before the journey commences. In implementing their mixed strategies, each participant will ultimately choose one of the pure strategies. In the actual play of the game, one and only one of these possible outcomes will occur. Holmes will either be killed or escape. That this was not fully appreciated by von Neumann and Morgenstern is clear from a footnote commenting on the outcome which I quote in full:

> The narrative of Conan Doyle - excusably - disregards mixed strategies and states instead actual developments According to these Sherlock Holmes gets out at the intermediate station and triumphantly watches Moriarty's special train going on to Dover. Conan Doyle's solution is the best possible under his limitations (to pure strategies), insofar as he attributes to each opponent the course which we found to be the most probable one (i.e. he replaces 60% probability with certainty). *It is, however, somewhat misleading that this procedure leads to Sherlock Holmes's complete victory, whereas, as we saw above, the odds (i.e. the value of a play) are definitely in favour of Moriarty. (Our result . . . yields that Sherlock Holmes is as good as 48% dead when the train pulls out from Victoria Station.)* (von Neumann & Morgenstern (1953) p178, fn 1. - emphasis added)

They then refer to an earlier suggestion of Morgenstern (1928) that the whole trip is unnecessary because the loser could be determined before the start!

To a modern game theorist this footnote is **extraordinary**. It confuses the expected payoff (in which there is a 48% chance *before the journey is undertaken* of Holmes's demise) with the actual outcome (in which Holmes either lives or dies). It makes no sense to talk of Holmes being 48% dead. Conan Doyle's account cannot be faulted on game theoretic grounds and Holmes would be well advised to insist on the game being played out.

This footnote suggests that von Neumann and Morgenstern did not immediately absorb the implications of the theory which they had developed. Of course it is easy with hindsight to see the fallacy in this footnote, to highlight the inconsistency with their theory. Nevertheless, in the light of the debate between Fréchet and von Neumann, it is interesting to note this difficulty that the authors had in fully comprehending their own work.

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