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DISCUSSION PAPER

Welfare Effects of Demographic Changes in a Ramsey Growth Model

Silvia Stiller

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Silvia Stiller

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Abstract

Population ageing and decline are inescapable facts for Germany which are expected to have - especially due to the labour force decrease - negative impacts on the economy. In this context it is often referred to the possibility of countering potentially negative economic effects of population ageing by a fertility increase or immigration of people at the working age. Of course, changing fertility and immigration are both suited to reduce population ageing. But they influence the population structure, labour supply and thus the economic conditions in substantially different way. This discussion paper focuses on one aspect which is related to this issue. It is examined by simulations how different demographic scenarios affect consumption development and thus average welfare of individual cohorts. The analysis is based on a Ramsey growth model and refers to the demographic structure in Germany. The model results imply that welfare of cohorts with a life expectancy up to 50 years is higher without immigration. Instead, when a consumer faces a life expectancy higher than 50 years or will be born in the future immigration has a positive influence on welfare. Independently of his life expectancy and year of birth a consumer is worse off if fertility rises.

Zusammenfassung

In Deutschland sind die Alterung und der drastische Rückgang der Bevölkerung unausweichlich. Im Zuge dieser Entwicklung werden, insbesondere wegen des Arbeitskräfterückgangs, negative Auswirkungen auf die Ökonomie erwartet. In diesem Zusammenhang wird häufig auf die Möglichkeiten verwiesen, den potentiellen negativen ökonomischen Effekten der Alterung durch Zuwanderung von Menschen im Erwerbsalter oder durch einen Fertilitätsanstieg entgegenzuwirken. Zwar sind sowohl höhere Fertilität als auch Zuwanderung geeignet, die demographische Alterung zu reduzieren. Aber Zuwanderung und steigende Fertilität sind aus ökonomischer Sicht nicht gleichwertig, weil sie den Bevölkerungsaufbau und das Arbeitsangebot sehr unterschiedlich beeinflussen. Dieses Diskussionspapier konzentriert sich auf einen hiermit verbundenen Aspekt. Es wird anhand von Simulationsrechnungen untersucht, wie sich unterschiedliche Bevölkerungsszenarios auf die Konsummöglichkeiten und deshalb die Wohlfahrt einzelner Kohorten auswirken. Die Analyse basiert auf einem Ramsey Wachstumsmodell und bezieht sich auf die demographische Situation in Deutschland. Die Ergebnisse zeigen, dass die Wohlfahrt von Kohorten mit einer Lebenserwartung bis 50 Jahre ohne Zuwanderung höher ist als mit Zuwanderung. Für Konsumenten mit einer höheren Lebenserwartung als 50 Jahre und für zukünftig geborene Kohorten, beeinflußt Zuwanderung die individuelle Wohlfahrt positiv. Unabhängig von seiner Lebenserwartung und seinem Geburtsjahr sinkt die Wohlfahrt eines Konsumenten, wenn die Fertilität steigt.

JEL classification: J 11, E 17, E 21
Key words: Demographic Forecast, Population Ageing, Consumption, Welfare

1 INTRODUCTION

Due to actual population structures Germany will face significant demographic changes during the next decades. First of all population numbers will enormously decline. At the same time there will be a dramatic shift in the age structure. Germany will witness the most incising ageing process among the members of the European Union (United Nations 1998). If there is neither immigration nor a fertility increase there will be roughly 52 million people in Germany in 2050, which means about 30 million less than in the year 2000. Simultaneously the proportion of elderly aged 65 and above will almost double and reach 28 % in the year 2050. The impending demographic changes will have – particularly due to labour force decline and ageing - far-reaching economic implications, for instance for public finances and the social security systems, the economy's production capacity, the private saving behaviour and per capita consumption.[1]

Population ageing is with the utmost probability foreseeable whereas its factual dimension and thus the magnitude of its economic repercussions crucially depend on future immigration and fertility levels. Therefore it is often referred to the possibility of countering potentially negative economic effects of population ageing by a fertility increase or immigration of people at the working age. Of course, rising fertility rates as well as immigration may slow down population ageing and decline. But they differently influence the population structure and labour supply. Consequently, rising fertility on the one hand and immigration on the other would affect the economic conditions in substantially different ways. There are various issues which are related to the specific economic impacts of different demographic patterns. Out of those this paper examines how different demography scenarios affect the average welfare of individual cohorts due to differences in consumption development.

The analysis focuses on two main channels through which population development might impact the consumption level and thus welfare. These are the dependency ratio and the interaction between labour force growth and required investment. With regard to those relations economic development is computed for various demographic scenarios by a simulation model based on a Ramsey framework modified to include demographic shifts. In doing so also scenarios of rising fertility are considered. This is in contrast to most former studies about macroeconomic repercussions of population ageing which focus on immigration. But undoubtedly, to make the discussion about the potential to cope with population ageing complete, dealing with the issue of rising fertility is indicated.

1 A brief survey of economic issues of population ageing is given by *Denton/Spencer* (1999).

Those considerations help evaluating whether the widely spread opinion that fertility only has to rise for compensating potentially negative effects of historical demographic trends is reasonable for the examined period of time.

From an analytical perspective the paper adds a welfare analysis to former simulation studies based on a Ramsey framework (e. g. *Cutler et al.* (1990), *Börsch-Supan* (1995b), *Schmidt/Straubhaar* (1996) and *Stiller* (2000b). Those former studies focus on the differences in consumption development among different population scenarios and not on welfare comparisons. In the following the simulation results for demographic scenarios are compared with regard to the average welfare of individual cohorts by a Hicksian measure of relative variation. This welfare measure clearly indicates that only looking at per capita consumption is not sufficient for judging about the welfare implications of a certain demographic scenario. Dealing with welfare effects for individual cohorts is motivated by the opinion that this a relevant aspect when discussing policy options related to population ageing.

The paper proceeds along the following lines. Section 2 describes historical and future population trends under various migration and fertility assumptions focusing on dependency and labour force development. Section 3 outlines the economic part of the simulation model. Basic results of the simulation model are briefly presented in section 4. Section 5 deals with the welfare implications of demographic changes. Section 6 concludes.

2 POPULATION DEVELOPMENT

2.1 Population Scenarios

The population model[2] forecasts the population structure by age and sex up to the year 2100. It is based on the age and sex specific German population structure of the year 1994. The demographic scenarios are constructed by combining different fertility and migration assumptions. Mortality patterns do not differ among the scenarios and are in accordance with those of the German Census Bureau forecast of 1994. The long-term life expectancy at birth of the female population is assumed to be 81.1 and that of the male population 74.6 years. In the demographic scenarios with migration only annual migration gains – no losses - are considered. There is a constant migration gain of 50.000 people (gradually decreasing to this level from 500.000 people immigration in

2 For a detailed description of the population model see *Stiller* (2000 a), chapter 2.

the base year) p.a. with Migration A, of 100.000 p.a. with Migration B, of 200.000 p.a. with Migration C and of 300.000 p.a. with Migration D. The assumed age and sex specific structure of the immigrants corresponds to those of the immigrants into Germany in 1992. According to these figures 60 % of the migrants are males. The average age of immigrants is 27 years which is roughly 10 years less than the average age of the German population in 1994.

The fertility scenarios are based on a cohort approach. In a "baseline scenario" a total cohort fertility rate of on average 1.41 children per woman is assumed. Furthermore, four scenarios of – compared to the "baseline scenario" - rising fertility are developed. In those scenarios the total fertility rate is 10 %, 15 %, 25 % and 45 % higher than in the "baseline scenario". There are nine demographic scenarios (see table 1). In the "baseline scenario" there is neither a fertility increase nor migration. In contrast to the "baseline scenario" the fertility scenarios assume higher fertility. In the migration scenarios it is abstracted from rising fertility but there is permanent immigration.

Table 1: The demographic scenarios

Scenario	Net migration gain until 2010	Net migration gain until 2030	Net migration gain until 2050	Long-term total fertility rate
Baseline scenario	0	0	0	1,41
"migration scenarios"				
Migration A	4 million	5 million	6 million	1,41
Migration B	2.9 million	4.9 million	6.9 million	1,41
Migration C	3.4 million	7.4 million	11.4 million	1,41
Migration D	5.1 million	11.1 million	17.1 million	1,41
"fertility scenarios"				
Fertility A	0	0	0	1,55
Fertility B	0	0	0	1,62
Fertility C	0	0	0	1,76
Fertility D	0	0	0	2,04

2.2 Historical and Prospective Demographic Trends

For describing historical and future demographic trends we concentrate on the labour force growth rate[3] and the dependency ratio since those demographic numbers are crucial for the economic development in the chosen model (see chapter 3). Assuming the

3 Age and sex specific labour force participation rates are assumed to be fixed at their levels in West Germany in 1994. According to these figures (see Statistisches Bundesamt 1995) 60 % of the female and 81,8 % of the male population belonged to the labour force.

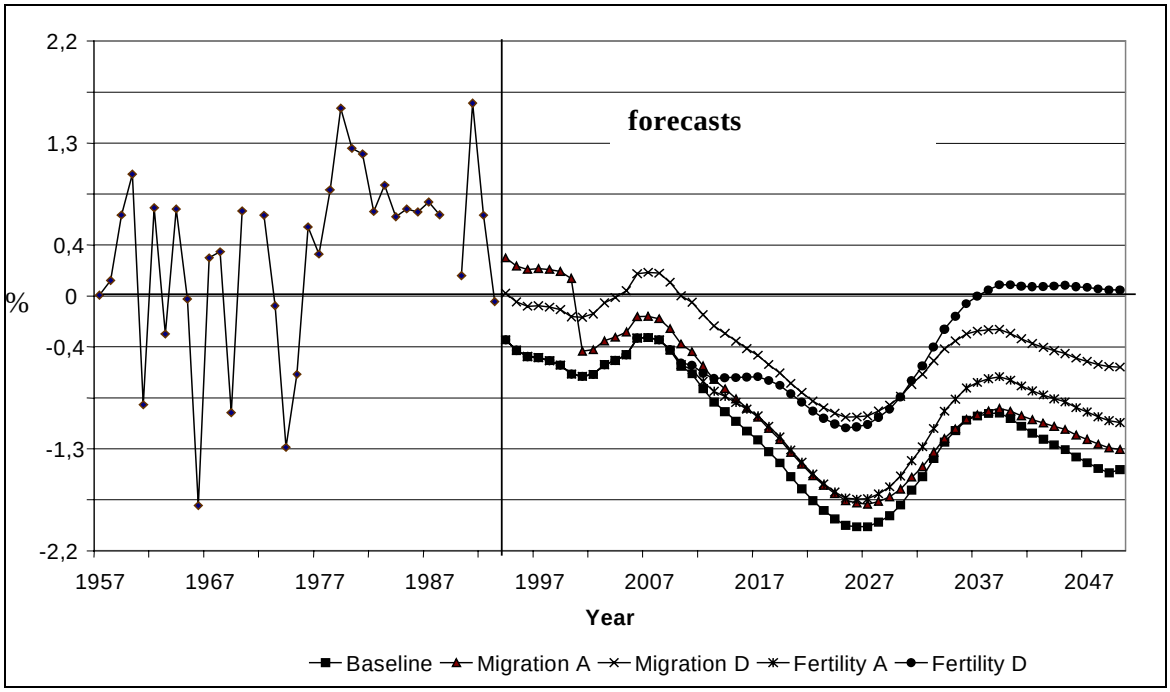
scenarios summarised in table 1 the fluctuations of the chosen demographic indicators are rather small after 2050. Thus considering their development until 2050 is sufficient.

Dependency ratios can be constructed in a number of different ways.[4] We define the dependency ratio (β) as the ratio among the whole population (B), which represents the number of people to be supported either by themselves or by others, and the labour force (L) (see equation 1). An increase in the dependency ratio implies an increase of consumers relative to workers and vice versa.

(1) $\beta = B / L$.

Prospective demographic changes will be strong compared to the status quo and population development will also clearly differ from its historical patterns. During the last four decades periods of labour force increase preponderated periods of decrease. Since 1977 the labour force has almost continuously been growing (see figure 1) as relatively large birth cohorts reached working age.

Figure 1: Labour force growth



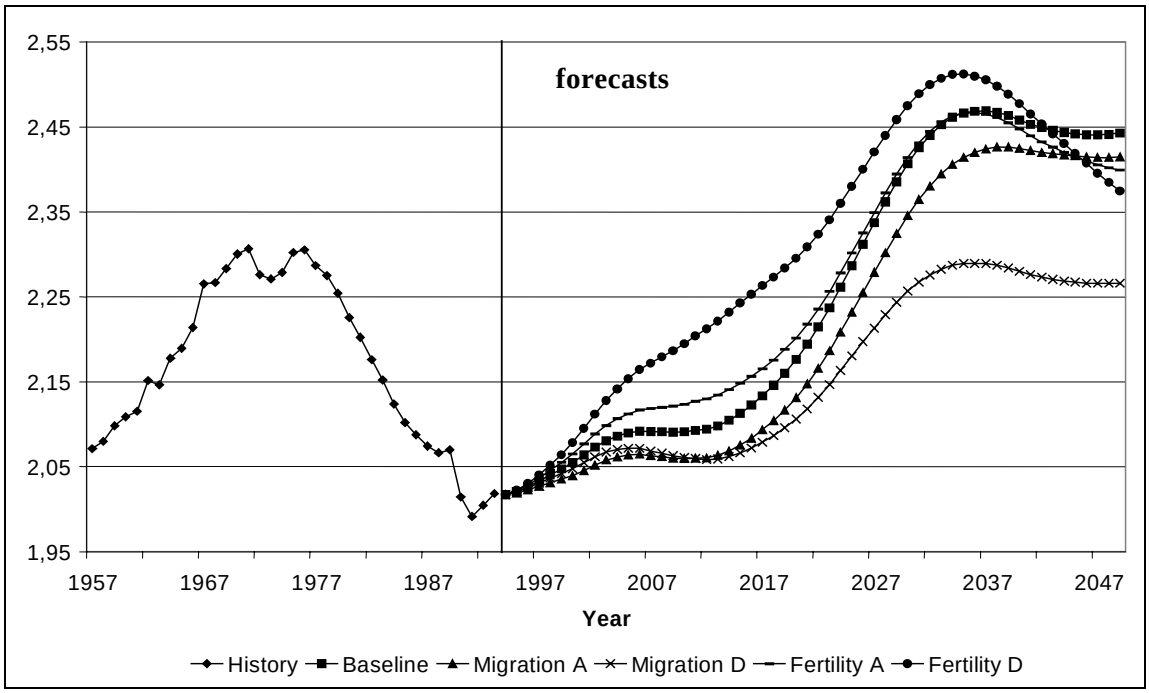
Source: Statistisches Bundesamt, Mikrozensus (different years) and own calculations (forecasts). Missing values when statistical concepts changed.

4 A more common concept is defining dependency as the ratio among the active and passive population. But this ratio is not suited for our analysis. Indeed the chosen definition depends on the objective of the analysis. *Denton/Spencer* (1999) p. 5 present different dependency concepts.

Instead, for the future a substantial labour force decline can be expected. The labour force will progressively decrease - at least until 2030 - even if there is permanent immigration or a fertility increase. The labour force increases again not earlier than in 2039 and only if we assume “fertility D”. Consequently, setting an end to the labour force decline requires a radical change of the fertility behaviour and a long period of time. Furthermore, even permanent immigration of 300.000 people a year who have a relatively juvenile age structure is not sufficient for opposing the impact of population ageing on labour supply.

The dependency ratio subsumes the population and labour force developments, which are both affected by historical demographic changes, i. e. falling birth rates on the one hand and rising life expectancies on the other. Dependency has continuously decreased during the two last decades since the population at the working age - mainly due to low birth rates – has grown more rapidly than the whole population. Thus, by historical standards dependency is low today (see figure 2).

Figure 2: Dependency Ratio



Source: Statistisches Bundesamt, Mikrozensus (different years) and own calculations (forecasts).

For the next decades it is most probably that with the expected population trends – a growing number of retiring people and a declining number of people entering the working force - dependency will sharply increase. During the whole forecasting period

lower rates of immigration imply higher dependency. The same is true for higher fertility rates until 2040. During the next decades dependency will gradually increase with rising fertility because of the rising share of children in the population which adds to the rising share of elderly. Even if fertility immediately increased it would strengthen labour supply not before two decades later. Therefore only after 2040 rising fertility will reduce dependency compared to the "baseline scenario". In the meantime the dependency ratio will be higher, not lower. Obviously transitions from low to high fertility entail a corresponding period of high dependency.[5] Those relations go back to the languishment of demographic processes.

The exact development of the key determinants of demographics, which are migration, fertility and life expectancy, are uncertain. Yet we can conclude from the population forecasts that there will be population ageing, an increase in dependency and labour force decline regardless if there is immigration or a fertility increase. Nevertheless, despite qualitative similarities there are in some cases significant quantitative differences among the fertility and migration scenarios. This is especially true for the amount of people at the working age. Immigration of relatively young people directly strengthens labour supply and thus the productive capacity. Instead, affecting the labour market by rising fertility involves a transition phase not shorter than 20 years. Furthermore, the implications rising fertility and migration have for dependency are contrary. Returning to higher fertility rates would worsen the dependency problem. Due to their substantially different impact on population development it is most probably that rising fertility on the one hand and permanent immigration on the other generate different economic repercussions. In the next section a long-term simulation model for calculating the economic development path which depends on the population's age structure is described. The aim of the analysis is not to forecast economic development. Rather the motivation for doing simulation analysis is to explore whether we can find an demographic impact on consumption development and thus on average welfare in a Ramsey framework.

5 *Weil* (1999), who analyses the relation among dependency and fertility by stable population models, confirms these results.

3 THE MODEL

There is a number of simulation studies dealing with the economic impacts of population ageing.[6] Some studies apply general equilibrium models[7] with overlapping generations that optimally choose life patterns of consumption and bequest (see for example *Auerbach et. al.* (1989), *Yoo* (1994 a), *Fougère/Mérette* (1999), *Miles* (1999), *Broer* (1999)). There are also, in the line of *Cutler et al.* (1990), studies which refer to a Ramsey[8] framework and thus abstract from the overlapping generation structure of the actual population (see for example *Börsch-Supan* (1993, 1994, 1995a, 1995b), *Schmidt/Straubhaar* (1996), *Birg/Börsch-Supan* (1999) and *Guest/McDonald* (1999)). The following simulation results are also based on a Ramsey growth model.[9] In general the more aggregated Ramsey model is much easier to handle than an overlapping generations model which is disaggregated by cohorts. But, under certain assumptions regarding preferences overlapping generations model and the Ramsey model lead to identical growth paths. In this case no insights concerning the interplay of demographic and economic developments are lost when applying the aggregated Ramsey framework.[10]

The Ramsey model is a dynamic macro framework. In the remainder it is referred to the model's closed economy version since the demographic impact on the long-term growth path is very clear then.[11] Unlike in *Ramsey* (1928) the population (B_t) and the labour force (L_t) are of different size as we consider an explicit age structure. The labour force grows at the rate n_t and to remind dependency is $\beta_t\ (= B_t / L_t)$. There is labour augmenting technical progress at a constant rate g. With A_t as the level of technology - its initial level (A_0) is normalised to one - the effective amount of labour (L_t^{eff}) at any point of time (t) is:

(2) $$L_t^{eff} = L_t \cdot A_t = L_t \cdot \exp(g \cdot t)$$

6 For an overview see OECD (1998), p. 110-112.
7 The contribution of *Auerbach/Kotlikoff* (1987) is the seminal work of this field of research.
8 The model goes back to *Ramsey* (1928).
9 For a detailed description of the simulation model see *Stiller* (2000 a).
10 See *Blanchard/Fischer* (1989), p. 106, *Maußner/Klump* (1996), p. 139 and *Börsch-Supan* (1995 b), p. 30-31. The identity only holds when assuming completely altruistic agents and a Barro-type clan utility function. Otherwise the growth path of a Ramsey model and an overlapping generations model deviate. *Yoo* (1994 b) compares simulation results of those two approaches for rather stylised demographic changes and points out the essential differences.
11 Closed economy means that Germany does not import or export capital. Nevertheless Germany "imports" labour by immigration. For the open economy version of the model see *Cutler* et. al. (1990) and *Börsch-Supan* (1993).

The output (Y_t), which is either consumed (C_t) or invested (I_t), is produced using labour and capital (K_t) which are fully employed at any point in time and paid by their marginal products (see Barro/Sala-i-Martin (1995), p. 68):

(3) $$Y_t = F(K_t, L_t, t) = F(K_t, L_t^{eff}) = C_t + I_t$$

The production function $[F(...)]$ satisfies the common neoclassical properties (see Barro/Sala-i-Martin (1995), p. 16) thus the model relations can be expressed per worker:

(4) $$f(k_t) = c_t \cdot \beta_t + i_i = c_t \cdot \beta_t + (n_t + d + g) \cdot k_t + \frac{dk_t}{dt}$$

In equation (4) c_t denotes per capita consumption, i_t investment, k_t capital stock and $f(k_t)$ output per worker. Gross investment subdivides in capital endowment for the growing efficient labour force ($(n_t + g) \cdot k_t$), replacement of depreciated capital ($d \cdot k_t$, d denotes the depreciation rate) and net investment per worker (dk_t / dt). The consumable amount of the production of one unit of effective labour – this is what is left after investment - is shared among workers and non-workers according to the dependency ratio. This relation implies that those in the population who are producers support themselves plus the non producing population (children and elderly). Rearranging (4) leads to the inter-temporal budget-constraint:

(5) $$\frac{dk_t}{dt} = f(k_t) - c_t \cdot \beta_t - (n_t + d + g) \cdot k_t .$$

In the model a benevolent social planner with perfect foresight maximises social welfare (W) at the beginning of the planning period for given capital endowment in the base year and with regard to the inter-temporal budget constraint[12, 13]:

(6) $$W = \int_{t=0}^{\infty} u(c_t) \cdot B_t \cdot \exp(-\rho \cdot t) dt .$$

Social welfare is the sum of all future per capita utility flows weighted by the population size (B_t). The "felicity function" $[u(...)]$ measures the instantaneous average per capita utility which only depends on per capita consumption (c) in t.[14] Discounting future

12 The optimal growth satisfies the transversality condition (see *Barro/Sala-i-Martin (1995)*, p. 65).

13 The solution technique for this type of dynamic optimisation problem is the optimal control theory (see *Silberberg* (1990), Chapter 18; *Chiang* (1992), Chapter 7 and *Gandolfo* (1995), Chapter 22).

14 The felicity function is increasing in (c) and concave (see *Barro/Sala-i-Martin* (1995), p. 65).

flows of utility implies that the society values the consumption of different generations at different rates. The higher the social time preference rate (ρ) the lower the contribution of a future generation's utility to the social welfare. We follow the common practice and assume a felicity function with a constant elasticity of inter-temporal substitution $(\sigma = 1/\theta)$:

$$u(c_t) = \frac{c_t^{(1-\theta)} - 1}{(1-\theta)} \quad for \quad \theta \neq 1, \tag{7}$$

$$= \ln c_t \qquad for \quad \theta = 1.$$

The welfare maximising per capita consumption path follows the Euler equation:

$$\frac{dc_t / dt}{c_t} = \sigma \cdot (r_t - \rho) \qquad \text{with } r_t = f'(k_t) - d. \tag{8}$$

The Euler equation implies rising (falling) per capita consumption as long as the net rate of return on capital (r_t) exceeds (falls short of) the social time preference. σ determines how sensitively per capita consumption reacts if the net rate of return and the social time preference rate deviate.

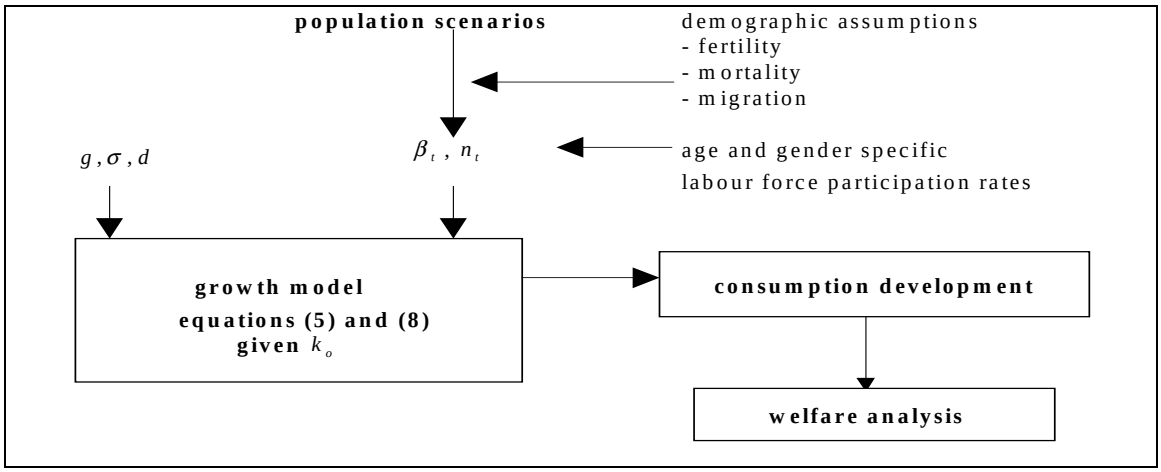
The dynamic properties of the optimal growth path are completely described by the time paths of capital (equation 5) and per capita consumption (equation 8). Demographics influence the model dynamics firstly via the adjustment of investment requirements, which affects the inter-temporal budget constraint, caused by the changing labour force growth rate. Secondly the level of average per capita consumption depends on the dependency ratio (see equation 4). Hence, the time paths of consumption and investment depend on future demographic patterns and thus the growth path can be expected to differ when the population structure changes.

Based on the model mechanisms the impact of population development on consumption development is analysed in a simulation model. Figure 3 outlines how demographic and economic developments are connected in the model. For the numerical examples assumptions regarding the exogenously economic parameters are necessary. Those take regard of the values chosen in the relevant literature[15]: $g = 1.41\,\%$, $d = 5\,\%$, $\sigma = 1$ and

15 See *Börsch-Supan* (1993), p. 34; *Börsch-Supan* (1994), p. 134; *Börsch-Supan* (1995 a), p. 224; *Börsch-Supan* (1995 b), p. 33; *Cutler et. al.* (1990), p. 23 und *Schmidt/Straubhaar* (1996), p. 403.

$\rho = 4\,\%$.[16] The production function is assumed to be of the Cobb Douglas type: $f(k_t) = \exp(g \cdot t) \cdot k_t^{0.3}$. β_t and n_t are also exogenous and follow directly from the population scenarios presented in section 2.

Figure 3: The simulation model



The time horizon of the simulation analysis is the year 2100. It is assumed that the demographic and economic terms reach steady state values at that point in time and are constant afterwards.[17] Furthermore, capital per worker in the base year – which is 1995 - is assumed to be at its steady state value. Under these assumptions and due to the model's saddle path stability the per capita consumption level in the base year, which will lead the economy to the steady state in 2100 is unique (see *Barro/Sala-i-Martin* (1995), p. 73). Furthermore, per capita consumption differs – due to the different developments of β_t and n_t - among the demographic scenarios. In order to determine the consumption level which allows the economic development according to the stable path the equations (5) and (8) are expressed per efficiency unit of labour since economic figures per efficiency unit are constant in the steady state which is advantageous for computing the stable growth path.[18] In order to find the scenario specific solutions for each demographic scenario a shooting algorithm is applied to equations (5) and (8) measured in efficiency units.[19]

16 The economic effects of demographic changes are not significantly altered when changing the assumptions regarding the values of the exogenously economic parameters (see *Stiller* 2000 a).

17 The model results until 2100 are only affected at the margin if we assume that the economy reaches its steady state later in the future.

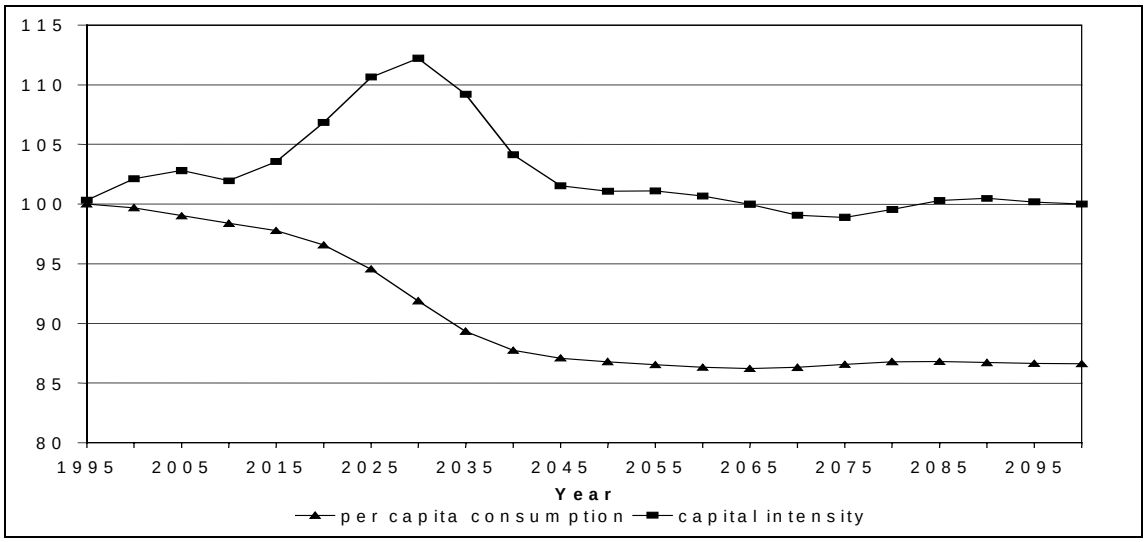
18 See *Stiller* (2000 b) p. 17.

19 The same method is applied by *Cutler* et. al (1990) and *Börsch-Supan* (1993, 1994, 1995 a, 1995 b).

4 ECONOMIC IMPACTS OF THE AGE STRUCTURE[20]

In the Ramsey model instantaneous utility and thus individual welfare only depend on the consumption level. Therefore the presentation of the simulation results is concentrated on per capita consumption. Corresponding to the model mechanisms the influence of the prospective demographic developments on per capita consumption are straightforward. Firstly, an ageing population tends to reduce the labour force (n_t is negative) and thus the investment requirements and needs for saving. This allows c. p. higher per capita consumption. Secondly, due to ageing dependency increases which c. p. lowers per capita consumption possibilities since one worker has to support the more consumers the higher dependency is. Thus the consumption effects of the decreasing labour force on the one hand and rising dependency on the other are opposed to each other. If per capita consumption is positively or negatively affected by population ageing depends on the relative weight of those opposing consumption effects. How the economy adjusts to demographic changes is reflected in the capital intensity and per capita consumption (exemplarily illustrated for the baseline case in figure 4).

Figure 4: Development of per capita consumption and capital intensity in the "baseline scenario", base year = 100, 1995 - 2100



20 As this paper focuses on quantifying the welfare effects of demographic changes the economic repercussions of population ageing in a Ramsey framework are only shortly summarised. For a detailed description of the macroeconomic feed-backs of demographic changes in the simulation model see *Stiller* (2000 a, 2000 b).

In figure 4 economic terms are measured in efficiency units in order to neutralise for technical progress. We did not figure out the growth effects merely caused by technical progress since this allows to illustrate clearly how consumption is affected by population ageing. Due to population ageing labour becomes the relative scarce production factor. Thus capital intensity initially increases, which leads to a decrease of the marginal product of capital and rising wages, and decreases afterwards. The steady state capital intensity per efficiency unit of labour is independent of demographic figures and thus is the same in the base year and in 2100.[21] Contrary, per capita consumption reaches a new steady state value in 2100 which is significantly lower than in the base year. As figure 4 illustrates consumption per capita in the "baseline scenario" – measured in efficiency units - would gradually decrease until 2050 and only slightly vary afterwards. Population ageing has a negative effect on average consumption possibilities due to the enormously increasing dependency.

Development in the "baseline scenario" indicates ageing without opposing demographic behaviour. The baseline scenario is the "worst case ageing scenario" since the proportion of elderly aged 65 an above will be highest in this scenario throughout the whole forecasting period.[22] Instead rising fertility and immigration work against ageing. The relative weight of the opposing consumption effects of population ageing crucially depends on the effective changes of β_t and n_t. Concerning the differences among the demographic scenarios the following relations hold with respect to economic adjustments:

- Investment requirements in the baseline case – because of higher labour force decrease - are during the whole simulation period lower than with immigration and after 2015 also lower than in the fertility scenarios. C. p. per capita consumption is the higher the less people immigrate and the lower fertility is (after 2015).
- Compared to the "baseline" case dependency decreases if people with a juvenile age structure immigrate and increases until 2040 if fertility rises. C. p. per capita consumption is the higher the more people immigrate and until 2040 the lower the more strongly fertility increases.

Since the development of n_t and β_t clearly differs among the population scenarios we expect per capita consumption and thus welfare to differ when varying the fertility and migration assumptions. Despite similarly qualitative reactions to population ageing in

21 For the steady state adjustments to demographic changes see *Stiller* (2000 a), pp. 150-164.
22 See *Stiller* (2000 a), p. 61.

all demographic scenarios there are clear quantitative differences. Figure 5 illustrates those by comparing the economy's year-by-year consumption possibilities. It should be mentioned that the quantitative differences among the demographic scenarios are independent of the rate of technical progress. Summarising the simulation results we conclude that in the middle and in the long terms per capita consumption is higher with immigration than without due to lower dependency (see figure 5 a, consumption in the baseline is defined as 100).[23] Instead rising fertility reduces per capita consumption compared to the "baseline scenario" for more than four decades since transition to a higher fertility involves a period of higher dependency. This cuts per capita consumption (see figure 5 b, consumption in the baseline case is defined as 100). The bigger part of the consumption differences among fertility and migration scenarios (see figure 5 c, consumption with "migration D" is defined as 100) is due to the – from the perspective of per capita consumption – more unfavourable development of dependency when assuming rising fertility. In contrast to the influence of dependency the impact of different investment requirements among the population scenarios on consumption differences is rather small.[24]

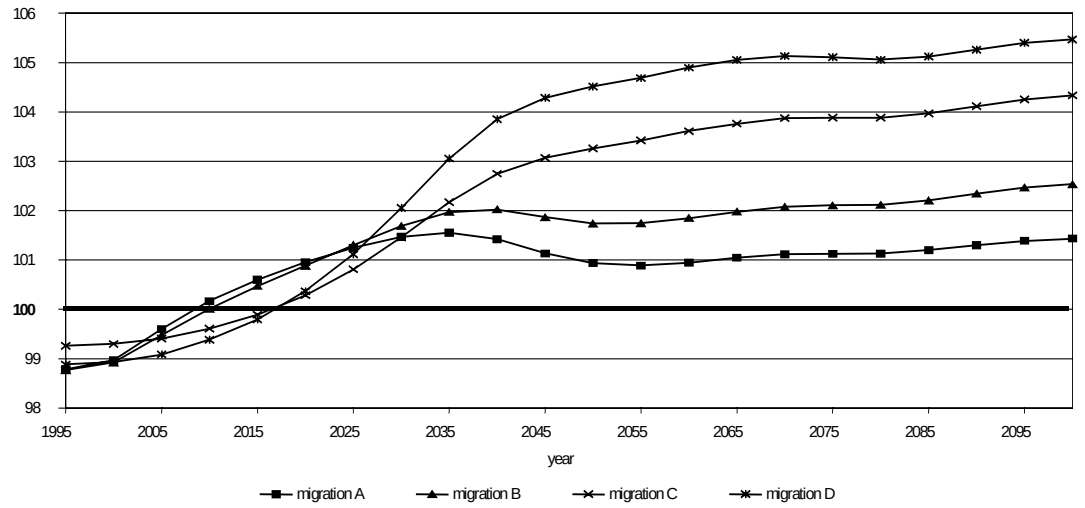
In the model the relation among demographic and economic developments was captured by the dependency ratio and the growth rate of the labour force. Of course, the repercussions of population ageing on the economy are more complex than considered in this paper. Nevertheless, the model results provide insights into the fundamental relationship between population and consumption development. In particular the model is well suited for understanding the key mechanisms which cause economic differences among scenarios of rising fertility and migration. In the next section we will turn to their welfare implications.

23 The same results can be found in *Cutler* et al. (1990) and. *Börsch-Supan* (1993, 1994, 1995 a, 1995 b). But those studies do not offer results concerning the economic effects of changing fertility rates.

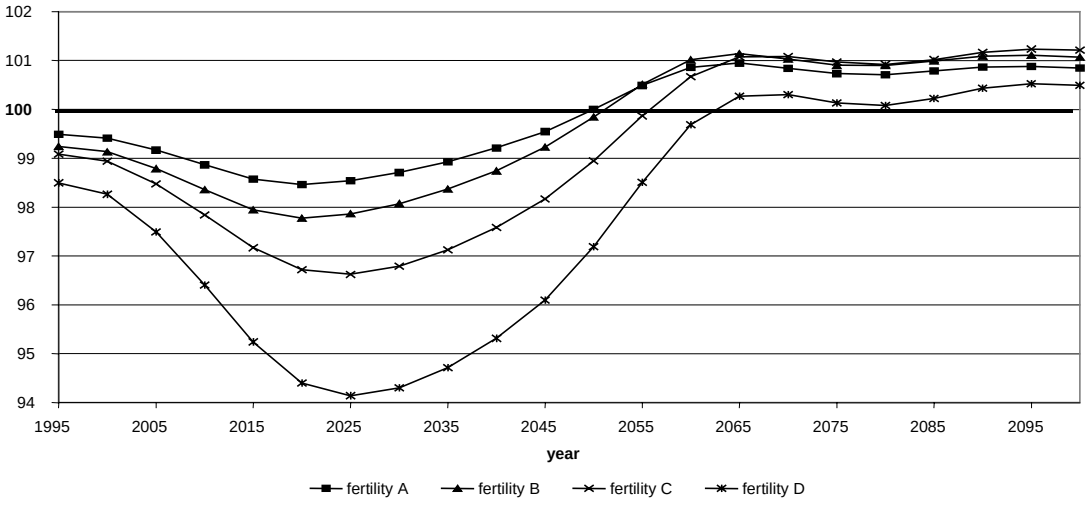
24 See *Stiller* (2000 a), pp. 150-164 who analyses the quantitative impact less investment on the one hand and higher dependency on the other have on per capita consumption.

Figure 5: Consumption development in different scenarios, 1995 - 2095

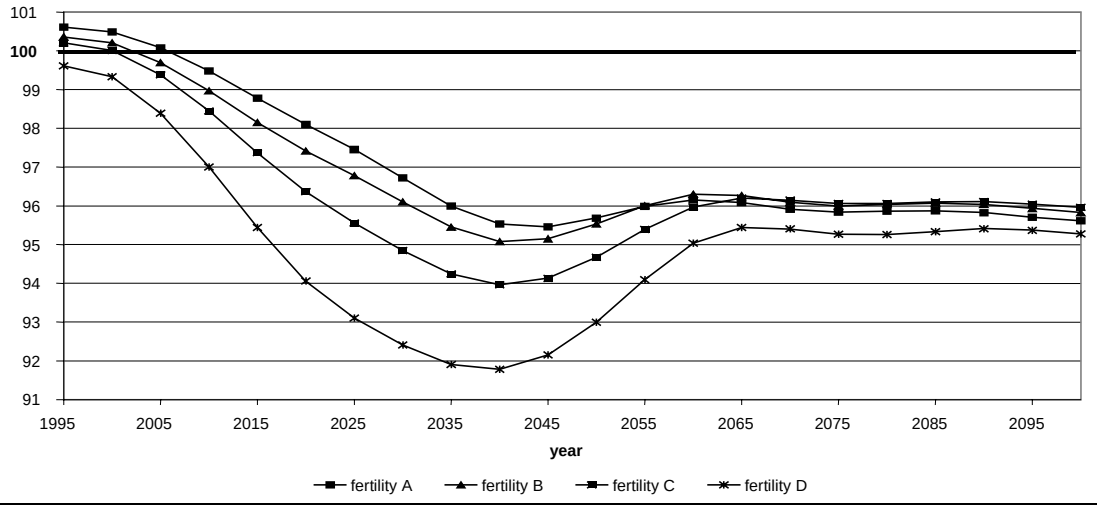
a) Per capita consumption in the migration scenarios (baseline = 100)



b) Per capita consumption in the fertility scenarios (baseline = 100)



c) Per capita consumption in the fertility scenarios (migration D = 100)



5 WELFARE EFFECTS OF DEMOGRAPHIC CHANGES

5.1 Welfare measure

In former quantitative studies concerning the demographic changes in a Ramsey framework the scenarios are evaluated by their periodical per capita consumption. *Börsch-Supan* (1993, p. 8) regards the year-by-year consumption possibilities as a welfare indicator: "Welfare of society is measured in terms of consumption per capita.". But as *Cutler et. al.* (1990, p. 54) correctly note, only looking at the per capita consumption path does not give any hint at the welfare effects of population development. Per capita consumption can be interpreted as an indicator for the average periodical standard of living but not as a welfare indicator in the inter-temporal context of the Ramsey model. In order to evaluate the welfare effects of demographic changes with regard to the Ramsey model one has to go beyond merely comparing the inter-temporal consumption levels and to take regard of the assumed utility function. The theoretically correct welfare indicators in the Ramsey framework are, as *Raffelhüschen* (1994, p. 145) points out, the Hicksian measures of relative variations. The advantage of these welfare indicators is that they allow welfare comparisons between individual cohorts on the basis of their lifetime utility. Therefore in the following a Hicksian measure of relative variation is applied for comparing the demographic scenarios.

In general the Ramsey framework offers two approaches for welfare measurement. The crucial issue is whether the total welfare of all future consumers or just their average welfare should be considered for comparing the demographic scenarios. This is an ethical question which cannot be resolved here.[25] We decided to follow the later concept. In the following it is evaluated how the individual average welfare, which is the average per capita lifetime-utility, is affected by the population development.

There are two standard measures of relative welfare change: The relative equivalent variation (REV) and the relative compensating variation.[26] We only look at the REV as both relative Hicksian welfare indicators are close to each other in this analysis. The welfare analysis compares the per capita lifetime utility among the demographic scenarios which is the sum of a consumer's discounted prospective utility flows:

25 The question of how to evaluate population size from a welfare perspective is discussed in *Razin/Sadka* (1995), pp. 37-71.

26 For another application of the relative Hicksian welfare measures see *Bröcker* (1997), pp. 8-9.

(9) $$v(S)=\sum_{t=0}^{n}u(c_t)\cdot(1+\rho)^{-t}=\sum_{t=0}^{n}\frac{c_t^{(1-\theta)}-1}{1-\theta}\cdot(1+\rho)^{-t}\ \text{with}\ S^{'}=(c_o,...,c_i,...,c_n).$$

S is the consumption path of a consumer who lives n more periods from the base year. For the welfare analysis we transform (9) into

(10) $$\tilde{v}(S)=\left[\sum_{t=0}^{n}c_t^{(1-\theta)}\cdot(1+\rho)^{-t}\right]^{\frac{1}{(1-\theta)}},$$

which is linearly homogenous:

(11) $$\tilde{v}(\lambda\cdot S)=\left[\sum_{t=0}^{n}(\lambda\cdot c_t)^{(1-\theta)}\cdot(1+\rho)^{-t}\right]^{\frac{1}{(1-\theta)}}=\lambda\cdot\tilde{v}(S).$$

For this functional form of the utility function the Hicksian measure of relative variation equals the percentage difference[27] of the future life time utility among two demographic scenarios. For deriving the *REV* we choose for expositional issues the "baseline scenario" as a reference case. S^0 denominates the per capita consumption path in the baseline scenario and $\tilde{v}(S^0)$ is the present value of future utility flows in this scenario. Analogous $\tilde{v}(S^i)$ measures the present value of utility with population scenario i.

Assuming a linearly homogenous utility function the *REV* indicates the yearly percentage change of per capita consumption which would be necessary in the "baseline scenario" for attaining the same utility as in scenario i:

(12) $$\tilde{v}\left((1+REV)\cdot S^0\right)=\tilde{v}(S^i)\Leftrightarrow REV=\frac{\tilde{v}(S^i)}{\tilde{v}(S^0)}-1.$$

According to equation (12) the *REV* measures in percent how much future utility is higher or lower in the scenario i than in the baseline scenario. The *REV* is positive if future utility is higher in the scenario i than in the "baseline scenario" and vice versa.

Life-time utility, our measure for average individual welfare, depends on the year in which the considered individual is born. Thus we have to differentiate among different birth cohorts for welfare comparison. For the welfare analysis we follow two ap-

27 See *Rutherford* (1995), p. 1.

proaches. Firstly, it is examined how the utility of a representative consumer of a cohort born in the past is affected by future population development. Secondly, the welfare effects of demographic changes for a representative consumer of a cohort which will be born in the future are analysed.

However, one might argue that the results regarding consumption development and thus regarding welfare would change when changing the objective function of the inter-temporal maximisation problem (see page 15). An alternative objective function in our case could be one that does not – in contrast to equation 6 - weight the average utility of different generations by the numbers of people in each generation. Then the life-time utility (V) of a representative member of the population living in $t = 0$ would be maximised, which is the discounted sum of its own future utility and per capita utility of his succeeding generations:[28]

(13) $$V = \int_0^{\infty} u(c_t) \cdot \exp(-\vartheta \cdot t) dt .$$

ϑ, the individual rate of pure time preference is positive. This implies that consumption in the present is preferred against future consumption. Furthermore, the higher ϑ, the lower is the contribution of future cohorts to life-time utility. Thus ϑ is and indicator for the degree of intergenerational altruism.

The per capita welfare effects would differ among the optimal growth path either belonging to the objective function expressed by equation 6 or by equation 13 if economic repercussions of population ageing would be substantially different among these models.[29] *Stiller* (2000 a) did a sensitivity analysis among an approach based on an inter-temporal utility function which weights the average utility by population size and one which does not. Her result is that weighting the average utility by population size or not has no significant influence on consumption development for the demographic scenarios considered in chapter two of this paper.[30] Thus welfare implications would not – at least in this study – be strongly affected by changing among those inter-temporal utility concepts.

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28 For example *Blanchard/Fischer* (1989) and *Schmidt/Straubhaar* (1996) apply this approach for solving the Ramsey problem.

29 *Canton/Meijdam* (1997) show that the macroeconomic effects of demographic changes may depend on the specification of the inter-temporal utility function. They are right in criticising that many authors do not comment on the chosen welfare function (see *Canton/Meijdam* (1997), p. 318) since the form of the welfare function may affect the economic development path.

30 See *Stiller* (2000 a), pp. 112-114 and pp. 194-196.

5.2 Welfare Comparisons

When evaluating the welfare effects for individuals born in the past we have to take regard of the fact that consumers born at different points in time have on average different remaining life spans over which their consumption level is affected by the future demographic development. Therefore the *REV* is calculated for different individual future life expectancies. When choosing this approach the *REV* (see table 2) firstly stresses in which way the welfare effects differ between individual cohorts which have deviating future life expectancies while assuming the same demographic scenario. Secondly it illustrates the welfare differences among demographic scenarios for individuals having the same average life expectancy.

Table 2: *REV*, "baseline scenario" as reference case

	Future life expectancy in years ...						
	5	**15**	**25**	**35**	**45**	**57**	**100**
Migration A	-1,2	-0,8	-0,4	-0,2	-0,1	0,0	0,1
Migration B	-1,2	-0,9	-0,5	-0,3	-0,1	0,0	0,2
Migration C	-0,7	-0,7	-0,5	-0,3	-0,1	0,1	0,3
Migration D	-1,1	-1,0	-0,8	-0,5	-0,2	0,0	0,4
Fertility A	-0,5	-0,7	-0,9	-1,0	-1,0	-1,0	-0,8
Fertility B	-0,8	-1,0	-1,3	-1,4	-1,4	-1,4	-1,2
Fertility C	-0,9	-1,2	-1,6	-1,9	-2,0	-2,0	-1,8
Fertility D	-1,5	-2,1	-2,7	-3,2	-3,4	-3,5	-3,2

Compared to the migration scenario the "baseline scenario" is preferable as long as a consumer's life expectancy in $t = 0$ is not higher than roughly 50 years. Among these the "baseline scenario" is the more advantageous the shorter the future life expectancy of a consumer is. The higher welfare in the "baseline scenario" is the outcome of two aspects which are decisive for the lifetime utility: the differing consumption paths between the demographic scenarios and discounting future utility flows.

Due to discounting the instantaneous utility of the near future contributes more to the overall utility than the consumption of later periods. As per capita consumption is initially higher in the "baseline scenario" than with immigration (see figure 5 a) the periods with relatively high contribution to overall utility are in favour of the "baseline scenario". Assuming a life expectancy of more than 50 years the higher instantaneous utility in the "baseline scenario" during the next decades cannot compensate for the medium and long-term higher instantaneous utility of later periods when assuming perma-

nent migration gains. Therefore, only when facing a relatively high future life expectancy migration has a positive influence on lifetime utility.

Independently of the assumed life expectancy a consumer is less well off if fertility rises compared to the "baseline scenario". Furthermore, the welfare differences between the "baseline scenario" and the fertility scenarios are the higher the higher the fertility increase is. Causal for the welfare differences between the fertility scenarios and the "baseline scenario" is the lower per capita consumption (see figure 5 b) in the fertility scenarios for about four decades. The welfare loss in the fertility scenarios compared to the "baseline scenario" is rising up to a life expectancy of roughly 60 year and amounts for example to a *REV* of –3,5 in the "fertility D" scenario.

If we choose the "migration D" scenario as the reference case for calculating the *REV* it becomes obvious that immigration – apart from some cohorts - is more advantageous from the perspective of lifetime utility than any of the fertility scenarios (see table 3). Those welfare differences are the higher the more strongly fertility increases. Furthermore, the welfare differences among the migration scenarios are rather small. This means that lifetime utility is not much affected by the amount of the annual migration gain. The reason for this is that in the near future per capita consumption increases with declining immigration because of lower investment requirements.

Table 3: *REV* **, "migration D" as reference case**

	future life expectancy in years ...						
	5	**15**	**25**	**35**	**45**	**57**	**100**
Baseline scenario	1,1	1,0	0,8	0,5	0,2	-0,0	-0,4
Migration A	-0,1	0,2	0,4	0,3	0,2	0,0	-0,3
Migration B	-0,1	0,1	0,3	0,3	0,2	0,0	-0,2
Migration C	0,4	0,3	0,3	0,2	0,1	0,0	-0,1
Fertility A	0,6	0,3	-0,1	-0,4	-0,7	-1,0	-1,2
Fertility B	0,3	0,0	-0,5	-0,9	-1,2	-1,4	-1,6
Fertility C	0,2	-0,2	-0,9	-1,4	-1,8	-2,0	-2,2
Fertility D	-0,4	-1,1	-2,0	-2,7	-3,2	-3,5	-3,6

In order to assess the welfare implications of demographic changes for individual future cohorts we assume that the individuals of each cohort face the same average life expectancy of 75 years. Again the "baseline scenario" is the reference case. Table 4 illustrates that for future cohorts lifetime utility is always higher with immigration than in the "baseline scenario". Those welfare differences are the more distinct the later in the fu-

ture an individual is born and the higher the immigration level is. Obviously the for the time being higher per capita consumption in the "baseline scenario" is not sufficient to compensate higher instantaneous utility with immigration in later years. Different from permanent immigration rising fertility has a negative influence on the lifetime utility of future cohorts. Although per capita consumption in the fertility scenarios in the long-term is the same as or even higher than in the "baseline scenario" (see figure 5 c) future cohorts will be better off with low fertility rates.

Table 4: *REV* **, "baseline scenario" as reference case**

Life expectancy = 75 years; year of birth ...						
	2000	**2005**	**2010**	**2015**	**2020**	**2025**
Migration A	0,5	0,8	1,0	1,1	1,2	1,3
Migration B	0,6	0,9	1,2	1,5	1,7	1,8
Migration C	0,6	0,9	1,3	1,7	2,2	2,6
Migration D	0,7	1,2	1,8	2,4	3,0	3,6
Fertility A	-0,9	-1,0	-1,0	-0,9	-0,7	-0,5
Fertility B	-1,4	-1,5	-1,5	-1,3	-1,1	-0,8
Fertility C	-2,1	-2,3	-2,4	-2,3	-2,1	-1,7
Fertility D	-3,8	-4,2	-4,4	-4,4	-4,1	-3,7

When choosing the "migration D" as the reference case we find for some scenarios noticeable welfare differences in favour of the "migration D" scenario. For future cohorts high immigration is more advantageous than the "baseline scenario" and rising fertility rates. The absolute value of the *REV* is the higher the later in the future a cohort is born and especially high in the case of a strong fertility increase. For example it is −7,0 with "fertility D" for cohorts born in 2025.

Table 5: *REV* **, "migration D" as reference case**

Life expectancy = 75 years; year of birth ...						
	2000	**2005**	**2010**	**2015**	**2020**	**2025**
Baseline scenario	-0,7	-1,2	-1,7	-2,3	-2,9	-3,4
Migration A	-0,2	-0,4	-0,7	-1,2	-1,8	-2,2
Migration B	-0,2	-0,3	-0,5	-0,8	-1,4	-1,6
Migration C	-0,2	-0,3	-0,5	-0,6	-0,8	-0,9
Fertility A	-1,6	-2,2	-2,7	-3,2	-3,6	-3,9
Fertility B	-2,1	-2,7	-3,2	-3,2	-3,9	-4,2
Fertility C	-2,8	-3,5	-4,1	-4,6	-4,9	-5,1
Fertility D	-4,5	-5,3	-6,1	-6,6	-6,9	-7,0

6 CONCLUSIONS

The German population and similarly the labour force will experience a considerable decline and degree of ageing in the decades ahead. Those demographic changes are about to come whether there is permanent immigration or fertility rates would recover again. Ageing will be lower with immigration or rising fertility and higher if life expectancy further increases.[31] There is no doubt that dependency will sharply increase and labour supply will decrease when large cohorts, i.e. the baby boom generations, will retire. Those inescapable demographic changes will affect the economic conditions. Labour supply is decisive for the economy's potential to produce. And dependency – assuming a ' pay as you go economy' - is relevant for difficulties a population has in supporting itself.

In this paper macroeconomic issues of a changing age structure were analysed in a Ramsey simulation model. Different from former simulation studies based on a Ramsey framework the paper does not mainly deal with the possibilities to offset, for example by by population-oriented policies, decreases in the standard of living in the course of population ageing policy. The main objective of the paper was to explore which impact different rates of immigration and fertility might have for the average welfare of individual cohorts. This was firstly motivated by the fact that political discussions about higher birth rates and labour-market oriented migration policy often neglect welfare issues. But welfare of different generations is differently affected by immigration and changing fertility behaviour. Secondly, in some former studies of economic repercussion of population ageing based on a Ramsey framework, per capita consumption development was wrongly interpreted as an indicator for the development of welfare. Instead in this paper a Hicksian welfare measure which takes regard of the assumed utility function was applied in order to compare the scenarios from the perspective of welfare. The REV subsumes future utility flows to a indicator which allows the welfare comparison of inter-temporal consumption paths. Although this indicator has shortcomings it should be preferred to judging about welfare effects of population development by merely comparing the year-by-year consumption level. The yearly per capita consumption indicates the average living standard of one period – not more and not less.

31 The influence of decreasing mortality on the growth path was not considered in the paper. Rising life expectancy would worsen the dependency problem, while leaving investments requirements unaffected, and therefore reduce average consumption and welfare for all generations. Indeed, in its latest population forecast the German Census Bureau (see Statistisches Bundesamt 2000) assumes higher life expectancies than in the previous forecast of 1994 (see *Sommer* 1994).

The model presented in this paper suggests that economic reactions to population ageing can be expected. Population shifts have – at least in the considered model – a substantial impact on the capital-labour-ratio which is crucial for economic development. How strongly demographic changes affect consumption development and thus the average level of utility crucially depends on the future immigration and fertility rates. Concerning the periodical standard of living - which is reflected in average per capita consumption – the model results suppose that immigration is and rising fertility is not suited for reducing the negative effects of the prospective demographic changes.

The *REV* makes obvious that immigration – despite higher consumption in the middle and in the long terms - is not for any cohort beneficial from the perspective of welfare. Individuals with a life expectancy up to 50 years are better off without immigration. For those cohorts stronger population ageing is beneficial since within their remaining life span those periods dominate in which less investment requirements outweigh the negative effects of rising dependency. Furthermore, despite clear differences in the consumption development, there are only small welfare differences with and without immigration for cohorts born in the past. This pictures changes when we look a future cohorts. They benefit the more from immigration the higher the annual immigration gains are. Furthermore, the *REV* suggests that future born cohorts as well as cohorts born in the past are negatively affected by rising fertility rates. Of course, relatively young cohorts, which would have to support a growing number of children, would realise a decline in their life-time utility compared to the baseline and to the migration scenarios. This is due to the fact that production has to be shared among the more people the higher fertility is which ends in lower utility levels.

When thinking about the potential to counterbalance the effects of population ageing it is sometimes referred to rising fertility and immigration as if affecting those demographic components would lead to the same results from an economic perspective. The results of this paper point out that rising fertility and permanent immigration are by no means substitutes with regard to offsetting undesirable economic effects of population ageing, in particular a decrease of the standard of living. Furthermore, we should take into account that the welfare of individual cohorts would be differently affected by higher immigration on the one hand and rising fertility rates on the other.

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