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# MULTIPLE SHIFTS IN THE PURE INVESTMENT DECISION

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# MULTIPLE SHIFTS IN THE PURE INVESTMENT DECISION

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## I

## Introduction

This paper presents a theory of how entrepreneurs choose between building a plant designed to operate at one shift or a plant designed to operate at multiple shifts on the basis of the existing technological and economic conditions given to them.

The technological characteristics that are considered in the analysis are those embodied in the production function: elasticity of substitution between labor and capital, degree of returns to scale and relative capital intensity of the processes. The economic characteristics are the relative price of factors of production, the cost differentials among shifts and the pecularities of the demand for the products of the plant. The basic behavioral postulate is that entrepreneurs maximize profits.

The choice between building a plant designed to operate at one shift or a plant designed to operate at multiple shifts has been called in a previous paper [5] the pure investment decision to differentiate it from the decision to work additional shifts in established plants. The basic difference between both problems is in the treatment of capital. For established plants there is a given stock of capital already purchased, which can be used in the additional shifts, while in the investment decision capital is a variable to be determined according to the type of plant that is built.

We have argued that the decision to work additional shifts in established plants and the investment decision are always inter-related: the construction of a new plant will be delayed if an additional shift in existing plants is more profitable and the additional shift will not be worked if it is more advantageous to build a new plant. Since this connection has been studied before, here we will assume that it has been decided to build a new plant and that the entrepreneur must only choose the number of shifts at which he will operate.

In the first section we will briefly review the theoretical literature on the subject of capital utilization that has dealt with this same problem and point out its fundamental weakness of assuming that the same amount of output will be produced with single and multiple shifts plants. The basic neoclassical model is then purged of this unjustified assumption and output is incorporated as a variable to be determined in the decision procedure that selects the type of plant to be built. The case of the Cobb-Douglas and CES production functions are then examined in detail.

All the conclusions presented are based on the assumption that there is substitution between capital and labor before the construction of the plant but not after it. This view implies that the entrepreneur can select between different techniques of production when he is planning his investment, but that once the machines are in place the instaneous capital-labor ratio is fixed.

The above assumption is used because it seems to be a good approximation to what happens in the real world, but it is not essential for the development of the theory. In fact in Section IV we present the investment choice for the Cobb-Douglas production function with the same ex-ante and ex-post elasticity of substitution. This is done to refute a thesis developed by Mary Ann Bailey and accepted by other authors [1/] that if you assume that there is no limit on the substitutability of capital for labor the existence of a wage differential among shifts is not sufficient to imply utilization rates at a single shift.

The conclusion of this paper is that the number of shifts in the Cobb-Douglas production function with the same ex-ante and ex-post elasticity of substitution depends exclusively on the scale parameter. With economies of scale plants will always be designed to work a single shift, while with diseconomies or constant returns to scale plants will always be designed to work at multiple shifts. Mary Ann Baily reached her results only because she assumes constant returns to scale.

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1/ Mary Ann Baily, "Capital Utilization in Kenya Manufacturing Industry", Unpublished Ph.D. dissertation, Department of Economics, Massachusetts Institute of Technology, Cambridge, December 1972,pp. 30-34; Winston Gordon C., "The Theory of Capital Utilization and Idleness", Department of Economics, Williams College (mimeo), Williamstown, Mass., March 1974, p. 16.

The number of shifts that the entrepreneur will plan to work depends on a specific relation between the technological and economic parameters facing him. If the economic parameters are the same throughout the economy, technological differences among industrial sectors can explain why in some of them multiple shifts are more prevalent than in others.

But the traditionally held belief that in industries with capital intensive technologies plants will he designed to work at multiple shifts while in labor intensive industries they will be designed to work at a single shift must be qualified. If two industries face the same economic variables, differences in the scale parameter of the production function and/or in the elasticity of substitution can induce the industry with labor intensive technology to operate at multiple shifts while the capital intensive one is working at a single shift. If the want to attribute to differences in the capital intensity all the explanatory power of differences in the number of shifts we must make very strong assumptions about elasticities of substitution and economies of scale throughout the economy.

Multiple shifts in an attractive policy because it is assumed that it increases employment and saves capital. The analysis demonstrates that this is not always so. A specific knowledge of the value of all the economic and technical parameters is required to determine precisely what will happen.

For a set of prices of the factors of production and wage premiums for late shifts, shift work could increase employment and save capital in some industries, have the opposite results in other ones and mixed results in a third group of industries. This indicates that an indiscriminate use of the policy will--at least in theory--not always be advantageous.[2/]

When the policy objective is the more restrictive one of increasing the labor-capital ratio of the new plants, we demonstrate that multiple shifts will always achieve this result in industries where the elasticity of substitution between labor and capital is less than $\log 2/\log(2+\theta)$, where $\theta$ is the wage premium for late shifts.

Economic policies to increase the number of shifts in the pure investment decision are related to changes in the relative prices of capital and labor and in the wage premium for late shifts. The effect of any of the possible policies is dependent on the technological characteristics of production. For example, when there are diseconomies of scale and the elasticity of substitution is greater than one, the traditionally advocated policy of increasing the relative price of capital will favor single shifts plants instead of multiple shift ones. A description of the appropriate policies in different situations is presented in the paper.

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2/ This remark is not valid for the case of established plants. There policies to increase the number of shifts will always increase employment.

II

## An Overview of the Literature on Capital Utilization

The pioneer work on capital utilization is that of Robin Marris [4]. He reports that in Great Britain industrial firms planned intentionally to leave capital idle as a rational part of their investment decision because there are extra costs associated with work at night and on the weekends. The extra costs may be due to institutional arrangements, such as social legislation restricting shift work by women and juveniles, or to the subjective evaluation of workers, who dislike nightwork and have difficulties in obtaining transport or sleeping at unusual hours.

Marris does not have data on actual utilization rates and uses as a proxy in each industrial sector the proportion of workers engaged in late shifts. Since his theory indicates that the fewer the number of workers per unit of capital in each plant the higher the utilization rate, this is a very poor index of utilization. The empirical side of his study is not very valuable.

Marris' theoretical model uses two factors of production: labor and capital. Ex-ante there is substitution between the factors of production, but once a technique is chosen capital must always be used by the same size work crew. In his analysis firms determine the capital stock and the utilization rate by maximizing the rate of return on the capital employed in the enterprise. It has been noted by Baily this is a "fundamental error. Maximizing the rate of return on capital does not lead to maximum profit except in the case where the amount of capital is fixed, but the essence of the problem is that the firm is choosing the optimal capital stock at the time when the investment decision is made" [1, p.11].

Marris may have been thinking in the distinction between owned and borrowed funds and that firms maximize then the rate of return on their owned funds. But if firms can borrow or lend, owned funds are not always equal to the capital stock. Although the first could be assumed fixed, the second one is not when we are considering an investment decision.

Gordon Winston [8,9] has reformulated Marris'analysis in terms of standard microeconomic theory. He assumes a neo-classical production function of the form Q = F(K,L), where K and L are flows of capital and labor services respectively per unit of time. The firm can produce a desired quantity of output per day in one or two shifts. It will choose that combination that minimizes costs.

Winston assumes that with two shifts the same size work crew must be used ex-post on both day and night shifts, that is, that the elasticity of factor servic substitution after the plant has been built is zero. Using a CES production function with constant returns to scale, the two shifts mode of operation will be selected whenever:

$$\left(1 + \frac{\theta}{2}\right)^{\tau-1} \left[1 - (2^{\tau-1} - 1)\left(\frac{\alpha}{1-\alpha}\right)^{\tau}\left(\frac{\omega}{2P_k}\right)^{\tau-1}\right] > 1 \text{ for } 0 \leq \tau < 1 \qquad (1)$$

$$\left(1 + \frac{\theta}{2}\right)^{\tau-1} \left[1 - (2^{\tau-1} - 1)\left(\frac{\alpha}{1-\alpha}\right)^{\tau}\left(\frac{\omega}{2P_k}\right)^{\tau-1}\right] < 1 \text{ for } 1 < \tau < \infty \qquad (2)$$

Where: $\tau$ = elasticity of factor service substitution ex-ante

$\alpha$ = CES parameter that indicates the capital intensity of the technology

$\theta$ = wage premium for night shifts

$\omega$ = wage rate

$P_k$ = cost of owning a unit of capital stock for a day

Winston's findings based on the above formula can be summarized in the following propositions:

1. The higher the relative price of capital services-given an elasticity of substitution less than one--the greater the need of reducing capital costs and the greater the likelihood that a multiple shifts plant will be chosen.
2. The higher the wage premium for night shifts--or other differential input prices--the greater the incentive to operate only during low costs periods and the greater the likelihood that a single shift plant will be chosen.
3. The more capital intensive the production process is the more important capital costs are and the greater the incentive to economize on them through multiple shifts.
4. The influence of factor prices on the optimum number of shifts depends on the elasticity of factor service substitution. With no substitution relative factor prices will have their full influence on the optimum number of shifts. With unitary elasticity factor

prices have no effect at all. With elasticities less than one the effect factor prices are those indicated in the first proposition. With elasticities greater than one the effect of factor prices on the number of shifts are the reverse of those stated in the first proposition.

Roger R. Betancourt and Christopher K. Clague [2] and Mary Ann Baily [1] use a model similar to that of Winston. In all of them output is given[3/], the ex-post elasticity of factor service substitution is zero and the firm minimizes costs of production. While Betancourt and Clague explicitly incorporate economies of scale, Baily incorporates a positive user cost of capital, a drop in productivity on the night shift and a transport parameter and deals with a minimum size of plant. They conclude that the existence of economies of scale, additional extra costs on the night shift and of a minimum size of a plant will increase the likelihood of choosing single shift plants.

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3/ R.R. Betancourt and C.K. Clague [2] relax the fixed output assumption at the end of their paper in a very peculiar fashion that never incorporates output as a variable to be determined in the maximization process. Their conclusion is that if differences in output are not large and if the decline in the cost function is not acute, the case of fixed output is a good approximation. This is a tautology that eliminates output from the analysis by assumption.

## III

## The Choice Between Plants Designed to Operate at Different Number of Shifts

A striking feature of the theoretical literature on capital utilization is that it has all been developed in the limited setting that compares cost of producing a given output with plants designed to operate at one and two shifts. Since there is always a differential in the prices of factors between day and night shifts, a justification for the assumption of equal output in both alternative plants should be given.

According to traditional economic theory the level of output is determined by the condition that marginal cost equal price.[4/] In competitive situations the assumption that output is the same with plants designed to operate at one and two shifts can only be justified if the marginal cost function is the same in both circumstances. It is obvious that this cannot in general be so in the presence of some wage differential between shifts in the multiple shifts plants. It could only be a special case for a certain specified value of the wage premium.

In defense of using the same output for one and two shifts plants it is sometimes argued that the capital-labor ratio is fixed ex-post. This conclusion does not follow. If the ex-post capital-labor ratio is fixed, the only alternative is to produce the same output in all shifts of the

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4/ In non competitive situations marginal cost must be equal to marginal revenue.

plant designed to operate at multiple shifts. Even if the production function is of the fixed coefficient type this does not imply that output with a single shift plant will be the same as output with a multiple shift plant: differences in capital and labor use and in economies of scale must be considered.

Another argument that is used to justify the assumption of a given output is that the production function has constant returns to scale. In that case output under competitive conditions is undertermined and it may seem sound to use the same amount in the comparison between investing in single and multiple shift plants. But this solution is an arbitrary way of disguising the ignorance of what would happen with constant returns to scale production functions.

In a discrete time analysis of optimal shift work the production function that has constant returns to scale is that of each shift individually. The first shift when working multiple shifts is always facing the same parameters as the single shift when working one. In using a given output for the comparison between the different plants, we are arbitrarily assuming that output will differ among them, which seems irrational. It may be better to assume another arbitrary distribution of output, like constant output per shift.

As will be explained later, for the choice between plants designed to operate at different number of shifts in the constant returns to scale case there is generally valid and not arbitrary solution. That is to compare average costs. With constant returns to scale average costs are constant. If between single and double shift plants they differ, the smallest value determines the plant to be built.

A general form of the production function with two factors is:

$$Q = F(K, L) \qquad (3)$$

where Q is output per unit of time, L is labor in man-hours per unit of time and K is capital services per unit of time.

To get capital services a stock of capital goods must be purchased by the firm and then used to produce the flow. On the other hand, the firms buy labor services directly and does not own the labor stock.[5/]

The firm must decide on the size of the capital stock and its rate of utilization. Both decisions are inter-related: the size of the capital stock will depend on its utilization and vice versa. Our analysis of utilization will be time discrete and we assume that the firm can work one, two or three shifts of "a" hours each.[6/] The analysis could also have been made in continu-

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5/ In some cases capital services can be bought directly by the firm (examples: computers, copying machines). Capital is then treated in the same way as labor.

6/ There is no substantial difference if the length of each shift is not the same, but the formulas become more complicated. If the unit of time is the day and maintenance and cleaning can be done while the plant is in operation, "a" is normally equal to 8 hours.

ous form, that is, making the firm choose a certain number of hours of daily work. But this does not seem to be the mose relevant case in the real world: workers in industrial plants are usually hired to work a fixed schedule and the wage rate is constant within that schedule.

We will also assume that capital services depend only on the hours of use of the given capital stock. The flow of capital services will then be the same in each shift.[7/] If a superscript represents the number of shifts worked and $K_s$ is the capital stock, the total flow of capital services in plants designed to work one and two shifts is respectively:[8/]

$$K^1 = a \cdot K_s^1 \tag{4}$$

$$K^2 = 2a \cdot K_s^2 \tag{5}$$

The price of output is "p", which we assume to be constant because we are dealing with a competitive firm. The wage rate for each man-hour is "$\omega_1$" in the first shift and "$\omega_2$" in the second shift. The second shift has a wage premium of "$\theta$" over the first

7/ In most cases it will always be optimal for the firm to produce within a shift at the maximum rate of capital utilization because this will lower the cost of production. Then the assumption that capital services depend only on the hours of use of the given capital stock is irrelevant. The only exception is the very peculiar case when the cost of using capital is increasing and dominates over the reduction in costs implied by using the already bought capital stock more hours.

8/ The case of plants designed to operate at three shifts is similar to that of two shifts plants and will not be described below.

shift. In other words:

$$\omega_2 = \omega_1 \ (1+\theta) \tag{6}$$

The purchase price of a unit of capital stock is "$P_k$". The daily interest rate is "i" and the daily depreciation rate and maintenance cost is "d"[9/]. When the capital stock is used one shift of "a" hours, the cost per hour of each unit of capital, that is the price of capital services:

$$r_1 = \frac{P_k(i+d)}{a} \tag{7}$$

Similarly, when two shifts are worked we have that the price of capital services is:

$$r_2 = \frac{P_k(i+d)}{2a} \tag{8}$$

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9/ Under this assumption depreciation and maintenance costs are a function of elapsed time and not related to the use of the capital stock. If we want to introduce use related costs of capital we make "$d_1$" the hourly depreciation and maintenance costs. Formulas (7) and (8) are then replaced by:

$$r_1 = \frac{P_k(i+a \ . \ d_1)}{a} \tag{7a}$$

$$r_2 = \frac{P_k(i+2a \ . \ d_1)}{2a} \tag{8a}$$

and (9) and (10) by:

$$\Pi^1 = PQ^1 - \omega_1 L^1 - P_k(i+a \cdot d_1) \cdot K_s^1 \tag{9a}$$

$$\Pi^2 = PQ^2 - \omega_1 L_1^2 - \omega_2 \cdot L_2^2 - P_k(i+2a \cdot d_1) K_s^2 \tag{10a}$$

Total profits for the two different plants in terms of the given prices of labor services and capital stock are:

$$\Pi^1 = p \cdot Q^1 - \omega_1 L^1 - P_k(i+d)K_s^1 \tag{9}$$

$$\Pi^2 = p \cdot Q^2 - \omega_1 L_1^2 - \omega_2 L_2^2 - P_k(i+d)K_s^2 \tag{10}$$

The behavioral postulate of maximization of profits enables us to get the optimal amounts of labor and capital services in terms of the unknown quantity of output of each plant and the given prices. For the plant designed to operate at one shift the values are:

$$L^1 = f_1(Q^1, \omega_1, P_k[i+d]) \tag{11}$$

$$K_s^1 = f_2(Q^1, \omega_1, P_k[i+d]) \tag{12}$$

For the plant to operate at two shifts:

$$L_1^2 = f_3(Q^2, \omega_1, \omega_2, P_k[i+d]) \tag{13}$$

$$L_2^2 = f_4(Q^2, \omega_1, \omega_2, P_k[i+d]) \tag{14}$$

$$K_s^2 = f_5(Q^2, \omega_1, \omega_2, P_k[i+d]) \tag{15}$$

Total costs for the different plants are:

$$C^1 = \omega_1 L^1 + P_k(i+d) \cdot K_s^1 \tag{16}$$

$$C^2 = \omega_1 L_1^2 + \omega_2 L_2^2 + P_k(i+d) \cdot K_s^2 \tag{17}$$

Replacing equations (11) to (15) in (16) and (17) total costs are expressed only in terms of output and the given prices:

$$C^1 = f_6(Q^1, \omega_1, P_k[i+d]) \tag{18}$$

$$C^2 = f_7(Q^2, \omega_1, \omega_2, P_k[i+d]) \tag{19}$$

Competitive firms select a level of output such that marginal cost equate the price of output. The output level of each of the two plants is then obtained from the equations:

$$\frac{dC^1}{dQ^1} = \frac{df_6\ (Q^1,\ \omega_1,\ P_k[i+d])}{dQ^1} = p \tag{20}$$

$$\frac{dC^2}{dQ^2} = \frac{df_7\ (Q^2,\ \omega_1,\ \omega_2,\ P_k[i+d])}{dQ^2} = p \tag{21}$$

Note that the single and the multiple shifts plants will produce the same quantity of output--as is assumed in the literature on capital utilization--only if the left hand side of equations (20) and (21) are equal. There is no reason for this to be so.

Replacing the solution of equations (20) and (21) in equation (11) to (15) factors used are expressed only in terms of the given prices and parameters of the production function. These values are introduced in equation (9) and (10) to get a numerical expression for profits in each of the plants. A comparison of these values will indicate what alternative will be preferred by entrepreneurs that must decide between building a plant designed to operate at one shift and a plant designed to operate at two shifts.

In the case of monopolistic markets, marginal cost is equated to marginal revenues to determine the level of output. Knowing the demand function, marginal revenue as a function of output is also known. "p" in equations (20) and (21) is replaced by a function and everything else remains unchanged.

While the general analysis indicates that there is a solution to the investment decision, the only way to get specific answers is to assume given the form of the production function. In the following sections we consider in detail the Cobb-Douglas and the CES production functions.

## IV

## The Pure Investment Decision in the Cobb-Douglas Production Function with Ex-Post Substitution Between Labor and Capital

The general form of the Cobb-Douglas production function with two factors of production is:

$$Q = A \cdot L^{\alpha} \cdot K^{\beta} \tag{22}$$

The optimal levels of output for plants designed to work at one and two shifts as a function of the given prices and the parameters of the production function and the demand function are derived in Appendix I for competitive and monopolistic market conditions and for fixed and variable ex-post capital-labor ratio.

Let us consider the general case when the ex-post capital-labor is not fixed and labor services in the second shift can be different from those of the first shift for plants working multiple shifts. The optimal levels of output for plants designed to work at one and two shifts under competitive conditions are respectively:

$$\bar{Q}_c^{1} = A \cdot a^{\beta} \cdot \mu_1^{\alpha} \cdot \left[\frac{p \cdot A \cdot \beta \cdot a \cdot \mu_1^{\alpha}}{P_k(i+d)}\right]^{\frac{\alpha + \beta}{1 - (\alpha+\beta)}} \tag{23}$$

$$\bar{Q}_c^{2} = A \cdot a^{\beta} \cdot \mu_1^{\alpha} \cdot (1+\rho)^{1-\alpha} \left[\frac{P \cdot A \cdot \beta \cdot a^{\beta} \cdot \mu_1^{\alpha} (1+\rho)^{1-\alpha}}{P_k(i+d)}\right]^{\frac{\alpha+\beta}{1- (\alpha+\beta)}} \tag{24}$$

where:

$$\mu_1 = \frac{\alpha}{\beta} \cdot \frac{P_k(i+d)}{\omega_1} \tag{25}$$

$$\rho = \left(\frac{\omega_1}{\omega_2}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{1}{1+\theta}\right)^{\frac{\alpha}{1-\alpha}} \tag{26}$$

$\mu_1$ and $\rho$ indicate relations between the parameters that have an economic meaning. $\mu_1$ is equal to the labor services-capital stock ratio of the plant designed to work at one shift (see Appendix I, equation 7). It represents the fundamental equality of the marginal rate of technical substitution and the input price ratio and it shows the optimal factor proportions when output changes and input prices remain fixed. $\mu_1$ times $(1+\rho)$ is equal to the labor services-capital stock ratio in the first shift of the plant designed to work at two shifts (see Appendix I, equation 27) and has a similar economic interpretation that $\mu_1$, but for this type of plant.

$\rho$ is the ratio between output in each shift--equal to the ratio between labor services in each shift--for the plant working two shifts (see Appendix I, equation 36).

The ratio between the optimal levels of output under multiple and single shift plants in competitive conditions is:

$$\lambda_1 = \frac{\bar{Q}_c^2}{\bar{Q}_c^1} = (1+\rho)^{\frac{1-\alpha}{1-(\alpha+\beta)}} \tag{27}$$

Given the parameters of the Cobb-Douglas production function ($\alpha$ and $\beta$) and the wage premium for the second shift ($\theta$), the above relation will compare output of a plant designed to operate at two shifts with output of a plant designed to operate at one shift. The condition for $\lambda_1=1$--as is assumed in the literature on capital utilization--is that

$\rho = o$, which implies an infinite wage premium in the second shift. In that case the multiple shift plant will obviously be unprofitable and the case is totally irrelevant.

Let us assume that the costs of labor are always less than the value of output ($\alpha < 1$) and that there is a positive wage premium ($\rho > 0$). With decreasing returns to scale ($\alpha + \beta < 1$) we will have that $\lambda > 1$ and the optimal output with plants designed to operate at two shifts is greater than output with plants designed to operate at one shift. 10/

The entrepreneur will build the plant with higher profits. If $\Pi^1$ are profits of the plant designed to work at one shift and $\Pi^2$ profits of the plant designed to work at two shifts, the two shift mode of operation will be chosen when:

$$\Pi^2 > \Pi^1 \tag{28}$$

Calling $R^1$ and $R^2$ to total revenues with one and two shifts respectively and $C^1$ and $C^2$ to total costs in the same cases, expression (28) is equivalent to:

$$R^2 - R^1 > C^2 - C^1 \tag{29}$$

where:

$$R^1 = p\,Q^1 \tag{30}$$

10/ The case of increasing returns to scale is not considered here because it is incompatible with perfect competition. Monopolistic markets are studied below.

$$R^2 = p\,Q^2 \tag{31}$$

$$C^1 = \omega_1 L^1 + P_k(i+d) \cdot K_s^1 \tag{32}$$

$$C^2 = \omega_1 L_1 + \omega_2 L_2^2 + P_k(i+d) \cdot K_s^2 \tag{33}$$

In Appendix I it is shown that total cost as a function of output in the Cobb-Douglas production function is:

$$C^1 = P_k(i+d)\ \frac{\alpha+\beta}{\beta}\left[\frac{Q^1}{A\cdot a^{\beta}\cdot \mu_1^{\alpha}}\right]^{\frac{1}{\alpha+\beta}} \tag{34}$$

$$C^2 = P_k(i+d)\ \frac{\alpha+\beta}{\beta}\left[\frac{Q^2}{A\cdot a^{\beta}\cdot \mu_1^{\alpha}\cdot (1+\rho)^{1-\alpha}}\right]^{\frac{1}{\alpha+\beta}} \tag{35}$$

From equation (27) we have that the relation between optimal outputs under two and one shift is:

$$\bar{Q}_c^2 = \lambda_1 \bar{Q}_c^1 \tag{36}$$

and therefore for the optimal levels of output:

$$\bar{R}^2 - \bar{R}^1 = p \cdot \bar{Q}_c^1\ (\lambda_1 - 1) \tag{37}$$

$$\bar{C}^2 - \bar{C}^1 = P_k(i+d)\ \frac{\alpha+\beta}{\beta}\left[\frac{\bar{Q}_c^1}{A\ .\ a^{\beta}.\ \mu_1^{\alpha}}\right]^{\frac{1}{\alpha+\beta}}\left[\left(\frac{\lambda_1}{(1+\rho)^{1-\alpha}}\right)^{\frac{1}{\alpha+\beta}} - 1\right] \tag{38}$$

When profits are maximized we must have $L^1 = \mu_1 K_s^1$. This indicates that:

$$\left[\frac{\bar{Q}_c^1}{A\cdot a^{\beta}.\mu_1^{\alpha}}\right]^{\frac{1}{\alpha+\beta}} = \bar{K}_s^1 \tag{39}$$

The plant designed to operate at two shifts will be chosen whenever:

$$\lambda_1 - 1 > \frac{P_k(i+d) \quad K_s^{-1}}{p \cdot Q_c^{-1}} \quad \frac{\alpha+\beta}{\beta} \left[ \left( \frac{\lambda_1}{(1+\rho)^{1-\alpha}} \right)^{\frac{1}{\alpha+\beta}} -1 \right] \tag{40}$$

$P_k \; (i+d)\bar{K}_s^{1}/p\bar{Q}_c^{1}$ is the share of capital costs in the value of output at the optimal level which is equal to β in the Cobb-Douglas production function. Replacing $(1+\rho)$ by its value in terms of $\lambda_1$ given by (27), the condition to invest in double shift plants is:

$$1 > \alpha+\beta \tag{41}$$

Therefore when there are decreasing returns to scale, profits of a plant designed to operate at two shifts are always greater than profits of a plant designed to operate at one shift.

Our first conclusion is that if the Cobb-Douglas production function has decreasing returns to scale and there is ex-post substitution between capital and labor and perfect competition it will always be advantageous to operate at multiple shifts, no matter what the relative price of factor inputs, the capital intensity or the wage differentials are. The elasticity of substitution--that is equal to 1--takes care of these factors through changes in the required input combination and the maximization of profits through changes in the optimal level of output.

The economic explanation of the above conclusion lies in the fact that the marginal product of labor approaches infinite as labor approaches zero: no matter what the premium in the second shift is, the marginal product of labor can always be greater than it and two shifts can always be advantageous. Of course this would not be true with some restrictions in the ex-post elasticity of substitution or with some minimum amount of labor to be used in the second shift.

Marginal costs are the first derivatives of equations (34) and (35). When there are constant returns to scale, marginal costs are constants and equal to average costs. Their values are:

$$MC^1 = \frac{P_k\ (i+d)}{\beta . A \cdot a^\beta \cdot \mu_1^\alpha} \tag{42}$$

$$MC^2 = \frac{P_k (i+d)}{\beta \cdot A \cdot a^\beta \cdot \mu_1^\alpha\ (1+\rho)^{1-\alpha}} \tag{43}$$

It is obvious that with a positive wage premium ($\rho > 0$) we have $MC^2 < MC^1$. The average costs of production with a multiple shift plant is smaller than the average cost of production of a single shift plant: the firm will always work multiple shifts if it finds it profitable to operate.11/ Of course, if $MC^2$ is greater than the constant price of output, no plant will be built at all.

Assuming now that the plant is facing a demand curve with elasticity "$\eta$" not equal to infinite, the optimal levels of output for plants designed to work at one and two shifts are respectively12/:

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11/ This result coincides with the one reported by Mary Ann Baily [1, pp. 31-34], although she uses the same output-which is not correct--and another method. Previously (equation 41) we have extended her conclusion to the case of diseconomies of scale and latter we will show that it is not valid for economies of scale. Mary Ann Baily only deals with the case of constant returns to scale.

12/ See equations (13a) and (35a) in Appendix I.

$$\bar{Q}_M^1 = A\cdot a^\beta\cdot\mu_1^\alpha\left[\frac{p_1(1-1/\eta_1)\cdot A\cdot\beta\cdot a^\beta\cdot\mu_1^\alpha}{P_k(i+d)}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}} \tag{44}$$

$$\bar{Q}_M^2 = A\cdot a^\beta\cdot\mu_1^\alpha\cdot(1+\rho)^{1-\alpha}\left[\frac{p_2\ (1-1/\eta_2)\ A\cdot\ \beta\cdot\ a^\beta\cdot\ \mu_1^\alpha\ \cdot(1+\rho)^{1-\alpha}}{P_k(i+d)}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}} \tag{45}$$

The use of the prices of output ($p_1, p_2$) and the elasticities of demand ($\eta_1$, $\eta_2$) in the above equations is only to simplify the expressions. In general the demand curve can be written as: $p=f(Q)$. Marginal revenues are then equal to $Q\cdot f^1(Q) + f(Q)$ and equal to: $p\cdot(1-1/\eta)$. Knowing the demand function and using the former relation for marginal revenues, equations (44) and (45) can be solved for Q as function of the parameters of the demand curve and not of the price of output or the elasticity of demand. Only these parameters are needed to get $\bar{Q}_M^1$ and $\bar{Q}_M^2$ .

The ratio between the optimal levels of output under multiple and single shift plants in monopolistic conditions is:

$$\lambda_2 = \frac{\bar{Q}_M^2}{\bar{Q}_M^1} = (1+\rho)^{\frac{1-\alpha}{1-(\alpha+\beta)}}\left[\frac{p_2\ (1-1/\eta_2)}{p_1\ (1-1/\eta_1)}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}} \tag{46}$$

We will assume that the demand curve has constant elasticity over the relevant range. The ratio between the optimal levels of output is then:

$$\lambda_2 = \frac{\bar{Q}_M^2}{\bar{Q}_M^1} = (1+\rho)^{\frac{1-\alpha}{1-(\alpha+\beta)}}\left[\frac{p_2}{p_1}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}} \tag{46a}$$

The difference in total revenues with two and one shift is now:

$$\bar{R}^2 - \bar{R}^1 = p_2\bar{Q}_M^2 - p_1\bar{Q}_M^1 = p_1\bar{Q}_M^1 \; (\lambda \cdot p_2/p_1 - 1) \tag{47}$$

The difference in total costs is as before:

$$\bar{C}^2 - C^1 = P_k(i+d) \cdot \bar{K}_s^1 \cdot \frac{\alpha+\beta}{\beta} \left[ \left( \frac{\lambda_2}{(1+\rho)^{1-\alpha}} \right)^{\frac{1}{\alpha+\beta}} - 1 \right] \tag{48}$$

The plant designed to operate at two shifts will be chosen whenever:

$$\bar{R}^2 - \bar{R}^1 > \bar{C}^2 - \bar{C}^1 \tag{49}$$

which is equivalent to:

$$\lambda_2 \cdot \frac{p2}{p1} - 1 > \frac{P_{k(i+d)} \cdot \bar{K}_s^1}{p1 \; \bar{Q}_M^1} \cdot \frac{\alpha+\beta}{\beta} \left[ \left( \frac{\lambda_2}{(1+\rho)^{1-\alpha}} \right)^{\frac{1}{\alpha+\beta}} - 1 \right] \tag{50}$$

$P_k(i+d)K_s^1/p_1\bar{Q}_M^1$ is equal to $\beta$ for the Cobb-Douglas production function. Replacing $(1+\rho)$ by its value in terms of $\lambda_2$ and $p_2/p_1$ given by (46a), the condition to invest in double shift plants is as before:

$$1 > \alpha+\beta \tag{51}$$

When there are decreasing returns to scale profits of a plant designed to work at two shifts are greater than profits of a plant designed to operate at one shift. Conversely, when there are increasing returns to scale profits of the single shift plant are greater than that of the double shift one. A priori this makes economic sense: if there are economies of scale it is convenient to take full advantage of them through the operation of a single shift plant, while if there are diseconomies of scale it is better to fraction production in different shifts so as not to operate in regions of higher costs.

Note that the above conclusions have been reached in the presence of ex-post substitution between capital and labor. We do not need fixed ex-post capital-labor ratio--a putty clay model--to account for the existence of single shift plants. 13/ Although the second shift uses less of the expensive factor--labor earning a wage premium--this never counterbalances the presence of economies of scale when the ex-post elasticity of substitution is one.

Shift work has usually been thought as a mean of increasing employment and saving capital. It is therefore interesting to find out if the plant designed to operate at two shifts will employ more labor and less capital than the potential (but less profitable) plant that could be designed to operate at one shift. 14/

From the equations in Appendix I that give optimal labor services as function of output and the parameters, we have:

$$\frac{L^2}{L^1} = \frac{L_1^2 + L_2^2}{L^2} = \frac{1}{1+\rho} \cdot \left( \frac{Q^2}{Q^1 (1+\rho)^{1-\alpha}} \right)^{\frac{1}{\alpha+\beta}} \left[ 1 + \left( \frac{\omega_1}{\omega_2} \right)^{\frac{1}{1-\alpha}} \right] \qquad (52)$$

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13/ This proposition was developed by Mary Ann Baily [1, pp. 33-34] and has been accepted by all other authors [2,5].

14/ Prof. Daniel M. Schydlowsky has commented that this type of comparison is not always the relevant one because the government must take some action to change the private decision and this makes the value of the the economic parameters differ for multiple and single shift plants. While this in general a valid qualification it is not applicable in the Cobb-Douglas case because there the decision depends only on technological characteristics. There is no sense in changing the relative price of factors or the wage premium. The government can only change private decisions by means of a lump-sum subsidy that does not affect the economic Parameters.

Since multiple shifts will only be profitable with decreasing returns to scale, it is fair to consider only the case of perfect competition. Using equation (27), at the optimal levels of output for double and single shift plants the ratio between labor in the former and the latter plants are:

$$\frac{\bar{L}^2}{\bar{L}^1} = (1+\rho)^{\frac{\beta}{1-(\alpha+\beta)}} \left[1 + \left(\frac{\omega_1}{\omega_2}\right)^{\frac{1}{1-\alpha}}\right] \tag{53}$$

With positive wage premium and $\alpha + \beta < 1$ (the condition for multiple shifts to be profitable), equation (53) is always greater than 1: shift work always increased employment in a Cobb-Douglas world.

Similarly:

$$\frac{K_s^2}{K_s^1} = \left[\frac{Q^2}{Q^1(1+\rho)^{1-\alpha}}\right]^{\frac{1}{\alpha+\beta}} \tag{54}$$

Again using equation (27), the ratio between capital stock in the double shift plant and the capital stock in the single shift plant at the optimal levels of output is:

$$\frac{\bar{K}_s^2}{\bar{K}_s^1} = \lambda = (1+\rho)^{\frac{1-\alpha}{1-(\alpha+\beta)}} \tag{55}$$

and shift work--if profitable--will not reduce the required capital stock. That is, shift work does not save capital in a Cobb-Douglas world.

Combining equations (53) and (55) we get the quotient between the capital-labor ratios in plants working two and one shifts:

$$\frac{\bar{L}^2/\bar{K}_s^2}{\bar{L}^1/\bar{K}_s^1} = \frac{1+\left(\frac{\omega_1}{\omega_2}\right)^{\frac{1}{1-\alpha}}}{1+\left(\frac{\omega_1}{\omega_2}\right)^{\frac{\alpha}{1-\alpha}}} \tag{56}$$

which with $\omega_2 > \omega_1$ and $0 < \alpha < 1$ is always greater than 1. The technology in a double shift plant will always be more labor intensive than that of a single shift plant, although shift work does not save capital.

Equations (53) and (54) are comparisons of the absolute amount of labor and capital stock. Since the plants produce different quantities of output this is not a completely valid comparison in all circumstances. If the government does not care of the fact that the production of the specified output will use more capital only because it works at multiple shifts, then it could be more interested in knowing what happens to the labor-output and capital-output ratio. This is obtained by multiplying the equations by $\bar{Q}^1/\bar{Q}^2$, whose value is given by equation (27). The following results:

$$\frac{\bar{L}^2/\bar{Q}^2}{\bar{L}^1/\bar{Q}^1} = \frac{1+\left(\frac{\omega_1}{\omega_2}\right)^{\frac{1}{1-\alpha}}}{1+\left(\frac{\omega_1}{\omega_2}\right)^{\frac{\alpha}{1-\alpha}}} \tag{57}$$

$$\frac{\bar{K}^2/\bar{Q}^2}{\bar{K}^1/\bar{Q}^1} = 1 \tag{58}$$

Double shift plants in a Cobb-Douglas world will always use more labor per unit of output and the same capital per unit of output than single shift plants.

V

## The Pure Investment Decision in the CES Production Function

The general form of the CES Production function with two factors of production is:

$$Q = \gamma \left[\alpha . K^{-\rho} + (1-\alpha) . L^{-\rho}\right]^{-\beta/\rho} \qquad (59)$$

where Q is output per unit of time, L is man-hours per unit of time and K capital services per unit of time. $\gamma$ is a scale parameter reflecting the efficiency of the technology, $\alpha$ indicates the capital intensity ($0 < \alpha < 1$), $\beta$ represents the degree of returns to scale and $\sigma = \frac{1}{1+\rho}$ is the elasticity of substitution between labor and capital.

The optimal levels of output for plants designed to work at one and two shifts as a function of the given prices and parameters of the production functions are derived in Appendix II for competitive and monopolistic product markets and fixed ex-post capital-labor ratio. 15/ When there is perfect competition the values are respectively:

$$\overline{Q}^{1}_{c} = \left[\frac{P \cdot \beta \cdot \gamma^{1/\beta} \cdot \left[\alpha \cdot a^{-\rho} \cdot \mu_1^{-\rho} + (1-\alpha)\right]^{-1/\rho}}{W_1 + Pk(i+d) \cdot \mu_1}\right]^{\beta/1-\beta} \qquad (60)$$

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15/ The case of non constant ex-post capital services-labor services ratio has a more complicated solution. Here we assume--as most authors do--that the crew size of machineries is constant.

$$\bar{Q}_c^2 = \left[ \frac{p \cdot \beta \cdot (2\gamma)^{1/\beta} \cdot [\alpha \cdot a^{-\rho} \cdot \mu_2^{-\rho} + (1-\alpha)]^{-1/\rho}}{w_1 + w_2 + P_k(i+d)\mu_2} \right]^{\beta/1-\beta} \tag{61}$$

where $\mu_1$ is the capital stock-labor services ratio of the single shift plant and $\mu_2$ half the capital stock-labor services ratio of the double shift plant, with values equal to:

$$\mu_1 = \left[ \frac{\omega_1 \cdot \alpha \cdot a^{-\rho}}{P_k(i+d)(1-\alpha)} \right]^{\frac{1}{1+\rho}} \tag{62}$$

$$\mu_2 = \left[ \frac{(\omega_1+\omega_2) \cdot \alpha \cdot a^{-\rho}}{P_k(i+d)(1-\alpha)} \right]^{\frac{1}{1+\rho}} \tag{63}$$

The ratio between the optimal levels of output under double and single shift plants in competitive conditions is:

$$\lambda_3 = \frac{\bar{Q}_c^2}{\bar{Q}_c^1} = 2^{1/(1-\beta)} \left[ \frac{(q \cdot a)^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} + 1}{[q(2+\theta)a]^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} + 1} \right]^{\frac{\beta}{(\beta-1)(\sigma-1)}} \tag{64}$$

where q is the relative prices between labor services and capital stock $q = \dfrac{\omega_1}{P_k(i+d)}$ and $\theta$ the wage premium for the second shift.

Given the parameters of the CES production function, the wage premium for the second shift and the relative prices of labor and capital, this relation will compare output of a plant designed to operate at two shifts with output of a plant designed to operate at one shift. The relation between both outputs will not be indedependent of the relative prices of the factors of production or the wage premium, as in the Cobb-Douglas case with non-fixed ex-post capital-labor ratio.

The entrepreneur will build the plant with higher profits. If $\Pi^1$ are profits of the plant designed to work at one shift and $\Pi^2$ profits of the plant designed to work at two shifts, the two shift mode of operation will be chosen when:

$$\Pi^2 > \Pi^1 \tag{65}$$

This expression is equivalent to:

$$R^2 - C^2 > R^1 - C^1 \tag{66}$$

where the R's represent total revenues and the C's total costs of the alternative plants.

In Appendix II it is shown that total cost as a function of output in the CES production function is:

$$C^1 = \frac{\omega_1 + P_k(i+d)\mu_1}{\gamma^{1/\beta}\,[\alpha\cdot a^{-\rho}\cdot \mu_2{}^{-\rho} + (1-\alpha)]^{-1/\rho}} \quad (Q^1)^{1/\beta} \tag{67}$$

$$C^2 = \frac{\omega_1 + \omega_2 + P_k(i+d)\cdot \mu_2}{(2\gamma)^{1/\beta}\,[\alpha\cdot a^{-\rho}\cdot\mu_2{}^{-\rho}+(1-\alpha)]^{-1/\rho}} \cdot (Q^2)^{1/\beta} \tag{68}$$

Using equations (60) and (61) we get that for the optimal levels of output:

$$\frac{\overline{C}^2}{\overline{C}^1} = \frac{\overline{Q}_c^2}{\overline{Q}_c^1} = \lambda_3 \qquad (69)$$

Since:

$$\overline{R}^2 = \rho\, \overline{Q}_c^2 = \rho \cdot \lambda_3 \cdot \overline{Q}_c^1 = \lambda_3 R^1 \qquad (70)$$

and $$\overline{C}^2 = \lambda_3 \overline{C}^1 \qquad (71)$$

the plant designed to operate at multiple shifts will be chosen whenever:

$$\lambda_3 > 1 \qquad (72)$$

which is equivalent to:

$$2^{\frac{1}{1-\beta}} \left[ \frac{(q \cdot a)^{1-\sigma} \left[\frac{\alpha}{1-\alpha}\right]^{-\sigma} + 1}{[q(2+\Theta)a]^{1-\sigma} \left[\frac{\alpha}{1-\alpha}\right]^{-\sigma} + 1} \right]^{\frac{\beta}{(\beta-1)(\sigma-1)}} > 1 \qquad (73)$$

If the plant designed to operate at multiple shifts will produce a greater output than single shift plants it will always be profitable to invest in the former one over the later one. If both optimal outputs are identical, both plants are equally profitable.[16/] A single shift plant is more profitable than a multiple shift if its optimal output level is greater.

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16/ G. Winston, R. Betancourt and C. Clague and M.S. Baily use identical output in both types of plants. They are able to infer conditions for the pure investment decision from a case of indifference only because one or both output levels are not the optimum. This is a fundamental error: entrepreneurs will always build a plant to produce what they consider to be the optimal output.

Assuming now that the plants are facing a demand curve with elasticity "$\eta$" not equal to infinite, the optimal levels of output for plants designed to work at one and two shifts are respectively:

$$\bar{Q}_m^1 = \left[ \frac{p_1(1-1/\eta_1)\cdot\beta\cdot\gamma^{1/\beta} \cdot [\alpha\cdot a^{-\rho} \cdot \mu_1^{-\rho} + (1-\alpha)]^{-1/\rho}}{\omega_1 + P_k(i+d)\cdot\mu_1} \right]^{\frac{\beta}{1-\beta}} \tag{74}$$

$$\bar{Q}_m^2 = \left[ \frac{p_2(1-1/\eta_2)\cdot\beta\cdot(2\gamma)^{1/\beta} [\alpha\cdot a^{-\rho} \cdot\mu_2^{-\rho} + (1-\alpha)^{-1/\rho}}{\omega_1 + \omega_2 + P_k(i+d) \cdot \mu_2} \right]^{\frac{\beta}{1-\beta}} \tag{75}$$

where $p_1(1-1/\eta_1)$ and $p_2(1-1/\eta_2)$ represent marginal revenues at the levels of output $\bar{Q}_m^1$ and $\bar{Q}_m^2$ respectively.

Note that the prices of output and the elasticities of demand are used in equations (74) and (75) only to indicate the parameters of the demand function. If we know this function, marginal revenue as a function of output is also known. This relation is replaced in the equations, which can then be solved for $\bar{Q}_m^1$ and $\bar{Q}_m^2$ in terms only of the given parameters and prices.

In the following analysis of monopolistic market conditions we will assume that the demand curve has constant elasticity over the relevant range, i.e., we will work with an arc elasticity of demand. In that situation, the ratio of marginal revenues at any two points is:

$$\frac{MR^2}{MR^1} = \frac{p_2}{p_1} = \left[ \frac{Q^2}{Q^1} \right]^{-1/\eta} \tag{76}$$

The ratio between the optimal levels of output under multiple and single shift plants will then be:

$$\lambda_4 = \frac{\bar{Q}_m^2}{\bar{Q}_m^1} = 2^{\frac{1}{1-\beta(1-1/\eta)}} \left[ \frac{(q \cdot a)^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} + 1}{[q(2+\theta)a]^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} + 1} \right]^{\frac{\beta}{[\beta(1-1/\eta)-1][\sigma-1]}} \tag{77}$$

From equations (67), (68), (74) and (75) the ratio between total cost of production under two and one shifts at the optimal level of output is:

$$\frac{\bar{C}^2}{\bar{C}^1} = \frac{p_2 \cdot \bar{Q}_m^2}{p_1 \cdot \bar{Q}_m^1} = \frac{p_2}{p_1} \cdot \lambda_4 \tag{78}$$

Since by definition:

$$\bar{R}^2 = p_2 \cdot \bar{Q}_m^2 = p_2 \cdot \lambda_4 \cdot \bar{Q}_m^1 = (p_2/p_1) \cdot \lambda_4 \cdot R_1 \tag{79}$$

the plant designed to operate at two shifts will be chosen whenever:

$$\frac{p_2}{p_1} \cdot \lambda_4 > 1 \tag{80}$$

Using equation (76) we conclude that the condition to build the two shifts plant is: 17/

$$\lambda_5 = \lambda_4^{1-1/\eta} > 1 \tag{81}$$

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17/ Comparing equations (64) and (77) we have that $\lambda_4 = \lambda_3^{\frac{1-\beta}{1-\beta(1-1/\eta)}}$ and the condition to build the two shifts plant can be written as:

$$\lambda_3^{\frac{(1-\beta)(1-1/\eta)}{1-\beta(1-1/\eta)}} > 1 \tag{81b}$$

or in terms of the given parameters:

$$2^{\frac{(1-1/\eta)}{1-\beta(1-1/\eta)}}\left[\frac{(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1}{[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1}\right]^{\frac{\beta(1-1/\eta)}{[\beta(1-1/\eta)-1][\sigma-1]}} \tag{77}$$

If the elasticity of demand is greater than one---which is a requirement for equilibrium in monopolistic markets,--$1-1/\eta$ is also always greater than one. A necessary condition to invest in the plant designed to work at two shifts is:

$$\lambda_4 > 1 \tag{81}$$

Again we conclude that if the plant designed to operate at multiple shifts will produce a greater output than the single shift plant it will always be profitable to invest in the former one over the later one. If both optimal output are identical, both plants are equally profitable. On the other hand, if optimal output of the single shift plant is greater, we need to look at the elasticity of demand for the individual firm to determine which plant will be built. With high values of this elasticity it is likely that the single shift plant will be preferred.

As can be noticed from equations (73) and (77), the pure investment choice between multiple and single shift plants depends on:

1. Relative factor prices ($q$)
2. Amplitude of the wage (or other cost) differential among shifts ($\theta$)
3. Relative capital intensity of the production process ($\alpha/1-\alpha$)
4. Elasticity of substitution between factors of production ($\sigma$)
5. Degree of returns to scale ($\beta$)

6. Length in time of each shift (a) 18/

7. Elasticity of demand for the output ($\eta$) if markets are non perfectly competitive.

Tables I to IV indicate the ratio between profits in double and single shift plants for different values of the elasticity of substitution, the degrees of returns to scale and the relative capital intensity of the production process. Tables I and II assume perfect competition and Tables III and IV a constant elasticity of demand for the product of the firm equal to 2. 19/ Tables I and III have been calculated with a relative price of labor services and capital stock of $q=1 \times 10^{10}$ and Tables II and IV with $q=1.0$. All tables assume that the wage premium in the second shift is $\theta=0.20$ and that the number of hours per shift is $a=8$ 20/. Whenever the reported value is greater than 1, profits in double shift plants are greater than profits in single shift plants and the former type of plant will be built.

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18/ This factor has been completely by-passed on all the literature on capital utilization. Gordon-Winston [5] did not consider it because of his special assumptions about the length of the production process: a full day or half a day. R. Betancourt and C. Clague [2] did not notice because they use the price of capital services in their analysis, which is not a datum given to the firm but depends on the number of shifts worked. The number of hours per shift is a parameter to the individual decision maker as fixed as any other parameter.

19/ If the elasticity of demand is "x", the reported value should be raised to $\frac{(1-1/x)(2-\beta)}{1-\beta(1-1/x)}$ to get the new ratio of profits.

20/ Although "q" if the price of labor services divided by the price per day of the capital stock, in the equations we always have the term "q.a", where "a" is a constant. This can be interpreted as the price of labor services divided by the price of capital services or as the wage per worker divided by the cost of capital per day.

# TABLE I

## Ratio Between Profits in Double and Single Shift Plants in Perfect Competition with "Low" Relative Price of Capital 1/

1. Degree of returns to scale: $\beta = 0.7$

| Relative capital intensity $\left(\frac{\alpha}{1-\alpha}\right)$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 1.601 | 1.601 | 1.630 | 4.049 | 10.078 | 10.079 |
| o.20 | 1.601 | 1.601 | 1.655 | 5.562 | 10.079 | 10.079 |
| 0.30 | 1.601 | 1.601 | 1.679 | 6.639 | 10.079 | 10.079 |
| 0.40 | 1.601 | 1.601 | 1.702 | 7.288 | 10.079 | 10.079 |

2. Degree of returns to scale: $\beta = 0.8$

| Relative capital intensity $\left(\frac{\alpha}{1-\alpha}\right)$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 1.366 | 1.366 | 1.409 | 6.701 | 31.991 | 32.000 |
| 0.20 | 1.366 | 1.366 | 1.446 | 11.871 | 31.997 | 32.000 |
| 0.30 | 1.366 | 1.366 | 1.481 | 15.639 | 31.998 | 32.0000 |
| 0.40 | 1.366 | 1.366 | 1.517 | 18.375 | 31.999 | 32.000 |

3. Degree of returns to scale: $\beta = 0.9$

| Relative capital intensity $\left(\frac{\alpha}{1-\alpha}\right)$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 0.848 | 0.848 | 0.909 | 30.378 | 1023.337 | 1024.000 |
| 0.20 | 0.848 | 0.848 | 0.964 | 109.986 | 1023.760 | 1024.000 |
| 0.30 | 0.848 | 0.848 | 1.018 | 204.501 | 1023.871 | 1024.0000 |
| 0.40 | 0.848 | 0.848 | 1.073 | 293.270 | 1023.908 | 1024.000 |

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1/ The Table gives the value of:

$$\lambda_3 = 2^{\frac{1}{1-\beta}} \left\{ \frac{[q \cdot a]^{1-\sigma} \left[\frac{\alpha}{1-\alpha}\right]^{-\sigma} + 1}{[q \cdot (2+\theta) \cdot a]^{1-\sigma} \left[\frac{\alpha}{1-\alpha}\right]^{-\sigma} + 1} \right\}^{\frac{\beta}{(\beta-1)(\sigma-1)}}$$

for a ratio between prices of labor per worker per hour and capital per day (q) of $1 \times 10^{10}$, a wage premium ($\theta$) of 0.20 and a number of hours per shift (a) of 8.

TABLE II

Ratio Between Profits in Double and Single Shift Plants in Perfect Competition With "High" Relative Price of Capital 1/

1. Degree of returns to scale: $\beta = 0.7$

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 1.856 | 1.871 | 1.888 | 1.897 | 1.920 | 1.953 |
| 0.20 | 1.874 | 1.980 | 2.130 | 2.226 | 2.472 | 3.065 |
| 0.30 | 1.885 | 2.063 | 2.352 | 2.555 | 3.116 | 4.139 |
| 0.40 | 1.893 | 2.132 | 2.560 | 2.877 | 3.772 | 5.325 |

2. Degree of returns to scale: $\beta = 0.8$

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 1.759 | 1.783 | 1.812 | 1.828 | 1.865 | 1.920 |
| 0.20 | 1.789 | 1.964 | 2.223 | 2.400 | 2.867 | 3.794 |
| 0.30 | 1.808 | 2.109 | 2.641 | 3.044 | 4.276 | 6.958 |
| 0.40 | 1.820 | 2.232 | 3.054 | 3.730 | 5.935 | 10.719 |

3. Degree of returns to scale: $\beta = 0.9$

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 1.499 | 1.543 | 1.597 | 1.632 | 1.709 | 1.825 |
| 0.20 | 1.556 | 1.920 | 2.537 | 3.014 | 4.531 | 8.445 |
| 0.30 | 1.592 | 2.254 | 3.738 | 5.145 | 11.056 | 33.065 |
| 0.40 | 1.618 | 2.560 | 5.184 | 8.131 | 23.112 | 87.407 |

1/ The Table gives the value of:

$$\lambda_3 = 2^{\frac{1}{1-\beta}} \left\{ \frac{[q\cdot a]^{1-\sigma}\left[\frac{\alpha}{1-\alpha}\right]^{-\sigma} + 1}{[q\cdot(2+\theta)\cdot a]^{1-\sigma}\left[\frac{\alpha}{1-\alpha}\right]^{-\sigma} + 1} \right\}^{\frac{\beta}{(\beta-1)(\sigma-1)}}$$

for a ratio between prices of labor and capital (q) of 1, a wage premium ($\theta$) of 0.20 and a number of hours in each shift (a) of 8.

## TABLE III

### Ratio Between Profits in Double and Single Shift Plants in Monopolistic Product Markets with "Low" Relative Price of Capital [1/]

1. Degree of returns to scale: $\beta$ = 0.7

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution | | | | | |
|---|---|---|---|---|---|---|
| | $\sigma$=0.1 | $\sigma$=0.5 | $\sigma$=0.9 | $\sigma$=1.1 | $\sigma$=1.5 | $\sigma$=2.0 |
| 0.10 | 1.115 | 1.115 | 1.119 | 1.381 | 1.704 | 1.704 |
| 0.20 | 1.115 | 1.115 | 1.123 | 1.486 | 1.704 | 1.704 |
| 0.30 | 1.115 | 1.115 | 1.127 | 1.547 | 1.704 | 1.704 |
| 0.40 | 1.115 | 1.115 | 1.131 | 1.581 | 1.704 | 1.704 |

2. Degree of returns to scale: $\beta$ = 0.8

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution | | | | | |
|---|---|---|---|---|---|---|
| | $\sigma$=0.1 | $\sigma$=0.5 | $\sigma$=0.9 | $\sigma$=1.1 | $\sigma$=1.5 | $\sigma$=2.0 |
| 0.10 | 1.053 | 1.053 | 1.059 | 1.373 | 1.782 | 1.782 |
| 0.20 | 1.053 | 1.053 | 1.063 | 1.510 | 1.782 | 1.782 |
| 0.30 | 1.053 | 1.053 | 1.068 | 1.581 | 1.782 | 1.782 |
| 0.40 | 1.053 | 1.053 | 1.072 | 1.624 | 1.782 | 1.782 |

3. Degree of returns to scale: $\beta$ = 0.9

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution | | | | | |
|---|---|---|---|---|---|---|
| | $\sigma$=0.1 | $\sigma$=0.5 | $\sigma$=0.9 | $\sigma$=1.1 | $\sigma$=1.5 | $\sigma$=2.0 |
| 0.10 | 0.985 | 0.985 | 0.991 | 1.364 | 1.878 | 1.878 |
| 0.20 | 0.985 | 0.985 | 0.997 | 1.533 | 1.878 | 1.878 |
| 0.30 | 0.985 | 0.985 | 1.002 | 1.623 | 1.878 | 1.878 |
| 0.40 | 0.985 | 0.985 | 1.006 | 1.676 | 1.878 | 1.878 |

4. Degree of returns to scale: $\beta$ = 1.1

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Subsittution | | | | | |
|---|---|---|---|---|---|---|
| | $\sigma$=0.1 | $\sigma$=0.5 | $\sigma$=0.9 | $\sigma$=1.1 | $\sigma$=1.5 | $\sigma$=2.0 |
| 0.10 | 0.824 | 0.824 | 0.831 | 1.334 | 2.154 | 2.154 |
| 0.20 | 0.824 | 0.824 | 0.839 | 1.595 | 2.154 | 2.154 |
| 0.30 | 0.824 | 0.824 | 0.845 | 1.736 | 2.154 | 2.154 |
| 0.40 | 0.824 | 0.824 | 0.851 | 1.802 | 2.154 | 2.154 |

5. Degree of returns to scale: $\beta = 1.2$

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 0.729 | 0.729 | 0.737 | 1.323 | 2.383 | 2.383 |
| 0.20 | 0.729 | 0.729 | 0.745 | 1.641 | 2.383 | 2.383 |
| 0.30 | 0.729 | 0.729 | 0.751 | 1.821 | 2.383 | 2.383 |
| 0.40 | 0.729 | 0.729 | 0.758 | 1.930 | 2.383 | 2.383 |

6. Degree or returns to scale: $\beta = 1.3$

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 0.622 | 0.622 | 0.630 | 1.302 | 2.694 | 2.694 |
| 0.20 | 0.622 | 0.622 | 0.639 | 1.697 | 2.694 | 2.694 |
| 0.30 | 0.622 | 0.622 | 0.646 | 1.929 | 2.694 | 2.694 |
| 0.40 | 0.622 | 0.622 | 0.653 | 2.080 | 2.694 | 2.694 |

1/ The Table gives the value of:

$$\lambda_5 = 2^{\frac{(1-1/\eta)}{1-\beta(1-1/\eta)}} \left[ \frac{(q\cdot a)^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} + 1}{[q(2+\theta)a]^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} + 1} \right]^{\frac{\beta(1-1/\eta)}{[\beta(1-1/\eta)-1][\sigma-1]}}$$

for a ratio between prices of labor per worker per hour and capital per day (q) of 1 x $10^{10}$, a wage premium ($\theta$) of 0.20, a number of hours per shift (a) of 8 and a constant elasticity of demand ($\eta$) 2.0.

# TABLE IV

## Ratio Between Profits in Double and Single Shift Plants in Monopolistic Product Markets with "High" Relative Price of Capital[1/]

1. Degree of returns to scale: $\beta = 0.7$

| Relative capital intensity ($\alpha/1-\alpha$) | Elasticity of Substitution $\sigma$=0.1 | $\sigma$=0.5 | $\sigma$=0.9 | $\sigma$=1.1 | $\sigma$=1.5 | $\sigma$=2.0 |
|---|---|---|---|---|---|---|
| 0.10 | 1.153 | 1.156 | 1.158 | 1.159 | 1.162 | 1.167 |
| 0.20 | 1.156 | 1.171 | 1.191 | 1.203 | 1.232 | 1.295 |
| 0.30 | 1.158 | 1.182 | 1.218 | 1.242 | 1.300 | 1.388 |
| 0.40 | 1.159 | 1.191 | 1.242 | 1.276 | 1.358 | 1.471 |

2. Degree of returns to scale: $\beta = 0.8$

| Relative capital intensity ($\alpha/1-\alpha$) | Elastiticy of Substitution $\sigma$=0.1 | $\sigma$=0.5 | $\sigma$=0.9 | $\sigma$=1.1 | $\sigma$=1.5 | $\sigma$=2.0 |
|---|---|---|---|---|---|---|
| 0.10 | 1.099 | 1.101 | 1.104 | 1.106 | 1.109 | 1.115 |
| 0.20 | 1.102 | 1.119 | 1.142 | 1.157 | 1.192 | 1.249 |
| 0.30 | 1.104 | 1.132 | 1.176 | 1.204 | 1.274 | 1.382 |
| 0.40 | 1.105 | 1.143 | 1.205 | 1.245 | 1.346 | 1.485 |

3. Degree of returns to scale: $\beta = 0.9$

| Relative capital intensity ($\alpha/1-\alpha$) | Elasticity of Substitution $\sigma$=0.1 | $\sigma$=0.5 | $\sigma$=0.9 | $\sigma$=1.1 | $\sigma$=1.5 | $\sigma$=2.0 |
|---|---|---|---|---|---|---|
| 0.10 | 1.037 | 1.040 | 1.043 | 1.046 | 1.050 | 1.056 |
| 0.20 | 1.041 | 1.061 | 1.088 | 1.105 | 1.147 | 1.214 |
| 0.30 | 1.043 | 1.077 | 1.127 | 1.161 | 1.244 | 1.374 |
| 0.40 | 1.045 | 1.089 | 1.161 | 1.210 | 1.330 | 1.501 |

4. Degree of returns to scale: $\beta = 1.1$

| Relative capital intensity ($\alpha/1-\alpha$) | Elasticity of Substitution $\sigma$=0.1 | $\sigma$=0.5 | $\sigma$=0.9 | $\sigma$=1.1 | $\sigma$=1.5 | $\sigma$=2.0 |
|---|---|---|---|---|---|---|
| 0.10 | 0.890 | 0.894 | 0.898 | 0.901 | 0.906 | 0.915 |
| 0.20 | 0.895 | 0.921 | 0.956 | 0.979 | 1.031 | 1.126 |
| 0.30 | 0.898 | 0.941 | 1.008 | 1.053 | 1.168 | 1.355 |
| 0.40 | 0.898 | 0.957 | 1.054 | 1.120 | 1.290 | 1.544 |

5. Degree of returns to scale: $\beta = 1.2$

| Relative capital intensity $(\alpha/1-\alpha)$ | $\sigma = 0.1$ | $\sigma = 0.5$ | $\sigma = 0.9$ | $\sigma = 1.1$ | $\sigma = 1.5$ | $\sigma = 2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 0.801 | 0.805 | 0.810 | 0.812 | 0.819 | 0.828 |
| 0.20 | 0.806 | 0.835 | 0.875 | 0.900 | 0.964 | 1.069 |
| 0.30 | 0.809 | 0.858 | 0.933 | 0.984 | 1.118 | 1.342 |
| 0.40 | 0.812 | 0.876 | 0.985 | 1.062 | 1.265 | 1.579 |

6. Degree of returns to scale: $\beta = 1.3$

| Relative capital intensity $(\alpha/1-\alpha)$ | $\sigma = 0.1$ | $\sigma = 0.5$ | $\sigma = 0.9$ | $\sigma = 1.1$ | $\sigma = 1.5$ | $\sigma = 2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 0.700 | 0.704 | 0.710 | 0.712 | 0.719 | 0.729 |
| 0.20 | 0.705 | 0.736 | 0.780 | 0.809 | 0.879 | 1.000 |
| 0.30 | 0.709 | 0.762 | 0.845 | 0.903 | 1.057 | 1.326 |
| 0.40 | 0.711 | 0.782 | 0.904 | 0.992 | 1.232 | 1.619 |

1/ The Table gives the value of:

$$\lambda_5 = 2^{\frac{1-1/\eta}{1-\beta(1-1/\eta)}} \left[ \frac{(q \cdot a)^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} + 1}{[q(2+\theta)a]^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} + 1} \right]^{\frac{\beta(1-1/\eta)}{[\beta(1-1/\eta)-1][\sigma-1]}}$$

for a ratio between prices of labor per worker per hour and capital per day (q) of 1, a wage premium ($\theta$) of 20%, a number of hours per shift (a) of 8 and a constant elasticity of demand ($\eta$) of 2.0.

The Tables make clear that a full knowledge of the specific values of <u>all</u> the technological and economic variables is necessary to find out when entrepreneurs will build plants designed to work at multiple shifts. No general rules can be deduced from information on only one of the relevant variables, as is done in some empirical studies that rely exclusively on capital intensity.

Assuming a given set of prices, can a very capital intensive technology result in a plant designed to work only one shift? The answer is a clear yes. From the data of Table III we infer that when the relative capital intensity is 0.40, the entrepreneur will choose a plant designed to work at a single shift over one designed to work at two shifts if the elasticity of substitution is low ($\sigma$=0.5) and there are small diseconomies of scale ($\beta$=0.9) or small economies of scale ($\beta$=1.1).

When we compare different firms each with a distinct technology--as must be done in the real world--the above conclusion is reasserted. Using again the data of Table III, plants with a capital intensity of 0.40 will be designed to work one shift if $\sigma$=0.5 and $\beta$=0.9, while plants with a capital intensity of 0.10 will be designed to work multiple shifts if $\sigma$=0.5 and $\beta$=0.8.

The above facts contradict the general conclusion of the <u>theoretical</u> literature on capital utilization which always describes a positive relation between capital intensity and investment in multiple shifts plants. It underscores the importance of defining the technological and economic characteristics before advancing conclusions on the investment decision between plants designed to operate at different number of shifts.

But if we were to establish <u>empirically</u> that only in technologically capital intensive industries entrepreneurs do plan to build multiple shifts plants, this would imply something about the technology that is available. In the specified case, the technology must have a very high elasticity of substitution between capital and labor or very big diseconomies of scale everywhere.

In a real world situation the range of the technological and economic parameters is more limited than what is indicated in the tables. The technological characteristics required for multiple shifts should then be fairly stable. For example, Table II indicates that when $\sigma=0.9$, $\beta=0.9$, $\alpha/1-\alpha=0.10$ multiple shift plants will be desirable if the relative price of labor and capital (q) is 1. For the same technological characteristics, multiple shifts will still be desirable for the lower price of capital indicated by q=1000 and q=1.000.000.

Taking the partial derivatives of the relative profitability of plants designed to operate at multiple and single shifts under perfectly competitive and monopolistic product markets--relations (73) and (81)--with respect to each of the parameters, we get the following results:

1. Relative prices between labor services and capital stock:

$$\frac{\partial\lambda_3}{\partial q} = \frac{\beta\cdot\lambda_3\cdot a^{1-\sigma}\cdot[\alpha\cdot q/(1-\alpha)]^{-\sigma}}{(1-\beta)[(q\cdot a)^{1-\sigma}\cdot(\alpha/1-\alpha)^{-\sigma}+1][[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1]}\left[1+\frac{(2+\theta)}{(2+\theta)^{\sigma}}\right] \tag{80}$$

$$\frac{\partial\lambda_5}{\partial q} = \frac{(1-1/\eta)\cdot\beta\cdot\lambda_5\cdot a^{1-\sigma}\cdot[\alpha\cdot q/(1-\alpha)]^{-\sigma}}{[1-\beta(1-1/\eta)][(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1][[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1]}\left[1-\frac{(2+\theta)}{(2+\theta)^{\sigma}}\right] \tag{81}$$

2. Wage premium in the late shift:

$$\frac{\partial\lambda_3}{\partial\theta} = \frac{\beta\cdot\lambda_3\cdot(2+\theta)^{-\sigma}\cdot(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}}{(\beta-1)[q(2+\theta)\cdot a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1} \tag{82}$$

$$\frac{\partial\lambda_5}{\partial\theta} = \frac{(1-1/\eta)\cdot\beta\cdot\lambda_5\cdot(2+\theta)^{-\sigma}(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}}{[\beta(1-1/\eta)-1]\;[q(2+\theta)a]^{1-\sigma}\cdot(\alpha/1-\alpha)^{-\sigma}+1} \tag{83}$$

3. Relative capital intensity:

$$\frac{\partial\lambda_3}{\partial(\alpha/1-\alpha)} = \frac{\beta\cdot\sigma\cdot\lambda_3\cdot(\alpha/1-\alpha)^{-\sigma-1}(q\cdot a)^{1-\sigma}[1-(2+\theta)^{1-\sigma}]}{(\beta-1)(1-\sigma)[(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1][[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1]} \tag{84}$$

$$\frac{\partial\lambda_5}{\partial(\alpha/1-\alpha)} = \frac{(1-1/\eta)\cdot\beta\cdot\sigma\cdot\lambda_5\cdot(\alpha/1-\alpha)^{-\sigma-1}(q\cdot a)^{1-\sigma}[1-(2+\theta)^{1-\sigma}]}{[\beta(1-1/\eta)-1](1-\sigma)[(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1][[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1]} \tag{85}$$

4. Number of hours per shift:

$$\frac{\partial\lambda_3}{\partial a} = \frac{\beta\cdot\lambda_3\cdot a^{-\sigma}\cdot q^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}[1-(2+\theta)^{1-\sigma}]}{(1-\beta)[(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1][[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1} \tag{86}$$

$$\frac{\partial\lambda_5}{\partial a} = \frac{(1-1/\eta)\cdot\beta\cdot\lambda_5\cdot a^{-\sigma}\cdot q^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}[1-(2+\theta)^{1-\sigma}]}{[1-\beta(1-1/\eta)][(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1][[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1} \tag{87}$$

In perfectly competitive product markets the plant must always be operating in a region of decreasing returns to scale ($\beta < 1$). An increase in the relative price of capital or a reduction in the relative price of labor--which is a policy often recommended for developing countries--will then increase the relative profitability of plants designed to operate at multiple shifts only if the elasticity of substitution is less than one. <u>But if the elasticity of substitution is greater than one, an increase in the relative price of capital will reduce the relative profitability of multiple shifts plants.</u>

The explanation of the above phenomenon lies in the fact--which will be proved later-- that with an elasticity of substitution greater than one the multiple shifts plant has a higher capital-labor ratio than the single shift plant. The higher relative price of capital affects then more the profits of the former one than those of the latter one.

In monopolistic product markets with constant elasticity of demand greater than one over the relevant range, an increase in the relative price of capital or a reduction in the relative price of labor will increase the relative profitability of plants designed to operate at multiple shifts only in the following circumstances:

1. If the elasticity of substitution is less than 1.0 and the degree of returns to scale (homogeniety) less than the inverse of $1-1/\eta$ $(\beta < 1/(1-1/\eta)$
2. If the elasticity of substitution is greater than 1.0 and the degree of returns to scale greater than the inverse of $(1-1/\eta)$.

Gordon Winston [5,6] has argued that with low values of the elasticity of substitution and relatively high capital prices, capital costs will be a large part of total costs and their reduction through multiple shifts plants will be more urgent. Here we note that this proposition must be qualified by the degree of returns to scale. Optimal output in multiple and single shifts plants is changing when the relative price of capital is altered. This influence must be considered in analyzing what type of plant is favored with the given change in the relative price of capital.

For example, if we consider Winston's case of a low elasticity of substitution $(\sigma < 1)$ and add the existence of some low economies of scale $(1-1/\eta > \beta > 1)$, the higher relative price of capital will reduce the optimal output to be produced in multiple shift plants relatively to that of single shift plants. With economies of scale this effect may completely cancel out the effect due to the lower capital-labor ratio in multiple shift plants and single shift plants can still be more profitable.

The effect of changes in the wage differential among shifts is independent of the elasticity of substitution but related to the degree of returns to scale. With decreasing returns to scale an increase in the wage differential will always reduce the relative profitability of the plant designed to work at multiple shifts. But with economies of scale greater than the inverse of $(1-1/\eta)$ in monopolistic product markets the higher wage differential will increase the relative profitability of the multiple shift plants.

The explanation of the above phenomenon lies in the decline in marginal cost that may take place in the multiple shift plant with large economies of scale as the more expensive factor (labor) is replaced by capital. A small decline in marginal cost increases output and the effect of this increase is compounded by the existence of large economies of scale.

A change in the length of the working schedule of each shift has results exactly equal to a change in the relative price of factors of production. If in equations (80) and (81) "a" is replaced by "q" and "q" by "a" we get equations (86) and (87). But it must be remembered that an increase in the number of hours worked per shift is equivalent to a decline in the price of capital stock and vice versa.

Therefore, in perfectly competitive product markets a decrease in the length of the working schedule of each shift will increase the relative profitability of multiple shifts plants when the elasticity of substitution is less than one, but not in the opposite case. In monopolistic productmarkets the same results will be obtained with an elasticity of substitution less than 1.0 and a degree of returns to scale less then the inverse of $(1-1/\eta)$.

Equations (84) and (85) indicate that the relative profitability of multiple shifts plants will increase as the technology is more capital intensive if there are decreasing returns to scale. In processes with economies of scale greater than the inverse of $(1-1/\eta)$ changes to more capital intensive technologies will increase the relative profitability of plants designed to work a single shift. These results hold irrespective of the elasticity of substitution between capital and labor.

It must be noted that the capital intensity we are talking about is that that is part of the ruling technology, that is a technological requirement of the productive process independent of the prevalent factor prices. It should not be confused with capital intensity as usually defined in terms of the quantity of capital relative to the quantity of labor used in the production process. [20/] Labor and capital

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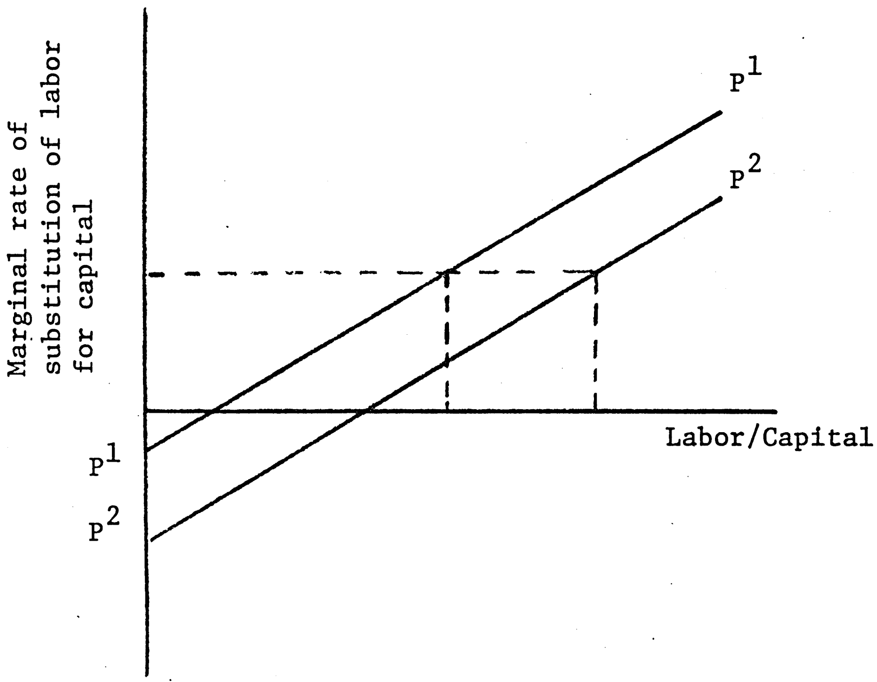
20/ For a further elaboration of this distinction see Murray Brown, On the Theory and Measurement of Technological Change, Cambridge: Cambridge University Press, 1966, pp. 15-17.

used are a result of the maximization procedure followed by the entrepreneur and not a given data. What is really relevant to the pure investment decision is to know the effect of the technologically given capital intensity of the production process on the choice between single and multiple shift plants. Of course we can compare afterwards the <u>resulting</u> capital-labor ratios, but it is absurd to assume them as given when we are deciding what plant to build.

In the real world it is difficult to say what part of the observed capital intensity is due to technological requirements of production and what to factor prices. Is a steel plant more capital intensive than a shoe factory because of technological requirements or because of relative prices of factors or because of both?

Those who think that the capital intensity of the technology is changing only as a reflection of factor prices can neglect the present discussion and procede directly to the comparison of the capital-labor ratios.

In the following figure we use a graph borrowed from Murray Brown [3]. Both axes are measured in logarithms. The lines show the logarithmic relation between the ratio of labor to capital and the marginal rate of substitution of labor for capital. $P^1P^1$ represents a more capital intensive technology than $P^2P^2$ because the marginal product of capital relative to that of labor is larger for any given labor-capital ratio




The marginal rate of substitution must be equal to the relative factor prices. When the available technology becomes more capital intensive for a certain plant- and all other parameters are fixed- the ratio of labor to capital decreases. Since the marginal product of capital is higher with more capital intensive technologies, the production process would use more capital relative to labor. This will affect the marginal cost function and the optimal level of output produced by the plant.

When the available technology becomes more capital intensive in the same amount for both multiple and single shifts plants, the changes in the labor-capital ratio are the same for both types of plants. Each plant has a lower labor-capital ratio but the relative ratio between the plants has not changed. But the changes in marginal cost at any level of output are not proportional. If there are economies of scale, marginal costs decrease more rapidly in single shift plant. Output and profits of this type of plant are greater relative to that of multiple shifts plants. With economies of scale the existence of a more capital intensive technology favor single shift plants. The opposite results may be obtained with diseconomies of scale.

The effect of changes in the degree of economies of scale is not unambiguous and depends on the specific values of all other economic and technical parameters. In monopolistic product markets the expression

$$Z = \ln \left[ \frac{(q.a)^{1-\sigma} \, (\alpha/1-\alpha)^{-\sigma} + 1}{[q(2+\theta)a]^{1-\sigma} \, (\alpha/1-\alpha)^{-\sigma} + 1} \right]^{1/\sigma-1} \qquad (88)$$

has to be compared in value to $(1-1/\eta)\ln 2$ to get an answer. If "Z" is greater than $(1-1/\eta)\ln 2$ then an increase in the degree of economies of scale of the technology will always favor multiple shifts plants. 21/

When there are constant returns to scale ($\beta=1$) and perfect competition the ratio between profits in double and single shift plants given by relation (73) is undetermined. But the pure investment decision in the specific instance of constant returns to scale can be solved without the improper assumption of equal output in the different plants that we have criticized. For this it should be noted that when there are constant returns to scale, marginal and average costs are both constant and equal. Entrepreneurs will always build the plant with the lower constant average costs.

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21/ This result contradicts one of the conclusions of R. Betancourt and C. Clague, op.cit., that when the technology of two process differs only in the scale parameter, the one with higher returns to scale will always be more favorable to single shift plants. There could be cases where this is not so.

Taking the first derivatives of equations (67) and (68) and introducing the fact that $\sigma = 1$, we get that marginal cost with single and double shift plants are respectively:

$$MC^1 = (1-\sigma) \cdot \gamma^{\sigma-1} \left[ \frac{p \cdot (1-\alpha)}{\omega_1} \right]^{\sigma} \left[\omega_1 + P_k (i+d) \mu_1\right] \tag{89}$$

$$MC^2 = (1-\sigma)(2\gamma)^{\sigma-1} \left[ \frac{p \cdot (1-\alpha)}{\omega_1 + \omega_2} \right]^{\sigma} \left[\omega_1 + \omega_2 + P_k (i+d) \mu_2\right] \tag{90}$$

The ratio between marginal (or average) costs under double and single shift plants is:

$$\lambda_6 = 2^{\sigma-1} \cdot \frac{[q \cdot (2+\theta) \cdot a]^{1-\sigma} \cdot \left(\frac{\alpha}{1-\alpha}\right)^{-\sigma} + 1]}{[q \cdot a]^{1-\sigma} \cdot \left(\frac{\alpha}{1-\alpha}\right)^{-\sigma} + 1]} \tag{93}$$

The values of $\lambda_6$ are tabulated in Table V. When the reported number is greater than 1.0 average costs in multiple shifts plants are greater than those in single shift plants and this last type of plant is preferred.

TABLE V

Ratio Between Average Costs in Double and Single Shift

Plants with Constant Returns to Scale 1/

1. "Low" Relative Price of Capital: $q=1 \times 10^{10}$

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 1.090 | 1.049 | 1.009 | 1.031 | 1.414 | 2.000 |
| 0.20 | 1.090 | 1.049 | 1.008 | 1.045 | 1.414 | 2.000 |
| 0.30 | 1.090 | 1.049 | 1.008 | 1.052 | 1.414 | 2.000 |
| 0.40 | 1.090 | 1.049 | 1.007 | 1.057 | 1.414 | 2.000 |

2. "High" Relative Price of Capital: q=1

| Relative capital intensity $\frac{\alpha}{1-\alpha}$ | Elasticity of Substitution $\sigma=0.1$ | $\sigma=0.5$ | $\sigma=0.9$ | $\sigma=1.1$ | $\sigma=1.5$ | $\sigma=2.0$ |
|---|---|---|---|---|---|---|
| 0.10 | 1.029 | 1.014 | 1.003 | 0.998 | 0.991 | 0.990 |
| 0.20 | 1.025 | 1.002 | 0.997 | 1.005 | 1.047 | 1.174 |
| 0.30 | 1.023 | 0.993 | 0.993 | 1.011 | 1.100 | 1.366 |
| 0.40 | 1.021 | 0.986 | 0.990 | 1.016 | 1.146 | 1.522 |

1/ The Table gives the value of:

$$\lambda_6 = 2^{\sigma-1} \frac{[q(2+\theta)a]^{1-\sigma}\left(\frac{\alpha}{1-\alpha}\right)^{-\sigma} + 1}{(q \cdot a)^{1-\sigma}\left(\frac{\alpha}{1-\alpha}\right)^{-\sigma} + 1}$$

for a wage premium ($\theta$) of 20% and a number of hours in each shift (a) of 8.

As indicated before, equations (62) and (63) give the capital stock-labor services ratio of the single shift plant and half the ratio of the double shift one respectively.[22/] From them we get that the relative labor-capital ratio is:

$$\frac{L^2/K_s2}{L^1/K_s1} = \frac{2}{(2+\theta)^{\sigma}} \qquad (94)$$

The relative labor-capital ratio between double and single shift plants is a function exclusively of the wage premium and the elasticity of substitution. We have used this fact before in explaining the influence of changes in the relative prices of factors of production and in the capital intensity of the technology.

With a positive wage differential, an elasticity of substitution greater than one will always imply that plants designed to operate at two shifts will be more capital intensive than single shift plants. Only if the elasticity of substitution is less than log 2/log(2+θ) will double shift plants be more labor intensive than single shift ones.

A more or less labor intensive production technique does not necessarily imply that employment is increased or decreased with multiple shifts plants because the amount of capital is generally different for dissimilar number of shifts in the investment decision. The marginal product of labor in the CES production function can be written in the following way for single and multiple shifts plants:

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22/ For the capital stock-labor stock ratio these relations have to be multiplied by the number of hours that each worker is employed, that is, the lenght of each shift "a". If "a" is the same for multiple and single shift plants, equation (94) is valid for the flow of labor services or the number of workers.

$$MPL^1 = \gamma^{-\rho/\beta} \cdot \beta \cdot (1-\alpha) \cdot (Q^1)^{1+\rho/\beta} \cdot (L^1)^{-\rho-1} \tag{95}$$

$$MPL^2 = (2\gamma)^{-\rho/\beta} \cdot \beta \cdot (1-\alpha) \cdot (Q^2)^{1+\rho/\beta} (La^2)^{-\rho-1} \tag{96}$$

Equating the marginal product of labor to the corresponding factor prices we get a relation between optimal labor services and output for any level of output:

$$L^1 = \left[ \frac{p_1 \cdot \gamma^{-\rho/\beta} \cdot \beta \cdot (1-\alpha) \cdot (Q^1)^{1+\rho/\beta}}{W_1} \right]^{\sigma} \tag{97}$$

$$L^2 = 2 \left[ \frac{p_2 \cdot (2\gamma)^{-\rho/\beta} \cdot \beta \cdot (1-\alpha) \cdot (Q^2)^{1+\rho/\beta}}{W_1 + W_2} \right]^{\sigma} \tag{98}$$

where "$p_1$" and "$p_2$" are the prevailing market prices for output when the plants works one and two shifts.

In perfectly competitive product markets the price of output is constant and we get the following relation between employment in the different plants:

$$\frac{L^2}{L^1} = \frac{2^{\frac{\sigma+\beta-1}{\beta}}}{(2+\theta)^{\sigma}} \left[ \frac{Q^2}{Q_1} \right]^{\sigma-(\sigma-1/\beta)} \tag{99}$$

At the optimal output level for each type of plant the above equation can be expressed in terms of the technological and economic parameters given to the decision maker. Using equation (73) shift work will increase employment if:

$$\frac{2^{\frac{2-\beta}{1-\beta}}}{(2+\theta)^{\sigma}} \left[ \frac{(q.a)^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} +1}{[q\ (2+\theta)a]^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} +1} \right]^{\frac{\sigma(\beta-1)+1}{(\sigma-1)(\beta-1)}} > 1 \qquad (100)$$

In monopolistic product markets with constant elasticity of demand "$\eta$" over the relevant range, the ratio of prices at any two points in the demand function can be expressed in terms of the corresponding levels of output. The relation between employment in the alternative plants is then:

$$\frac{L^2}{L^1} = \frac{2^{\frac{\sigma+\beta-1}{\beta}}}{(2+\theta)^{\sigma}} \left[ \frac{Q^2}{Q^1} \right]^{\sigma(1-1/\eta) + (1-\sigma)/\beta} \qquad (101)$$

Replacing $Q^2/Q^1$ at the optimal output levels by its value given gy (77) this relation can be expressed in terms of the given technological and economic parameters. Shift work increases employment if:

$$\frac{2^{\frac{2-\beta+1/\eta(\beta-1)}{1-\beta(1-1/\eta)}}}{(2+\theta)^{\sigma}} \left[ \frac{(q\cdot a)^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} +1}{[q(2+\theta)a]^{1-\sigma} (\alpha/1-\alpha)^{-\sigma} +1} \right]^{\frac{\sigma\ [\beta(1-1/\eta)-1]\ +1}{[\beta(1-1/\eta)-1](\sigma-1)}} > 1 \qquad (102)$$

Equations (99) to (102) are comparisons of the <u>absolute</u> number of workers in double and single shift plant and do not adjust for the fact that each plant produces a different quantity of output. If the interest is in the labor-output ratio the following relations should be used:

$$\frac{L^2/Q^2}{L^1/Q^1} = \frac{2^{\frac{\sigma+\beta-1}{\beta}}}{(2+\theta)^{\sigma}} \left[\frac{Q^2}{Q^1}\right]^{(\sigma-1)(1-1/\beta)}$$

for competitive conditions (103)

$$\frac{L^2/Q^2}{L^1/Q^1} = \frac{2^{\frac{\sigma+\beta-1}{\beta}}}{(2+\theta)^{\sigma}} \left[\frac{Q^2}{Q^1}\right]^{\sigma(1-1/\eta)+\frac{1-\sigma}{\beta}-1}$$

for monopolistic conditions (104)

where $Q^2/Q^1$ can be replaced by their value in term of the technological and economic parameters given by (73) and (77).

A similar analysis can be done for the stock of capital. The resulting ratios of capital stock in double and single shifts plants are the following:

1. For perfectly competitive product markets:

$$\frac{K_s^2}{K_s^1} = 2^{(\sigma-1)/\beta} \left[\frac{Q^2}{Q^1}\right]^{\sigma-(\sigma-1/\beta)} \quad (105)$$

$$\frac{Ks^2/Q^2}{Ks^1/Q^1} = 2^{(\sigma-1)/} \left[\frac{Q^2}{Q^1}\right]^{(\sigma-1)(1-1/\beta)} \quad (106)$$

2. For monopolistic product markets:

$$\frac{K_s^2}{K_s^1} = 2^{(\sigma-1)/\beta} \left[\frac{Q^2}{Q^1}\right]^{\sigma(1-1/\eta)+1-\sigma/\beta} \quad (107)$$

$$\frac{Ks^2/Q^2}{K_s^1/Q^1} = 2^{(\sigma-1)/\beta} \left[\frac{Q^2}{Q^1}\right]^{\sigma(1-1/\eta)+\frac{1-\sigma}{\beta}-1} \quad (108)$$

where again $Q^2/Q^1$ can be replaced in term of the technological and economic parameters.

The above results underscore the fact that it is not warranted that multiple shifts will always increase employment or save capital. A specific knowledge of the value of all the economic and technical parameters is required to determine precisely when this will happen. When some policy planned to increase the likelihood that entrepreneurs invest in multiple shift plants is pursued, some of the new multiple shifts plants <u>may</u> employ less workers and use more capital than the alternative single shift plant or than the single shift plant that would be built without the policy. Off course this is a theoretical result whose practical relevance in a given situation must be ascertained.

# VI

## Conclusion

The choice between building a plant designed to operate at one shift or a plant designed to operate at multiple shifts depends on the interaction between the technological characteristics of production and the economic variables facing the decision maker. We have explained in general how this choice is made and developed the specific formula that related the decision to the parameters embodied in a Cobb-Douglas or CES production function, to the prices of factors of production and to the differential in wages among shifts. The cases of perfectly competitive product markets and of a downward sloping demand curve facing the firm are considered.

For a given set of values of the economic variables, the ex-ante elasticity of substitution between capital and labor and the degree of returns to scale are crucial technological characteristics. If the technology available has very high elasticity of substitution the likelihood of choosing a multiple shift plant is increased. Economies of scale will in general favor single shift plants. But there are no general rules to relate the choice of plants to any value of these parameters. A high elasticity of substitution may be compensated by economies of scale or by appropriate values of the other parameters and vice versa.

Several emprical studies have stated that multiple shifts are more common in capital intensive industries. These studies fail to distinguish between capital intensity that is a technological requirement of production and capital intensity that is chosen by the rational entrepreneur. This distinction is crucial if we interpret the empirical results as implying that in capital intensive sectors plants will be designed to work at multiple shifts.

The empirical studies define capital intensity as the ratio between the quantity of capital and the quantity of labor employed in the production process. This capital intensity is a result of the maximization analysis that the entrepreneur undertakes before building a plant. It depends on the prices of factors of production and on the technology available and cannot be assumed to be a basic parameter of a certain industrial sector. Making a statement on this definition of capital intensity does not get into the factors distinguishing one industry from another one. 23/

We have calculated the ratio between capital and labor in multiple and single shift plants. Assuming that the prices of factors production are the same but that the technology differs among sector,

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23/ Nevertheless this capital intensity is useful in explaining why an established plant does not find profitable to work multiple shifts. For these analysis see my paper "Economic Analysis of Multiple Shifts in Established Plants". Center for Latin American Development Studies, Boston University (mimeo), Boston, Mass., April, 1974.

the ratios are:

$$\frac{K^1}{\overset{2}{L}} = \left[\frac{\omega \cdot \alpha_1 \cdot a^{-\rho_1}}{P_k(i+d) \cdot (1-\alpha_1)}\right]^{\sigma_1} \qquad \text{for single shift plants}$$

$$\frac{K^2}{L^2} = 2\left[\frac{\omega \cdot (2+\theta) . \alpha_2 \cdot a^{-\rho_2}}{P_k(i+d) \cdot (1-\alpha_2)}\right]^{\sigma_2} \qquad \text{for double shift plants}$$

From these equations we can deduce the conditions for a plant that works two shifts to have a higher capital-labor ratio than a single shift plant. These conditions are satisfied in a broad range of circumstances. If the elasticities of substitution between capital and labor and the capital intensity are equal, $K^2/L^2$ will always be greater than $K_1/L_1$. Assuming similar values of the technological capital intensity we need an elasticity of substitution in the sectors working one shift that is greater than that of sectors working two shifts to get a higher capital-labor ratio in former sectors. This is not very likely to be a common case because, as we noted before, the higher the elasticity of subsitution the greater the likelihood of having multiple shifts plants.

What is more interesting to note is that it is theoretically possible for a single shift plant to have a lower capital-labor ratio than a multiple shift one and more capital intensive requirements of production <u>at the same time</u>. A statement about the capital-labor ratio is completely misleading in this case.

Therefore the proposition that multiple shifts plants will be built in sector with a higher capital-labor ratio is not very meaningful because it does not consider the real causes of building the multiple shift plants. It only makes an assertion about the effects of multiple shifts. In many practical cases--where we compare sectors with very similar values of the elasticity of substitution and the technological capital intensity--it is very close to a tautology: because we built the multiple shifts plants we have by definition a higher capital-labor ratio.

Considering the case where the technologies in different sectors are identical in everything but the technological capital intensity, it is not <u>always</u> true that multiple shifts operations will be more probably in the more capital intensive sectors. The scale parameter has an important influence in this respect. If there are substantial economies of scale we could in theory have a situation where the capital intensive sector would work a single shift while the labor intensive would work multiple shifts. But with diseconomies of scale or small economies of scale--the likely values in the real world--it will be true that multiple shift plants will exist in the more technologically capital intensive sectors.

Several economic policies that may be used to increase the number of shifts worked act through changes in the economic parameter given to the entrepreneurs that must choose between building a plant designed to operate at multiple shifts or a plant designed to operate at one shift. The effect of these economic policies depends on the technological characteristics of production. Table VI indicates if multiple or single shift plants will be favored in the pure investment decision with different policy alternatives, given the technological conditions of production.

TABLE VI

Types of Plants Favored in the Pure Investment Decision with Different Economic Policies

| Policies | Degree of returns to scale | Elasticity of Substitution $\sigma < 1$ | $\sigma = 1$ | $\sigma > 1$ |
|---|---|---|---|---|
| Increase in the relative price of capital | $\beta < 1$ | Multiple shifts | No influence 2/ | Single shift |
| | $\beta = 1$ 1/ | Multiple shifts | No influence 2/ | Multiple shifts |
| | $1 < \beta < 1/(1-1/\eta)$ | Multiple shifts | No influence 2/ | Single shift |
| | $\beta > 1/(1-1/\eta)$ | Single shift | No influence 3/ | Multiple shifts |
| Decrease in the relative price of capital | $\beta < 1$ | Single shift | No influence 2/ | Multiple shifts |
| | $\beta = 1$ 1/ | Single shift | No influence 2/ | Single shift |
| | $1 < \beta < 1/(1-1/\eta)$ | Single shift | No influence 2/ | Multiple shifts |
| | $\beta > 1/(1-1/\eta)$ | Multiple shifts | No influence 3/ | Single shift |
| Decrease in the wage premium for late shifts | $\beta < 1$ | Multiple shifts | No influence 2/ | Multiple shifts |
| | $\beta = 1$ 1/ | Single shift | No influence 2/ | Multiple shifts |
| | $1 < \beta < 1/(1-1/\eta)$ | Multiple shifts | No influence 2/ | Multiple shifts |
| | $\beta > 1/(1-1/\eta)$ | Single shift | No influence 3/ | Single shift |
| Increase in the wage premium for late shifts | $\beta < 1$ | Single shift | No influence 2/ | Single shift |
| | $\beta = 1$ 1/ | Multiple shifts | No influence 2/ | Single shift |
| | $< \beta < 1/(1-1/\eta)$ | Single shift | No influence 2/ | Single shift |
| | $\beta > 1/(1-1/\eta)$ | Multiple shifts | No influence 3/ | Multiple shifts |

1/ The results reported in this raw correspond to the case of perfect competition. The case of monopolistic markets and constant returns to scale ($\beta=1$) have the same results as that of $\beta < 1$ or of $1 < \beta < 1/(1-1/\eta)$.

2/ Multiple shifts plants will always be built.

3/ Single shift plants will always be built.

If the technology available has an elasticity of substitution between capital and labor equal to one (Cobb-Douglas production function) a change in the relative price of factors of production or in the wage premium will have no effect at all on the choice between building a plant designed to work one shift or a plant designed to work multiple shifts.

Given the above mentioned technology the type of plant to be built will depend exclusively on the degree of economies of scale. With diseconomies of scale or economies of scale lower than $1/(1-1/\eta)$ --where $\eta$ is the price elasticity of demand for the product of the firm--only plants designed to work multiple shifts will be built. The firm will adjust its employment of labor and capital to the relative price of factors of production and the wage premium, but these parameters cannot change the type of plant that will be preferred. If economies of scale are greater than $1/(1-1/\eta)$ entrepreneurs will always build single shift plants.

Of more interest is the general case where the elasticity of substitution between capital and labor is not one. Then changes in the relative price of factors of production or in the wage premium have an effect on the choice between building a multiple or a single shift plant. The influence of changes in the wage premium depend only on the value of the scale parameter, while the influence of changes in the relative prices of factors will depend on both the elasticity of substitution and the scale parameter.

If empirically it were established that the degree of returns to scale is less than $1/(1-1/\eta)$ then a policy of decreasing the wage premium for workers in the late shifts will always be favorable to multiple shifts plants. If at the same time it is established that the elasticity of substitution is less than one, increases in the relative price of capital will also be favorable to multiple shifts plants. On the other hand, if the elasticity of substitution is greater than one--and the degree of returns to scale less than $1/(1-1/\eta)$--then an increase in the relative price of capital will be propitious to single shift plants.

Institutional arrangements and/or labor legislation will determine the length of the working schedule in each shift. When the number of hours per shift is decreased the relative profitability of building multiple shifts plants will increase if the degree of returns to scale is less than $1/(1-1/\eta)$ and the elasticity of substitution less than one. It should be noted that this change is equivalent to an increase in the relative price of capital.

Shift work has usually been thought of as a mean of increasing employment and saving capital. Since the alternative multiple and single shift plants have in general different levels of output we must distinguish if we want these results for each each plant or in some relative sense, as measured by the labor-output and the capital-output ratio. In any case, in the pure investment decision these results will not <u>always</u> be forthcoming. Only a complete knowledge of the specific values of all the economic and technical parameters will determine if the desired objectives will be obtained in a given circumstance.

We have found values of the absolute and relative number of workers and capital stock of multiple and single shift plants. They are given in the following formulas:[24]

$$\frac{L^2}{L^1} = \frac{2^{\frac{2-\beta+1/\eta(\beta-1)}{1-\beta(1-1/\eta)}}}{(2+\theta)^{\sigma}} \left[ \frac{(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1}{[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1} \right]^{\frac{\sigma[\beta(1-1/\eta)-1]+1}{[\beta(1-1/\eta)-1](\sigma-1)}}$$

$$\frac{L^2/Q^2}{L^1/Q^1} = \frac{2^{\frac{(1-1/\eta)(1-\beta)}{1-\beta(1-1/\eta)}}}{(2+\theta)^{\sigma}} \left[ \frac{(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1}{[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1} \right]^{\frac{\sigma[\beta(1-1/\eta)-1]+1-\beta}{[\beta(1-1/\eta)-1](\sigma-1)}}$$

$$\frac{K_s^2}{K_s^1} = 2^{(1-1/\eta)/[1-\beta(1-1/\eta)]} \left[ \frac{(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1}{[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1} \right]^{\frac{\sigma[\beta(1-1/\eta)-1]+1}{[\beta(1-1/\eta)-1](\sigma-1)}}$$

$$\frac{K_s^2/Q^2}{K_s^1/Q^1} = 2^{-1/\eta[1-\beta(1-1/\eta)]} \left[ \frac{(q\cdot a)^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1}{[q(2+\theta)a]^{1-\sigma}(\alpha/1-\alpha)^{-\sigma}+1} \right]^{\frac{\sigma[\beta(1-1/\eta)-1]+1-\beta}{[\beta(1-1/\eta)-1](\sigma-1)}}$$

When the policy objective is the more limited one of having a greater labor-capital ratio in all plants that are built, shift work

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24/ When these formulas are written in terms of the parameters they are are strictly valid if the multiple shifts plants are made more profitable through policies that did not affect the optimum employment of factors of production in the single shift plants. Examples of such policies are changes in the wage premium for late shifts and export subsidies, tax rebates and lump-sum transfers tied to work multiple shifts. When the policies imply changes in the relative price of factors of production the parameter "q" should be different in the numerator and denominator of the equations and the term inside the brackets is multipled by the change in the prices.

will achieve this result for any degree of economies of scale if the elasticity of substitution is less than log 2/log $(2+\theta)$, where $\theta$ is the wage premium for late shifts. Since this premium is not likely to be greater than 50%, we should encourage multiple shifts in all sectors where this elasticity is less than 0.75. If to promote multiple shifts the wage premium is effectively reduced, the limit on the elasticity rises, but we never should encourage multiple shifts in sectors with a value greater than 1.0.

A policy of change in the relative prices of factors of production needs some caution because it makes the employment of factors different in the single shift plant before and after the change was introduced. Nevertheless if the elasticity of substitution is less than one and the degree of returns to scale less than $1/(1/1-\eta)$ the recommended change to encourage multiple shifts is an increase in the relative price of capital (see Table VI). If this is done, the capital-labor ratio in the single shift plant is always lower after the change in relative prices than before. Since the multiple shifts plant will have a lower capital-labor than the single shift plant after the increase in the relative price of capital, it will also have a lower capital-labor ratio than before this increase.

In summary, there seems to be a good theoretical case for the application of a policy of shift work in sectors where the elasticity of substitution is less than 0.75. There we will always increase the labor-capital ratio of the plants that are built if they are designed to work multiple shifts instead of a single shift. Leaving aside the cases of great economies of scale in the technology available, any policy instrument is desirable to get these results.

We must note that this paper presents a theory identifying the forces that determine when multiple shifts will be advantageous and when profit maximizing entrepreneurs will invest in these type of plants. Professor Paul Rosenstein-Rodan has commented that "nature does not imitate Cezanne". It is not likely that any individual plant will have a Cobb-Douglas, CES or any other well defined production function. They may be a more realistic representation of aggregates, since deviations from them in individual cases may cancel out. This is enough for our purposes. Our hope is that Cezanne was trying to represent nature.

# APPENDIX I

## The Optimum Output When Working One and Two Shifts in the Cobb-Douglas Production Function

### A. Optimum Output for One Shift

The production function is:

$$Q^1 = A(L^1)^{\alpha} \;.\; (K^1)^{\beta} \tag{1}$$

where $Q^1$ is output per day, $L^1$ is labor in man hours per day and $K^1$ capital services per day.

The firm works one shift of "a" hours. If $K_s{}^1$ is the capital stock we have that:

$$K^1 = a\, K_s{}^1 \tag{2}$$

The price of output is "p", the wage rate "$W_1$" and the price of capital services is:

$$r_1 = \frac{Pk(i + d)}{a} \tag{3}$$

where "Pk" is the purchase price of a unit of capital stock, "i" the daily interest rate and "d" the daily depreciation rate and maintenance costs.

Profits are:

$$\pi^1 = p \;.\; A \;.\; a^{\beta} \;.\; (L^1)^{\alpha} \;\; (K_s{}^1)^{\beta} \; - W_1 L^1 - Pk(i + d)K_s{}^1 \tag{4}$$

When profits are maximized the following conditions are met:

$$\frac{\partial \Pi^1}{\partial L^1} = p \cdot A \cdot a^{\beta} \cdot \alpha \cdot (L^1)^{\alpha-1} (Ks^1)^{\beta} - W_1 = 0 \qquad (5)$$

$$\frac{\partial \Pi^1}{\partial Ks^1} = p \cdot A \cdot a^{\beta} \cdot \beta \cdot (L^1)^{\alpha} \cdot (Ks^1)^{\beta-1} - Pk(i+d) = 0 \qquad (6)$$

The above equations indicate that each factor should be hired up to the point where their marginal revenue product is equal to its price.

From (5) and (6):

$$\frac{L^1}{Ks^1} = \frac{\alpha}{\beta} \cdot \frac{Pk(i+d)}{w_1} = \mu_1 \qquad (7)$$

This is the equation of the expansion path for this type of production function: it shows the optimal factor proportions when output changes but input prices remain constant. It indicates the fundamental equality of the marginal rate of technical substitution and the input price ratio. For the Cobb-Douglas production function the expansion path will be a straight line no matter what the degree of returns to scale are.

Replacing (2) and (7) in (1) we get the optimal amount of capital stock as a function of output:

$$Ks^1 = \left[ \frac{Q^1}{A.a^{\beta} \cdot \mu_1^{\alpha}} \right]^{\frac{1}{\alpha+\beta}} \qquad (8)$$

Replacing (8) in (7) we get the optimal amount of labor services as a function of output:

$$L^1 = \mu_1 \left[ \frac{Q^1}{A.a^{\beta}.\mu_1^{\alpha}} \right]^{\frac{1}{\alpha+\beta}} \qquad (9)$$

Equations (8) and (9) are valid for any level of output. They are the input-output space equivalent of the expansion path.

Total cost of production is:

$$C^1 = W_1 L^1 + r_1 K^1 = W_1 L^1 + Pk\ (i+d).\ Ks^1 \qquad (10)$$

Replacing (8) and (9) in (10) we get total cost as a function of output:

$$C^1 = \left[ \frac{Q^1}{A.a^{\beta}.\mu_1^{\alpha}} \right]^{\frac{1}{\alpha+\beta}} .\ Pk(i+d) \left[ \frac{\alpha+\beta}{\beta} \right] \qquad (11)$$

Marginal cost of production is:

$$MC^1 = \frac{Pk(i+d)}{\beta A.a^{\beta}.\mu_1^{\alpha}} \left[ \frac{Q^1}{A.a^{\beta}.\mu_1^{\alpha}} \right]^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} \qquad (12)$$

The optimal level of output in competitive conditions $(\overline{Q}_c^{\,1})$ is obtained when marginal cost equals the constant price of output. Therefore it is:

$$\overline{Q}_c^{\,1} = A.a^{\beta}.\mu_1^{\alpha}\left[\frac{p.A.\beta.a\beta.\mu_1^{\alpha}}{Pk(i+d)}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}} \qquad (13)$$

Note that output is undetermined when $\alpha + \beta = 1$. When there are constant returns to scale in competitive conditions profits are maximized when output increases indefinetely.

When there are economies of scale ($\alpha + \beta > 1$), the level of output determined by equation (13) is that of minimum profits. This is indicated by the second order conditions that are not developed here. To maximize profits the competitive firm must again increase output to infinite.

It is a familiar proposition in economic theory that perfect competition is incompatible with economies of scale. Therefore this case must assume a demand curve for the product of the plant with elasticity not equal to infinite. Knowing the demand function, the marginal revenue function is also known. The optimal level of output under these conditions is obtained when marginal cost equals marginal revenue.

When the demand curve is: $p = f(Q)$, marginal revenues is: $MR = Q \,.\, f'(Q) + f(Q) = p(1-1/\eta)$, where $\eta$ is the elasticity of demand. Marginal revenue depends only on the level of output and the parameters of the demand function. Equating (12) with any of these formulations we can find the optimal level of output.

For expository purposes, the optimal level of output under monopolistic conditions ($\overline{Q}_M{}^1$) will be written as:

$$\overline{Q}_M{}^1 = A \cdot a^{\beta} \cdot (\mu_1)^{\alpha} \left[ \frac{p_1 \, (1-1/\eta_1) \, . \, A \, . \, \beta \, . \, a^{\beta} \, . \, \mu_1{}^{\alpha}}{Pk \, (i + d)} \right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}} \tag{13a}$$

where "$p_1$" and "$\eta_1$" are the price and the elasticity of demand when quantity $\overline{Q}_M{}^1$ is sold and are determined concurrently with it.

## B. Optimal Output for Two Shifts

In the general case, the ex-post capital-labor ratio is not fixed and labor services in the second shift can be different from those of the first shift. The production functions for the first and second shift are respectively:

$$Q_1{}^2 = A \, . \, (L_1{}^2)^{\alpha} \, . \, (K_a{}^2)^{\beta} \tag{14}$$

$$Q_2{}^2 = A \, . \, (L_2{}^2)^{\alpha} \, . \, (K_a{}^2)^{\beta} \tag{15}$$

where $L_1{}^2$ and $L_2{}^2$ are labor services in each shift and $K_a{}^2$ the constant capital services per shift. If $K_s{}^2$ is the capital stock we have:

$$K_a{}^2 = aK_s{}^2 \tag{16}$$

and total capital services are:

$$K^2 = 2K_a{}^2 = 2aK_s{}^2 \tag{17}$$

The price of capital services is:

$$r_2 = \frac{Pk(i+d)}{2a} \tag{18}$$

Profits for the firm are:

$$\Pi^2 = p \cdot (Q_1{}^2 + Q_2{}^2) - W_1 L_1{}^2 - W_2 . L_2{}^2 - Pk(i+d) . K_s{}^2 \tag{19}$$

When profits are maximized, the following conditions are met:

$$\frac{\partial \Pi^2}{\partial L_1{}^2} = p.A.a^{\beta}.\alpha \cdot (L_1{}^2)^{\alpha-1} \cdot (K_s{}^2)^{\beta} - W_1 = 0 \tag{20}$$

$$\frac{\partial \Pi^2}{\partial L_2{}^2} = p.A.a^{\beta}.\ \alpha \cdot (L_2{}^2)^{\alpha-1} \ (K_s{}^2)^{\beta} - W_2 = 0 \tag{21}$$

$$\frac{\partial \Pi^2}{\partial K_s{}^2} = p.A.a^{\beta}.\beta \left[ (L_1{}^2)^{\alpha} + (L_2{}^2)^{\alpha} \right] \cdot (K_s{}^2)^{\beta-1} - Pk(i+d) = 0 \tag{22}$$

Again these equations indicate that each factor is hired up to the point when its marginal revenue product is equal to its price, but they consider the fact that the same amount of capital is used in both shifts and that aggregate profits and not profits per shift are maximized.

From (20) and (21) we get:

$$L_2{}^2 = \left[ \frac{W_1}{W_2} \right]^{\frac{1}{1-\alpha}} L_1{}^2 \tag{23}$$

The above equation relates labor services in the first and second shift when there is a wage differential between them. Since in general $W_2 > W_1$ and $0 < \alpha < 1$, we have that $L_2^2 < L_1^2$. If labor services per worker are the same in both shifts, the optimum number of workers in the second shift will always be less than that of the first shift.

Replacing (23) in (22):

$$\frac{\partial \Pi^2}{\partial K_s^2} = p.A.a^{\beta}.\beta \;.\; (L_1^2)^{\alpha} \left[ 1 + \left[\frac{W_1}{W_2}\right]^{\frac{\alpha}{1-\alpha}} \right] (K_s^2)^{\beta - 1} - Pk(i+d) = 0 \qquad (24)$$

We define:

$$\rho = \left[\frac{W1}{W2}\right]^{\frac{\alpha}{1-\alpha}} \qquad (25)$$

If $\theta$ is the wage premium for the second shift over the first shift:

$$\rho = \left[\frac{1}{1+\theta}\right]^{\frac{\alpha}{1-\alpha}} \qquad (26)$$

From (20) and (24):

$$\frac{L_1^2}{K_s^2} = \frac{\alpha}{\beta} \cdot \frac{Pk(i+d)}{W_1(1+\rho)} = \frac{\mu_1}{(1+\rho)} \qquad (27)$$

Replacing (27) in (23):

$$L_2^2 = \frac{\mu_1}{(1+\rho)} \left[\frac{W_1}{W_2}\right]^{\frac{1}{1-\alpha}} . K_s^2 \qquad (28)$$

Equations (27) and (28) represent now the expansion path and the fundamental equality of the marginal rate of technical substitution and the input price ratio. The first equation is in terms of workers in the first shift and the second one in terms of workers in the second shift. They are valid for any level of output and indicate the appropriate input mix to maximize profits given the price of inputs.

Replacing (27) and (28) in $Q^2 = Q_1{}^2 + Q_2{}^2$ we get the optimal amount of capital stock under two shifts as a function of output:

$$K_s{}^2 = \left[\frac{Q^2}{A.a^{\beta}.\mu_1{}^{\alpha}\ (1+\rho)^{1-\alpha}}\right]^{\frac{1}{\alpha+\beta}} \tag{29}$$

Replacing (29) in (27) and (28) we get the optimal amount of labor services in the first and second shift as a function of output:

$$L_1{}^2 = \frac{\mu\ 1}{(1+\rho)}\left[\frac{Q2}{A.a^{\beta}.\mu_1{}^{\alpha}.\ (1+\rho)^{1-\alpha}}\right]^{\frac{1}{\alpha+\beta}} \tag{30}$$

$$L_2{}^2 = \frac{\mu_1}{(1+\rho)}\left[\frac{W_1}{W_2}\right]^{\frac{1}{1-\alpha}}.\left[\frac{Q^2}{A.a^{\beta}.\mu_1{}^{\alpha}(1+\rho)^{1-\alpha}}\right]^{\frac{1}{\alpha+\beta}} \tag{31}$$

Total cost of production is:

$$C^2 = W_1\ L_1{}^2 + W_2L_2{}^2 + r_2K^2 = W_1L_1{}^2 + W_2L_2{}^2 + Pk(i+d)K_s{}^2 \tag{32}$$

Replacing (29), (30) and (31) we get total cost as a function of output:

$$C^2 = \left[\frac{Q_2}{A.a^{\beta}.\mu_1^{\alpha}\ (1+\rho)^{1-\alpha}}\right]^{\frac{1}{\alpha+\beta}} .\ Pk(i+d) \left[\frac{\alpha+\beta}{\beta}\right] \qquad (33)$$

Marginal cost of production is:

$$MC^2 = \frac{Pk(i+d)}{\beta .A.a^{\beta}.\mu_1^{\alpha}.\ (1+\rho)^{1-\alpha}} \cdot \left[\frac{Q_2}{A.a^{\beta}.\mu_1^{\alpha}.\ (1+\rho)^{1-\alpha}}\right]^{\frac{1-(\alpha+\beta)}{(\alpha+\beta)}} \qquad (34)$$

The optimal level of output in competitive conditions ($\overline{Q}_c^{\,2}$) is obtained when marginal cost equals the constant price of output. Therefore it is:

$$\overline{Q}_c^{\,2} = A.a^{\beta}.\mu_1^{\alpha}.\ (1+\rho)^{1-\alpha} \left[\frac{p.A.\beta .a^{\beta}.\mu_1^{\alpha}.\ (1+\rho)^{1-\alpha}}{Pk(i+d)}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}} \qquad (35)$$

Note again that when there are constant returns to scale ($\alpha+\beta=1$), the optimal level of output under two shifts defined by equation (35) is undetermined. Also when there are economies of scale ($\alpha+\beta>1$), the level of output given by that equation is the one of minimum profits. Assuming a demand curve for the product of the firm with elasticity "$\eta$", the optimal level of output under monopolistic conditions ($\overline{Q}_M^{\,2}$) is:

$$\overline{Q}_M^{\,2} = A\ .\ a^{\beta}\ .\ \mu_1^{\alpha}\ .\ (1+\rho)^{1-\alpha} \left[p_2(1-1/\eta).A.\beta.a^{\beta}.\mu_1^{\alpha}(1+\rho)^{1-\alpha}\right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}}$$

(35a)

The wage premium for the second shift will determine how the optimal level of aggregate output is divided among the two shifts. Dividing (15) by (14):

$$\frac{Q_2{}^2}{Q_1{}^2} = \left[\frac{L_2{}^2}{L_1{}^2}\right] = \left[\frac{W_1}{W_2}\right]^{\frac{\alpha}{1-\alpha}} = \rho \qquad (36)$$

This relation is valid in any market condition--perfect competition or monopoly--so long as the ex-post capital-labor ratio is not fixed.

When there is no ex-post substitution between factors of production, labor services in the second shift must be equal to that of the first shift. Therefore the level of production is the same in each shift and total production is:

$$Q^2 = 2 \,.\, A \,.\, (La^2)^{\alpha} \; (Ka^2)^{\beta} \qquad (37)$$

where $La^2$ and $Ka^2$ are the constant capital and labor services per shift.

Using equation (17), total production as a function of the capital stock is:

$$Q^2 = 2 \,.\, A \,.\, (La^2)^{\alpha} \,.\, (2aK_s{}^2)^{\beta} \qquad (38)$$

Profits for the plant are:

$$\pi^2 = p \,.\, Q^2 - W_1 La^2 - W_2 La^2 - Pk(i+d)K_s{}^2 \qquad (39)$$

When profits are maximized, the following conditions are met:

$$\frac{\partial\pi^2}{\partial La^2} = 2^{\beta+1} \,.\, p \,.\, A \,.\, a^{\beta} \,.\, \alpha \,.\, (La^2)^{\alpha-1} (K_s{}^2)^{\beta} - W_1 - W_2 = 0 \qquad (40)$$

$$\frac{\partial \pi^2}{\partial K_s^2} = 2^{\beta+1} \cdot p \cdot A \cdot a^{\beta} \cdot \beta \cdot (La^2)^{\alpha} \cdot (Ks^2)^{\beta} - Pk(i+d) = 0 \qquad (41)$$

The above equations indicate that each factor is hired up to the point where its marginal revenue product is equal to its price. The specific formulation takes into account the fact that the plant operates at two shifts and that there is no ex-post substitution.

From (41) and (40) we get:

$$\frac{La^2}{K_s^2} = \frac{\alpha}{\beta} \cdot \frac{Pk(i+d)}{W_1 + W_2} \qquad (42)$$

or

$$\frac{La^2}{K_s^2} = \frac{\alpha}{\beta} \cdot \frac{Pk(i+d)}{W_1(1+\theta)} = \frac{\mu_1}{(1+\theta)} \qquad (43)$$

Equation (43) indicates now the expansion path and the fundamental equality of the marginal rate of technical substitution and the input price ratio.

Replacing (43) in (38) we get the optimal amount of capital stock under two shifts and no ex-post substitution as a function of output:

$$K_s^2 = \left[ \frac{Q^2}{2^{\beta+1} . a^{\beta} . A . \mu_1^{\alpha} (1+\theta)^{1-\alpha}} \right]^{\frac{1}{\alpha+\beta}} \qquad (44)$$

Replacing (44) in (43) we get the optimal amount of labor services per shift as a function of output:

$$L_a^2 = \frac{\mu_1}{(1+\theta)} \left[ \frac{Q^2}{2^{\beta+1} . a^{\beta} . A . \mu_1^{\alpha} . (1+\theta)^{1-\alpha}} \right]^{\frac{1}{\alpha+\beta}} \qquad (45)$$

Total cost of production is:

$$C^2 = W_1 La^2 + W_2 La^2 + Pk(i+d)K_s^2 \qquad (46)$$

Replacing (44) and (45) in (46) we get total cost as a function of output:

$$C^2 = \left[ \frac{Q^2}{2^{\beta+1} . a^{\beta} . A . \mu_1^{\alpha} . (1+\theta)^{1-\alpha}} \right]^{\frac{1}{\alpha+\beta}} . \ Pk(i+d) . \left[ \frac{\alpha(2+\theta)+\beta(1+\theta)}{\beta(1-\theta)} \right] \qquad (47)$$

Marginal cost of production is:

$$MC^2 = \frac{Pk(i+d) \ [\alpha(2+\theta)+\beta(1+\theta)]}{\beta . 2^{\beta+1} . a^{\beta} . A . \mu_1^{\alpha} . (1+\theta)^{2-\alpha} \ (\alpha+\beta)} \cdot \left[ \frac{Q^2}{2^{\beta+1} . a^{\beta} . A . \mu_1^{\alpha} \ (1+\theta)^{1-\alpha}} \right]^{\frac{1-(\alpha+\beta)}{(\alpha+\beta)}} \qquad (48)$$

The optimal level of output in competitive conditions ($\overline{Q}_c^{\,2}$) is obtained when marginal cost equals the constant price of output. Therefore it is:

$$\overline{Q}_c^{\,2} = \left[ \frac{p . A . \beta . 2^{\beta+1} . a^{\beta} \ \mu_1^{\alpha} \ (1+\theta)^{2-\alpha} . (\alpha+\beta)}{Pk(i+d) \ [\alpha(2+\theta) + \beta(1+\theta)]} \right]^{\frac{\alpha + \beta}{1-(\alpha+\beta)}} . \ 2^{\beta+1} . A . a^{\beta} . \mu_1^{\alpha} \ (1+\theta)^{1-\alpha} \qquad (49)$$

Assuming a demand curve for the product of the firm with elasticity "$\eta$", the optimal level of output under this monopolistic conditions $(\bar{Q}_M^2)$ is now:

$$\bar{Q}_M^2 = \left[ \frac{p_2(1-1/\eta_2).A.\beta.2^{\beta+1}.a^{\beta}.\mu_1^{\alpha}.(1+\theta)^{2-\alpha}.(\alpha+\beta)}{Pk(1+d)\ [\alpha(2+\theta)+\beta(1+\theta)]} \right]^{\frac{\alpha+\beta}{1-(\alpha+\beta)}} .2^{\beta+1}.A.a^{\beta}.\mu_1^{\alpha} . (1+\theta)^{1-\alpha} \qquad (50)$$

# APPENDIX II

## The Optimum Output When Working One and Two Shifts in the CES Production Function

### A. Optimum Output for One Shift

The production function is:

$$Q^1 = \gamma \left[ \alpha (K^1)^{-\rho} + (1-\alpha) \; (L^1)^{-\rho} \right]^{-\beta/\rho} \qquad (1)$$

where $Q^1$ is output per day, $L^1$ is labor in man-hours per day and $K^1$ capital services per day.

The firm works one shift of "a" hours. If $K_s{}^1$ is the capital stock we have that:

$$K^1 = a \, . \, K_s{}^1 \qquad (2)$$

The price of output is "p", the wage rate "$W_1$" and the price of capital services is:

$$r_1 = \frac{Pk(i+d)}{a} \qquad (3)$$

where "Pk" is the purchase price of a unit of capital stock, "i" the daily interest rate and "d" the daily depreciation rate and maintenance costs.

Profits for the firm are:

$$\Pi^1 = p \cdot \gamma \left[ \alpha . \; a^{-\rho} \; . \; (K_s{}^1)^{-\rho} + (1-\alpha) \; . \; (L^1)^{-\rho} \right]^{-\beta/\rho} - W_1 L^1 - Pk(i+d)K_s{}^1 \qquad (4)$$

When profits are maximized the following conditions are met:

$$\frac{\partial \Pi^1}{\partial L^1} = p \cdot \gamma \cdot \beta \cdot (1-\alpha) \left[ \alpha \cdot a^{-\rho} \cdot (K_s{}^1)^{-\rho} + (1-\alpha)\ (L^1)^{-\rho} \right]^{-\beta/\rho\ -1} (L^1)^{-\rho-1}$$

$$- W_1 = 0 \qquad (5)$$

$$\frac{\partial \Pi^1}{\partial K^1} = p \cdot \gamma \cdot \beta \cdot \alpha \cdot a^{-\rho} \left[ \alpha \cdot a^{-\rho} \cdot (K_s{}^1)^{-\rho} + (1-\alpha)(L^1)^{-\rho} \right]^{-\beta/\rho\ -1} (K_s{}^1)^{-\rho-1}$$

$$- Pk(i+d) = 0 \qquad (6)$$

The above equations indicate that each factor should be hired up to the point where their marginal revenue product is equal to its purchase price.

From (5) and (6):

$$\frac{L^1}{K_s{}^1} = \left[ \frac{W_1 \cdot \alpha \cdot a^{-\rho}}{Pk(i+d) \cdot (1-\alpha)} \right]^{-1/1+\rho} = \frac{1}{\mu_1} \qquad (7)$$

This is now the equation of the expansion path. It shows the optimal factor proportions under any market condition when output changes but input prices remain constant. It indicates the fundamental equality of the marginal rate of technical substitution and the input price ratio. For the CES production function the expansion path is also a straight line through the origin.

Replacing (2) and (7) in (1) we get the optimal amount of labor services as a function of output and the given prices of inputs:

$$L^1 = \frac{(Q^1)^{1/\beta}}{\gamma^{1/\beta} \, [\alpha \, . \, a^{-\rho} \, . \, \mu_1^{-\rho} + (1-\alpha)]^{-1/\rho}} \tag{8}$$

Replacing (8) in (7) we get the optimal amount of capital stock as a function of output and the given prices of inputs:

$$K_s{}^1 = \mu_1 \frac{(Q^1)^{1/\beta}}{\gamma^{1/\beta} \, [\alpha \, . \, a^{-\rho} \, . \, \mu_1^{-\rho} + (1-\alpha)]^{1/\rho}} \tag{9}$$

Equations (8) and (9) are valid for any level of output. They are the input-output space equivalent of the expansion path. They indicate the optimal amount of factors of production for any output given the prices of all the factors of production.

Total cost of production is:

$$C^1 = W_1 \, . \, L^1 + r_1 \, . \, K^1 = W_1 \, . \, L^1 + Pk(i+d) \, . \, K_s{}^1 \tag{10}$$

Replacing (8) and (9) in (10) total cost is expressed as a function of output:

$$C^1 = \frac{(Q^1)^{1/\beta}}{\gamma^{1/\beta} \, [\alpha \, . \, a^{-\rho} \, . \, \mu_1^{-\rho} + (1-\alpha)]^{-1/\rho}} \, [W_1 + Pk(i+d) \, . \, \mu_1] \tag{11}$$

Marginal cost of production is:

$$MC^1 = \frac{[W_1+Pk(i+d).\mu_1]}{\beta . \gamma^{1/\beta} [\alpha.a^{-\rho}.\mu_1^{-\rho}+(1-\alpha)]^{-1/\rho}} .(Q^1)^{1-\beta/\beta} \qquad (12)$$

The optimal level of output in competitive conditions ($\overline{Q}_c{}^1$) is obtained when marginal cost equals the constant price of output. Its value is:

$$\overline{Q}_c{}^1 = \left[ \frac{p.\beta.\gamma^{1/\beta}.[\alpha.a^{-\rho}.\mu_1^{-\rho}+(1-\alpha)]^{-1/\rho}}{W_1 + Pk(i+d) . \mu_1} \right]^{\beta/1-\beta} \qquad (13)$$

Note that output is undetermined when there are constant returns to scale ($\beta$=1). In this case and with perfect competition profits are maximized when output increases indefinetely.

When there are economies of scale ($\beta$>1), the level of output determined by equation (13) is that of minimum profits. This is indicated by the second order condition that is not developed here. To maximize profits the competitive firm must again increase output to infinite.

The case of economies of scale must be considered in the context of a demand curve for the product of the plant with elasticity not equal to infinite. The price of output is not given to the firm and the level of production is determined at the point where marginal cost (equation 12) is equal to marginal revenue. Knowing the demand function the marginal revenue function is also known.

If "$\eta$" is the elasticity of demand, marginal revenues are equal to $p(1-1/\eta)$, where $p=f(Q)$ represents the demand function. The optimal level of output under monopolistic conditions ($\bar{Q}_M^{\,1}$) can be written as:

$$\bar{Q}_M^{\,1} = \left[ \frac{p_1(1-1/\eta_1) \;.\; \beta \;.\; \gamma^{1/\beta} \left[\alpha . a^{-\rho} . \mu_1^{-\rho} + (1-\alpha)\right]^{-1/\rho}}{W_1 + Pk(i+d)\,\mu_1} \right]^{\beta/1-\beta} \tag{14}$$

where "$p_1$" and "$\eta_1$" are the price and the elasticity of demand when quantity $\bar{Q}_M^{\,1}$ is sold and are determined concurrently with it.

## B. Optimal Output for Two Shifts

The production functions for the first and second shifts are respectively:

$$Q_1^{\,2} = \gamma \left[ \alpha (Ka^2)^{-\rho} + (1-\alpha) \; (L_1^{\,2})^{-\rho} \right]^{-\beta/\rho} \tag{15}$$

$$Q_2^{\,2} = \gamma \left[ \alpha (Ka^2)^{-\rho} + (1-\alpha) \; (L_2^{\,2})^{-\rho} \right]^{-\beta/\rho} \tag{16}$$

where $L_1$ and $L_2$ are labor services in each shift and Ka the constant capital services per shift.

With fixed ex-post capital services-labor services ratio [1/] we must have:

$$L_1^{\,2} = L_2^{\,2} = La^2 \tag{17}$$

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1/ In the CES production function the general case of non constant ex-post capital services-labor services ratio has a very complicated solution and is not presented here. Most authors do assume that the crew size of machinery is constant, which is equivalent to our case.

and

$$Q_1^2 = Q_2^2 \tag{18}$$

Total labor services are:

$$L^2 = L_1^2 + L_2^2 \tag{19}$$

and total production:

$$Q^2 = Q_1^2 + Q_2^2 = 2\gamma \left[\alpha (Ka^2)^{-\rho} + (1-\alpha)\ (La^2)^{-\rho}\right]^{-\beta/\rho} \tag{20}$$

Let us call $K_s^2$ the stock of capital. Since each shift is of "a" hours, capital services per shift are:

$$Ka^2 = a \,.\, Ks^2 \tag{21}$$

and the total capital services are:

$$K^2 = 2Ka^2 = 2aKs^2 \tag{22}$$

The price of capital services is:

$$r_2 = \frac{Pk(i+d)}{2a} \tag{23}$$

Profits for the firm are:

$$\Pi^2 = p.Q^2 - (W_1+W_2)\ .\ La^2 - Pk(i+d)\ .\ K_s^2 \tag{24}$$

When profits are maximized the following conditions are met:

$$\frac{\partial \Pi^2}{\partial La^2} = 2p.\gamma\beta.(1-\alpha)\left[\alpha.a^{-\rho}.(Ks^2)^{-\rho} + (1-\alpha)(La^2)^{-\rho}\right]^{-\beta/\rho-1}(La^2)^{-\rho-1}$$

$$-(W_1+W_2) = 0 \qquad (25)$$

$$\frac{\partial \Pi^2}{\partial Ks^2} = 2p.\gamma.\beta.\alpha.a^{-\rho}\left[\alpha.a^{-\rho}.(Ks^2)^{-\rho} + (1-\alpha)(La^2)^{-\rho}\right]^{-\beta/\rho-1}(Ks^2)^{-\rho-1}$$

$$-Pk(i+d) = 0 \qquad (26)$$

These equations indicate that each factor is hired up to the point where its marginal revenue product is equal to its price, but they consider the fact that the same amount of each factor is used on both shifts, that labor in the second shift receives a wage premium and that aggregate profits and not profits per shift are maximized.

From (25) and (26) the labor-capital ratio in each shift is:

$$\frac{La^2}{Ks^2} = \left[\frac{(W_1+W_2).\alpha.a^{-\rho}}{Pk(i+d)(1-\alpha)}\right]^{-1/1+\rho} = \frac{1}{\mu 2} \qquad (27)$$

and the total labor-capital ratio is:

$$\frac{L^2}{Ks^2} = \frac{2La^2}{Ks^2} = 2\left[\frac{(W_1+W_2).\alpha.a^{-\rho}}{Pk(i+d).(1-\alpha)}\right]^{-1/(1+\rho)} = \frac{2}{\mu 2} \qquad (28)$$

Equations (27) and (28) represent the expansion path and the fundamental equality of the marginal rate of technical substitution and the input price ratio. They are valid for any level of output and indicate the appropriate input mix to maximize profits given the price of inputs.

Replacing (27) in (20) we get the optimal amount of labor services per shift as a function of output:

$$La^2 = \frac{(Q^2)^{1/\beta}}{(2\gamma)^{1/\beta}\,[\alpha.\, a^{-\rho}\, .\, \mu_2^{-\rho} + (1-\alpha)]^{-1/\rho}} \qquad (29)$$

Replacing (29) in (27) we get the optimal amount of capital stock as a function of output:

$$K_s^{\,2} = \mu_2 \frac{(Q^2)^{1/\beta}}{(2\gamma)^{1/\beta} \cdot [\alpha\, .a^{-\rho}\, .\mu_2^{-\rho} + (1-\alpha)]^{-1/\rho}} \qquad (30)$$

Again equations (29) and (30) are valid for any level of output. They are the input-output space equivalent of the expansion path.

Total cost of production is:

$$C^2 = (W_1 + W_2)\ .\ La^2 + Pk(i+d)\ .\ Ks^2 \qquad (31)$$

Replacing (29) and (30) in (31) we get total cost as a function of output:

$$C^2 = \frac{(Q^2)^{1/\beta}}{(2\gamma)^{1/\beta} \cdot [\alpha . a^{-\rho} . \mu_2^{-\rho} + (1-\alpha)]^{-1/\rho}} [W_1+W_2+Pk(i+d).\mu_2] \tag{32}$$

Defferentiating we obtain marginal cost:

$$MC^2 = \frac{[W_1+W_2+Pk(i+d).\mu_2\ ]}{\beta . (2\gamma)^{1/\beta} . [\alpha . a^{-\rho} . \mu_2^{-\rho} + (1-\alpha)]^{1/\rho}} \cdot (Q^2)^{\frac{1-\beta}{\beta}} \tag{33}$$

The optimal level of output in competitive conditions ($\overline{Q}_c{}^2$) is obtained when marginal cost equals the constant price of output. Its value is:

$$\overline{Q}_c{}^2 = \left[ \frac{p. \ \beta \ . \ (2\gamma)^{1/\beta} \ . \ [\alpha . a^{-\rho} . \mu_2^{-\rho} + (1-\alpha)]^{-1/\rho}}{W_1+W_2+Pk(i+d).\mu_2} \right]^{\beta/1-\beta} \tag{34}$$

When there are constant returns to scale ($\beta$=1) and perfect competition in the product market output must be increased without limit to maximize profits : the above equation is undetermined.

When there are economies of scale ($\beta$>1) the level of output given by equation (34) is that of minimum profits. This is indicated by the second order conditions. Assuming a demand curve for the product of the firm with elasticity "$\eta$", the optimal level of output under monopolistic conditions ($\overline{Q}_M{}^2$) is:

$$\overline{Q}_M^{\,2} = \left[ \frac{p_2(1-1/\eta_2).\ \beta\ .(2\gamma)^{1/\beta}\ [\alpha.a^{-\rho}\ .\mu_2^{-\rho} + (1-\alpha)]^{-1/\rho}}{W_1 + W_2 + Pk(i+d).\ \mu_2} \right]^{\beta/1-\beta} \tag{35}$$

where "$p_2$" and "$\eta_2$" are the price and the elasticity of demand when quantity $\overline{Q}^2_M$ is sold and are determined concurrently with it.

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