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A STATISTICAL SHIFT-CHOICE MODEL OF CAPITAL UTILIZATION

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A STATISTICAL SHIFT-CHOICE MODEL OF CAPITAL UTILIZATION

Roberto Abusada-Salah*

1. INTRODUCTION

Economic theory has traditionally focused on problems dealing with the level of utilization of productive capacity within the framework of short-run macro models. Thus, underutilization has almost invariably been associated with Keynesian aggregate demand deficiencies. Nevertheless, low utilization levels have characterized secular situations in many economies notably those of some developing countries.

Much of the work on capital utilization within the theory of production stems from the work by Marris which views low levels of utilization a part of rational entrepreneurial decisions in the light of day-night wage rate differentials, marked economies of scale in large plants and oligopolistic market structures.1/ Thus the theory of capital utilization appears as an added dimension of the theory of the production functions. Empirical tests of the theoretical models rest, therefore, on the ability to generate parameters of a production function consistent with the given industrial structure.

Empirical observation suggests however that in addition to the

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* I thank Christopher Clague and Daniel Schydlowsky for most valuable comments.

1/ Robin Marris, The Economics of Capital Utilization: A Report on Multiple-Shift Work, Cambridge University Press, 1964.

determinants of the level of utilization that emerge from the theory of production functions, there can be a number of other factors present in the causation of the utilization phenomenon that might have no relationship to the structure of production. Such is the case of any institutional characteristic of the labor market limiting shift work. Further, some very basic features of particular production processes like the requirement that they be continuously operated, are not explicitly captured by any estimable specification of the production function, yet when present they will obviously determine the observed level of utilization. The same is true for other determinants of the utilization level such as constraints in the availability of inputs.

Capacity Utilization as a Discrete Choice

The utilization of installed productive capacity is one of many economic choices having lumpy characteristics. Thus, the number of shifts that a given industrial plant can work can be either one, two or three. There can clearly be alternative levels of intensity of use (or speed) within any given number of shifts; even time utilization can be stretched resorting to the use of overtime work, but the basic long-run utilization decision reflects itself in the choice of one of three alternative shifting modes. Further, institutionalized rules make it extremely difficult to hire labor services for less than a full work period.

Most theoretical models of shift choice do, in fact typically identify the conditions under which a firm will move from one shift-alternative to another.[1/] The breaking point is generally a function of the profitability of the alternatives which in turn depend on the characteristics of the production function in the face of day-night input costs fluctuations and other cost elements of discrimination between shifting modes.

Therefore, an appropriate specification for a utilization variable in the context of both theory and empirical observation would be one which coincides with the trichotomous alternative.

Many empirical studies on the causes of under utilized capacity have attempted to elaborate a measure of capacity use which is constructed against a potential use level somehow defined.[2/] This index is then used as the dependent variable in regression analyses. This practice has however several drawbacks. First, since the index is

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1/ See M.A. Baily, "Capital Utilization in Kenya Manufacturing Industry", Ph.D. thesis, M.I.T., January 1974. Betancourt and Clague, "An Economic Analysis of Capital Utilization", (mimeo), Univ. of Maryland, revised 1973. P. Millan, "The Intensive Use of Capital in Industrial Plants: Multiple-Shift as an Economic Option", Ph.D. thesis, Harvard University 1975. D. Schydlowsky, "Influencia del Mercado Financiero Sobre la Utilizacion de Capacidad Instalada". Desarrollo Economico (Buenos Aires). July-September 1974. G. Winston, "Capital Utilization and Employment: A Neo-Classical Model of Optimal Shift Work", (mimeo) Williams College, December 1973.

2/ See Y.C. Kim and J.K. Kwon, Capital Utilization in Korean Manufacturing 1962-71: Its Level, Trend, and Structure, Seoul, Korea, Korea Industrial Development Research Institute, 1973; F. Thoumi, "The Utilization of Fixed Industrial Capital in Colombia: Some Empirical Findings" (mimeo) World Bank, Washington D.C., Dec. 1973; G. Winston "Capital Utilization and Economic Development", Economic Journal, 81 (1971) conceptually bounded between 0 and 100 percent, there is always the difficulty of interpreting predicted values outside that range. Second, a basic specification error is involved in defining as continuous a variable that is in fact discrete.

Clearly, there exist many ways in which a discrete utilization variable can be rendered continuous. Thus, if we look at number of hours that industrial establishments operate per day one should expect a clear cluster around 8, 16 and 24 hours. 1/ If however one considers a year instead of a day then the bunching around three points will be less marked due to variance in the number of days per annum that each plant works. Further, if we weigh the number of hours that a plants remains open by the number of workers operating each process within each shift and/or the level of production within every shift; we shall be confronted with an even more continuous-like utilization variable.

Although the above procedure might be valid for the elaboration of some capacity use index it is clearly inappropriate to study the determinants of long run utilization decisions because such an index would include the effect of seasonal demand fluctuations which influence the number of days that the entrepreneur will keep the plant open. In addition if some of the weights mentioned before are used the index

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1/ Appendix figure 5 presents a histogram of hours worked by plants in Peru. The figure shows an almost total clustering around 8, 16 and 24 hours. The frequency of firms operating 9 and 10 hours shows overtime work.

will incorporate other factors such as imbalances in the production line, differences in productivity etc. thus making inter-firm comparisons extremely difficult.

Besides being conceptually more appealing, the use of a categorical measure of capacity use (number of shifts), means that there will be no need to wrestle with the problem of defining maximum capacity utilization. In this framework, econometric models of capital utilization are required to classify any given sample into three shift categories given the values of a set of independent variables for every firm. The dependent variable in such models can be therefore interpreted as a probability distribution describing the odds that a given firm be found in each category.

This paper represents an attempt to formulate an empirical model that incorporates elements of both production function models of capital utilization as well as other determinants derived from empirical observation.

Our analytical category in the treatment of the utilization variable will be the number of shifts that the plant operates. Given the trichotomous characteristic of our dependent variable the estimation is carried out using multinomial-multivariate logit regression.

The next section is devoted to the discussion of the multinomial logit model making use of our shift-work example. Also basic reference is made to the linear probability and probit models. In addition, there is a discussion of the estimating procedure and a measure of goodness-of-fit.

Section 3 deals with the main explanatory variables of the model and presents the results of the estimation for a set of data which consists of information on 1102 Peruvian manufacturing firms. An analysis of the impact of the explanatory variables at different levels is performed in a sensitivity fashion and a summary of the prediction results are presented. Finally, the performance of the logit model is compared with that of its linear counterpart. Section 4 comments on the main findings and results.

II. DISCRETE CHOICE MODELS

The Linear Probability Model

The elements of the probability distribution can be estimated independently through OLS by a set of three linear equations having dependent variables which take the value of zero or one.

$$P_{jt} = \sum_{i=1}^{h} \beta_{ij} X_{it} + \mu_t \qquad (1)$$

$$j = 1, 2, 3$$

$$\text{Where } P_{jt} = \begin{cases} 1 \text{ if } j \text{ shifts are worked by plant } t \\ 0 \text{ otherwise} \end{cases}$$

X_i are the independent variables affecting shift work and

μ is a random disturbance term

The problems found in the estimation of OLS equations with zero/one dependent variables have been adequately discussed in the econometric literature.1/ Briefly, the main difficulties deal with the fact that linear equations are unbounded and hence the [0,1] interval (required for the interpretation of a proportion as a probability) cannot be guaranteed in the estimation of the dependent variable.

1/ See for example Goldberger, Econometric Theory, New York; Wiley, 1964.

The other difficulty is certainly more fundamental and relates to the fact that the disturbance term is not homoskedastic thus negating the OLS method.1/ Therefore, estimation would have to be carried out through generalized least squares techniques which while correcting for heteroskedasticity, cannot solve the interval problem.

An interesting feature of the linear probability model is that, while it is possible for the estimated probabilities p_j to fall outside of the [0,1] range, their sum will be one if a constant term is included in the data set.2/

1/ Since p_t can either be zero or one, μ_t will be either $-\sum_{i=1}^{h} \beta_i X_i$ or $1 - \sum_{i=1}^{h} \beta_i X_i$. If the expected value of μ_t, $E\mu_t$ is zero then the it's variance is $E\mu_t^2 = E_{pt}(1-E_{pt})$ which is dependent on the values of the X's.

2/ This can be seen when we consider the individual equations in model (1).

$$p_{1t} = X_t\beta_1 + \varepsilon$$

$$p_{2t} = X_t\beta_2 + \nu$$

$$p_{3t} = X_t\beta_3 + \mu$$

The LES of β_j's are

$$\hat{\beta}_1 = (X'X)^{-1} X'p_1$$

$$\hat{\beta}_2 = (X'X)^{-1} X'p_2$$

$$\hat{\beta}_3 = (X'X)^{-1} X'p_3$$

The sum of $\hat{p}_j$'s is $X_t\hat{\beta}_1 + X_t\hat{\beta}_2 + X_t\hat{\beta}_3$ or

$[X(X'X)^{-1} X'] 1$ since the sum of p_j's equals a vector of ones. (One of the three shift-alternatives is always present)

$[X(X'X)^{-1}X']\,1 = 1$ iff 1 is in the space spanned by columns of X (which is clearly true if X includes a constant term)

Proof: (I owe suggestions for this proof to Mr. David Jones)

Consider the data matrix in partioned form

$$X = [1 \mid X_o]$$

then
$$(X'X) = \begin{pmatrix} 1'1 & 1'X_o \\ X_o'1 & X_o'X_o \end{pmatrix}$$

and
$$(X'X)^{-1} = \begin{pmatrix} (1'1)^{-1} + (1'1)^{-1}1'X_oQ^{-1}X_o'1(1'1)^{-1} & -(1'1)^{-1}1'X_oQ^{-1} \\ -Q^{-1}X_o'1(1'1)^{-1} & Q^{-1} \end{pmatrix}$$

where $Q = (X_o'X - X'1(1'1)^{-1}1'X)$

postmultiplying by $\begin{bmatrix} 1' \\ X_o' \end{bmatrix}$ and premultiplying by $[1 \mid X_o]$, we get

$$\begin{pmatrix} (1'1)^{-1}1' + (1'1)^{-1}1'X_oQ^{-1}X_o'1(1'1)^{-1}1' - (1'1)^{-1}1'X_oQ^{-1}X_o' \\ -Q^{-1}X_o'1(1'1)^{-1}1' + Q^{-1}X_o' \end{pmatrix}$$

$$1[(1'1)^{-1}+(1'1)^{-1}1'X_oQ^{-1}X_o'1(1'1)^{-1}1'-(1'1)^{-1}1'X_oQ^{-1}X_o']$$

$$+ X_o[-Q^{-1}X_o'1(1'1)^{-1}1' + Q^{-1}X_o^{-1}]$$

postmultiplying by 1 and arranging terms. (Notice that $(1'1)^{-1} = \frac{1}{T}$ where T is the number of observations).

$$\frac{1}{T}\, 11'(I + X_o Q^{-1} X_o' \frac{1}{T} 11' - X_o Q^{-1} X')1 - X_o Q^{-1} X_o' \frac{1}{T} 11'\, 1 + X_o Q^{-1} X_o' 1$$

$$= \frac{1}{T}\, 11'(1 + X_o Q^{-1} X_o' 1 - X_o Q^{-1} X' 1) - X_o Q^{-1} X_o' 1 + X_o Q^{-1} X_o'\, 1 = 1$$

An alternative method is that of probit analysis which assumes that the critical level of the independent variable determining the "shift event" is normally distributed. The specification is

$$P_j(x) = F(\beta_o + \beta X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta_{jo} + \beta_j x} e^{-z^2/2} dz \qquad (2)$$

Where $P_j(x)$ is the conditional probability of the shift event; x is the independent variable.

This transformation guarantees the [0.1] interval for P(x) with the argument varying in the [-∞,∞] range.1/

Probit analysis has had extensive application in the biological sciences and typically probit functions have been specified, as in equation (2), as having only one explanatory variable, the reason for this being the fact that most notable applications of probit biology dealt with univariate specifications. In addition multivariate probit poses formidable computational difficulties.2/

1/ For a discussion of probit analysis see D. Finney Probit Analysis, Cambridge University Press, 1952. Also H. Theil, Principles of Econometrics, Wiley, 1971.

2/ Some of these problems are mentioned in J.E. Grizzle "Multivariate Logit Analysis", Biometrics, Dec. 1971. Also see S.H. Walker and D.B. Duncan "Estimation of the Probability of an event as a Function of Several Independent Variables", Biometrika, 1967. In both papers logit analysis is used as an approximation of probit analysis as suggested by Finney (op. cit.)

Logit Analysis

Another transformation similar to that performed in probit analysis can be conducted without the normality assumption[1/], where the probability of finding the t^{th} firm working j-shifts is related to the explanatory variables by the following function.

$$P_{jt} = \frac{\exp\left(\sum_{i=1}^{h} \beta_{ij} X_{it}\right)}{\sum_{j=1}^{J} \exp\left(\sum_{i=1}^{h} \beta_{ij} X_{it}\right)} \qquad (3)$$

where J is the number of shift-categories (three) and h is the number of independent variables. Our problem would then involve the estimation of three equations (one for each shift event) which can be expressed in the following manner. First, normalize with respect to the "three-shifts" category making β_{i3} constant, thus (3) can be written as

$$\frac{P_{jt}}{P_{3t}} = \exp\left(\sum_{i=1}^{h} \alpha_{ij} X_{it}\right) \qquad (4)$$

$$j = 1, \ldots, J - 1$$

$$\text{or} \quad j = 1,2$$

In effect, normalization can be carried out by setting the β_{i3}'s equal

1/ Actually for the treatment of the problem at hand it is more appropriate to disregard normality since there is no reason for assuming that the critical values of the variables explaining the shift event for each observation should be normally distributed.

to zero and (4) written as

$$\frac{p_{jt}}{p_{3t}} = \exp(\sum_{i=1}^{h} \beta_{ij} X_{it}) \tag{5}$$

Therefore, after normalization there are only two equations to estimate

$$\frac{p_{1t}}{p_{3t}} = \exp(\sum_{i=1}^{h} \beta_{i1} X_{it})$$

or

$$\ln \frac{p_{1t}}{p_{3t}} = \sum_{i=1}^{h} \beta_{i1} X_{it} \tag{6}$$

and

$$\ln \frac{p_{2t}}{p_{3t}} = \sum_{i=1}^{h} \beta_{i2} X_{it} \tag{7}$$

The left hand side of equations (6) and (7) are called logits and vary in the range $-\infty, \infty$ as p_{ij} moves between 0 and 1. Equations (6) and (7) also permit us to solve for p_{1t}, p_{2t} and p_{3t} since the three add up to 1--for every firm one of the alternatives is realized--therefore,

$$p_{1t} = \frac{\exp(\sum_{i=1}^{h} \beta_{i1} X_{it})}{1 + \exp(\sum_{i=1}^{h} \beta_{i1} X_{it}) + \exp(\sum_{i=1}^{h} \beta_{i2} X_{it})} \tag{8}$$

$$p_{2t} = \frac{\exp(\sum_{l=i}^{h} \beta_{i2} X_{it})}{1 + \exp(\sum_{i=1}^{h} \beta_{i1} X_{it}) + \exp(\sum_{i=1}^{h} \beta_{i2} X_{it})} \tag{9}$$

$$P_{3t} = \frac{1}{1 + \exp\left(\sum_{i=1}^{h} \beta_{i1} X_{it}\right) + \exp\left(\sum_{i=1}^{h} \beta_{i2} X_{it}\right)} \qquad (10)$$

Notice that in our example, the absence of information (i.e. $\sum_{i=1}^{h} \beta_i X_i = 0$, for all j's) implies simply that all shift alternatives have the same probability and equal to one third ($e^0/1+e^0+e^0$).

Estimation

The difficulty in the estimation of equations (6) and (7) stems from the fact that the odds in the left hand side of equations (6) and (7) are not directly observable and hence they would have to be obtained first by constructing a contingency table where the probability of the shift-event j is the relative frequency of firms that have the characteristics X_i appearing in the j^{th} cell.

This method of estimation is readily applied when the X's are categorical, or dummy, variables.1/ In our case, however, as in the majority of economic applications, most explanatory variables are of the continuous kind.

1/ Extensive work on different estimating methods of models involving relationships among categorical data has been done by L. Goodman, see for example "The Analysis of Multinomial Contingency Tables: Stepwise Procedures and Direct Estimation Methods for Building Models for Multiple Classifications", Technometrics, Vol. 13 No.1, Feb. 1971.

One solution to this problem is to categorize the continuous variables, thus making possible the construction of a contingency table. Notwithstanding its validity, the method produces a score of empty or nearly-empty cells except when the number of observations is very large and the number of variables is small.

An alternative way to solve the estimation problem is through the maximum likelihood method where the estimated β_{ij}'s are those maximizing the likelihood function.1/ Remembering that this function is the joint density function and that individual densities are expressed in (8), (9) and (10),

$$L = \prod_{t=1}^{T} \prod_{j=1}^{J} \left[\frac{\exp(\sum_{i=1}^{h} \beta_{ij} X_{it})}{\sum_{j=1}^{J} \exp(\sum_{i=1}^{h} \beta_{ij} X_{it})} \right]^{Q_{jt}} \qquad (11)$$

where $Q_{jt} = \begin{cases} 1 \text{ when the firm works } j \text{ - shifts.} \\ 0 \text{ otherwise.} \end{cases}$

If observations in our problem are ordered so as to have first all single-shift firms (T_1 in all), followed by all firms working two shifts (T_2) and finally all firms working three shifts ($T - T_1 - T_2$) then equation (11) can be written,

$$L = \prod_{t=1}^{T_1} \frac{e^{z1}}{1 + e^{z1} + e^{z2}} \prod_{t=T_1+1}^{T_2} \frac{e^{z2}}{1 + e^{z1} + e^{z2}} \prod_{t=T_2+1}^{T} \frac{1}{1 + e^{z1} + e^{z2}} \qquad (12)$$

1/ A discussion of the MLE of the binomial logit model can be found in S. Warner, Stochastic Choice of Mode in Urban Travel: A Study in Binary Choice, Northwestern Univ. Press, 1962.

where $zj = \sum_{i=1}^{h} \beta_{ij} X_{it}$ $\qquad j = 1, 2$

and the log-likelihood function is

$$\ln L = \sum_{t=1}^{T_1} [z_1 - \ln(1 + e^{z_1} + e^{z_2})] + \sum_{t=T_1+1}^{T_2} [z_2 - \ln (1 + e^{z_1} + e^{z_2})]$$

$$+ \sum_{t=T_2+1}^{T} - \ln (1 + e^{z_1} + e^{z_2}) \tag{13}$$

$$= \sum_{t=1}^{T_1} z_1 + \sum_{t=T_1+1}^{T_2} z_2 - \sum_{t=1}^{T} \ln (1 + e^{z_1} + e^{z_2}) \tag{14}$$

The necessary conditions for a maximum are obtained by getting the vector of first derivatives of lnL with respect to β_{ij}.

$$\frac{\partial \ln L}{\partial \beta_{ij}} = \sum_{t=s}^{S} Xij - \sum_{t=1}^{T} Xij \frac{e^{z_j}}{1 + e^{z_1} + e^{z_2}} = 0 \tag{15}$$

$$\text{where } s,S = \begin{cases} 1, \ldots, T_1 \text{ when } j = 1 \\ T_1+1, \ldots, T_2 \text{ when } j = 2 \end{cases}$$

or

$$\sum_{t=s}^{S} X_{ij} - \sum_{t=1}^{T} X_{ij} P_{jt} = 0 \tag{16}$$

The iterative procedure in our estimation uses the Newton-Raphson method which corrects the set of trial values for the β_{ij} coefficients by computing linear adjustments using the vector of first derivatives of lnL and the matrix of second derivatives.1/ The adjustment is computed as

$$\varepsilon_j = - \left[\frac{\partial \ln L}{\partial \beta_j} \right] \left[\frac{\partial^2 \ln L}{\partial \beta_j^2} \right]^{-1} \qquad (17)$$

ε_j are added to β_j and the process is then repeated until (15) is approximated.

Goodness-of-Fit Measures

When the dependent variable is, as in this case, a probability distribution, the R^2 statistic is inappropriate as a measure of goodness-of-fit. The main reason is the added dimension of correctness or incorrectness of the predicted value for each element of the distribution. Moreover, the R^2 statistic, while having some validity for a dichotomous case where only one logit equation is fitted, cannot be computed when the probability distribution comprises more than two alternatives.

In this paper we will use a two measures of goodness-of-fit based on

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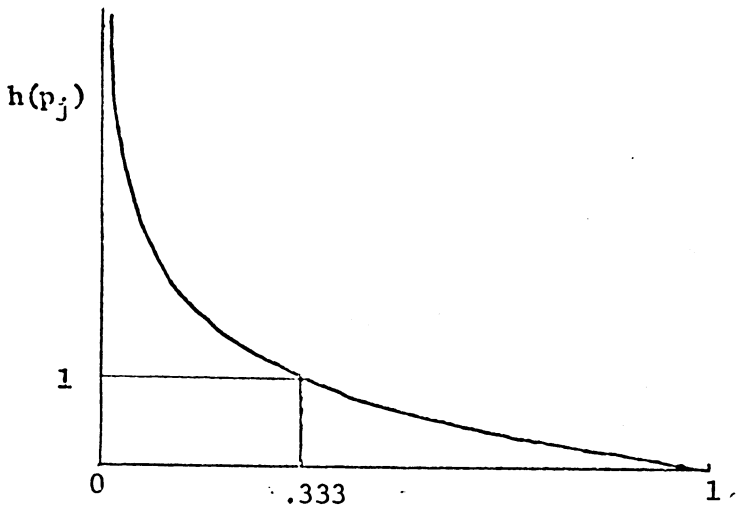
1/ The estimation is preformed with the "Computer Programs for Cross-Section Data - Version 3" by M.G. Kohn.

the theory of information.[1/]

For each element of the probability distribution the information content can be expressed as a decreasing function $h(p_j) = \log \frac{1}{p_j}$ of the probability having the shape depicted in fig. 1.

Figure 1



$h(p_j)$ measures the level of "surprise" if j would actually take place. Clearly if p_j is close to one and in fact the shift-event j takes place, there will be little or no suprise. The unit in which the level of surprise is measured depends on the logarithmic base. Notice that in our case we could use logarithms of base 3 which would yield 1 unit when $p_j = 1/3$ (an even probability for all events).

For every predicted probability distribution we can calculate the "expected surprise" as

$$H_t = p_1 \log \frac{1}{p_1} + p_2 \log \frac{1}{p_2} + p_3 \log \frac{1}{p_3} \qquad (18)$$

1/ The basic concepts for the construction of these measures are found in Theil op.cit.

or

$$Ht = -[p_1 \log p_1 + p_2 \log p_2 + p_3 \log p_3] \qquad (19)$$

H is called the entropy of a distribution and will take its maximum value when all pj's are equal. Again this value will be 1 when the log-base is 3.

For the sample as a whole, we can calculate an expected surprise, on average prior amount of uncertainty, in the absence of any knowledge about the independent variables as the entropy which corresponds to the pj's given by the marginal distribution (i.e. the proportions of pj in the sample).

$$\xi\ (P) = -[T_1/T \log(T_1/T) + T_2/T \log(T_2/T) + p_3 \log(T_3/T) \qquad (20)$$

where T_j is the number of firms working j shifts.

Now we can ask by how much is this average level of uncertainty reduced when the independent variables are introduced. In order to answer this question consider again equation (11) which defines the likelihood function. For the trichotomous case equation (11) can be written as

$$L = \prod_{t=1}^{T} p_1(X_{it})^{Q_{1t}} \ p_2(X_{it})^{Q_{2t}} \ p_3(X_{it})^{Q_{3t}} \qquad (21)$$

and the log-likelihood is

$$\text{Log } L = \sum_{t=1}^{T} [Q_{1t} \log p_1(X_{it}) + Q_{2t} \log p_2(X_{it}) + Q_{3t} \log p_3(X_{it})] \quad (22)$$

Notice that the maximization of the log-likelihood function is equivalent to the minimization of total uncertainty. Also notice that if we divide equation (22) by the number of observations in the sample a measure of minimum average uncertainty is obtained, thus

$$\xi(p/X_i) = \frac{-\log L}{T} \quad (23)$$

Hence a goodness-of-fit measure analogous to the statistics based on variance can be defined as

$$\xi = \frac{\xi(p) - \xi(p/X_i)}{\xi(p)} \quad (24)$$

ξ is the percent reduction in uncertainty or the relative amount of information gained after the independent variables are introduced.[1/] ξ will vary between zero and one. Clearly, for ξ to be one it is required that every event is predicted with probability of 1.

Another measure of goodness-of-fit was recently suggested by Betancourt and Clague.[2/] This measure, called $\overline{I}$, is constructed by

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1/ The ξ coefficient is totally analogous to the uncertainty coefficient used in the analysis of contingency tables. For a discussion of the uncertainty coefficient see William Hays Statistics for Psychologists, Holt, Rinehart and Winston, 1963.

2/ Betancourt and Clague, "An Econometric Analysis of Capital Utilization" (mimeo), University of Maryland, May 1974.

calculating H_t in equation (18) for each observation. Then, for each case the amount of information is taken as

$$I_t = 1 - H_t \qquad (25)$$

or

$$I_t = 1 - H_t/H_{(1/3)}$$

if the log-base is not 3

where $H_{(1/3)}$ is the entropy of the "even-odds" distribution.

Finally the total amount of correct and incorrect information is obtained as

$$Ic = \Sigma_t I_t \qquad \text{for all correct predictions} \qquad (26)$$

and $$I_I = \Sigma_t I_t \qquad \text{for all incorrect predictions}$$

and $\overline{I}$ is then defined as

$$\overline{I} = \frac{(Ic - I_I)}{T} \qquad (27)$$

$\overline{I}$ will take values between -1, when all actual events had a predicted probability of zero, and 1 when actual events were predicted with total certainty ($p = 1$).

Although based on the same principles, ξ has the advantage of being independent of the criterion used to establish the level of the predicted probability beyond which the information is considered correct or incorrect. Further, ξ is more readily interpreted since it simply represents the percent reduction in uncertainty in an analogous meaning to that which is attributed to the statistics based on variance.

An Ordinal Measure of Fit [1/]

The measures of goodness-of-fit that have been presented before (ξ and $\bar{I}$) while being appropiate for the logit model, do not take into consideration "how far" are predictions away from the actual shift-event. This is a major drawback in the case where the events that the model predicts are ordered. Thus, if a firm operates a single shift and a particular model predicts it to be working three shifts, then an alternative model that predicts the same firm to be working two shifts can be considered better since the distance from the actual event is smaller. The purpose of this note is to present a measure of fit suitable to be applied in cases where ordered alternatives are present.

$$\text{Let} \quad Q_{jt} = \begin{cases} 1 \text{ if j-shifts are worked by firm t} \\ 0 \text{ Otherwise} \end{cases}$$

$$j = 1, \ldots, J$$

$$t = 1, \ldots, T$$

1/ The measure of fit presented here was suggested to me by Professor Thomas A. Louis of the Department of Mathematics at Boston University. This measure is the subject of a forthcoming paper by Louis.

Let also p_j be the marginal probability of j shifts

$$p_j = \frac{Tj}{T} \tag{28}$$

where Tj is the number of firms operating j shifts.

Define the cumulative distribution of p_j as

$$Oj = p_1 + \ldots + p_j \quad ; \quad O_J = 1 \tag{29}$$

also the cumulative distributions of Q_{jt} as

$$S_{jt} = Q_{j1} + \ldots + Q_{jt} \; ; \; S_{Jt} = 1 \tag{30}$$

Let

$$d_t^M = \sum_{j=1}^{J} (S_{jt} - O_j)^2 \tag{31}$$

Then the marginal distance function is

$$d^M = \sum_{t=1}^{T} d_t^M \tag{32}$$

For the choice model distance, let p_{jt} be the probabilities generated by the model for the t^{th} firm

$$O_{jt} = p_{1t} + \ldots + p_{jt} \; ; \; O_{Jt} = 1 \tag{33}$$

Then the choice model's distance is

$$d_t = \sum_{j=1}^{J} (S_{jt} - O_{jt})^2 \tag{34}$$

and

$$d = \sum_{t=1}^{T} d_t \tag{35}$$

a distance measure of fit can then be defined as

$$\delta = \frac{d^M - d}{d} \tag{36}$$

δ is then the percent reduction in distance when the independent variables in the model are introduced. The range for δ goes from 0 (when the model fails to reduce distance) to 1 (when the model drives the distance to zero.) The δ measure will obviously be useful when the model at hand has three or more alternatives.

It should be stressed that in the actual computation of the δ's for the conditional logit and linear trichotomous models, we used the p_{jt}'s generated by those models. Strictly, the p_{jt} should be generated by a minimum distance method so as to produce the best δ.

3. A STATISTICAL SHIFT-CHOICE MODEL FOR THE PERUVIAN INDUSTRIAL SECTOR

The data used in the estimation of the model corresponds to firms with more than 19 workers which answered the 1971 industrial census questionnaire. The total number of firms in the more-than-19- workers category is 1474. The number of firms that entered the estimation is 1102. 291 firms were excluded because they had incomplete information, and in addition 81 more firms were eliminated as having unreliable value added or capital stock figures.[1/]

The 1102 firms included account for the bulk of the Peruvian modern industrial production representing approximately 74% of total manufacturing output and 55% of the employment generated by non-cottage industry.

In addition the author conducted 40 in-depth interviews with general and production managers in an equal number of firms.

The characteristics of firms which enter the model are:

(i) Size. It is hypothesized that as the size of the firm increases, the organizational problems of shiftwork diminish. The extreme example of these difficulties would be the case of cottage industry where continuous operation would entail no physical rest. Further, large

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1/ The criterion of exclusion was: value added/capital stock > 10.

firms are bound to have administrative structures which are less dependent on family work-force for night-time supervisory positions.

In the construction of a variable to represent size in a utilization model one has to take into consideration the fact that most variables used for this purpose, such as number of workers, production or value added, are affected in their magnitude by utilization itself and thus this reverse causality has to be eliminated to avoid the tautological relationship between shift work and size.

One alternative is to divide the size variable by the number of shifts that the plant works. This correction would however be too great in view of the fact that not all shifts are of the same size. The appropriate measure would then have to be "size in the first shift", as opposed to average shift-size.

The census does not provide any information on shift-size differentials which are known to be great.1/ Therefore presently we will use shift size differential data in terms of workers published by the United Nations for 5 countries.2/

Thus the size variable used will be value added adjusted for the number of shifts (i.e. value added in the first shift).

1/ The differences in plants of the interview sample were as large as 5 to 1 in terms of workers employed in different shifts.

2/ United Nations Industrial Development Organization, Profiles of Manufacturing Establishments. The computed correction coefficient used is 1.6 when the plant works 2 shifts and 2.2 when it works 3. The coefficients imply second and third shifts equal in size and 40% smaller than the first shift. The tabulations of the 5 country shift-size differentials (UNIDO data) were kindly made available to me by Christopher Clague. The correction coefficients are then taken as an average.

(ii) Capital Intensity. In all theoretical models of capital utilization there exists a positive relationship between capital use and the capital-labor ratio or the rental-wage ratio. The result is very much intuitive since the penalties for maintaining idle capital are much greater the larger the capital intensiveness of the process.

In our model we use a ratio of the capital stock to the number of workers corrected for the number of shifts in the manner explained above. The figure of capital stock used corresponds to the book value adjusted by accounting depreciation and revaluation of assets.

(iii) Capital Productivity. Variations in this variable are presumably very important since they respond to both differences in the intensity of non-capital inputs as well as the level of profits. Since the variable is measured as the ratio of value-added on the first shift to capital stock, material inputs are excluded and therefore variations in the labor-intensity of the process, the amounts of working capital used and the profits generated become the main factors affecting average productivity.1/

(iv) Continuous Processes. All plants having some production processes with continuous characteristics were assigned a dummy value of 1. The presumption being that a plant having continuous processes, whatever

1/ For an analysis of the relation between working capital and shift work see D. Schydlowsky, "Influencia del Mercado Financiero sobre la Utilizacion de Capacidad Instalada", op cit. Also C. Clague "The Profitability of Shift-Work in Imperfect Capital Markets" (mimeo) University of Maryland, July 1975 their relative importance will force solutions on some organizational problems limiting shift work.[1/] Of course, plants with mainly continuous processes are likewise differentiated by the same dummy.

(v) Foreign Capital. Firms having any participation of foreign capital in ownership were assigned a dummy value of 1, in order to investigate whether there is a higher preference for shift work embodied in foreign financial capital.

(vi) Market Share. To analyze the effect of market structure on utilization, a variable was constructed as a ratio of the firm's sales to total industry (3-digit old ISIC) sales.

In addition to the above variables, quadratic terms for the "capital productivity" and "capital intensity" variables were entered in the model to account for parabolic influence of these variables especially in the case of the two-shift category whose probability one would expect to be influenced by continuous variables in a U-shaped fashion.

Finally, 15 different industry constants (2-digit ISIC-Rev.1) were entered in the model.[2/]

1/ The dummy was assigned on an industry basis at a very disaggregated level. The industries chosen are (ISIC Revision 1) 205, 207, 209, 213, 271, 311, 321, and 334.

2/ The five remaining constants had to be excluded as causing non-convergence of the iterative procedure.

Estimation Results

The results of the estimation based on the 1102 plants are shown in table 1. As opposed to linear regression coefficients which can be readily interpreted, the coefficients of a logit model need first to be converted into the probability distribution which their given values imply. This is so because the dependent variable in every logit model equation (two in our case) is not a probability of an event but the log-odds of one event expressed in terms of another event. Under these circumstances the coefficient of a given independent variable for one equation has no complete meaning without reference at least to the value of the coefficients in the other equations. In addition, logit model coefficients will obviously have differential impact depending on the value taken by the corresponsing independent variable unless, of course, the independent variable in question takes only one non-zero value as in the case of dummies.

In order to analyze the impact of an independent variable in the model we shall set the coefficients of all other variables equal to zero, and the impact of a given variable will be viewed as modifying a uniform distribution which is used as a base line and is described by equations (8), (9), and (10) when all β_{ij} are zero.[1/]

In the case of the two dummy variables in our model (continuous process and foreign capital) the implied conditional probability dis-

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1/ $$p_1 = p_2 = p_3 = \frac{e^0}{1 + e^0 + e^0} = \frac{1}{3}$$

TABLE 1

MULTINOMIAL LOGIT PROBABILITY MODEL OF SHIFT CHOICE. DEPENDENT VARIABLES ARE THE LOG-ODDS OF ONE SHIFT AND TWO SHIFTS WITH RESPECT TO THREE SHIFTS. STANDARD ERRORS IN PARANTHESIS

Independent Variable	$\ln P_1/P_3$	$\ln P_2/P_3$
Constant a/	5.97 (1.26)	3.11 (1.29)
Size (thousands of soles)	-0.878×10^{-5} (0.295×10^{-5})	-0.829×10^{-5} (0.318×10^{-5})
Capital Intensity (thousands of soles)	-0.367×10^{-2} (-0.609×10^{-3})	0.238×10^{-3} (0.942×10^{-3})
Capital Productivity	0.615 (0.242)	0.445 (0.269)
Continuous Processes	-1.86 (0.387)	-2.09 (0.460)
Foreign Capital	-0.354 (0.276)	0.298) (0.274)
Market Share	-1.55 (1.20)	-0.111 (1.247)
Capital Intensity (squared)	0.414×10^{-6} 0.759×10^{-7}	-0.600×10^{-6} (0.509×10^{-6})
Capital Productivity (squared)	-0.345×10^{-1} (0.350×10^{-1})	-0.238×10^{-1} 0.382×10^{-1}

$\xi = .334$

$\bar{I} = .321$

$\ln L = -677.37$ $\delta = .439$

a/ Model includes 15 additional industry constants tributions are readily obtained using equations (8), (9) and (10).[1/] The distributions for the two dummies are as follows:

	p_1	p_2	p_3
Continuous Processes	.12	.10	.78
Foreign Capital	.23	.44	.33

In the case of both variables their effect on the distributions is to tilt them towards multiple shifting.

For the continuous variables the procedure used to assess their influence is similar except that it is repeated for several values of the variable. In the case of those variables having quadratic terms, the latter are included in the calculation of the probability distribution while all the rest are, again, set at zero. The sensitivity analysis is conducted in such a way as to start with values of the variable small enough to render almost equal probabilities for the three shift-events. In figures 2-5 we graph the effects on the probability distribution of increasing the value of one dependent variable from a small arbitrary value-12% of its mean-to a large value: 8 times its mean.

The effect of the size variable on the probability distribution is to monotonically increase the probability of three shifts while p_2 and p_1 diminish in the same fashion. Notice that for the extreme value of size(on the right of fig. 2) the plant works 3 shifts with

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1/ Therefore,

$$p_1 = \frac{e^{\beta 1}}{1 + e^{\beta 1} + e^{\beta 2}}, \quad p_2 = \frac{e^{\beta 2}}{1 + e^{\beta 1} + e^{\beta 2}}, \quad p_3 = \frac{1}{1 + e^{\beta 1} + e^{\beta 2}}$$

where β_j is the coefficient of the non-zero variable for the jth shift-category.

a probability higher than .70. The result is consistent with the shift pattern of the relatively few firms in that size bracket operating in Peru. In addition, the exercise shows almost equal chance for non-three shift plants.

The case of the capital intensity variable is rather interesting since it shows a markedly higher probability of multiple shifts than of single shift even at mild levels of capital intensity. Beyond the level of the mean the probability of two shifts ceases to grow and starts to diminish joining the single-shift trend. Notice that the mean level is relatively low for modern manufacturing.1/ At that level the probability of working one shift is already very small (.12).

The patterns of influence of the capital productivity variable (Fig. 4) seem puzzling at first glance. The reason for the reversing of the trends beyond certain critical value of the variable is, however, obvious when we consider the fact that such a value is greater than 8.0 (meaning a value-added-per-year figure in excess of 8 times the total value of the assets) and that the coefficient of the quadratic term (having opposite sign) is included in the calculation. Furthermore, as mentioned earlier, plants with extremely high values for this variable were excluded from the sample. Therefore, in the relevant range, the variable behaves as hypothesized: The larger the capital productivity

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1/ The exchange rate is: 1 sol = .02 dollars, approx.

Fig. 2

Effects on the Predicted Conditional Probability Distribution of Shiftwork Caused by the Increase in Plant Size Assuming no Influence of Other Independent Variables

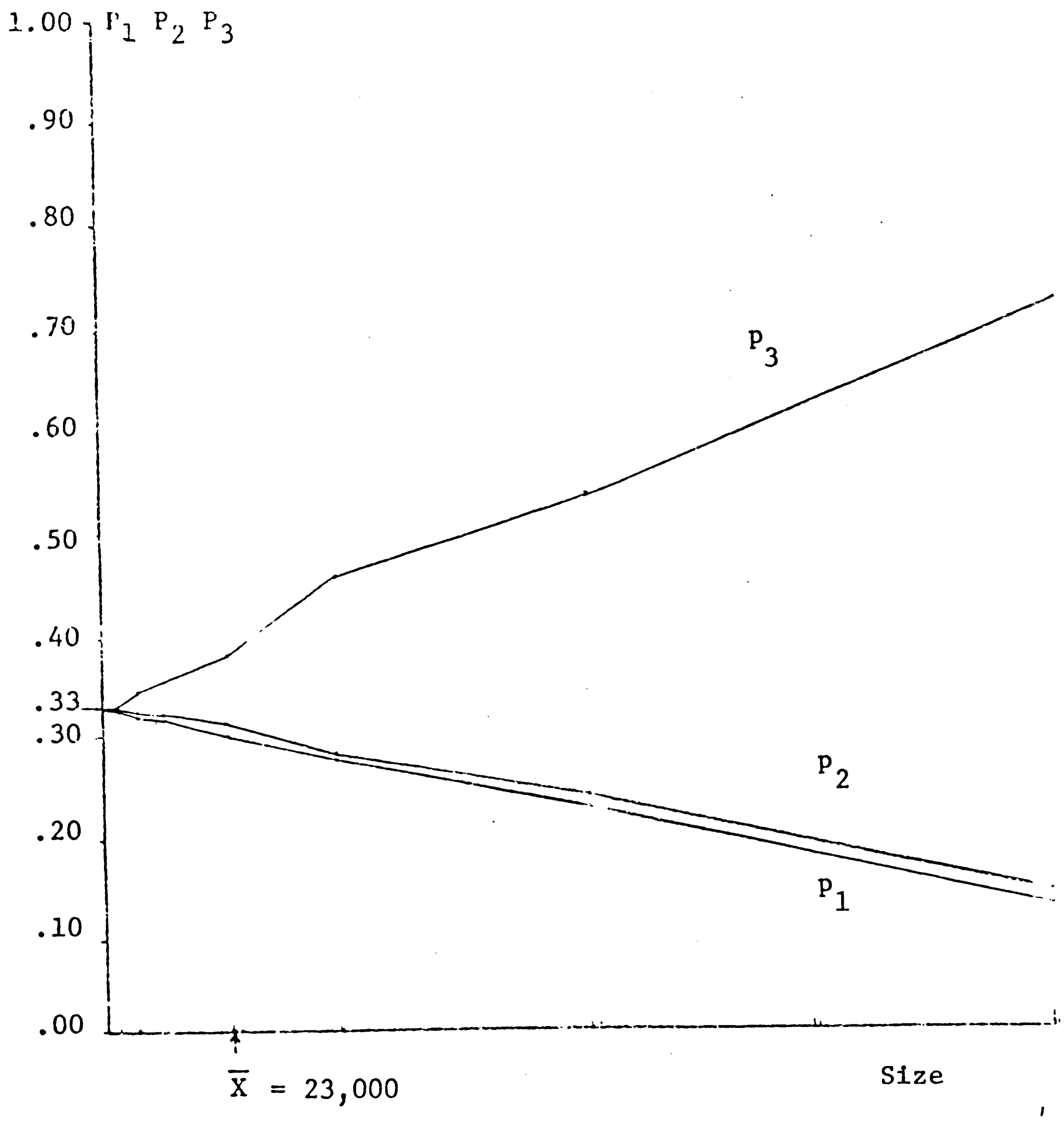


Fig. 3

Effects on the Predicted Conditional Probability Distribution of Shiftwork Caused by the Increase in Capital Intensity Assuming no Influence of other Independent Variables

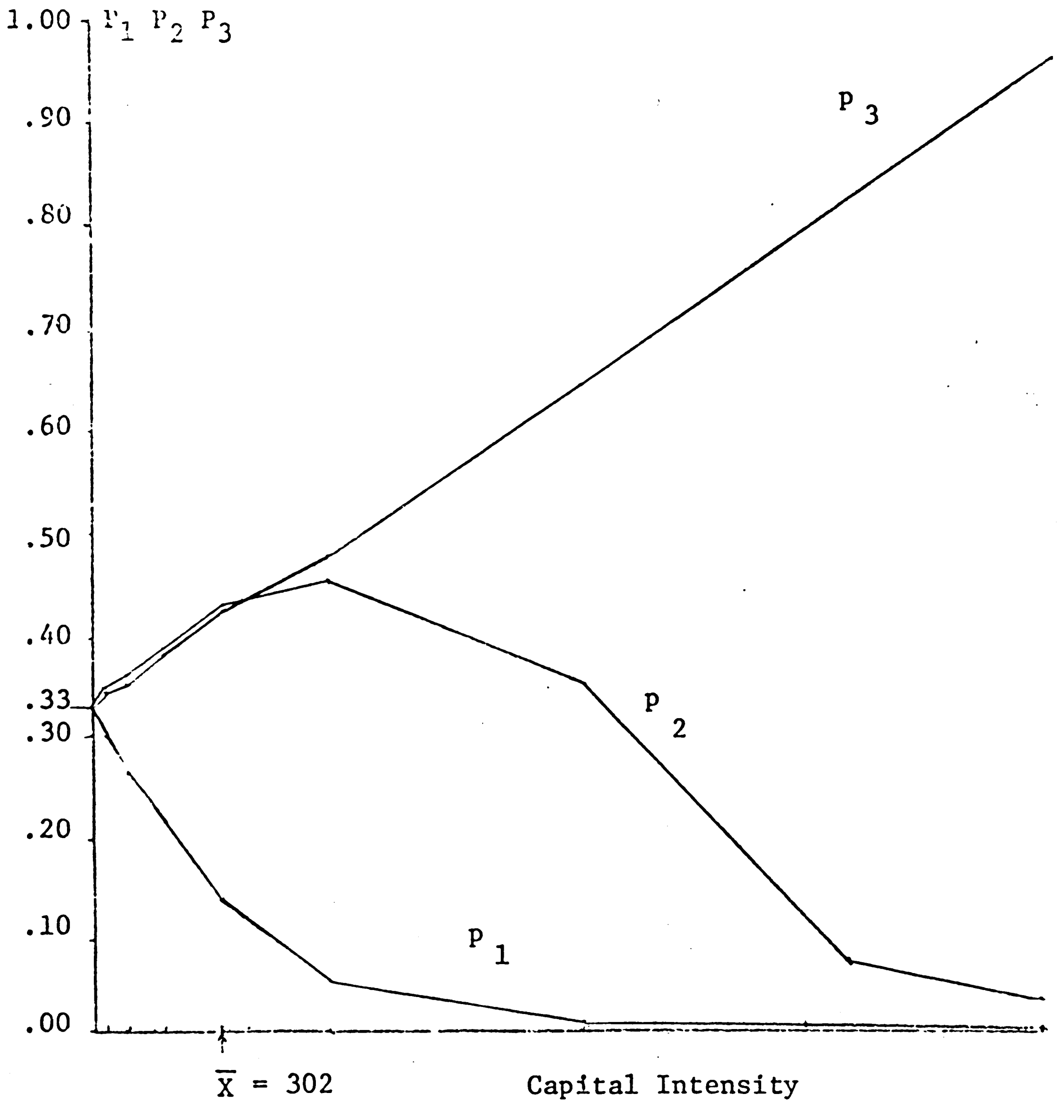


Fig. 4

Effects on the Predicted Conditional Probability Distribution of Shiftwork Caused by the Increase in Capital Productivity Assuming no Influence of other Independent Variables

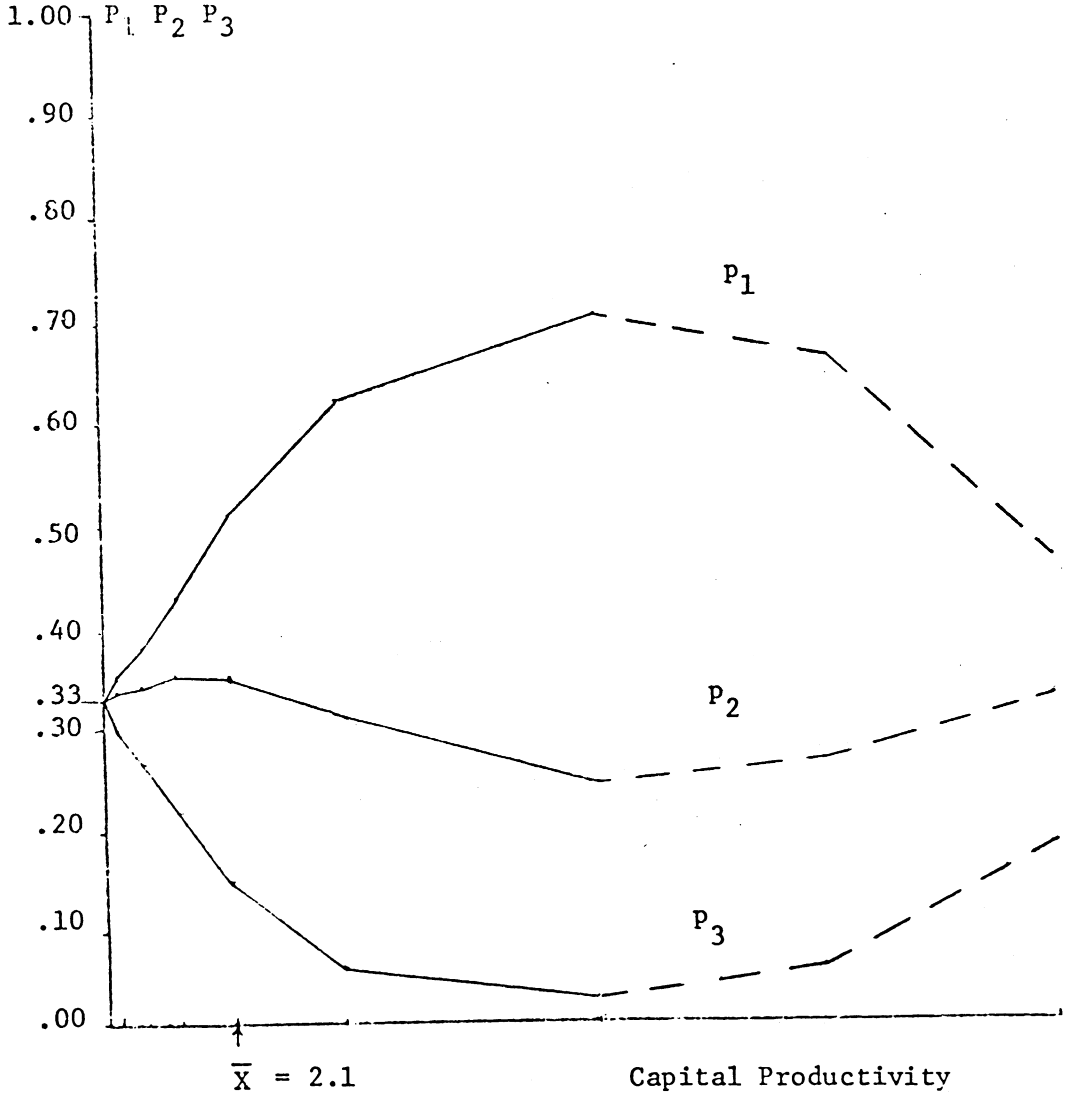
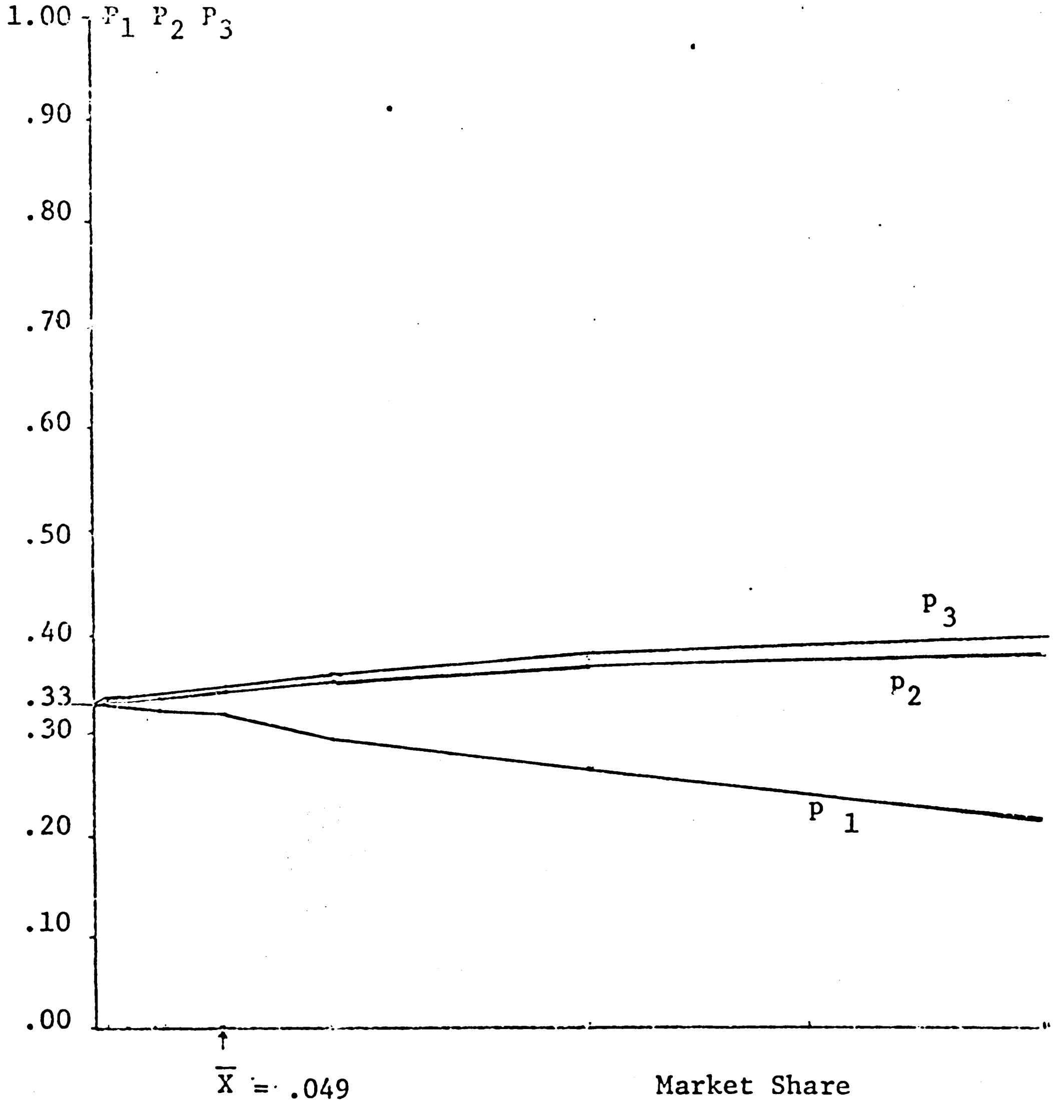


Fig. 5

Effects on the Predicted Conditional Probability Distribution of Shiftwork Caused by the Increase in Market Share Assuming no Influence of Other Independent Variables



(reflecting the intensity of non-capital inputs such as labor and working capital) and the higher the profit level, the lower the probability of multiple shifting.

Finally, the effect of the market share variable appears to be not very dramatic. However the probability of working one shift for the monopolist is less than .10. This result contradicts partial evidence for the U.S. on the positive relation between excess capacity and tight oligopolistic market structures. 1/

The current sensitivity exercise, while giving an idea about the effect of any given variable when its level is changed, has the drawback of assuming all others at specific constant levels (zero in this case). The same exercise was therefore repeated assuming that all continuous variables are set at the level of their mean except for the one that is being analyzed and dummies are alternatively turned "on" or "off". These results are shown in appendix tables 2-5. The base for these exercises will now be no longer $p_j = 1/3$ but whatever distribution emerges from the logit equations with the value of the variable being analyzed set at zero and the rest at the level of their mean.

In this case the value taken by the dummy variables turns out to be of decisive importance. When both are "off" the experiment yields a pattern similar to the one depicted on figures 1-4, although, being the starting base different (biased against single shifting) the values

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1/ See F. and L. Esposito "Excess Capacity and Market Structure", Review of Economics and Statistics, May 1974. Their analysis is, however, based on utilization data based on preferred vs. actual operating rates which miss the shift-work phenomenon.

of the distributions are markedly different. When, on the other hand, the dummies are turned "on", multiple shifting is almost certainly guaranteed: two shifts when the continuous processes dummy is zero, three shifts when it is one and three shifts also when both the foreign and continuous processes dummies are one. Again three shifts are guaranteed when only the continuous processes dummy is one. One must however bear in mind that only a small proportion in the sample have "turned on" dummies.

Predictive Performance

In the context of a probabilistic model, predictive performance has to be defined in terms of the ability of the model to discriminate effectively among the possible events given the values of each observation's characteristics.

Out of the 1102 observations in the sample the model predicted the actual shift-event in 833 cases (75.6%). The value of ξ is .334 Prediction results are summarized in Table 2 which orders the data horizontally according to the actual number of shifts that each plant operates; and vertically according to the number of shifts that the conditional logit model predicts.

The model performs very well in predicting the one and three shifts categories and rather poorly at predicting two shifts. Out of 806 plants which the model predicts as working one shift, 80.3% are actually in that category. Also a large proportion (66.8%) of firms predicted as working three shifts, do so; however only 42.9% of firms predicted as working two shifts do in fact work that number of shifts.

The two-shift category emerges there as having very fuzzy characteristics. The model in fact "overshoots" the one-shift category in 159 cases and 104 of them are in fact operating two shifts which would suggest that these firms have characteristics no different from those working one shift. Further, the model "under-shoots" the three-shift case and predicts two-shifts in 21.9% of the cases actually working three shifts.

All this of course suggests that our three analytical categories are in the real world ranked with respect to most characteristics with the two shift category lying between one and three shifts. It is, nevertheless, possible to envisage a model which would contain elements that are highly discriminatory between both one and two shifts as well as two and three shifts. These charactersitics would in fact have to replicate the double-dichotomy pattern separating two from one and three shifts.

In the analysis of capacity utilization only very sharp day-night input costs fluctuations could provide for such a discriminatory variable. In the Peruvian setting no such variables can be justified

TABLE 2

CLASSIFICATION OF THE CONDITIONAL LOGIT MODEL PREDICTIONS ACCORDING

TO ACTUAL OBSERVATIONS.

PERCENTAGES IN PARANTHESIS

	OBSERVATIONS			
MODEL PREDICTION	One Shift	Two Shifts	Three Shifts	Total
Predicts one shift	647 (80.3)	104 (12.9)	55 (6.8)	806 (73.1)
Predicts two shifts	9 (18.4)	21 (42.9)	10 (38.8)	49 (4.4)
Predicts three shifts	28 (11.3)	54 (21.9)	165 (66.8)	247 (22.4)
Total	684 (62.1)	179 (16.2)	239 (21.7)	1102 (100.0)

except perhaps for the implicit wage rate for which there is evidence of mild fluctuations. 1/

Finally, a factor leading to the lack of good prediction in the two-shift case is the fact that in the estimation procedure only 16.2% of the sample belonged to that category. This is in addition reflected in the relatively larger standard errors of the two-shift logit equation coefficients.

The Dichotomous Case

The difficulties mentioned above led us to the estimation of a model where the two and three-shifts categories are lumped together. The estimated equation and a summary of the prediction results are shown in tables 3 and 4a-b. Signs behave in the same manner as in the trichotomous case replicating as expected, the pattern of the P_1/P_3 log-odds equation. The prediction qualities of the dichotomous model are obviously better but on the other hand the underlying specification is inferior since a dichotomy is being forced upon what is really a trichotomy. The goodness-of-fit value is as expected very similar (.368).

In the dichotomous case the effect of the two dummies in the model seems to be quite strong. The implied multiple shift probability is

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1/ See P. Millan, op.cit.

TABLE 3

COEFFICIENTS OF THE BINOMIAL LOGIT PROBABILITY MODEL OF SHIFT CHOICE. DEPENDENT VARIABLE IS THE LOG-ODDS OF SINGLE SHIFTING WITH RESPECT TO MULTIPLE SHIFTING ($\ln P_1/P_2$ or 3) STANDARD ERRORS IN PARANTHESIS.

INDEPENDENT VARIABLE	
CONSTANT[a/]	2.56 (0.411)
Size (thousand of soles)	-0.372×10^{-5} (0.218×10^{-5})
Capital Intensity	-0.343×10^{-2} (0.555×10^{-3})
Capital Productivity	0.416 (0.169)
Continuous Processes	-1.00 (0.315)
Foreign Capital	-0.508 (0.220)
Market Share	-1.56 (0.980)
Capital Intensity (squared)	0.375×10^{-6} (0.701×10^{-7})
Capital Productivity (squared)	-0.244×10^{-1} (0.213×10^{-1})

$\xi = .368$

$\bar{I} = .323$

$\ln L = -462.72$

a/ Model includes 15 additional industry constants

TABLE 4a

CLASSIFICATION OF THE CONDITIONAL DICHOTOMOUS LOGIT MODEL PREDICTIONS

ACCORDING TO ACUTAL OBSERVATIONS.

PERCENTAGES IN PARANTHESIS

	OBSERVATIONS			
MODEL PREDICTION	One Shift	Two Shifts	Three Shifts	Total
Predicts one Shift	606 (83.0)	89 (12.2)	35 (4.8)	730 (66.2)
Predicts more than one Shift	78 (21.0)	90 (24.2)	204 (54.8)	372 (33.8)
Total	684 (62.1)	179 (16.2)	239 (21.7)	1102 (100.0)

4b

MODEL PREDICTIONS	One Shift	Two Shifts	Three Shifts	Total
Wrong Prediction	78 (38.6)	89 (44.1)	35 (17.3)	202 (18.3)
Correct Prediction	606 (67.3)	90 (10.0)	204 (22.7)	900 (81.7)
Total	684 (62.1)	179 (16.2)	239 (21.7)	1102 (100.0)

.73 for the continuous process dummy and .62 for foreign capital. A graphical analysis similar to that depicted in figures 2 to 5 is preformed for the dichotomous case. This is shown in figures 1 to 4 in the appendix.

The model predicts the right event in 900 (81.7%) of the 1102 cases but, as it was the case before, the prediction is more accurate when the firm works either one or three shifts. Only half of two-shift cases are predicted correctly.[1/]

Finally, a dichotomous logit equation was estimated without the two-shift observations to see the effect of the removal of this category on the effectiveness of the model in discriminating among the "extreme" categories. As expected, the results show coefficients with smaller standard errors than in previous cases (see tables 5-6) and a much better fit ($\xi = .510$). This model discriminates between the one and three-shift categories in 825 out of the total 923 cases. Further, in 661 cases the prediction is performed with a probability greater than .8.

A Comparison Between the Trinomial Logit and Linear Models

Despite its theoretical difficulties the linear probability model has often been used with or without correcting for heteroskedasticity.

1/ When a lower degree of uncertainty is imposed on the definition of "correct prediction" the two-shift firms are again the most affected. If we require a probability higher than .8 for a correct prediction the number of successfully predicted two shift cases falls by 52% while in the one and three shift cases it diminishes 31 and 41 percent respectively.

TABLE 5

COEFFICIENTS OF THE BINOMIAL LOGIT PROBABILITY MODEL OF SHIFT CHOICE BETWEEN ONE AND THREE SHIFTS. DEPENDENT VARIABLE IS $\ln P_1/P_3$

STANDARD ERRORS IN PARANTHESIS

INDEPENDENT VARIABLE	
Constant[a/]	5.38 (1.24)
Size (thousand soles)	-0.113×10^{-4} (0.383×10^{-5})
Capital Instensity (thousand soles)	-2.59×10^{-2} (0.537×10^{-3})
Capital Productivity	0.762 (0.250)
Continuous Processes	-2.06 (0.417)
Foreign Capital	-0.285 (0.310)
Market Share	-1.58 (1.36)
Capital Intensity (squared)	-0.130×10^{-5} (0.396×10^{-6})
Capital Productivity (squared)	-0.507 (0.337)

$\xi = .510$

$\bar{I} = .438$

$\ln L = -257.59$

a/ Model includes 15 additional industry constants.

TABLE 6

CLASSIFICATION OF THE CONDITIONAL LOGIT MODEL PREDICTIONS EXCLUDING THE SECOND SHIFT CATEGORY ACCORDING TO ACTUAL OBSERVATIONS. PERCENTAGES IN PARANTHESIS

	ACTUAL OBSERVATIONS		
MODEL PREDICTION	One Shift	Three Shifts	Total
One Shift	649 (91.2)	63 (8.8)	712 (77.1)
Three Shifts	35 (16.6)	176 (83.4)	211 (32.9)
Total	648 (74.1)	239 (25.1)	923 (100.0)

A common argument is that a linear function is a good approximation of the logit function at non-extreme points. This contention is correct but any model with powerful explanatory variables is bound to generate predicted probabilities lying precisely in the neighborhood of the extreme points where the linear model not only does not approximate the hypothetical true probability function but predicts values which can no longer be interpreted as probabilities.

In order to test the relative quality of the linear probability model we estimated the set of equations (1). The results of the estimation of the three equations are presented in table 7. The measure of goodness-of-fit was calculated to provide a common ground for comparison. Thus it can be seen that the value of this summary measure is lower in the linear case (.284) than that of the conditional logit model. To calculate ξ and $\bar{I}$ in the linear case, negative and, greater-than-one predictions were constrained[1/] to the [0,1] interval and therefore the value reported overestimates the quality of the fit. R^2- statistics are also reported for each equation to provide an idea on the relative order of magnitude of R^2 ξ and $\bar{I}$. The two shift equation performance is relatively rather poor since a linear relation is being forced over a structure which is curvilinear on account of all variables except those having quadratic terms. These variables however, turned out to be insignificant in that particular equation.

1/ The percentage of predictions falling outside the [0,1] range is 11.6%, 5.2% and 21.6% for the single, double and triple-shift equations respectively.

TABLE 7

LINEAR PROBABILITY MODEL OF SHIFT CHOICE. DEPENDENT VARIABLE EQUALS ONE WHEN j-SHIFTS ARE WORKED, ZERO OTHERWISE. t-VALUES IN PARENTHESIS.

Independent Variable	One Shift	Two Shifts	Three Shifts
Constant a/	0.816 (17.5)	0.139 (3.19)	0.045 (1.11)
Size (thousands of soles)	-0.519×10^{-6} (1.83)	-0.726×10^{-7} (0.273)	0.592×10^{-6} (2.42)
Capital Intensity (thousands of soles)	-0.359×10^{-3} (7.28)	0.194×10^{-4} (0.420)	0.340×10^{-3} (7.98)
Capital Productivity	0.103 (4.85)	-0.0382 (1.93)	-0.064 (3.52)
Continuous Processes	-0.19 (4.11)	-0.128 (2.89)	0.323 (7.88)
Foreign Capital	-0.100 (3.16)	0.102 (3.53)	-0.439×10^{-2} (0.164)
Market Share	-0.255 (1.78)	0.113 (0.846)	0.141 (1.14)
Capital Intensity (squared)	0.459×10^{-7} (5.92)	-0.605×10^{-8} (0.834)	0.399×10^{-7} (5.96)
Capital Productivity (squared)	-0.826 (3.30)	0.279×10^{-2} (1.19)	0.548×10^{-2} (2.54)
R^2	0.400	0.092	0.380
F	31.33	4.93	29.00

$\xi = .289$

$\bar{I} = .284$ $\delta = .391$

a/ Model includes 15 additional industry constants.

The predictive performance of the linear model is also summarized in table 8 which is directly comparable with table 2 which was constructed for the logit model. From these two tables, it can readily be concluded that the logit model performs substantially better than its linear counterpart. The difficulties encountered in the prediction of the two-shift category are magnified in the linear case as can be expected of the poor fit that the linear two-shift-equation yields. In terms of accuracy the logit model also performs better in predicting the actual event with a lower level of uncertainty, thus its higher ξ value.

Finally it should be noted that while there are enormous differences in the time consummed in actual computing procedures, the logit model does not require an unduly large amount of time.[2/]

2/ The computing time for the trichotomous model with 24 variables is 13 min. on an IBM 370/145 system; convergence of the procedure took 9 iterations. When the 15 industry constants are dropped computing time is less than one min. and convergence requires 7 iterations.

TABLE 8

CLASSIFICATION OF THE LINEAR PROBABILITY MODEL PREDICTION ACCORDING TO ACTUAL OBSERVATION

PERCENTAGES IN PARENTHESIS

MODEL PREDICTION	OBSERVATIONS One Shift	Two Shifts	Three Shifts	Total
Predicts one Shift	652 (77.1)	118 (13.9)	76 (9.0)	846 (76.8)
Predicts two Shifts	4 (20.0)	10 (50.0)	6 (30.0)	20 (1.8)
Predicts three Shifts	28 (11.9)	51 (21.6)	157 (66.5)	236 (21.4)
Total	684 (62.1)	179 (16.2)	239 (21.7)	1102 (100.0)

4. CONCLUSIONS

The main object of this paper has been the introduction of an empirical model of capital utilization. The model yields results which are consistent with the basic results of theoretical production function models of utilization. Prominent among these results is the strong influence of capital intensity and capital productivity in determining the choice of the shift alternatives. The positive relationship between size and multiple shifting was established. Size seems however, to positively influence the choice of the three-shifts alternative since the probability of both one and two shifts diminishes with size reflecting no substantial difference in behavior between single and double shifts with respect to the size dimension. The market share variable, on the other hand, tends to lump the two and three shifts categories together and even though the positive relation proved to be relatively weak, the sensitivity exercises suggest a low probability of single shifting in the case of firms operating in tight-oligopoly or monopolistic market structures.

The role of capital intensity in the model appeared to be in close accord to intuition. Both two and three shifts seem the most likely choices even at mild levels of capital intensity and as these levels increase the three shifts choice becomes by far the dominant alternative. In the case of the capital productivity variable the results confirm the hypothesis that firms having a high yield in terms of value added per unit of capital reflecting their relative intensive use of non-capital inputs as well as the realization of large profits will very rarely operate three shifts.

Finally, the effect of the continuous processes dummy proved to be decisive. Plants having continuous processes tend to operate three shifts even if the relative importance of these processes within the plant is not very high. The foreign capital dummy has a similar influence except that in this case it enhances the chances of the two-shifts alternative being chosen (given of course that the plant has no continuous processes.) The patterns of influence of the continuous process and foreign dummies together with the importance of the size variable, underline dramatically the importance of organizational elements in the determinants of the shifting mode.

The comparisons of the statistical techniques showed that logit analysis, in addition to being conceptually more appropriate to deal with the problem at hand, provided better predictive power to the model, both in terms of a higher frequency of correct predictions as well as a lower degree of uncertainty

APPENDIX TABLE 1

CONDITIONAL PROBABILITY DISTRIBUTION OF p_j FOR DIFFERENT VALUES OF ONE EXPLANATORY VARIABLE WHERE ALL OTHERS ARE SET EQUAL TO ZERO

VALUE	SIZE			CAPITAL INTENSITY		
	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.33	.33	.34	.30	.35	.35
$\overline{X}/4$	.33	.33	.34	.27	.37	.36
$\overline{X}/2$	.32	.32	.36	.22	.39	.39
$\overline{X}$	.31	.31	.38	.14	.43	.42
$2\overline{X}$	.28	.29	.43	.06	.45	.49
$4\overline{X}$	.23	.24	.53	.01	.34	.64
$8\overline{X}$	.13	.15	.72	.01	.04	.95

VALUE	CAPITAL PRODUCTIVITY			MARKET SHARE		
	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.36	.34	.30	.33	.33	.33
$\overline{X}/4$	.38	.34	.27	.33	.33	.34
$\overline{X}/2$	.43	.35	.22	.33	.34	.34
$\overline{X}$	.51	.34	.14	.32	.34	.34
$2\overline{X}$	.62	.31	.07	.30	.35	.35
$4\overline{X}$	.70	.26	.03	.27	.36	.37
$8\overline{X}$	.48	.33	.19	.22	.38	.40

APPENDIX TABLE 2

CONDITIONAL PROBABILITY DISTRIBUTION OF p_j FOR DIFFERENT VALUES OF ONE EXPLANATORY VARIABLE WHEN ALL OTHER CONTINUOUS VARIABLES ARE SET AT THE LEVEL OF THEIR MEAN AND DUMMIES EQUAL TO ZERO

	SIZE			CAPITAL INTENSITY		
VALUE	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.24	.53	.23	.44	.37	.19
$\overline{X}/4$	.24	.53	.23	.40	.39	.20
$\overline{X}/2$	.24	.52	.24	.34	.44	.22
$\overline{X}$	.23	.51	.26	.23	.51	.26
$2\overline{X}$	.22	.48	.30	.10	.57	.32
$4\overline{X}$	.18	.42	.30	.03	.49	.47
$8\overline{X}$	.12	.29	.59	.00	.08	.91

	CAPITAL PRODUCTIVITY			MARKET SHARE		
VALUE	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.13	.42	.45	.24	.50	.26
$\overline{X}/4$	.14	.43	.41	.24	.50	.26
$\overline{X}/2$	.17	.47	.36	.24	.50	.26
$\overline{X}$	.23	.51	.26	.23	.51	.26
$2\overline{X}$	.32	.53	.15	.22	.51	.27
$4\overline{X}$	.41	.51	.08	.19	.53	.28
$8\overline{X}$	.21	.47	.32	.15	.55	.30

APPENDIX TABLE 3

CONDITIONAL PROBABILITY DISTRIBUTIONS OF p_j FOR DIFFERENT VALUES OF OUR EXPLANATORY VARIABLE WHEN ALL OTHER CONTINUOUS VARIABLES ARE SET AT THE LEVEL OF THEIR MEAN; THE CONTINUOUS PROCESS DUMMY IS ZERO AND THE FOREIGN DUMMY IS ONE

	SIZE			CAPITAL INTENSITY		
VALUE	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.15	.64	.21	.31	.50	.19
$\overline{X}/4$	.15	.64	.21	.28	.52	.20
$\overline{X}/2$	.15	.63	.22	.22	.56	.21
$\overline{X}$	.14	.62	.24	.15	.61	.23
$2\overline{X}$	.14	.59	.27	.06	.66	.28
$4\overline{X}$	.12	.52	.36	.02	.57	.41
$8\overline{X}$	.08	.36	.56	.00	.11	.89

	CAPITAL PRODUCTIVITY			MARKET SHARE		
VALUE	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.08	.51	.41	.15	.61	.23
$\overline{X}/4$	.09	.53	.37	.15	.61	.23
$\overline{X}/2$	.11	.57	.32	.15	.62	.23
$\overline{X}$	.15	.62	.23	.14	.62	.24
$2\overline{X}$	.21	.66	.13	.14	.62	.24
$4\overline{X}$	.28	.65	.07	.12	.63	.25
$8\overline{X}$	.13	.57	.29	.09	.65	.26

APPENDIX TABLE 4

CONDITIONAL PROBABILITY DISTRIBUTIONS OF p_j FOR DIFFERENT VALUES OF ONE EXPLANATORY VARIABLE WHEN ALL OTHER CONTINUOUS VARIABLES ARE SET AT THE LEVEL OF THEIR MEAN; THE CONTINUOUS PROCESS DUMMY IS ONE AND THE FOREIGN DUMMY IS ZERO

	SIZE			CAPITAL INTENSITY		
VALUE	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.11	.20	.69	.22	.15	.63
$\overline{X}/4$	.11	.19	.70	.20	.15	.65
$\overline{X}/2$	.10	.19	.71	.16	.16	.68
$\overline{X}$	.10	.17	.73	.10	.17	.73
$2\overline{X}$	.08	.15	.77	.03	.17	.79
$4\overline{X}$	.06	.10	.83	.01	.11	.88
$8\overline{X}$	.02	.05	.92	.00	.01	.99

	CAPITAL PRODUCTIVITY			MARKET SHARE		
VALUE	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.04	.10	.86	.11	.17	.72
$\overline{X}/4$	.05	.11	.84	.10	.17	.72
$\overline{X}/2$	.06	.13	.81	.10	.17	.72
$\overline{X}$	.10	.17	.72	.10	.17	.73
$2\overline{X}$	.19	.25	.56	.09	.17	.73
$4\overline{X}$	.31	.30	.38	.08	.17	.74
$8\overline{X}$	.08	.14	.78	.06	.18	.76

APPENDIX TABLE 5

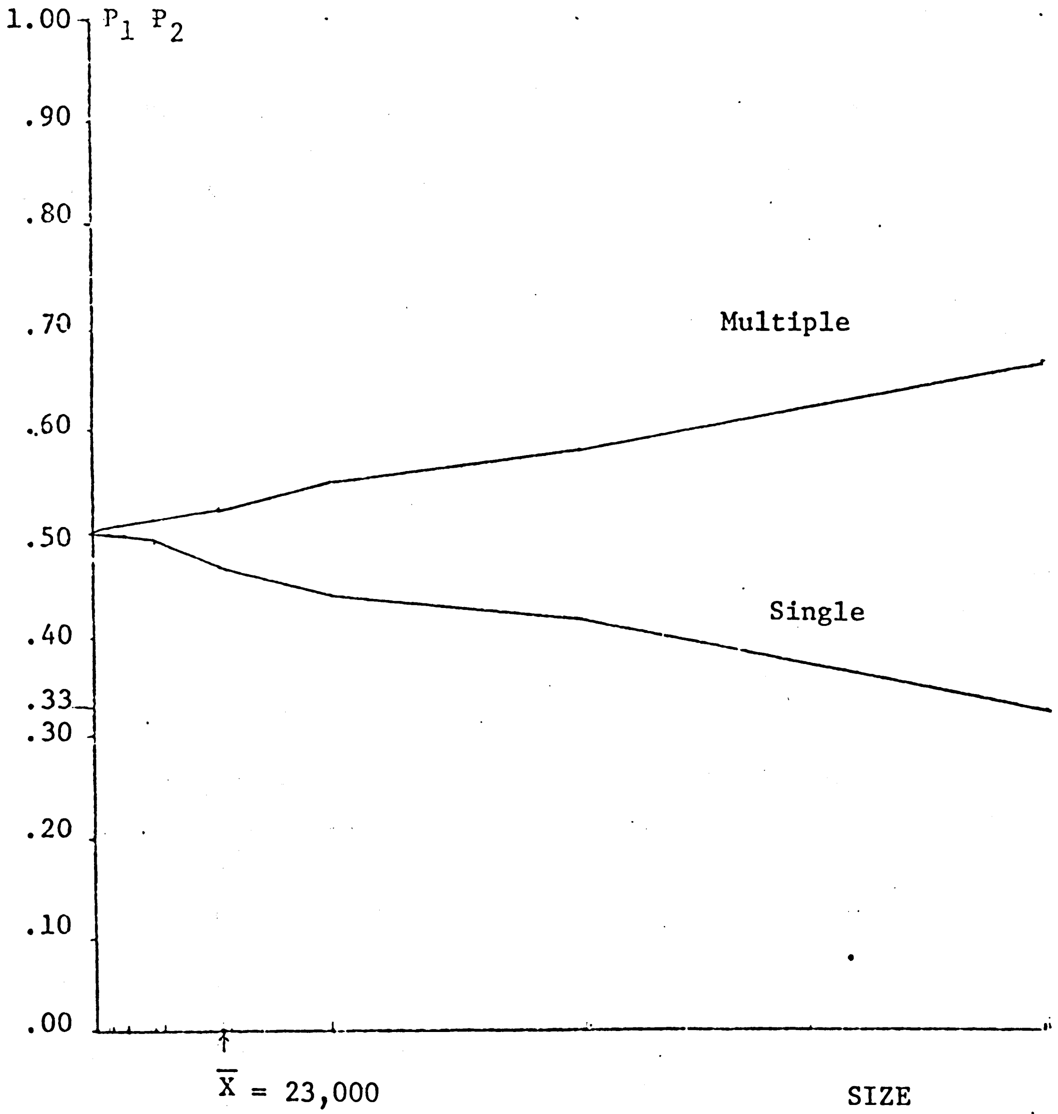
CONDITIONAL PROBABILITY DISTRIBUTION OF p_j FOR DIFFERENT VALUES OF ONE EXPLANATORY VARIABLE WHEN ALL OTHER CONTINUOUS VARIABLES ARE SET AT THE LEVEL OF THEIR MEAN AND DUMMIES EQUAL ONE

	SIZE			CAPITAL INTENSITY		
VALUE	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.08	.26	.66	.16	.20	.64
$\overline{X}/4$	.08	.25	.67	.14	.21	.65
$\overline{X}/2$	.07	.24	.68	.11	.22	.67
$\overline{X}$	.07	.23	.71	.07	.23	.71
$2\overline{X}$	.06	.20	.75	.03	.22	.75
$4\overline{X}$	.04	.14	.81	.01	.15	.85
$8\overline{X}$	.02	.07	.91	.00	.02	.98

	CAPITAL PRODUCTIVITY			MARKET SHARE		
VALUE	p_1	p_2	p_3	p_1	p_2	p_3
$\overline{X}/8$	.03	.13	.84	.07	.22	.70
$\overline{X}/4$	.03	.14	.82	.07	.23	.70
$\overline{X}/2$	.04	.17	.79	.07	.23	.70
$\overline{X}$	.07	.22	.71	.07	.23	.71
$2\overline{X}$	.13	.33	.54	.06	.23	.71
$4\overline{X}$	.22	.41	.37	.05	.23	.72
$8\overline{X}$	.05	.18	.76	.04	.23	.73

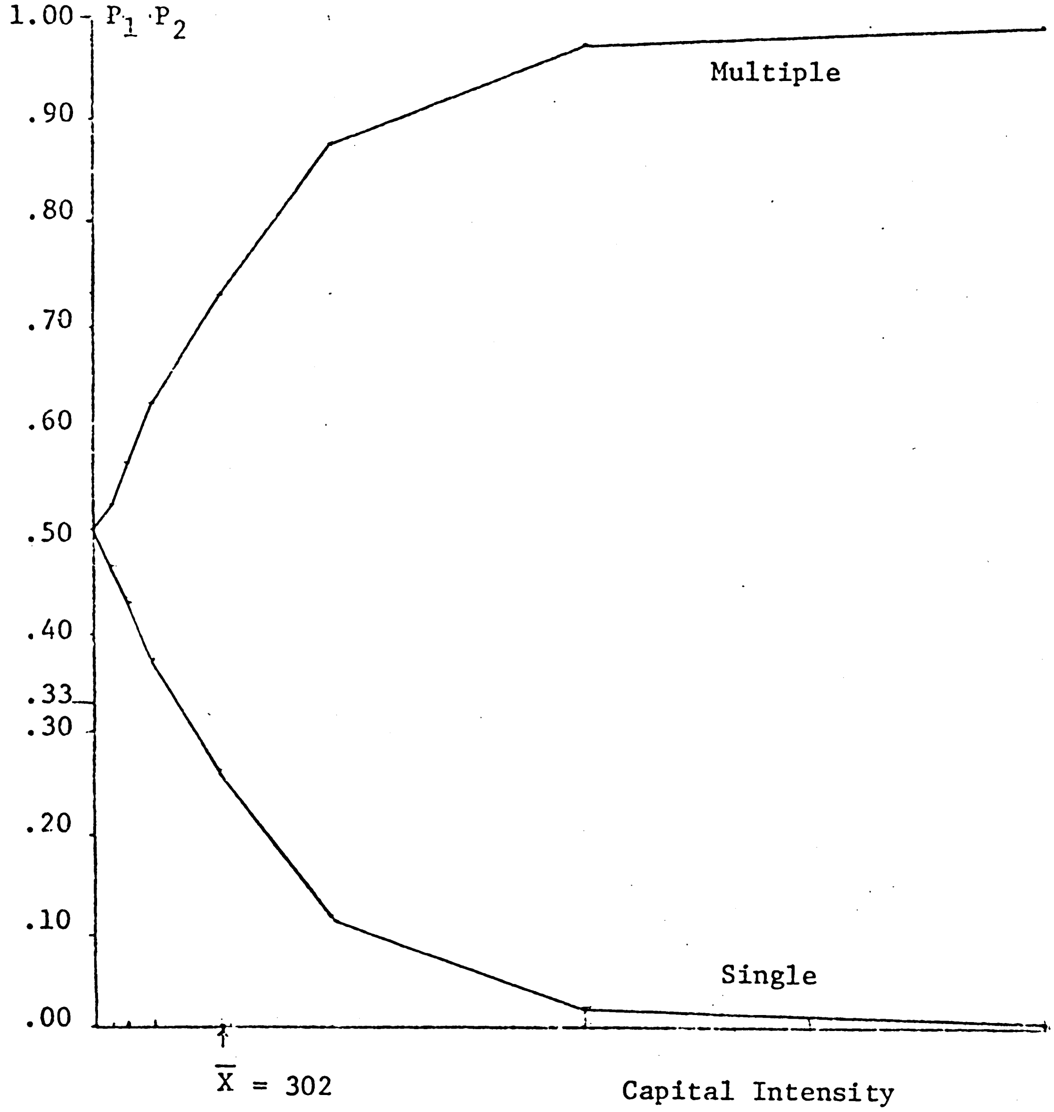
Appendix Fig. 1

Dichotomous Case: Effects on the Predicted Conditional Probability Distribution of Shiftwork Caused by the Increase in Plant Size Assuming no Influence of Other Independent Variables



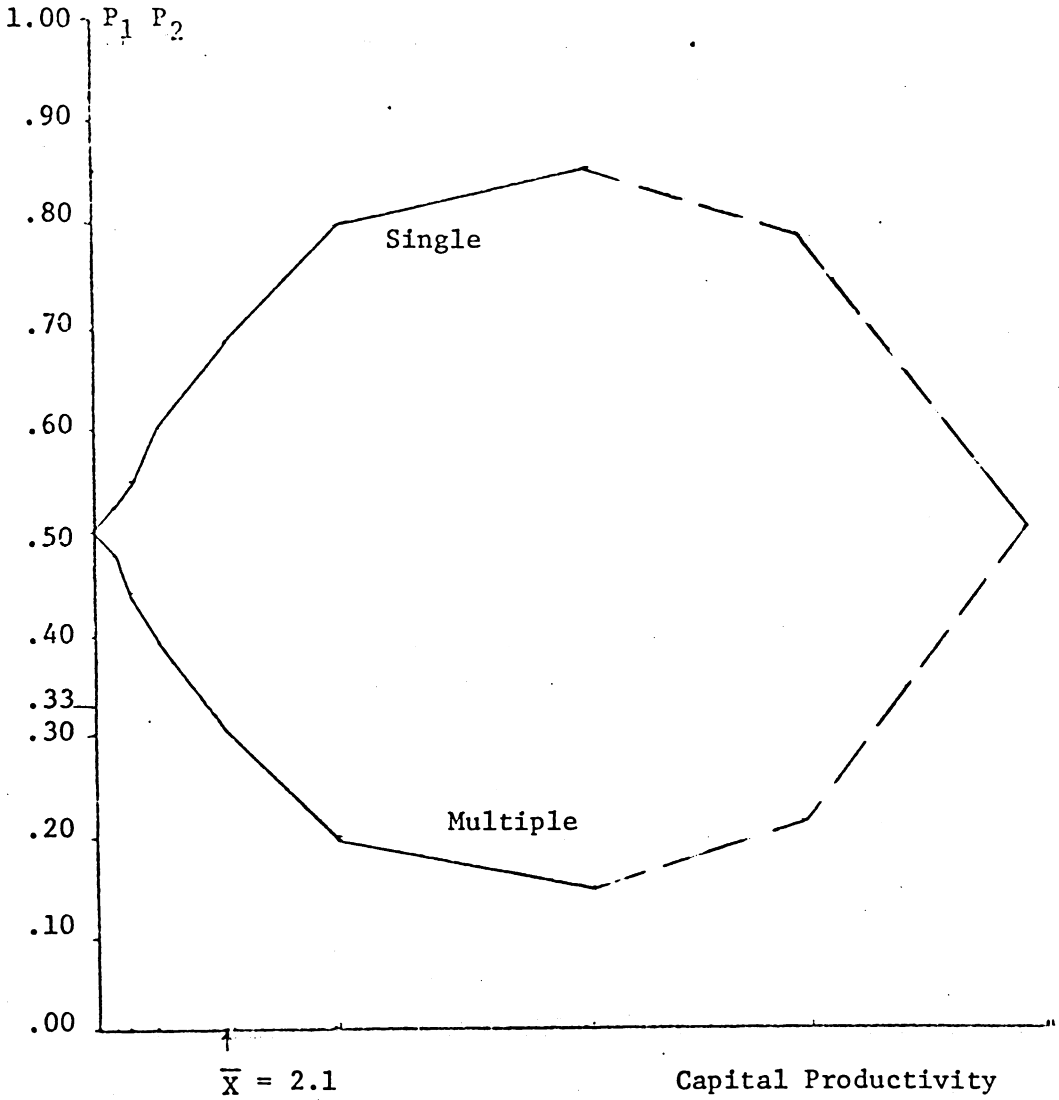
Appendix Fig. 2

Dichotomous Case: Effects on the Predicted Conditional Probability Distribution of Shiftwork Caused by the Increase in Capital Intensity Assuming no Influence of Other Independent Variables



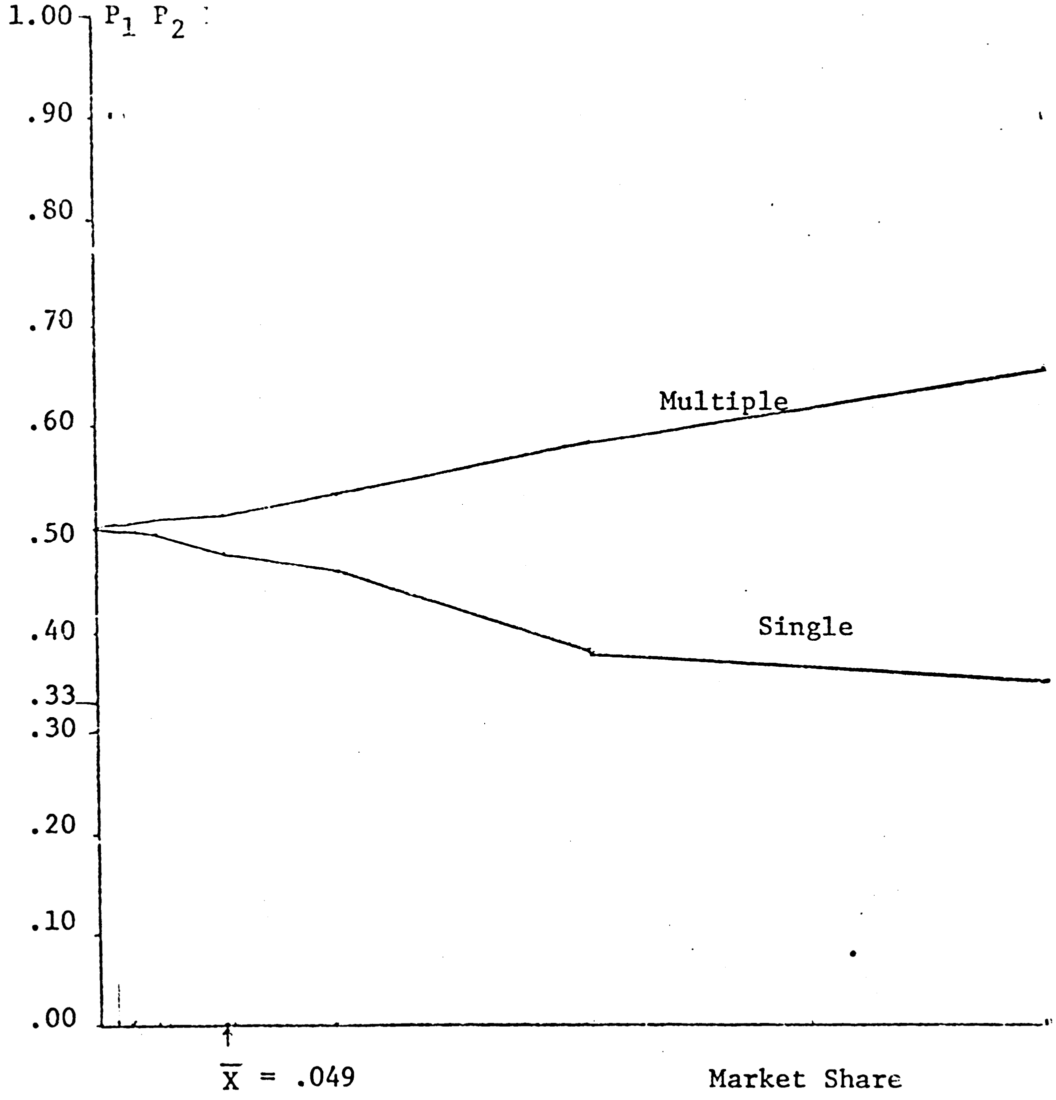
Appendix Fig. 3

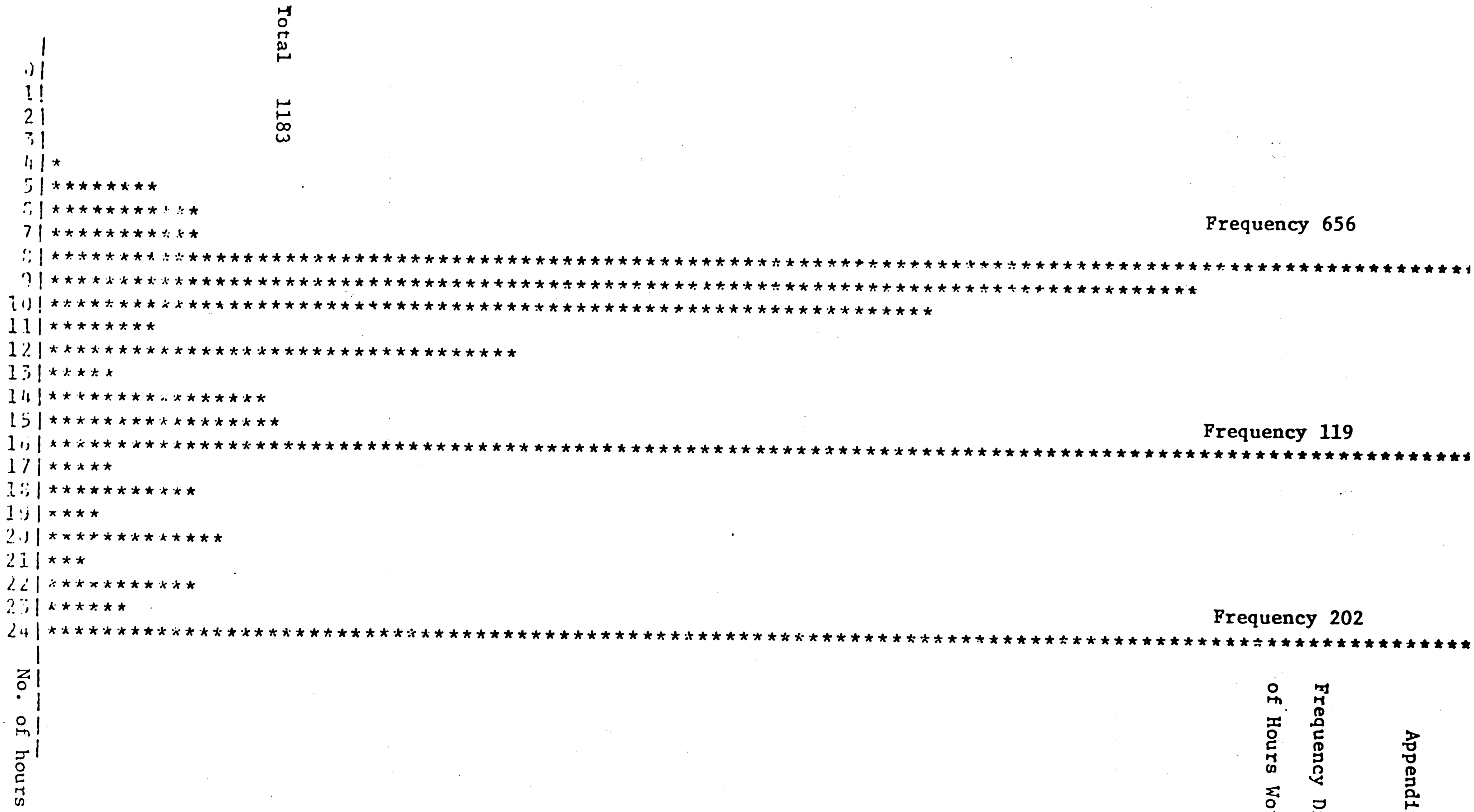
Dichotomous Case: Effects on the Predicted Conditional Probability Distribution of Shiftwork Caused by the Increase in Capital Productivity Assuming No Influence of other Independent Variables



Appendix Fig. 4

Dichotomous Case: Effects on the Predicted Conditional Probability Distribution of Shiftwork Caused by the Increase in Market Share Assuming no Influence of Other Independent Variables





Appendix Fig. 5

Frequency Distribution of Hours Worked by Plants