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METHODOLOGY FOR THE EMPIRICAL ESTIMATION OF SHADOW PRICES

by

Daniel M. Schydlowsky

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I

The Shadow Prices to be Estimated

This methodology is designed for the empirical estimation of shadow prices for use in the evaluation of projects under the realistic conditions in which project evaluation and implementation is carried out in Latin America.

The companion piece entitled "Project Evaluation in Economies in General Disequilibrium" discusses in some detail the macroeconomic framework within which projects are evaluated and implemented in Latin America as well as the implications of that particular environment for the design of project evaluation methodology and the definition of the relevant shadow prices. In consequence, only a brief outline of this environment will be presented here and the relevant points will be elaborated as necessary.

The basic feature of evaluation of projects in Latin America is that it applies to a very limited number of projects only. Systematic project evaluation does not apply to all public projects nor does it apply exclusively to public projects. Some public projects are subject to evaluation scrutiny by the planning agency, while a large

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[1]This paper has been prepared in the context of a consulting agreement with the Inter-American Development Bank.

number of others are executed by independent agencies in the public sector or by parts of the central government not subject to the administrative review of the planning body. On the other hand, a number of private projects are presented to the planning authorities as a preliminary step to the request of tax concessions, preferential credit, or other incentive measures specified in the relevant legislation. Under these circumstances, it is appropriate to talk of the evaluated sector and the unevaluated sector, where the former comprises those projects in the public sector and in the private sector which are subject to investment evaluation review whereas the unevaluated sector comprises all remaining investment projects. Although the evaluated sector can be expected to grow over time, at the moment it is still rather small comprising at most half of public sector investment and a quarter of private sector investment.

Investments in the evaluated sector are to be chosen in a way which maximizes the benefit from the investment. The objective function is taken to be the present value of the future consumption stream generated by the projects. Investment projects outside of the evaluated sector are decided on a variety of grounds; presumably those in the private sector are chosen in order to maximize the profits of the investors whereas those in the public sector are selected in whatever way is traditional.

All projects as indeed the economy as a whole make their current operating decisions at market prices. This includes the operation of the projects in the evaluated sector. Although these projects will be chosen at shadow prices, they will later operate and maximize profits

or their administrative objectives at market prices.

As a result of all the foregoing, the projects in the evaluated sector are truly marginal to the economy's operation as a whole and indeed are likely to be fairly marginal to the aggregate investment of the country. The costs of factors and the value of products in the evaluated sector can then be taken as the marginal costs and marginal benefits to the rest of the economy of either giving up the factors or having the products. In turn, this implies that market prices, such as they are, determine in large measure the shadow prices of the factors and products for the evaluated sector. At the same time, market prices in the economy as a whole reflect government policy which is thus reflected directly in the shadow prices used in the evaluated sector.

Macroeconomic policies are assumed to be set, and quite realistically, on considerations not directly related to project choice in the evaluated sector. Thus, the exchange rate is fixed on considerations relating to the general behavior of the balance of payments, in particular the cost of producing in traditional export industries and adjusted to the rate of domestic price inflation. Free trade is not adopted; import duties and export taxes, if any, are maintained although the structure of such import duties and export taxes may change over time. Furthermore, the regulations on active and passive interest rates of the banking system are set at traditionally acceptable rates; labor policy is determined on humanistic and social considerations relating to the minimum wage, job security, social security, etc. Finally, the structure of the tax system as well as the levels of its various rates respond to revenue needs rather than being designed specifically as an

incentive structure with optimal allocation properties.

The lack of responsiveness of the general policy measures to their implications for investment choice makes the shadow prices which should be used in the evaluative sector very explicitly second best shadow prices. As a result, a set of second best shadow prices will exist for each set of market prices in the economy as a whole. In turn, each set of macropolicies will determine a set of market prices; thus, for each set of macropolicies there will correspond a set of shadow prices. Naturally, if the macroeconomic policies became optimal and market prices therefore became optimal as well, the shadow prices to be used in the evaluated sector will be identical to these market prices. Such a first best situation is assumed not to occur within the planning horizon. This paper thus addresses itself specifically to the calculation of second best shadow prices.

II

The Second Best General Disequilibrium Shadow Price of Foreign Exchange

The value of a unit of foreign exchange, call it a dollar, depends fundamentally on the way in which an additional dollar is absorbed or released by the economy as a whole, since the evaluated sector essentially buys from or sells to the rest of the economy the foreign exchange it needs or produces.

The economy can absorb an additional dollar[1] in one of four manners: (a) it can increase the reserves of foreign exchange; (b) it

[1]Except in unusual cases the release of a dollar will be precisely the negative of an absorption of a dollar.

can increase its net capital outflow or decrease its net capital inflow; (c) it can increase imports; and (d) it can reduce exports. In the presence of non-optimal macroeconomic policies and market distortions, the social marginal utility of a dollar in each of these uses will be different and thus the particular way in which the economy distributes marginal foreign exchange among these uses is critical to the value of a unit of additional foreign exchange availability.

Absorption of additional project-generated foreign exchange in reserves is a rather rare situation since usually reserve levels are determined with regard to macroeconomic parameters and implemented by macroeconomic policy. It could well happen, however, that for a certain period a country will be systematically accumulating part of its new foreign exchange receipts arising from projects. This is usually such a short-run situation that it seems appropriate to disregard it in the project analysis context.

The use of additional foreign exchange to affect the net capital flow is a much more common situation. What is involved is essentially a decision on the part of the economy to use some of its newly available foreign exchange to import financial paper, i.e. to make a foreign investment. This is so whether the explicit form is that of the purchase of financial obligations issued by others, i.e. the case of a conventional capital outflow, or when the adjustment takes the form of a reduction in net capital inflow, i.e. an increase in the reservation demand for own indebtedness. It is possible therefore to see this use of foreign exchange as a special case of the import of a capital good yielding a private product equal to the interest rate earned or the

interest payment saved. Empirically, however, the relationship at the margin between this use of foreign exchange and the incremental foreign exchange availability is highly unstable. In consequence, we shall exclude it from the remainder of the analysis.

The third form of absorbing foreign exchange, i.e. an expansion of imports, is the most common and most important one. In the presence of a greater availability of foreign exchange, imports can rise as a result of one of two mechanisms: (i) the relative price of imports can fall, thus increasing the purchase of importables; and (ii) the domestic aggregate demand can rise thus leading to an outward shift of the demand curve for imports with the consequent increase in the quantity imported at constant prices. In the Latin American context where tariff reductions are rare and balances of payments are managed to a large extent through adjustments in the aggregate levels of demand, the second mechanism is the more important one, with the income elasticities determining the marginal import mix.

The absorption of additional foreign exchange through a reduction of exports can be the result of a reduction in the production of exportables, an increase in the consumption of exportables or both. A reduction of exports arising from a fall in production requires a lowering in the relative price of export goods with regard to other goods. This in turn implies a revaluation of the exchange rate as a result of additional foreign exchange availability. We regard it as an empirical reality in Latin America, however, that exchange rates are set basically with reference to the cost of export production in existing export industries. Thus such a revaluation of the exchange rate can be ruled

out and no change in the production of exportables will arise. In consequence, an increase in the availability of foreign exchange can only be absorbed by a reduction in exports at a constant real exchange rate if there is domestic consumption of exportables and such consumption has an income elasticity greater than zero. In such a situation, the domestic absorption of exportables will increase pari passu with the absorption of imports arising from an increase in aggregate demand. It should be noted, however, that the majority of Latin American countries do not have significant home demands for their major export products at this time, although this may well change as industrial exports become increasingly important in the export bill.

In summary, then, we conclude that in practice additional foreign exchange is absorbed through an expansion of imports. A marginal dollar is therefore worth as much as the marginal imports on which it is spent. The marginal import mix, in turn, will be a function of market prices, income and the respective elasticities.

In order to derive a summary expression for the value of a marginal dollar, it is useful to disaggregate the marginal expenditure of foreign exchange into four categories: (i) import of consumption goods, (ii) import of intermediate goods, (iii) import of capital goods. The value of a marginal dollar will then be equal to the weighted sum of a marginal dollar's worth of imports of each of these three categories. In addition, however, it is necessary to take into account that additional imports imply additional resources for the government in the nature of additional tax collections. If the fiscal peso has a value different

from the private peso, there is an additional benefit or cost arising from the transfer to the treasury caused by the availability of an additional dollar. In symbols:

$$P\$_t = R_t \alpha_t \qquad (1)$$

where

$P\$$ = shadow price of foreign exchange

R = financial exchange rate

α = premium of shadow rate

t = time subscript (set equal to 0, i.e. the present period, unless otherwise mentioned)

$$\alpha = m_c Pm_c + m_1 Pm_1 + m_K Pm_K + t_m PG \qquad (2)$$

where m_c is the marginal share of expenditure on consumption imports from total foreign exchange expenditure.

m_I is the marginal share for imports of intermediate goods.

m_K is the marginal share for capital goods.

These marginal shares are to be defined in foreign exchange terms. The corresponding P_m's measure the marginal social utility of dollars spent on each of these categories and in turn reflect the present value of the consumption equivalent of these imports.

t_m is the average incidence of tariffs and other taxes on imports per dollar of additional foreign exchange expenditure and PG is the excess of the social marginal utility of a fiscal peso over the social marginal utility of a private peso, i.e. the net gain in welfare arising from the shift of a peso from the private to the public sector.

The price of imported consumption goods, Pm_c, is equal to the price the consumer pays for a dollar's worth of imported consumption goods of a composition determined by the marginal import propensities of consumption goods, less the real cost to the economy of bringing such goods from the port to his hands. Under normal importing and marketing conditions, the price to the consumer would be formed from the elements shown in equation (3):

$$Pm_c^{cons} = \text{CIF} + \text{duties} + \text{port charges} + \text{transportation costs} + \text{wholesaler's margin} + \text{retailer's margin} + \text{sales taxes.} \quad (3)$$

If port charges, transportation costs and wholesaler's and retailer's margins are regarded as real costs, and if sales taxes are distributed between the various elements comprising the joint cost of putting the imported consumer goods in the hands of the consumer, the value of a dollar's worth of imported consumer's goods will be as in equation (4):

$$\begin{aligned} Pm_c &= \text{CIF} + \text{duties} + \text{pro-rata of sales tax} \quad (4) \\ &= 1 + tm_c + (1 + tm_c)s_c \\ &= (1 + tm_c)(1 + s_c) \end{aligned}$$

where tm_c = weighted average of tariffs on consumer goods imports, i.e. ex-post tariff incidence = $\Sigma t_j m_j^c$, where m_j^c are marginal shares of individual consumer import goods in total imports of consumer goods.

The social marginal utility of intermediate imports results from two elements: (i) the social marginal product of such imported intermediate goods and (ii) the social marginal product of previously unemployed domestic factors of production complementary to imports which become employed as a result of the economy's expanded level of activity pursuant to the aggregate demand adjustment necessary to absorb the imports. Equation (5) shows these two elements in symbolic form:

$$Pm_I = (1+tm_I) + \frac{dGNP}{dM_I}\left(1-\frac{dL^e}{dGNP} - \frac{dL^n}{dGNP}\ \frac{PL}{w} - \frac{dtmI}{dGNP}\right) \qquad (5)$$

The first term in this equation shows the private demand price for intermediate imports which is a partial measure of the marginal productivity of such imports. The second term measures the value of the increase in overall output arising from the adjustment in aggregate demand. Notice that from the increase in GNP due to the availability of more imports are deducted the value of wages of previously employed labor $\left(\frac{dL^e}{dGNP}\right)$, as well as that part of the wage bill of unemployed labor $\left(\frac{dL^n}{dGNP}\right)$ which represents a real cost $\left(\frac{PL}{w}\right)$ and the tariffs on intermediate good imports which are already counted in the first term. The second term of equation (5) could also have been built up from the cost side by simply adding together the excess of wages over marginal social cost of labor for unemployed labor, the value of profits, depreciation, interest and taxes other than on intermediate inputs which would arise out of the higher level of activity. The underlying and very plausible assumption here is that all factors other than skilled labor were under- or unemployed.

The GNP increase measured by the second term of equation (5) will accrue in part to government, thus an adjustment for this fiscal effect is needed. The base for the new tax revenue will be the increase in money income, thus new tax revenue will be:

$$\frac{dGNP}{dM_I}\left(1 - \frac{dL^e}{dGNP}\right) t_d \qquad (5.1)$$

where t_d is the marginal tax ratio of all domestic taxation on income (revenue from import duties is excluded).

Incorporating this fiscal effect in (5) yields (6):

$$Pm_I = (1+tm_I) + \frac{dGNP}{dM_I}\left[(1 - \frac{dL^e}{dGNP})(1+t_d PG) - \frac{dL^n}{dGNP}\frac{PL}{w} - \frac{dtm_I}{dGNP}\right] \qquad (6)$$

Inspection of the second term of equation (6) will show that it measures the welfare increase of additional availability of intermediate imports in terms of income and not consumption equivalents. A further adjustment to consumption equivalents is therefore indicated. This adjustment is incorporated into equation (7):

$$Pm_I = (1+tm_I) + \frac{dGNP}{dM_I}\left[(1- \frac{dL^e}{dGNP})\ (c+sPK+t_d PG) - \frac{dL^n}{dGNP}\frac{PL}{w} - \frac{dtm_I}{dGNP}\right] \qquad (7)$$

where c = marginal propensity to consume

s = marginal propensity to save

$c+s \equiv 1$

PK = shadow price of investment

It is important to note that the new money income generated is equivalent to the increase in GNP + minus the wage bill of previously employed workers. It is this amount that must be distributed between consumption and savings. The deduction for the leisure of previously unemployed workers which is measured by $\frac{dL^e}{dGNP} \cdot \frac{PL}{w}$ does not signify a reduction in new money income and thus does not affect the new consumption and savings, but only represents a deduction in welfare terms from the new availability of goods and services.[1]

The use of equation (6) or equation (7) in project evaluation depends on the manner in which the reinvestment and fiscal effects are taken into account. If a special calculation of the reinvestment implication is made, then equation (6) is appropriate and the use of (7) would imply double counting. On the other hand, if no special reinvestment or fiscal calculation is made, then the reinvestment and fiscal effects must be built into each of the factor and output prices and the use of (7) becomes imperative as does the inclusion of reinvestment and fiscal effects into the shadow price of labor. In this analysis, it will be assumed from now on that a specific calculation of the reinvestment effects is undertaken and thus we will use expression (6) in all further developments.

The price of a dollar's worth of imported capital goods is given by the present value of the future consumption stream arising from its

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[1]Note that equation (6) incorporates the fiscal effect of the current year, whereas PK in (7) incorporates the fiscal effect in future years. Thus no double counting occurs.

importation. This present value can be derived from the private price of such capital good imports and the social value of the private stream of benefits resulting, i.e. from the shadow price of capital. In Formula (8) the first term converts the CIF price of imported capital goods to the landed equivalent, i.e. the private purchase price of such goods. The second term is the shadow price of investment which is by definition equivalent to the present value at shadow prices of the future income stream generated by one peso of investment at market prices.

$$Pm_K = (1 + tm_K)P_K \tag{8}$$

Bringing together the various components of the shadow price of foreign exchange we obtain equation (9):

$$\begin{aligned} \alpha = {} & m_c(1 + tm_c) + m_c s_c && \text{consumption imports} \\ & + m_I(1 + tm_I) + m_I \frac{dGNP}{dM_I} \left[(1-\frac{dL^e}{dGNP})(1+t_d PG) - \frac{dL^n}{dGNP}\frac{PL}{w} - \frac{dtm_I}{dGNP}\right] && \text{intermediate imports} \\ & + m_k(1 + tm_K) P_K && \text{capital imports} \\ & + t_m PG && \text{fiscal transfer} \end{aligned} \tag{9}$$

Inserting the definition of the marginal share of intermediate imports in total imports (10) into (6) we obtain equation (11) for the value of intermediate imports:

$$\frac{dGNP}{dM_I} = \frac{dGNP}{dM}\quad\frac{dM}{dM_I} = \frac{dGNP}{dM}\quad\frac{1}{M_I} \tag{10}$$

$$m_I Pm_I = m_I\,(1+tm\,) + \frac{dGNP}{dM}\left[(1 - \frac{dL^e}{dGNP})(1+t_d PG) - \frac{dL^n}{dGNP}\frac{PL}{w} - \frac{dtm_I}{dGNP}\right] \tag{11}$$

Substituting (11) in (9) and summing up we obtain as the shadow price of foreign exchange:

$$\alpha = \sum_i m_i(1+t_m) + m_c s_c + \frac{dGNP}{dM}\left[(1-\frac{dL^e}{dGNP})(1+t_d PG) - \frac{dL^n}{dGNP}\frac{PL}{w} - \frac{dtm_I}{dGNP}\right]$$

$$+ m_k(1+tm_k)(PK-1) + \sum_i m_i t_i PG \qquad (12)$$

$$P\$ = R_o + R_o\left[\sum m\ tm + m_c s_c + \frac{dGNP}{dM}\left[(1-\frac{dL^e}{dGNP})(1+t_d PG) - \frac{dL^n}{dGNP}\frac{PL}{w} - \frac{dtm_I}{dGNP}\right]\right.$$

$$\left. + m_k(1+tm_k)(Pk-1) + \sum m\ tmPG\right] \qquad (13)$$

The corrections to be made to the market price of foreign exchange in order to derive the shadow price can now be interpreted as follows: the first term in the bracket corresponds to the adjustment for the difference between the private demand price and the CIF price of imports; the second term corrects for the difference between demand and supply prices induced by sales taxes on final consumption goods; the third term adjusts for the macroeconomic impact of the additional availability of intermediate goods as well as all the taxes attached to additional output; the fourth term corrects for the difference between the private demand price and supply price of imported capital goods as well as for the additional value of investment over consumption in the private sector, and; the fifth term takes into account the additional value of the transfer of fiscal resources from the private to the public sector.

III

Numerical Estimation of the Second Best General Disequilbrium Shadow Price of Foreign Exchange

Estimation of the shadow price of foreign exchange requires deriving estimates for each of the elements in equation (13). These can be surveyed one by one:

(1) $\Sigma m_i t_i$

This number represents the marginal incidence of import duties as a proportion of imports. Two major estimation methods can be used:

(i) $\frac{\Delta \text{ Tariff revenue}}{\Delta \text{ Imports}}$ measured for discrete periods of six months or a year and taken for two to three different periods and then averaged; or

(ii) performing a regression analysis of the form $T = a + bM$ where T = tariff revenue and M = total imports

It should be noticed that in the second method distortions will be introduced if rates have been changing during the period for which the estimated data is obtained. The problem encountered here is a traditional one in measuring the elasticity of the tax system: with historical data only the ex-post elasticity can be derived and it becomes very difficult to separate the effect of the change in the rates from the effect of the change in the base. It would therefore seem desirable to undertake the estimation by both methods and to compare the results. In addition, it could also be useful to calculate for comparison the average tariff incidence for some recent years

although the additional calculation effort involved in taking first differences rather than absolute levels is so small as to make the additional accuracy of the marginal calculation very inexpensive.

(2) $m_c s_c$:

This item represents the average incidence of indirect taxes (e.g. sales taxes) on imported consumer goods weighted by the importance of such imported consumer goods in total imports. It is unlikely that data will be available in a form which allows estimation of the product involved. It will therefore become necessary to estimate each element separately.

"m_c" can be estimated either by taking $-\frac{\Delta M_c}{\Delta M}$ or by running a regression of the form: $M_c = a + bM$. This regression is not directly subject to the kind of bias that affects the tax regression discussed under point (1). Nonetheless, changes in relative prices arising out of tariff changes would influence the regression coefficients and would make them less applicable for current estimates or projections the more changes in relative prices there have been. From this point of view, once again the separate formulation of first differences for a recent number of years or six-month periods seem preferable.

For s_c two alternative estimation procedures are available: (i) use the tax rates as legislated and add onto them an estimate of the monopoly or monopsony/margins; and (ii) analyze the marketing process of imports in order to divide the ratio of domestic to CIF price into its various components: taxes, port charges, transportation, etc., etc.

(3) dGNP/dM:

Here again the ratio of first differences for a number of recent periods or a regression of the form GNP = a + bM could be employed. Once again, as well, the structural transformation of the economy is likely to make the regression analysis of rather dubious value, unless the process of structural transformation is a smooth one and could be expected to continue as it has in the past.

(4) $dL^e/dGNP$, $dL^n/dGNP$:

Both these parameters can be estimated by the same methods as the previous one. It will probably be convenient to define employed and unemployed labor force according to skill, with the skilled labor force presumed employed and the unskilled one presumed unemployed at the margin. This assumption as well as the number of labor categories used can be refined in accordance with the availability of data.

The computation of the shadow price of labor will be discussed later, and its divisor the market wage rate for unskilled labor can be determined either from labor market information or from the average payment to existing unskilled labor.

(5) $dtm_I/dGNP$:

This measure can be decomposed into the marginal rate of import taxes on intermediate goods and the marginal increase of intermediate good imports per unit of GNP. Both the total ratio and each of the component ratios can be estimated by the first difference or the regression method. It is advisable to use all six versions of estimating this ratio and compare the results for stability.

(6) If equation (6) is to be estimated, the coefficients c and s, are necessary as well. These can be estimated by the first difference method or by the regression method. In using the latter, however, it is necessary to take due account of the simultaneity. Thus if we estimate

$$C = c_o + cGNP$$

$$S = s_o + sGNP$$

then we must recall the constraint $c + s \equiv 1$. Only if the regression algorithm builds in this constraint will we get consistent estimates.

$$(7) \quad m_k (1+tm_k) = m_k + m_k tm_k = \frac{\Delta M_k}{\Delta M} + \frac{\Delta T_k}{\Delta M}$$

where T_k is tariff revenue from the import of capital goods.

Estimation can proceed on the basis of the left-most or the right-most expression. In the former case m_k and tm_k are estimated separately by either the first difference method or the regression method. In the latter alternative, the direct estimation of $m_k tm_k$ is undertaken either with the first difference method or by specifying a regression of the form $T_k = a + bM$.

(8) PK, PG

The estimation of these shadow prices is discussed in later sections.

Section IV

The Rate of Discount

It will be recalled that the basic measure of benefits of the project is the stream of consumption over time which the project originates. Similarly, the cost of the project is taken to be the stream of consumption foregone through withdrawing for use in the project investment resources and current factors of production which would otherwise have been used elsewhere. Since the streams of consumption equivalent to the benefits and to the costs occur over time, a discount rate is needed which will make consumption in different periods commensurable.

Since it has been assumed that the savings and investment market is not in equilibrium, the marginal rate of transformation of consumption in one period into consumption in another period will differ from the marginal rate of substitution between consumption in different periods as viewed by the typical consumer. The marginal rate of transformation of consumption in one period into consumption of another period or set of periods is given by the various projects in which the investment resources can be used. Thus project evaluation is precisely the choice between different points on the marginal rate of transformation curve by evaluating the differing consumption streams resulting. Since consumption is taken as the measure of benefit because it is a measure of consumer welfare, it follows that consumption in different periods must be made commensurable on the basis of an adjustment reflecting the utility trade-off between

consumption in different periods as seen by the consumers themselves. This implies adopting a discount rate based on the marginal rate of substitution of consumption in different periods.

The most direct way of measuring such a rate is to observe the behavior of savers. If it is known at what rate people will save and it is further assumed that they are rational in their behavior, savers will adjust the quantities consumed and saved in such a way as to equate their marginal rate of substitution to the market interest rate facing them. Observation of the market interest rate, even if this is a regulated rate and not equal to the marginal rate of return on investment, will yield an adquate measure of the marginal rate of substitution of savers. With a fractioned market, it would still be true that the savers in each market would equate their marginal rate of substitution of consumption between different periods to the particular rate facing them. Unfortunately, the derivation of an aggregate discount rate from such observations is well nigh impossible for several reasons:

(a) A substantial part of savings goes directly from the saver into investment in the saver's own enterprise, without passing through any formal money market. As a result, no interest rate is directly observable. This means, however, that the interest rate to which a substantial segment of savers equate their marginal rate of substitution across time is not observable.

(b) A good many would-be savers have no access to savings instruments yielding a positive interest rate, due to the underdevelopment

of the financial institutions. As a result, no interest rate or savings are observed for them. Nonetheless, these would-be savers have a marginal rate of substitution across time, albeit unobservable.

(c) Even if all rates were observable, it would be necessary to construct a weighted average. If these weights are derived from information on quantities actually saved, the resultant marginal rate of substitution would reflect the distribution of savings. This distribution is, however, considerably more skewed than the distribution of income, and the latter is usually regarded as more unequal than desirable. In addition, however, weighting by the contribution to savings would leave out entirely the preference of all those individuals in the society who are would-be savers but do not save because they are confronted by a zero or negative interest rate.

Thus both on grounds of empirical implementability as well as in view of the distribution that is implicit in it, an average of market interest rates is a very unsatisfactory way of deriving the aggregate marginal rate of substitution across time.

A preferable and more feasible approach starts from the consumption side. If it is borne in mind that the decision to consume reflects precisely the same determinants as the decision to save, it becomes obvious that the information content of observations on either should be the same. Nonetheless, the determination of the marginal rate of substitution across time from the consumption side has several advantages:

(a) Information on consumption covers all individuals, whether they save or not. Thus, even those would-be savers who are not actual

savers because they are confronted with zero or negative interest rates are included in the observations.

(b) Aggregate information on consumption incorporates the distribution of consumption which is substantially more equal than the distribution of income, precisely because the distribution of savings is more skewed than the distribution of income. In circumstances in which it is regarded as desirable to improve the equality of the income distribution, the use of data which includes the more equal distribution of consumption is clearly advantageous.

The relationship between the consumption decision and the marginal rate of substitution across time can be formalized by making explicit the determinants of the relative value of consumption in different periods, i.e. the time preference of consumers. Following Boehm-Bawerk,[1] the two elements determining time preference can be defined as: (1) the inherent discount arising from futurity, or what Frisch[2] has called "prospective shortening" and which others call pure time preference; and, (2) the effect of higher income in the future which makes all goods available later relatively less valuable than goods available earlier.

Following Eckstein,[3] these two factors can be used to derive an aggregate marginal rate of substitution as follows:

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[1] Eugene V. Boehm-Bawerk, Positive Theory of Capital.

[2] R. Frisch, "Dynamic Utility," Econometrica, July 1964.

[3] Otto Eckstein, "Investment Criteria for Economic Development and the Theory of Intertemporal Welfare Economics," QJE, Febuary 1957.

Define:

W = social welfare

C = aggregate consumption

p = population

ϵ = elasticity of the marginal utility of per capita consumption.

Using a constant elasticity function, the marginal utility of consumption in period t can be written as

$$\frac{dW_t}{dC_t} = \left(\frac{C_t}{p_t}\right)^{-\epsilon} \tag{14}$$

Defining the rate of pure time preference as d, we have

$$\frac{dW_o}{dC_t} = \left(\frac{C_t}{p_t}\right)^{-\epsilon} \frac{1}{(1+d)^t} \tag{14.1}$$

If the growth of population is written as δ and the growth of consumption as g, then

$$\frac{dW_o}{dC_t} = \left(\frac{1+g}{1+\delta}\right)^{-\epsilon t} \left(\frac{C_o}{p_o}\right) (1+d)^{-t} \tag{14.2}$$

$$\frac{\dfrac{dW_o}{dC_t}}{\dfrac{dW_o}{dC_{t+1}}} = \left(\frac{1+g}{1+\delta}\right)^{\epsilon} (1+d) \tag{14.3}$$

The left hand side of (14.3) is the ratio of marginal utilities of consumption in different periods, i.e. their marginal rate of substitution. Thus the social time preference is:

$$STP = \left(\frac{1+g}{1+\delta}\right)^{\epsilon} (1+d)^{-1} - 1 \tag{14.4}$$

Whereas the foregoing analysis is couched in terms of the representative consumer, this formulation has been generalized by Feldstein[1] to hold for any function given a constant distribution of income. This more general function is as follows:

$$STP = (1+\delta)^{1-\phi}\left(\frac{1+g}{1+\delta}\right)^{-\epsilon}(1+d) - 1 \qquad (15)$$

where δ, g, ϵ, d are as before and ϕ represents the exogenous effect of greater population (when $\phi = 1$ there is no effect).

It should be noted that of these parameters the rate of population growth, the rate of per capita growth of consumption as well as the elasticity of the marginal utility of income are directly estimable. The population parameter as well as the pure social time preference are not directly observable. These last two, however, are offsetting effects. Thus if, for example, ϕ has a value of .7, it will precisely offset a pure time preference 5 percent. One might thus with some justification ignore both effects and work merely with the remainder.

The elasticity of marginal utility of income can, under the assumption of an additional utility function, be broken down into parameters reflecting the price and income elasticities and the budget share of the demand for various goods as follows:[2]

$$\epsilon = \frac{E_i(1-a_iE_i)}{\epsilon_{ii}+a_iE_i} \qquad (16)$$

where E_i = income elasticity for good i

ϵ_{ii} = price elasticity for good i

a_i = budget share of good i

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[1] M. Feldstein, "The Derivation of Social Time Preference Rates," Kyklos, 1965, Fasc. 2.

[2] R. Frisch. "A Complete Scheme for Computing all Direct and Cross Demand Elasticities in a Model with Many Sectors." Econometrica, April 1959.

Section V

Numerical Estimation of the General Disequilibrium Rate of Discount

Numerical estimation most conveniently starts from formula (15) and the assumption that the exogenous population effect and the pure time preference cancel out. Then (15) reduces to

$$STP = \left(\frac{1+g}{1+\delta}\right)^{-\epsilon} - 1 \qquad (17)$$

and only estimate for the growth of aggregate consumption and population and for the elasticity of the marginal utility of consumption are needed.

(1) $\frac{1+g}{1+\delta}$

This ratio is the rate of growth of per capita consumption. It can be estimated directly from national accounts data or its component rates can be estimated separately. If it is expected that future rates will differ from past rates, then the historical growth rates must be adjusted accordingly before use in (17).

The historical rates can be estimated in two ways:

(i) a continuous growth rate can be assumed:

$$\frac{C_t}{P_t} = \frac{C_o}{P_o} \epsilon^{\left(\frac{1+g}{1+\delta}\right) t} \qquad (18)$$

The corresponding least squares estimating equation is a logarithmic transformation:

$$\log\left(\frac{C_t}{P_t}\right) = \log\left(\frac{C_o}{P_o}\right) + \left(\frac{1+g}{1+\delta}\right) t \qquad (18.1)$$

The regression coefficient of the second independent variable, time, will then be the estimated growth rate.

(ii) A discrete rate of growth can be assumed as an approximation to (18) and an average of annual percentage increases can be used.

(2) ϵ

The elasticity of the marginal utility of income can most simply be estimated by applying the Frisch formula cited above on previously estimated elasticity and budget share data. It should be noted, however, that use of independently estimated demand equations for the derivation of the elasticity of the marginal utility of income is very likely to yield a different estimate for each equation used. It would then become necessary to average the various estimates obtained. This might be an unweighted average or an average weighted by the share of consumption spent on each of the products.

Alternatively, a single estimate for the elasticity of the marginal utility of consumption can be obtained directly from consumption data by constraining the estimating equations to yield a single value for this parameter. Such a result can be obtained by using particular forms of the demand equations, such as Stone's linear expenditure system,[1] Barten's almost additive preference system,[2] or Powell's additive

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[1] R. Stone, "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand," Economic Journal, Sept. 1964.

[2] A. P. Barten, "Consumer Demand Functions under Conditions of Almost Additive Preferences," Econometrica, Jan./Apr. 1964.

preference ... system.[1] It is also possible to undertake direct non-linear estimation of double-logarithmic demand functions in which the univalued elasticity of the marginal utility of consumption parameter is substituted directly into the estimating equations. In this approach, one starts from the conventional double-logarithmic elasticity formulation:

$$\log C_i = a_o + E_i \log Y + \epsilon_{ii} \log P_i \tag{19}$$

The price elasticity is then replaced by its definitional equation (16) in terms of income elasticity, budget share, and elasticity of the marginal utility of income. Carrying out the substitution, the following estimating system is obtained:

$$\log C_i = a_o + E_i \log Y + \left(-E_i a_i + \frac{E_i - a_i t_i}{\epsilon}\right) \log P_i \tag{19.1}$$

$$\log C_i = a_o + E_i (\log Y - a_i \log P_i) \frac{E_i - a_i E_i^2}{\epsilon} \log P_i \tag{19.2}$$

The estimation of (19.2) by least squares is complicated because of the squared term in E_i, however an iterative procedure on ϵ can be used for single equation non-linear least squares in which the E_i are estimated for different assumed values of ϵ. That ϵ which yields the overall minimum residual sum of squares is then selected as the best estimate.

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[1] A. Powell, "A Complete System of Consumer Demand Equations for the Australian Economy Fitted by a Model of Additive Preferences," Econometrica, July 1966.

It should be noted that the quality of estimate could be improved if consumption data were available by income stratum. In this eventuality, it would be possible to estimate an elasticity of the marginal utility of income for each size stratum of income and then aggregate by the desired income distribution. It is probable that the resulting parameter would be different from the one obtained from the assumption of a representative consumer.

VI

The Shadow Price of Investment

The value of investable resources in the programmed sector is taken as the present value of the stream of consumption these same resources would have generated in the nonprogrammed sector. As a result, this value will depend on the use to which investment funds are put at the margin in the unprogrammed sector as well as on the consumption streams generated by each of the uses.

In order to derive the shadow price of investment, it is necessary to correct the private evaluation of such funds by the following elements:

(1) direct and indirect taxes. Indirect taxes are excluded from producer's revenue since they must be directly paid to the treasury. At the same time, the value of the output to the consumers includes the payment of all the indirect taxes; thus producer evaluation of the output understates its marginal utility. In consequence, correction from private to social value entails adding in all indirect taxation. Similarly, the rate of return of relevance to the private sector is after corporate taxes. For society as a whole, however, such taxes are transfer payments that may increase or decrease social welfare depending on the shadow price of fiscal resources. From the point of view of production, however, the productivity of investment clearly includes the product transferred to government through corporate taxes.

(2) protection of outputs and inputs. Private profitability calculations are made inclusive of the effect of import duties whether these produce increases in the product price (when they affect imports competing with the output) or whether they raise the cost of inputs. From society's point of view, goods which are import substituting or export goods should be valued at their foreign exchange value adjusted for the shadow price of foreign exchange, with tariffs regarded as transfer items.

(3) difference between shadow and market price of labor. Whereas private profitability calculations take the cost of labor at the market wage, from society's point of view the shadow price is appropriate. Thus, the difference between these two prices must be added to the private profitability to obtain the social rate of return.

(4) differences in the discount rate. Whereas the private sector will determine the present value of investment on the basis of a market determined interest rate, given the imperfections in the capital markets, society's rate will differ from the private rate. As a result, the annual private flow adjusted for the previous items must be discounted at the social rate.

(5) reinvestment. Given the divergence between private and social discount rates, the part of the project returns which are reinvested have a per unit consumption equivalent greater than 1. As a result, it is necessary to adjust the marginal social productivity of capital defined in income terms by the expected reinvestment in order to obtain the consumption equivalent.

In order to develop estimation formulae for the adjustments to be made to the private rate of return, it is useful to think of investment in the non-programmed sector as proceeding on a wide front in a substantially balanced fashion directed towards the production of two kinds of goods: non-traded goods and traded goods. The former are all those commodities produced for domestic use which do not substitute for existing imports. Traded goods, in turn, are exports and commodities produced for domestic use which were previously imported. Furthermore, it is useful to recall the basic input-output identity which states that the market value of any product is equal to the market cost of all direct and indirect inputs:

$$X_i \equiv \sum_j r_{ji} M_j (1+tm_j) + \sum_j r_{ji} L_j + \sum_j r_{ji} Int_j + \sum_j r_{ji} \pi_j \qquad (20)$$

X_i = output at producer prices of sector i

M_j = import requirement at CIF prices per unit of output of j

L_j = labor requirement per unit of output j

π_j = profit before corp. taxes and depreciation per unit of output of j

tm_j = tariffs on j

r_{ji} = direct and indirect input requirement of commodity j per unit of i.

Making use of this identity it is possible to write the private profits generated in the nontraded goods sector as follows:

$$\pi^{NT} = X^{NT} - \Sigma r_{jNT} M_j (1+tm_j) - \Sigma r_{jNT} L_j - \Sigma r_{jNT} Int_j \qquad (21)$$

where NT = non-traded.

In turn, social profits will result from the difference between the value of output at user prices (i.e. including indirect taxes) and input costs at shadow prices. Furthermore, the profit on own and borrowed capital should be taken together and thus interest payments should be summed to profits. The emerging expression is as follows:

$$\text{Soc } \pi^{NT} = (1+ts)X^{NT} - \Sigma r_{jNT}M_j \frac{P\$}{R} - \Sigma r_{jNT}L_j \frac{PL}{w} \qquad (22)$$

where ts = indirect taxes

Note that the value of output is now stated at buyers' prices, i.e. including indirect taxation, whereas imported inputs are valued at the shadow price of foreign exchange and labor is valued at its shadow price. Inserting (21) in (22) and making use of the basic definitional equation of the shadow price of foreign exchange,
(1) $P\$ = R\alpha$ we obtain:

$$\text{Soc } \pi^{NT} = \pi^{NT} + tsX^{NT} + \Sigma r_{jNT}M_j(tm_j - \alpha + 1) + \Sigma r_{jNT}L\left(1 - \frac{PL}{w}\right) \qquad (23)$$

A similar derivation can be undertaken for the determination of annual social profits from traded goods. Equation (24) and (25) define the private and social profits in traded good production.

$$\pi^T = X^T - \Sigma r_{jT}M_j(1+tm_j) - \Sigma r_{jT}L_j - \Sigma r_{jT}Int_j \qquad (24)$$

$$\text{Soc } \pi^T = X^T \frac{P\$}{R(1+t_T)} - \Sigma r_{jT}M_j \frac{P\$}{R} - \Sigma r_{jT}L_j \frac{PL}{w} \qquad (25)$$

where t_T is the average nominal protection on traded output.

Replacing (24) in (25) and making use of the basic shadow price of foreign exchange definitional equation, we obtain:

$$Soc\ \pi^T = \pi^T + X^T\left(\frac{\alpha}{1+t_T} - 1\right) - \sum r_{jT}M_j(\alpha - 1 - tm_j) + \Sigma r_{jT}L_j\left(1 - \frac{PL}{w}\right) + \Sigma r_{jT}Int_j \quad (26)$$

Aggregating both kinds of social profits we obtain an expression for annual social profit from traded and nontraded production in terms of the respective private profits and the corresponding adjustments:

$$Soc\pi = Soc\pi^{NT} + Soc\ \pi^T = \pi^{NT} + \pi^T + tsX^{NT} + X^T\left(\frac{\alpha}{1+t_T} - 1\right) + \quad (27)$$

$$\Sigma r_{jNT}M_j(tm_j - \alpha + 1) + \Sigma r_{jT}M_j(tm_j - \alpha + 1) + \Sigma r_{jNT}L_j\left(1 - \frac{PL}{w}\right) + \Sigma r_{jT}L_j\left(1 - \frac{PL}{w}\right)$$

$$+ \Sigma r_{jNT}Int_j + \Sigma r_{jT}Int_j$$

However,

$$\sum_j r_{jNT}M_j + \sum_j r_{jT}M_j = \sum_{NT}^{I} \sum_j r_{ji}M_j = \sum_i M_i = M \quad (28)$$

$$\sum_j r_{jNT}M_j(tm_j - \alpha + 1) + \sum_j r_{jT}M_j(tm_j - \alpha + 1) = \sum_i M_i tm_i + M(1 - \alpha) \quad (28.1)$$

$$\sum_j r_{jNT}L_j + \sum_j r_{jT}L_j = \sum_i L_i = L \quad (28.2)$$

$$\sum_j r_{jNT}Int_j + \sum_j r_{jT}Int_j = \sum_i Int_i = Int \quad (28.3)$$

$$Soc\pi = \pi + tsX^{NT} + X^T\left(\frac{\alpha}{1+t_T} - 1\right) - \left[M(\alpha - 1) - \sum_i M_i t_i\right] \quad (29)$$

$$+ L\left(1 - \frac{PL}{W}\right) + Int.$$

Equation (29) says that annual social profits are equal to private profits plus indirect taxation on the output of nontraded commodities plus the excess in the value of shadow priced traded output over market priced traded output minus the excess of the value of imports at shadow over market prices plus the excess of the wage bill at market over shadow prices and plus expenditure on interest.

Adjustment for the fiscal effects must now be included. Corporate profit tax will be paid on private profits and such transfer payments will generate additional welfare according to the shadow price of fiscal resources; indirect taxes on non-traded goods will have a like effect; taxes on labor income as well as interest income must be taken into account as well. Since the shadow price of foreign exchange already includes the fiscal effect, arising from charges in foreign exchange availability, no further adjustment is needed for the revaluation of the output of traded commodities or the use of imports. The following equation incorporates these adjustments.

$$\text{Soc } \pi = \pi(1+tx_{PG}) + ts X^{NT}(1+P_G) + X^{T}\left(\frac{\alpha}{1+t_T} - 1\right) - \left[M(\alpha-1) - \sum_i M_i tm_i\right] \qquad (30)$$

$$+ L\left(1+tx_w P_G - \frac{P_L}{w}\right) + Int(1+tx_i P_G)$$

where tx = corporate profits tax

tx_w = income tax on wages

tx_i = income tax on interest earnings

We now convert expression (30) into a multiplicative adjustment expressing the annual social rate of return as a proportion of the annual private rate of return as follows:

$$\frac{\text{Soc } \pi}{\text{private I}} = \frac{\text{Soc } \pi}{\pi} \cdot \frac{\pi}{\text{priv. I}} = \frac{\text{Soc } \pi}{\pi} \cdot r \qquad (31)$$

Define

$$\beta = \frac{\text{Soc } \pi}{\pi}$$

hence $\frac{\text{soc } \pi}{\text{private I}} = \beta r$

$$\beta = 1+txPG + \frac{X^{NT}}{\pi} (ts+tsPG) + \frac{X^{T}}{\pi} \left(\frac{\alpha}{1+t_T} - 1\right) - \left[\frac{M}{\pi} (\alpha - 1) - \frac{\Sigma_i M_i tm_i}{\pi}\right] \qquad (32)$$

$$+ \frac{L}{\pi} \left(1+tx_w PG - \frac{PL}{w}\right) + \frac{Int}{\pi} (1+tx_i PG)$$

We can now define present value of a peso of private investment in income terms, i.e. excluding the readjustment of consumption arising from reinvestment. Making use of the social time preference, and assuming the private rate of return before taxes, r is constant over time whereas the adjustments to social profits are different every year, we have:

$$PK(Y) = \sum_t (1+STP)^{-t} r\beta_t$$

$$= \sum_t (1+STP)^{-t} r \left\{1 + txPG_t + \frac{X^{NT}}{\pi} (ts+tsPG_t) + \frac{X^{T}}{\pi} \left(\frac{\alpha_t}{1+t_T} - 1\right)\right.$$

$$- \left[\frac{M}{\pi} (\alpha_t - 1) - \frac{\Sigma M_i tm_i}{\pi}\right] + \frac{L}{\pi} \left(1+tx_w PG_t - \frac{PL}{w}\right) \qquad (33)$$

$$\left. + \frac{Int}{\pi} (1+tx_i PG_t)\right\}$$

Equation (33) assumes that all tax ratios and profit intensities are constant over time and the only changes occur in the shadow prices

of government resources and foreign exchange. Naturally, a more general formulation with the other elements varying as well is equally conceivable. Assuming the shadow prices of government and foreign exchange approximate the market equivalents at a steady rate over time, it is possible to simplify this expression considerably.

$$\alpha_t = \alpha_o (1+\omega)^{-t}$$

$$PG_t = PG_o (1+\omega)^{-t} \tag{34}$$

$$PK(Y) = r \sum_t (1+STP)^{-t} \left\{ 1 + \frac{X^{NT}}{\pi} (ts) - \frac{X^T}{\pi} + \frac{M}{\pi} + \frac{\Sigma M_i tm_i}{\pi} \right. \tag{35}$$

$$\left. + \frac{L}{\pi} \left(1 - \frac{PL}{w}\right) + \frac{Int}{\pi} \right\} + \sum_t (1+STP)^{-t} \left\{ txPG_o (1+\lambda)^{-t} \right.$$

$$+ \frac{X^{NT}}{\pi} tsPG_o (1+\omega)^{-t} + \frac{X^T}{\pi(1+t_T)} \alpha_o (1+\omega)^{-t} \cdot \frac{M}{\pi} \alpha_o (1+\omega)^{-t}$$

$$\left. + \frac{L}{\pi} tx_w PG_o (1+\omega)^{-t} + \frac{Int}{\pi} tx_i PG_o (1+\omega)^{-t} \right\}$$

$$PK(Y) = r \sum_t (1+STP)^{-t} \left\{ 1 + \frac{X^{NT}}{\pi} (ts) - \frac{X^T}{\pi} + \frac{M}{\pi} \right. \tag{35.1}$$

$$\left. + \frac{\Sigma M_i tm_i}{\pi} + \frac{L}{\pi} \left(1 - \frac{PL}{w}\right) + \frac{Int}{\pi} \right\}$$

$$+ r \sum_t (1+STP)^{-t} (1+\omega)^{-t} \left\{ PG_o \left(tx + \frac{X^{NT}}{\pi} ts + \frac{L}{\pi} tx_w \right. \right.$$

$$\left. \left. + \frac{Int}{\pi} tx_i \right) + \alpha_o \left(\frac{X^T}{\pi(1+t_T)} - \frac{M}{\pi} \right) \right\}$$

Making use of the usual perpetuity assumption (35.1) solves to the following:

$$PK(Y) = \frac{r}{STP}\left[1 + \frac{X^{NT}}{\pi}(ts) - \frac{X^{T}}{\pi} + \frac{M}{\pi} + \frac{\Sigma M_i tm_i}{\pi} + \frac{L}{\pi}\left(1 - \frac{PL}{w}\right) + \frac{Int}{\pi}\right] + \frac{r}{STP + \omega + \epsilon STP}\left[PG_o\left(tx + \frac{X^{NT}}{\pi} ts + \frac{L}{\pi} tx_w + \frac{Int}{\pi} tx_i\right) + \alpha_o\left(\frac{X^{T}}{\pi(1+t_T)} - \frac{M}{\pi}\right)\right] \tag{36}$$

$$PK(Y) = \frac{r}{STP}\left[1 + \frac{X}{\pi}\left[(1-d)ts - d\right] + \frac{M}{\pi} + \frac{M_i tm_i}{\pi} + \frac{L}{\pi}\left(1 - \frac{PL}{w}\right) + \frac{Int}{\pi}\right] + \frac{r}{STP + \omega + \epsilon STP}\left[PG_o\left(tx + ts(1-d)\frac{X}{\pi} + \frac{L}{\pi} tx_w + \frac{Int}{\pi} tx_i\right) + \alpha_o\left(\frac{d}{1+t_T}\frac{X}{\pi} - \frac{M}{\pi}\right)\right] \tag{36.1}$$

where $d = \frac{X^{T}}{X^{T}+X^{NT}} = \frac{X^{T}}{X}$

Adjusting for reinvestment can now be undertaken by segregating from the annual benefits the proportion s which gets reinvested and assigning it the value of its consumption equivalent PK.

If PK is assumed constant over time, the derivation is fairly simple. Starting from the income value of investment specified in (35.1), we find the following:

Define

$$A = 1 + \frac{X^{NT}}{\pi}(ts) - \frac{X^{T}}{\pi} + \frac{M}{\pi} + \frac{\Sigma M_i tm_i}{\pi} + \frac{L}{\pi}\left(1 - \frac{PL}{w}\right) + \frac{Int}{\pi} \qquad (37)$$

$$B = G_o\left(tx + \frac{X^{NT}}{\pi}\, ts + \frac{L}{\pi}\, tx_w + \frac{Int}{\pi}\, tx_i\right) + \alpha_o\left(\frac{X^{T}}{\pi(1+t_T)} - \frac{M}{\pi}\right)$$

then:

$$PK(Y) = r\sum_t (1+STP)^{-t} A + r\sum_t (1+STP)^{-t} (1+\omega)^{-t} B \qquad (37.1)$$

$$PK = r\sum_t (1+STP)^{-t} A(1-s) + r\sum_t (1+STP)^{-t} AsPK + \qquad (38)$$

$$r\sum_t (1+STP)^{-t}(1+\omega)^{-t} B(1-s) + r\sum_t (1+STP)^{-t} (1+\omega)^{-t} BsPK$$

$$PK\left[1 - r\sum_t (1+STP)^{-t} As - r\sum_t (1+STP)^{-t} (1+\omega)^{-t} Bs\right] = \qquad (39)$$

$$r\sum_t (1+STP)^{-t} A(1-s) + r\sum_t (1+STP)^{-t} B(1-s)(1+\omega)^{-t}$$

$$PK = \frac{r\sum_t (1+STP)^{-t} A(1-s) + r\sum_t (1+STP)^{-t}(1+\omega)^{-t} B(1-s)}{1 - r\sum_t (1+STP)^{-t} AS - r\sum_t (1+STP)^{-1}(1+\omega)^{-t} Bs} \qquad (40)$$

$$PK = \frac{r(1-s)\left[\frac{A}{STP} + \frac{B}{STP + \omega + \omega STP}\right]}{1 - rs\left[\frac{A}{STP} + \frac{B}{STP + \omega + \omega STP}\right]} \qquad (41)$$

It should be noted, however, that a PK constant over time is very unlikely in the presence of changes in the other shadow prices, particularly P$ and PL.

Section VII

Numerical Estimation of the Second Best General Disequilbrium Shadow Price of Investment

Estimation of the shadow price of investment requires deriving estimates of the various elements of equation (41). These are discussed in what follows:

(1) r

This number represents the rate of return to investment in the unprogrammed sector before corporate taxes. It can be derived from balance sheets or from national account data. If the former are used, it is important to take into account firms that invested and made losses or went out of business, as well as those that thrived. Furthermore, own capital stock of the firms analyzed must be supplemented by their borrowed capital since for our purposes total investment is a more appropriate base than equity for measuring the private rate of return. Finally, an adjustment for risk may be required, particularly if firms with losses are underrepresented.

(2) s

The marginal propensity to save has already been measured when constructing the estimate of the shadow price of foreign exchange. See Section III, point (6).

(3) STP

The estimation of the social time preference has been discussed at length in Section V.

(4) ω

This number is the annual rate at which the ratio between the shadow and market rates of foreign exchange (α) and the shadow price of fiscal funds (PG) decline to zero. It can be estimated by guesstimating the percentage by which the gap will have narrowed over a ten-year period and then converting the result to an annual rate with a present value table.

For example, if $\alpha_{10} = .7\,\alpha_o$

Then $\alpha_o(1+\omega)^{10} = .7\,\alpha_o$

$(1+\omega)^{10} = .7$

$\omega = .03$

(5) Elements in A and B:

(i) X^{NT}, X^T

The increase in gross output as measured in the national accounts is divided into three categories: (a) increase in export production, (b) new production substituting for items previously imported, and, (c) the rest. The sum of the first two items constitutes new production of traded goods and the third items constitutes the production of non-traded goods.

(ii) $\frac{1}{\pi}$ (X^{NT}, X^T, M, $\Sigma M_i tm_i$, L, Int)

The marginal ratios to profits of output, imports, tariff revenue, wage bill and interest payments can be ascertained either directly if suitable input-output data or national accounts data is available or

can be constructed as a product of two ratios, e.g.

$$\frac{\Delta M}{\Delta \pi} = \frac{\Delta M}{\Delta X} \cdot \frac{\Delta X}{\Delta \pi} \tag{42}$$

Estimation can be undertaken by the first difference method or the regression method. See Section III.

(iii) tx, ts, tx_w, tx_i

The tax rates are determined either from the legislated rates or from the ex-post observed marginal ratio of tax collections to the respective tax base. In the latter case, the first difference method is preferable to the regression method. See Section III, point (1)(ii).

(iv) PL, w

The shadow price of labor is taken as estimated according to Section VIII. The wage rate is taken as observed from national accounts or survey data. Notice that it may be desirable to disaggregate into skilled/unskilled or employed/unemployed categories.

Section VIII

The Second Best General Disequilibrium Shadow Price of Labor and Its Numerical Estimation

In the analysis of labor markets, it is customary to distinguish between the market for skilled labor and the market for unskilled labor. Furthermore, under the disequilibrium conditions of Latin America, it is useful to further distinguish between that segment of the labor market which is protected in the sense of entry being restricted and the wage rate in that market being maintained substantially higher than the wage rate in the rest of the economy, from an unprotected urban sector and from an unprotected rural sector.

If there is an increase in the demand for skilled labor, this increase will be satisfied by drawing on unskilled labor and training it. Thus there is a flow from the unskilled to the skilled sector of the labor market as demand in the latter increases. Similarly, if there is an increase in demand for labor in the protected sector, the unprotected urban sector gets drawn on as supplier. Furthermore, labor flows typically from the unprotected rural sector into the unprotected urban sector. If there is migrational balance between these various parts of the labor market, then any increase in demand produced in the protected sector will transmit itself through the free urban sector into the unprotected rural sector. Under the conditions existing in Latin America, however, no such migrational equilibrium seems to exist. Labor flows from the unprotected rural into the unprotected urban sector whether or not the expected income of the migrators exceeds their income in the rural sector. Migration is caused as much by

expulsion from the rural sector as attraction into the urban sector. As a consequence, a substantial amount of unemployment exists in the unprotected urban sector. Under these conditions, an increase in demand in the protected urban sector will transmit itself to the unprotected urban sector but will not be further transmitted into the rural sector.

In project evaluation, the shadow price of labor is used to cost an input of labor; thus we're always dealing with the opportunity cost of withdrawing labor from other activities and putting it to work in the project being evaluated. From this point of view, any urban project creating a demand for labor, whether it be situated in the protected or unprotected sector, will imply an increased demand for labor in the unprotected sector. The opportunity cost of drawing labor from that sector depends on the degree of unemployed urban labor existing. If unemployment in the urban unprotected sector is significant, the opportunity cost of using labor in urban projects is equal to the wage at which such unemployed laborers would be willing to accept employment. It should be noticed that this will hold whether or not the project is in the protected urban sector.

If the project is in the rural sector, then the relevant shadow price of labor is the marginal productivity of such labor in other rural occupations. The average income of rural laborers may well overestimate such marginal productivity if the arrangements governing the distribution of the rural product are not based on competitive labor markets.

The shadow price of skilled labor is composed of the shadow price of unskilled labor and the cost of training. In turn, the latter is measured by the skilled premiums currently paid to skilled workers over their unskilled cohorts. Skill premiums may well vary between the protected and unprotected urban sectors, since part of the protection in the former may take the form of widened skill premiums. In that case, the premiums from the unprotected sector would be an appropriate measure of the shadow skill premium. In empirical investigation, however, it must be borne in mind that the type of skill in the protected sector is likely to be different from the skill in the unprotected sector.

Numerical estimation of the price at which workers are willing to work as well as the skill premiums can be obtained almost exclusively on the basis of survey data. For consistency checks, it is useful to include in the sample to be questioned both employed and unemployed workers as well as the same skill categories in the protected and unprotected sectors.

Section IX

The Second Best General Disequilibrium Price of Fiscal Revenue and Its Numerical Estimation

When the distribution of income between the public and private sector is not optimal, the shift of a marginal unit of funds between these two sectors will change the level of welfare in the economy. If the level of government revenue is suboptimal, the shift from the private to the public sector will increase welfare and when the level of government revenue is hyperoptimal then the converse effect will ensue.

Defining the consumption equivalents of a peso's worth of private expenditure and government expenditure respectively, we obtain the following expressions:

$$PE = (1 - sp) + sp\ PK \tag{43}$$

$$GE = (1 - sg)\ Cg + sg\ PK_g \tag{44}$$

where: sp, sg = marginal savings ratios of private and public sectors, respectively

Cg = private consumption equivalent of government consumption

PK_g = private consumption equivalent of government investment.

Taking the difference between the two consumption equivalents, we obtain:

$$PG = GE - PE = (1-sg)\ Cg-(1-sp) + sgPK_g - spPK \qquad (45)$$

It will be noticed that this difference is affected by differences is the savings propensities, differences in the value of government and private consumption, Cg, and differences between the consumption equivalents of government and private investment. If a proportionality factor is introduced for this last element, equation (45) can be simplified:

$$PK_g = \lambda PK \qquad (46)$$

$$PG = (1-sg)\ Cg-(1-sp) + PK(sg\ \lambda - sp) \qquad (47)$$

The relative value of government and private consumption is a hotly debated subject. On the one hand, it is often argued that public goods are in too short supply compared to private goods. This argument would claim that Cg is substantially above one. On the other hand, it is as often argued that much of government expenditures is wasted. In that case, Cg would be substantially less than one. In the absence of any readily available way of measuring the true value of this parameter, we can assume that there is on balance no difference between a peso's worth of private consumption and a peso's worth of public consumption, i.e. Cg =1. In that case (47) can be further simplified as follows:

$$PG = sp - sg + PK\ (sg\ \lambda - sp) \qquad (48)$$

We can now see that the shadow price of public funds is equal to the excess of the private over the public propensity to save plus the shadow price of investment multiplied by the aforementioned difference as adjusted by the difference between the productivity of public and private investment.

The marginal propensities to save of both sectors can be estimated from national accounts data by either the regression or the first difference technique. It should be noted, however, that the standard classification rules used in the national accounts include as government consumption expenditure a number of items that would more appropirately be considered investment, i.e. education, public health. For the purpose of measuring the shadow price of fiscal goods, it is preferable to reassign such government expenditures to the investment category.

The relative value of public and private investment, λ, can be estimated on the basis of a comparison between the benefit-cost ratios of government investment projects in different sectors and the shadow price of investment in the private sector as derived in Section VII. In line with the adjustment of the previous paragraph, it is appropirate to include in the calculation of the returns to investment in the public sector, the returns from expenditure on education, public health, etc.