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Interest rate volatility prior to monetary union under alternative pre-switch regimes

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Hamburgisches Welt-Wirtschafts-Archiv (HWWA)
Hamburg Institute of International Economics
Öffentlichkeitsarbeit
Neuer Jungfernstieg 21 - 20347 Hamburg
Telefon: 040/428 34 355
Telefax: 040/428 34 451
e-mail: hwwa@hwwa.de
Internet: http://www.hwwa.de/

Bernd Wilfling
Telefon: 040/42834-446
e-mail: bernd.wilfling@hwwa.de

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Summary

The volatility of interest rates is relevant for many financial applications. Under realistic assumptions the *term structure of interest rate differentials* provides an important prediction of the *term structure of interest rates*. This paper derives the term structure of differentials in a situation in which two open economies plan to enter a monetary union in the future. Two systems of floating exchange rates prior to the union are considered, namely a free-float and a managed-float regime. The volatility processes of arbitrary-term differentials under the respective pre-switch arrangements are compared. The paper elaborates the singularity of extremely short-term (i.e. instantaneous) interest rates under extensive leaning-against-the-wind intervention policies and discusses empirical issues.

Zusammenfassung

Die Volatilität von Zinssätzen ist für viele Anwendungen aus den Bereichen der Finanz- und Geldtheorie von Bedeutung. Unter realistischen Bedingungen liefert die *zeitliche Struktur von Zinsdifferentialen* eine wichtige Vorhersage für die *Zinsstruktur*. Diese Arbeit leitet die Zinsdifferentialstruktur in einer Situation her, in der zwei offene Volkswirtschaften in der Zukunft eine Währungsunion bilden wollen. Es werden zwei alternative Wechselkursregime vor der Währungsunion betrachtet und zwar ein „Reiner" sowie ein „Gemanagter Float". Das Papier vergleicht die Volatilitätsprozesse von Zinsdifferentialen beliebiger Fristigkeit unter den verschiedenen Wechselkurssystemen. Ferner stellt die Arbeit die singuläre Stellung extrem kurzfristiger (sogenannter instantaner) Zinssätze unter intensiven „leaning-against-the-wind" Interventionspolitiken heraus und diskutiert empirische Aspekte.

JEL classifications: E43, F31, F33, C52

Key words: Interest rate volatility, term structure, exchange rate arrangements, intervention policy, stochastic processes

1 Introduction

Interest rates of different maturities are among the most fundamental prices determined in financial markets. Up to now, many models have been put forward to explain their dynamic behaviour and there still are massive ongoing research activities in the field. This enormous scientific attention may be explained by the significance of interest rates in many economic disciplines. For example, in macroeconomics interest rates play a crucial role in the transmission of monetary policy to inflation and real growth. Moreover, in many models longer-term interest rates affect capital movements or saving, investment and consumption decisions. In finance, interest rates and their term structure are of primary importance for most subareas. For instance, the volatility of arbitrary-term interest rates—which is the main concern of this paper—is a key variable in the valuation of contingent claims such as interest-rate-options as well as in the selection of optimal hedging strategies for risk-averse investors.

An explanation of the *term structure of interest rates* has long been a topic of major concern for economists. The most famous strands of theory are the *expectations*, the *liquidity preference*, the *market segmentation*, and the *preferred habitat* hypotheses. Another widely celebrated approach by Cox, Ingersoll and Ross (1985) is to gain access to the term structure by using an intertemporal general equilibrium asset pricing model. Although there exists an extensive body of literature on both, theoretical and empirical findings on the term structure, the topic is still far from being settled.[1]

In recent years there have been several attempts to analyze the term structure of interest rates under particular exchange rate arrangements. The fundamental works are Svensson (1991a, b) who explores the *term structure of interest rate differentials* (i.e. the differential between pairs of domestic and foreign interest rates with the same time to maturity) in a monetary flex-price exchange rate target zone model of the Krugman-(1991)-type. Svensson (1991b, p. 90) offers the following justification for considering the *term structure of interest rate differentials* as a shortcut to analyzing the *term structure of interest rates*:[2]

> For a small open economy which faces an exogenous term structure of world interest rates, the domestic term structure of interest rates follows the term structure of interest rate differentials. The term structure of interest rate differentials is related via interest parity conditions to the term structure of expected currency depreciation, if domestic and world capital markets are

[1] For an overview see e.g. Cuthbertson (1996, pp. 207) and the literature cited there.

[2] See also Kempa *et al.* (1999, p. 818) for further comments on this justification.

sufficiently integrated. The term structure of expected currency depreciation can be explicitly characterized in an exchange rate target zone.

It is the aim of this paper to analyze the term structure of interest rate differentials in a situation in which a small open economy faces a future entrance into a monetary union with its partner countries. In practice, the introduction of a common currency is typically initiated by a switch in exchange rate regime from a more or less floating system to an arrangement of completely fixed rates. For example, the introduction of the euro was cogently linked to the irreversible and complete fixing of the EMU-countries' bilateral exchange rates at their central parities from the European Exchange Rate Mechanism from 1 January 1999 onwards. Consequently, the stylized models developed in the subsequent sections may be used in future investigations to study the evolution of interest rates prior to EMU.

The general setup of this paper is in many respects similar to that in the aforementioned work of Svensson (1991b). But, as will be shown below, in contrast to the problem addressed in his article, there exist analytically closed-form solutions of the term structure of interest rate differentials under an anticipated entrance into a monetary union.[3] And more than that, such closed-form solutions exist under alternative exchange rate regimes prior to monetary union, namely under a free-float as well as under a managed-float pre-switch regime. This advantage can be exploited to pursue two objectives: First, it is possible to derive and explore the volatility of interest rate rate differentials of arbitrary maturities thus providing a shortcut to analyzing the volatility processes of arbitrary-term interest rates. And second, the results under the alternative pre-switch regimes can be compared. These insights may then provide a useful tool for a broad range of financial applications (e.g. for interest-rate-sensitive security valuation).

The paper is organized as follows: Section 2 briefly reviews some previous results essential for further considerations, puts forward the general model setup and derives closed-form solutions of the term structure of interest rate differentials under free-float and managed-float pre-switch regimes. Sections 3 provides the regime comparison elaborating the main differences in the interest rate volatility processes under the respective regimes. It also discusses empirical issues and implications for financial applications. Section 4 offers some concluding comments.

[3]Under a credible target zone of the Krugman-type, the term structure of interest rate differentials can be expressed as a solution to a partial differential equation with appropriate smooth-pasting conditions. Svensson (1991b, pp. 97) uses two numerical methods to analyze this solution.

2 Preliminaries, previous results, model setup

2.1 Exchange rate dynamics

We consider a world with two open economies under perfect capital mobility in which the domestic economy is assumed to be small. Now, let the political authorities of the two economies decide to create a monetary union in the future. On the analogy of Stage III of EMU, the authorities therefore announce at date t_A to irreversibly fix the exchange rate from the future date t_S onwards (i.e. $t_A < t_S$) at the specific parity s.

To assess the exchange rate dynamics under such a time-contingent switch in exchange rate regime, it is convenient to consider the well-known monetary exchange rate model with flexible prices in continuous time. In this equilibrium model with rational expectations, the logarithmic spot rate at time t, $x(t)$, equals the sum of two components: (a) an exogenously given fundamental $k(t)$, which may be viewed as a collection of all economic and/or political factors that financial markets deem to be important for current exchange rate valuation, and (b), a speculative component representing the agents' expectations about future changes in the currency value. These two elements of currency pricing may be formalized as

$$x(t) = k(t) + \alpha \cdot \frac{E\left[dx(t)|\phi(t)\right]}{dt}, \quad \alpha > 0. \tag{1}$$

In Eq. (1), $E[\cdot|\cdot]$ denotes the expectation operator conditional on the information set $\phi(t)$ which contains all information available to market participants at time t.[4]

In the monetary flex-price model the fundamental k is an aggregate of given macroeconomic variables such as domestic and foreign money supplies and outputs. The dynamic structure imposed on k prior to the fixed-rate system represents the explicit regime of floating exchange rates prior to monetary union. In this paper, we consider two alternative pre-switch regimes of floating rates, namely a free-float and a managed-float system, respectively. A pure free-float pre-switch system (subsequently denoted by the subscript FF), in which the monetary authorities refrain from any interventions in foreign exchange markets, is typically modelled by letting the fundamental evolve

[4] In Eq. (1), $E[dx(t)|\phi(t)]/dt$ is an abbreviation of $\lim_{s\downarrow 0}\left\{E[x(t+s)|\phi(t)] - x(t)\right\}/s$. Since x denotes the natural logarithm of the nominal exchange rate, it follows immediately that $E[dx(t)|\phi(t)]/dt$ represents the expected (instantaneous) rate of change in the nominal exchange rate. In the monetary flex-price exchange rate model, the parameter α is the semielasticity of money demand with respect to a short-term interest rate. In Eq. (1), α may simply be viewed as a parameter weighting the fundamental component against the speculative motives for currency valuation.

over time as a Brownian motion, i.e.

$$dk(t) \equiv dk_{FF}(t) = \sigma \cdot dz(t), \quad t < t_S, \tag{2}$$

with $\sigma > 0$ denoting the infinitesimal standard deviation and $dz(t)$ the increment of a standard Wiener process.[5]

In contrast to this, it may be desirable to model a managed-float exchange rate system (denoted by MF) prior to monetary union, i.e. to explicity allow for central bank interventions which aim at stabilizing the exchange rate near some specified target parity. For simplicity, assume that the target parity prior to the regime switch is s (i.e. the later fixing parity). According to Eq. (1), it seems consistent for the central banks to prevent the fundamental from wandering too far away from the exchange rate target parity. This leaning-against-the-wind policy is adequately modelled by a mean-reverting Ornstein-Uhlenbeck process with stochastic differential

$$dk(t) \equiv dk_{MF}(t) = \eta \cdot [s - k_{MF}(t)] \cdot dt + \sigma \cdot dz(t), \quad t < t_S. \tag{3}$$

In Eq. (3), the parameter $\eta \geq 0$ represents the speed with which the fundamental k_{MF} tends to revert towards s after a temporary deviation. Therefore, η may be interpreted as a measure for the willingness and/or the capability of the central banks to stabilize the exchange rate x near the target parity by appropriate interventions in foreign exchange markets.

In conjunction with the pre-switch regime-dependent specifications of the fundamental in Eqs. (2) and (3), the general law of exchange rate evolution from (1) represents a stochastic differential equation. This can be solved by stochastic integration techniques and the imposition of adequate economic constraints which correctly reflect financial markets' anticipations of future exchange rate regime switching. Following Wilfling and Maennig (2001) the equilibrium exchange rate path under a free-float pre-switch system consists of the two branches

$$x_{FF}(t) = k_{FF}(t), \quad \text{for } t < t_A, \tag{4}$$

and

$$x_{FF}(t) = \left[1 - \mathrm{e}^{(t-t_S)/\alpha}\right] \cdot k_{FF}(t) + \mathrm{e}^{(t-t_S)/\alpha} \cdot s, \quad \text{for } t \in [t_A, t_S). \tag{5}$$

[5] A Brownian motion may be considered a continuous-time analog of a random walk. Mathematically, a Brownian motion belongs to the class of diffusion processes whose dynamics is represented by stochastic differentials which—on the basis of all information available at present date t—determine the probabilistic nature of changes in the stochastic process in the infinitesimally close future (cf. Karlin and Taylor, 1981, Chap. 15).

Along similar lines, Wilfling (2001, pp. 104) derives the saddlepath under a managed-float as

$$x_{MF}(t) = \frac{1}{1+\alpha\eta} \cdot k_{MF}(t) + \frac{\alpha\eta}{1+\alpha\eta} \cdot s, \quad \text{for } t < t_A, \tag{6}$$

and

$$x_{MF}(t) = s + \left[1 - \mathrm{e}^{(1+\alpha\eta)\cdot(t-t_S)/\alpha}\right] \cdot \frac{k_{MF}(t) - s}{1+\alpha\eta}, \quad \text{for } t \in [t_A, t_S). \tag{7}$$

For the time after t_S the exchange rate will be fixed by assumption so that for both pre-switch regimes one finds

$$x_{FF}(t) = x_{MF}(t) = s \tag{8}$$

for all $t \geq t_S$.

2.2 The interest rate differential

Following the setups in Svensson (1991a, b) let $i^*(t,\tau)$ denote the foreign nominal interest rate on a pure foreign-currency discount bond purchased at time t which matures at time $t+\tau$. The small domestic economy cannot affect the foreign rate $i^*(t,\tau)$ by economic policy, but has to accept the foreign interest rate as exogenously given. The corresponding nominal domestic interest rate on a pure domestic-currency discount bond will accordingly be denoted by $i(k(t),t,\tau)$. Note that the realization of the fundamental k—representing either the free-float pre-switch regime according to Eq. (2) or the managed-float pre-switch system according to Eq. (3)—is among the arguments of the domestic interest rate i.

Further suppose that international investors consider the home and foreign bonds as perfect substitutes and assume perfect international capital mobility. Under this setting the following form of the uncovered interest parity condition should hold at all points in time:

$$ID(k(t),t,\tau) \equiv i(k(t),t,\tau) - i^*(t,\tau) = \frac{E[x(t+\tau)|\phi(t)] - x(t)}{\tau}. \text{[6]} \tag{9}$$

[6] To make the right-hand side of Eq. (9) plausible, let $X(t)$ denote the exchange rate in levels, i.e. $x(t) = \ln[X(t)]$. Furthermore, assume $X(\cdot)$ to be a deterministic function differentiable with respect to time. Then, for infinitesimal future dates, the uncovered interest parity condition claims that the interest rate differential equals the (expected) change in the exchange rate, i.e.

$$i(t) - i^*(t) = \frac{dX(t)/dt}{X(t)} = \frac{d\ln[X(t)]}{dt} = \lim_{\tau\to 0} \frac{x(t+\tau) - x(t)}{\tau}.$$

Generalizing from infinitesimal to finite future dates and taking conditional expectations motivates

One special case included in Eq. (9) concerns the interest rate differential for so-called instantaneous bonds, i.e. for bonds with infinitesimally short time to maturity. These are defined by letting $\tau \to 0$ and signify extremely short-term interest rates (e.g. overnight rates). Bearing in mind Footnote 4, the corresponding interest rate differential is obtained from Eq. (9) as

$$ID(k(t), t, 0) \equiv \lim_{\tau \downarrow 0} ID(k(t), t, \tau) = \frac{E\left[dx(t)|\phi(t)\right]}{dt}. \qquad (10)$$

The equilibrium exchange rate paths (4) to (8) and the uncovered interest parity conditions (9) and (10) now allow us to compute closed-form solutions of interest rate differentials for arbitrary terms τ under both pre-switch regimes.[7] In particular, consider first the free-float case. For the time prior to the announcement date, i.e. for $t \in [0, t_A)$, agents believe the free-float system to be permanent forever. In this case the differential for arbitrary term τ equals zero on the whole time domain, i.e.

$$ID_{FF}(k_{FF}(t), t, \tau) = 0 \quad \text{for all } t \in [0, t_A), \tau \geq 0. \qquad (11)$$

During the interim period $[t_A, t_S)$ the analytical form of interest rate differentials is no longer independent of the term τ. To be more explicit, the instantaneous interest rate differential for $t \in [t_A, t_S)$ is found to be

$$ID_{FF}(k_{FF}(t), t, 0) = \mathrm{e}^{(t-t_S)/\alpha} \cdot \frac{s - k_{FF}(t)}{\alpha}, \qquad (12)$$

while the differentials for strictly positive terms $\tau > 0$ evolve along two analytically distinct equilibrium paths which are separated from each other by the date $t_S - \tau$:

$$ID_{FF,1}(k_{FF}(t), t, \tau) = \mathrm{e}^{(t-t_S)/\alpha} \cdot \frac{1 - \mathrm{e}^{\tau/\alpha}}{\tau} \cdot [k_{FF}(t) - s] \quad \text{for } t \in [t_A, t_S - \tau), \qquad (13)$$

and

$$ID_{FF,2}(k_{FF}(t), t, \tau) = \left[1 - \mathrm{e}^{(t-t_S)/\alpha}\right] \cdot \left[\frac{s - k_{FF}(t)}{\tau}\right] \quad \text{for } t \in [t_S - \tau, t_S). \qquad (14)$$

By analogous arguments, the managed-float counterparts of the interest rate differentials (11) to (14) may be derived. Making use of the exchange rate equilibrium path

the right-hand side of Eq. (9).

[7]The computations involve basic principles of stochastic calculus. The main difficulty consists of calculating the expected exchange rate $E[x(t+\tau)|\phi(t)]$. But closed-form solutions of these expected values follow from well-known formulae for conditional expectations of Brownian motions and Ornstein-Uhlenbeck processes (see e.g. Karlin and Taylor, 1981, Chap. 15).

(6), the differential for $t \in [0, t_A)$ obtains as

$$ID_{MF}(k_{MF}(t), t, \tau) = \frac{s - k_{MF}(t)}{1 + \alpha\eta} \cdot \frac{1 - \mathrm{e}^{-\eta \cdot \tau}}{\tau}. \tag{15}$$

In contrast to its free-float counterpart (11), the permanent managed-float differential (15) depends on the current fundamental $k_{MF}(t)$ and on the term τ. For zero-term the instantaneous differential follows by means of standard calculus:

$$ID_{MF}(k_{MF}(t), t, 0) = \frac{\eta \cdot [s - k_{MF}(t)]}{1 + \alpha\eta}. \tag{16}$$

For the interim period $[t_A, t_S)$ it is straightforward to derive the instantaneous differential as

$$ID_{MF}(k_{MF}(t), t, 0) = \left[\frac{s - k_{MF}(t)}{1 + \alpha\eta}\right] \cdot \left[\eta + \frac{1}{\alpha} \cdot \mathrm{e}^{(1+\alpha\eta)\cdot(t-t_S)/\alpha}\right], \tag{17}$$

while the two branches of the equilibrium differential paths for $\tau > 0$ are given by

$$ID_{MF,1}(k_{MF}(t), t, \tau) = \left[\frac{k_{MF}(t) - s}{1 + \alpha\eta}\right] \cdot \left\{\frac{\mathrm{e}^{-\eta \cdot \tau} - 1}{\tau} + \left[\frac{1 - \mathrm{e}^{\tau/\alpha}}{\tau}\right] \cdot \mathrm{e}^{(1+\alpha\eta)\cdot(t-t_S)/\alpha}\right\} \tag{18}$$

for $t \in [t_A, t_S - \tau)$ and

$$ID_{MF,2}(k_{MF}(t), t, \tau) = \frac{1}{\tau} \cdot \left[\frac{s - k_{MF}(t)}{1 + \alpha\eta}\right] \cdot \left[1 - \mathrm{e}^{(1+\alpha\eta)\cdot(t-t_S)/\alpha}\right] \tag{19}$$

for $t \in [t_S - \tau, t_S)$, respectively.

Finally, it remains to specify the interest rate differential dynamics for the time after t_S under each pre-switch regime. According to Eq. (8) the exchange rate will be fixed forever at the preannounced parity s from t_S onwards. Hence, for both pre-switch regimes, Eqs. (9) and (10) clearly provide

$$ID_{FF}(k_{FF}(t), t, \tau) = ID_{MF}(k_{MF}(t), t, \tau) = 0 \quad \text{for all } t \geq t_S, \tau \geq 0. \tag{20}$$

3 Regime-comparison and implications

3.1 Measuring the volatility of interest rate differentials

In order to measure volatility it is convenient to draw on the concept of the so-called infinitesimal variance of interest rate differentials. As will become evident below, this dispersion measure—subsequently denoted by $\nu^2_{\{ID(k(t),t,\tau)\}}$—explicitly depends on the present date t. Loosely speaking, it approximates the variance of interest rate differential realizations from the infinitesimally close future conditional upon all information included in the current information set $\phi(t)$.[8]

One attractive feature of the time-varying infinitesimal variance $\nu^2_{\{ID(k(t),t,\tau)\}}$ is that it can be computed conveniently by the well-known Ito-lemma under very mild assumptions. To be more explicit, consider the diffusion process $\{y(t), t \geq 0\}$ which is assumed to be a function of the fundamental k and the time index t, i.e.

$$y(t) = F(k, t). \tag{21}$$

Recall that throughout this paper the dynamics of the fundamental k determines the pre-switch exchange rate regime. For this, k is modelled either by a Brownian motion with stochastic differential (2), or by an Ornstein-Uhlenbeck process with differential (3) so that either $k = k_{FF}$ or $k = k_{MF}$. A common feature of both processes is that their infinitesimal variances are equal and constant over time, namely $\nu^2_{\{k_{FF}(t)\}} = \nu^2_{\{k_{MF}(t)\}} = \sigma^2$ (see for example Wilfling 2001, p. 203). Now Ito's lemma states the following: if the defining function $F(\cdot,\cdot)$ in Eq. (21) is twice differentiable with respect to k and once differentiable in t, then the infinitesimal variance of the transformed process $\{y(t)\}$ is given by

$$\nu^2_{\{y(t)\}} = \left[\frac{\partial F}{\partial k}(k(t),t)\right]^2 \cdot \nu^2_{\{k(t)\}} = \left[\frac{\partial F}{\partial k}(k(t),t)\right]^2 \cdot \sigma^2. \tag{22}$$

Via the rule (22) it is possible to derive analytically closed-form expressions of the infinitesimal variances of all interest rate differentials from the previous section. In particular, focussing on the interim period $[t_A, t_S)$ and assuming a free-float pre-switch exchange rate regime, the infinitesimal variances of the instantaneous differential (12) and both differential branches (13) and (14) for strictly positive terms $\tau > 0$ follow directly by the Ito-rule (22) as

$$\nu^2_{\{ID_{FF}(k_{FF}(t),t,0)\}} = \sigma^2 \cdot \left[\frac{e^{(t-t_S)/\alpha}}{\alpha}\right]^2, \tag{23}$$

$$\nu^2_{\{ID_{FF,1}(k_{FF}(t),t,\tau)\}} = \sigma^2 \cdot \left[e^{(t-t_S)/\alpha} \cdot \frac{1-e^{\tau/\alpha}}{\tau}\right]^2, \tag{24}$$

$$\nu^2_{\{ID_{FF,2}(k_{FF}(t),t,\tau)\}} = \sigma^2 \cdot \left[\frac{1-e^{(t-t_S)/\alpha}}{\tau}\right]^2. \tag{25}$$

[8] See for example Karlin and Taylor (1981, pp. 159).

In the same manner the infinitesimal variances of the interest rate differential paths (17), (18) and (19) under a managed-float for $t \in [t_A, t_S)$ may be derived as

$$\nu^2_{\{ID_{MF}(k_{MF}(t),t,0)\}} = \left[\frac{\sigma}{1+\alpha\eta}\right]^2 \cdot \left[\eta + \frac{1}{\alpha} \cdot \mathrm{e}^{(1+\alpha\eta)\cdot(t-t_S)/\alpha}\right]^2, \tag{26}$$

$$\begin{aligned}\nu^2_{\{ID_{MF,1}(k_{MF}(t),t,\tau)\}} &= \left[\frac{\sigma}{1+\alpha\eta}\right]^2 \\ &\quad \times \left[\frac{\mathrm{e}^{-\eta\cdot\tau}-1}{\tau} + \left(\frac{1-\mathrm{e}^{\tau/\alpha}}{\tau}\right) \cdot \mathrm{e}^{(1+\alpha\eta)\cdot(t-t_S)/\alpha}\right]^2,\end{aligned} \tag{27}$$

$$\nu^2_{\{ID_{MF,2}(k_{MF}(t),t,\tau)\}} = \left[\frac{\sigma}{1+\alpha\eta}\right]^2 \cdot \frac{1}{\tau^2} \cdot \left[1 - \mathrm{e}^{(1+\alpha\eta)\cdot(t-t_S)/\alpha}\right]^2. \tag{28}$$

As mentioned above, all infinitesimal variances in the Eqs. (23) to (28) explicitly depend on the time index t. Consequently, it seems justified to speak of variance or volatility paths. Besides the above volatility paths for $t \in [t_A, t_S)$ it is straightforward to derive the infinitesimal variances of interest rate differentials for the time before t_A and after t_S. Since the volatility of interest rates under permanent free-float and/or managed-float systems (i.e. for $t < t_A$) are only of secondary concern for the rest of this paper, we refrain from giving explicit formulae here.[9] For $t \geq t_S$ it evident from Eq. (20) that the variances of arbitrary-term interest rate differentials vanish completely under both pre-switch regimes. This result is most intuitive since the interest parity conditions (9) and (10) imply constant zero-differentials under a system of fixed exchange rates.

3.2 Pre-switch-regime comparison

3.2.1 The free-float regime

Figure 1 displays the evolution of infinitesimal variances for alternative terms τ during the interim period $[t_A, t_S)$ under a free-float pre-switch regime. The following relations are easy to verify and reveal some striking features of the variance paths (23), (24) and (25):

[9] For reasons of argument at a later stage, it should be noted that according to Eq. (11) the volatility path of any arbitrary-term differential is constantly equal to zero under a permanent free-float. In contrast to that, the formulae under a permanent managed-float are slightly more complex as can be seen from the Eqs. (15) and (16).

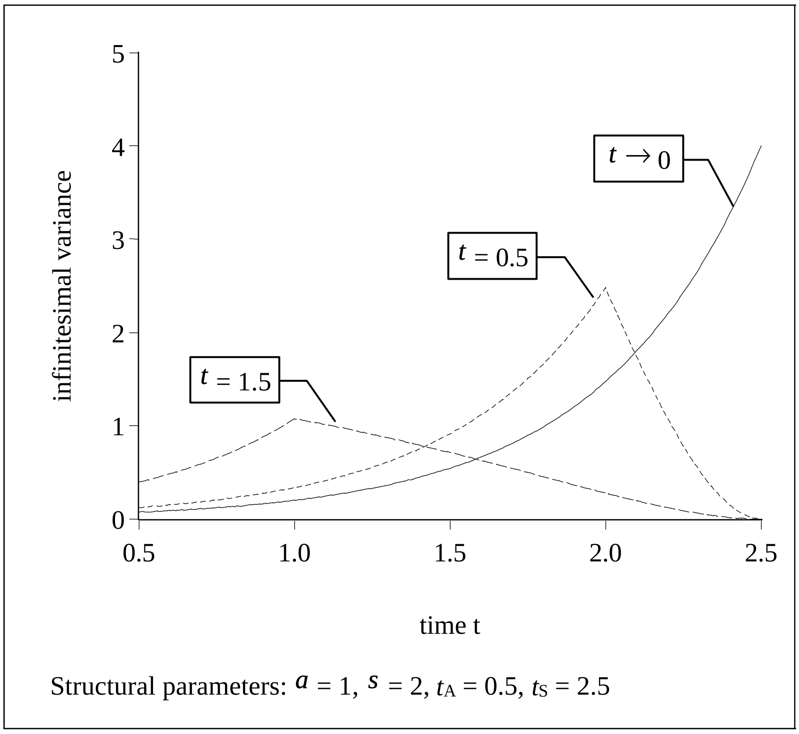

Figure 1: Infinitesimal variance paths under a free-float pre-switch regime

$$\frac{\partial \nu^2_{\{ID_{FF}(k_{FF}(t),t,0)\}}}{\partial t} > 0, \quad \frac{\partial \nu^2_{\{ID_{FF,1}(k_{FF}(t),t,\tau)\}}}{\partial t} > 0, \quad \frac{\partial \nu^2_{\{ID_{FF,2}(k_{FF}(t),t,\tau)\}}}{\partial t} < 0, \tag{29}$$

$$\lim_{t \uparrow t_S - \tau} \nu^2_{\{ID_{FF,1}(k_{FF}(t),t,\tau)\}} = \lim_{t \downarrow t_S - \tau} \nu^2_{\{ID_{FF,2}(k_{FF}(t),t,\tau)\}}, \tag{30}$$

$$\lim_{t \uparrow t_S} \nu^2_{\{ID_{FF}(k_{FF}(t),t,0)\}} = \frac{\sigma^2}{\alpha^2}, \tag{31}$$

$$\lim_{t \uparrow t_S} \nu^2_{\{ID_{FF,2}(k_{FF}(t),t,\tau)\}} = 0. \tag{32}$$

The derivatives in (29) imply that differential variances are (a) increasing over time for zero-term differentials during the whole interim period $[t_A, t_S)$, (b) increasing for strictly positive terms on the interval $[t_A, t_S - \tau)$, and (c) decreasing for $\tau > 0$ on $[t_S - \tau, t_S)$. This monotonic volatility behaviour raises some economic questions two of which will be tackled here. (1) Why does the announcement of future time-contingent exchange rate fixing entail a general increase in differential volatility for arbitrary terms as opposed to the zero-volatility paths under a permanent free-float system (cf. Footnote 9)? (2) Why does the differential volatility for strictly positive terms vanish completely at t_S while the variance of instantaneous interest rate differentials does not?

The first question is easy to answer. Under a permanent free-float system the exchange rate x_{FF} coincides exactly with the fundamental k_{FF} according to Eq. (4). But the fundamental k_{FF} evolves along the (driftless) Brownian motion (2) so that the expected rate of change of k_{FF} is zero over any time interval of finite length. Hence, the expected rate of change in the exchange rate equals zero, too. Consequently, under the interest parity conditions (9) and (10), the interest rate differentials constantly equal zero and have zero-variance before t_A. Now, triggered by the announcement at t_A of future exchange rate fixing at t_S, the exchange rate x_{FF} jumps from the equilibrium path (4) on the saddlepath (5). In this phase, the exchange rate is a time-dependent average of the stochastic fundamental k_{FF} and the fixing parity s. The evolution of the exchange rate thus depends—with changing weights over time—on both, the conditional distribution of the fundamental as well as on the deterministic parity s. The same is clearly true for the expected rate of change in the exchange rate and thus for the interest rate differential. Hence its positive volatility. In the end, the volatility increases in interest rate differentials result from the fact that the exchange rate x_{FF} has to leave its stationary saddlepath (4) in order to ensure an arbitrage-free transition into the fixed-rate system at date t_S and that the interest rates have to react accordingly.

The second question can also be justified on grounds of an arbitrage argument. For this, consider first the differentials (12) and (14) for which one finds

$$\lim_{t \uparrow t_S} ID_{FF}(k_{FF}(t), t, 0) = \frac{1}{\alpha} \cdot \left[s - \lim_{t \uparrow t_S} k_{FF}(t) \right]$$

and

$$\lim_{t \uparrow t_S} ID_{FF,2}(k_{FF}(t), t, \tau) = 0.$$

Note that the first limit for instantaneous interest rate differentials is different from zero with probability 1.[10] Obviously, strictly positive-term domestic and foreign interest rates adjust completely during the interim period while instantaneous interest rates in general do not. An explanation for this difference is as follows: if interest rates for strictly positive term $\tau > 0$ did not adjust completely among the two economies at date t_S, there would be room for riskless profits by buying the domestic and selling the foreign bond (or vice versa) infinitesimally shortly before t_S. These riskless-profit

[10]To be mathematically precise: all limits of the stochastic processes $\{k_{FF}\}$ and $\{k_{MF}\}$ draw on the concept of convergence with probability 1.

transactions can only be ruled out by a complete adjustment of domestic and foreign interest rates. But the necessity of complete interest rate equalization implies a volatility reduction towards zero. On the other hand, no such arbitrage opportunities exist for instantaneous bonds because at any date $t \in [t_A, t_S)$ the time to maturity, $t + dt$, always lies within the interim period. Hence, for instantaneous interest rates there is no need for a complete equalization at the date of transition into the fixed-rate system. Thus the stochastic fundamental k_{FF} still has a significant impact on the interest rate differential. Consequently, according to Eq. (31), there remains the strictly positive infinitesimal variance σ^2/α^2 for $t \to t_S$.

Apart from the evolution of interest rate volatility over time, it is important for many financial applications to analyze the impact of the term τ on the volatility at any point in time. As will be shown below, there is a clear-cut relationship between the degree of volatility and the term τ under a free-float pre-switch regime. Before elaborating this, it proves convenient to consider the following auxiliary function of the term τ:

$$f(\tau) = \frac{1 - \mathrm{e}^{\tau/\alpha}}{\tau}.$$

Note the following properties of f:

1. $f(\tau) < 0$ for all $\tau > 0$,

2. $f'(\tau) \equiv df(\tau)/d\tau < 0$ for all $\tau > 0$.

Via these properties the partial derivatives of the volatility paths (24) and (25) with respect to τ obtain as

$$\frac{\partial \nu^2_{\{ID_{FF,1}(k_{FF}(t),t,\tau)\}}}{\partial \tau} = 2 \cdot \sigma^2 \cdot \left[\mathrm{e}^{(t-t_S)/\alpha}\right]^2 \cdot f(\tau) \cdot f'(\tau) > 0, \tag{33}$$

$$\frac{\partial \nu^2_{\{ID_{FF,2}(k_{FF}(t),t,\tau)\}}}{\partial \tau} = -\frac{2}{\tau} \cdot \nu^2_{\{ID_{FF,2}(k_{FF}(t),t,\tau)\}} < 0. \tag{34}$$

These relations establish the following clear-cut result: Along the $ID_{FF,1}$-path, interest rate differentials for shorter terms exhibit lower volatility than differentials for longer terms. The reverse is true along the $ID_{FF,2}$-path where differentials for longer terms exhibit less volatility than those for shorter terms.

There is, however, one aspect which needs some attention when comparing differentials with alternative terms. To illustrate, consider the terms τ_1 and τ_2 with $\tau_1 < \tau_2$.[11]

[11] For simplicity assume further that both terms are less than the length of the interim period (i.e. $\tau_1 < \tau_2 < t_S - t_A$). This guarantees that both interest rate differentials in fact have $ID_{FF,1}$- and $ID_{FF,2}$-branches.

Eq. (33) clearly provides

$$\nu^2_{\{ID_{FF,1}(k_{FF}(t),t,\tau_1)\}} < \nu^2_{\{ID_{FF,1}(k_{FF}(t),t,\tau_2)\}}$$

for $t \in [t_A, t_S - \tau_2)$ while for $t \in [t_S - \tau_1, t_S)$ the derivative (34) establishes

$$\nu^2_{\{ID_{FF,2}(k_{FF}(t),t,\tau_2)\}} < \nu^2_{\{ID_{FF,2}(k_{FF}(t),t,\tau_1)\}}.$$

On the interval $(t_S - \tau_2, t_S - \tau_1)$ the τ_1-differential is still on its $ID_{FF,1}$-branch while the τ_2-differential has already reached its $ID_{FF,2}$-branch. From the relations in (29) and (30) it follows directly that the τ_2-volatility path necessarily crosses the τ_1-volatility path once from above on this interval (see the terms $\tau_1 = 0.5$ and $\tau_2 = 1.5$ in Fig. 1).

3.2.2 The managed-float regime

Figure 2 depicts the variance paths (26) to (28) during the interim period under a managed-float pre-switch regime for the alternative terms $\tau = 1, \tau = 0.5$ and $\tau \to 0$. At first glance the volatility paths exhibit striking similarities to their counterparts under a free-float. The volatility paths (26) and (27) for instantaneous and $ID_{MF,1}$-differentials seem increasing on their admissible domains while the $ID_{MF,2}$-variance path (28) is decreasing over time and obviously tends to zero for $t \to t_S$. Indeed, it is straightforward to verify the relations (29) to (32)—which are valid under a free-float pre-switch regime—also for the managed-float volatility paths (26) to (28).

Among these relations only the validity of (31) is somewhat surprising. Evidently, at the moment of transition into the fixed-rate system, the volatility of instantaneous differentials, which equals σ^2/α^2, is independent of the parameter η. In other words, shortly before t_S instantaneous rates are always subject to exactly the same degree of volatility regardless of the intensity of central bank interventions.

Further evidence of the singularity of instantaneous interest rates as opposed to strictly positive-term rates is provided by the following result: Consider a situation in which the central banks are willing (and able) to defend the parity s by any necessary amount of intervention at any date during the interim period $[t_A, t_S)$. Put differently, the authorities *de facto* implement a fixed-rate system at the announcement date t_A. This extreme willingness to intervene is reflected by letting $\eta \to \infty$ in the volatility paths (26) to (28). It is easy to check that for $\tau > 0$ the $ID_{MF,1}$- and $ID_{MF,2}$-variance paths (27) and (28) constantly shrink to zero, i.e.

$$\lim_{\eta \to \infty} \nu^2_{\{ID_{MF,1}(k_{MF}(t),t,\tau)\}} = 0 \quad \text{and} \quad \lim_{\eta \to \infty} \nu^2_{\{ID_{MF,2}(k_{MF}(t),t,\tau)\}} = 0 \tag{35}$$

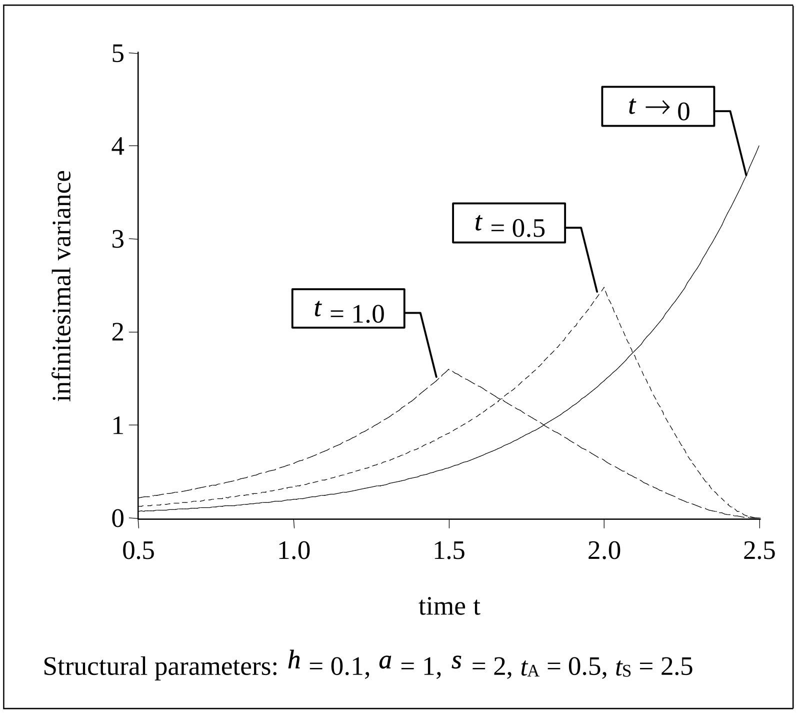

Figure 2: Variance paths under a managed-float pre-switch regime ('low' η-value)

for all admissible $t \in [t_A, t_S)$. On the other hand, the instantaneous volatility path (26) yields

$$\lim_{\eta \to \infty} \nu^2_{\{ID_{MF}(k_{MF}(t),t,0)\}} = \frac{\sigma^2}{\alpha^2}$$

for all $t \in [t_A, t_S)$. This establishes another interesting result: Instantaneous differentials remain volatile during the interim period even under an infinitely high degree of intervention while differentials for strictly positive terms become certain (i.e. have zero-variances) under the same intervention policy.

Finally, we address the relation between the term τ and differential volatility. First, recall that under a free-float pre-switch regime there exists a unique volatility ranking for strictly positive terms: Along the $ID_{FF,1}$-path longer terms imply higher volatility than shorter terms while the reverse is true along the $ID_{FF,2}$-path. Under a managed-float an analogous result may only be derived analytically for the $ID_{MF,2}$-variance path (28):

$$\frac{\partial \nu^2_{\{ID_{MF,2}(k_{MF}(t),t,\tau)\}}}{\partial \tau} = -\frac{2}{\tau} \cdot \nu^2_{\{ID_{MF,2}(k_{MF}(t),t,\tau)\}} < 0$$

for all $t \in [t_S - \tau, t_S)$ and arbitrary values of the intervention parameter η. Hence, in accordance with (34), under a managed-float longer-term differentials are less volatile than shorter-term differentials along the $ID_{MF,2}$-path.

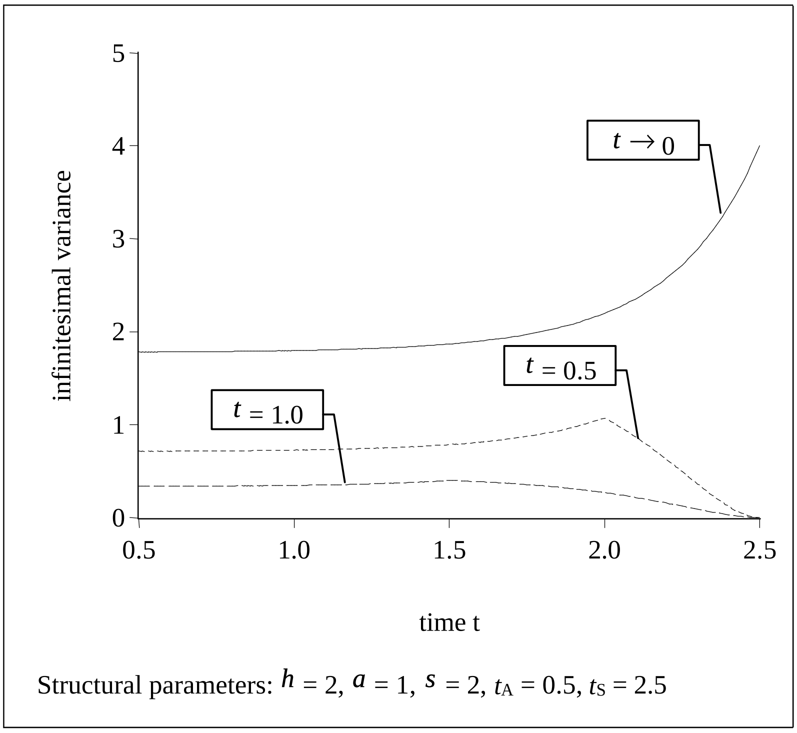

Figure 3: Variance paths under a managed-float pre-switch regime ('high' η-value)

But in contrast to the free-float case no clear-cut relation between differential volatility and the term $\tau > 0$ exists any more along the $ID_{MF,1}$-path. This becomes evident from the Figures 2 and 3. The 'low' η-value of 0.1 in Figure 2 leads to the same $ID_{MF,1}$-volatility structure as under a free-float with higher variances for longer terms. In contrast to this, the 'high' η-value of 2.0 in Figure 3 *ceteris paribus* entails exactly the reverse volatility ranking with respect to τ.

Obviously the intervention parameter η plays a crucial role for the relation between differential volatility and the term. The figures give rise to the conjecture that there may exist the following unique relation between τ and differential volatility along the $ID_{MF,1}$-path: Depending on the (constant) level of intervention, the $ID_{MF,1}$-variance paths for longer terms lie always either completely above or completely below those for shorter terms. Figure 4 shows that this last conjecture is definitely false in general. The line shown depicts the difference between two $ID_{MF,1}$-variance paths which—all else equal—only differ in the term τ. Evidently, the difference-line crosses the zero-line indicating a crossing of the respective variance paths.

A further conjecture may stem from the relation in (35). For any $\tau > 0$ there is a pointwise volatility convergence towards zero at every interim date $t \in [t_A, t_S)$ if the central banks' willingness to intervene becomes maximal (i.e. for $\eta \to \infty$). From this

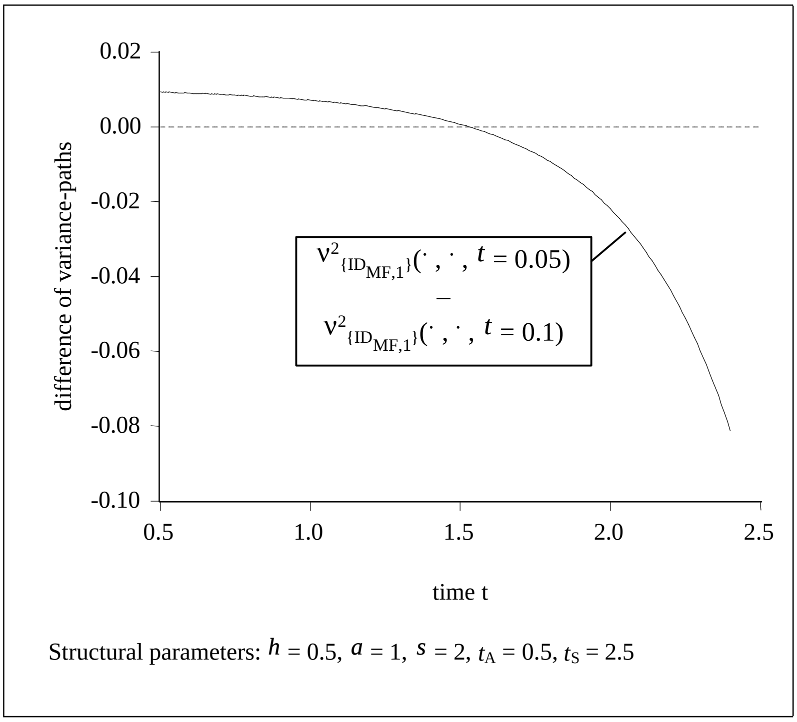

Figure 4: Crossing variance-paths with alternative terms

one might be inclined to think that an increase in η, *ceteris paribus*, possibly entails a decrease in the variance paths on their admissible domains. Figure 5 shows three complete variance paths which only differ in their intervention parameters. Obviously, the two paths generated with $\eta_1 = 1.0$ and $\eta_2 = 1.5$ cross so that the above conjecture does not hold in general.

3.3 Implications and empirical issues

Financial models which capture the volatility of interest rates serve at least two purposes (cf. Chan et al. 1992, p. 1210). First, the volatility of (primarily short-term) rates is a key variable for the valuation of contingent claims such as interest rate options. Second, the level of term structure volatility plays a crucial role in the selection of optimal hedging strategies for risk-averse investors. Therefore, it seems warranted to assess the above results on interest rate variability empirically and to make use of this knowledge in financial applications.

The econometric treatment of the volatility processes of interest rate differentials turns out a difficult matter from a technical point of view. A suitable estimation procedure should cover all unknown parameters from the differential paths (12) to (14) and (17) to (19). In the most general setting, these unknown parameters are α, σ and

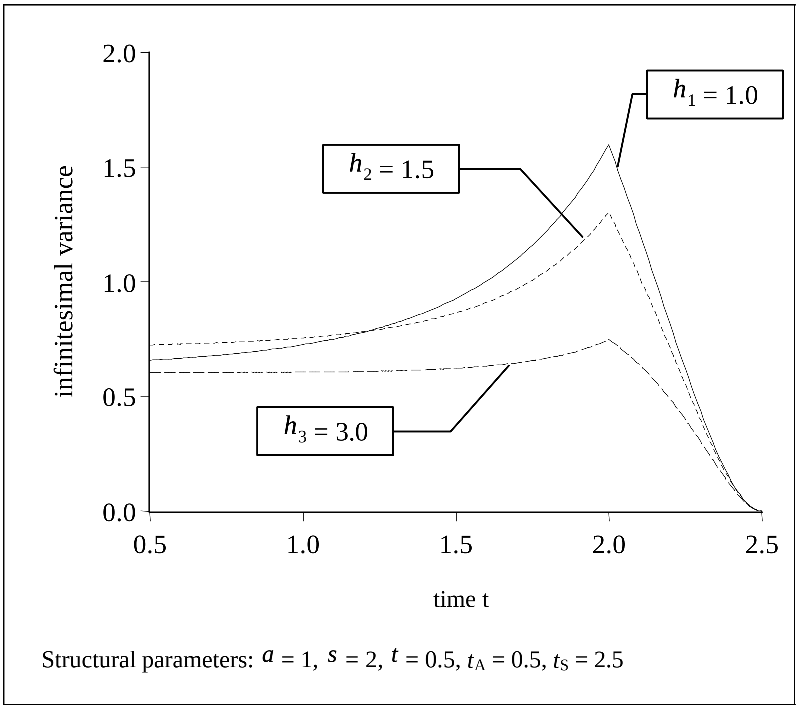

Figure 5: Crossing variance-paths for alternative η-values

the intervention parameter η.

Several methods of estimating continuous-time processes from discrete data have been proposed in the literature. One technique is the method of Simulated-Moments (SM-estimation) which is described in McFadden (1989), Pakes and Pollard (1989) or Lee and Ingram (1991). The use of this method has been proposed by Smith and Spencer (1992) and applied by Lindberg and Söderlind (1992, 1994) to estimate structural parameters in continuous-time models of exchange rate target zones. However, Wilfling (2001, pp. 190) argues that—in contrast to target zones—the application of the SM-technique for estimating the above interest rate differentials seems highly questionable. The econometric reason is that all interest rate differential paths directly depend on the time index t. Thus, suitably discretized counterparts of the continuous paths, which are necessary to perform SM-estimation, are non-stationary and hence non-ergodic. But both properties—stationarity and ergodicity—are essential to guarantee statistical consistency of the SM-estimators and to derive their asymptotic distributions. For alternative methods of estimating continuous-time processes from discrete data see Hansen *et al.* (1998), Darolles and Gouriéroux (2001), Elerian et al. (2001) and the literature cited there. However, the statistical properties of these alternative approaches are far from being settled by now. Applications as well as statistical comparisons of the estimation procedures mark a challenging task for future econometric research.

Apart from these statistical difficulties the complete term structures of interest rate differentials developed and analyzed here provide several useful qualitative insights into interest rate dynamics prior to a monetary union. For example, further properties of the interest rate equalization process at the end of the interim period may be investigated from the ID_2-paths (14) and (19) under each pre-switch regime.

An additional result on interest rate volatility under a managed-float that has not been explicitly mentioned so far is depicted in Figure 5. The solid and the shortly dashed variance paths provide an example of the impact of central bank intervention activities—primarily aimed at stabilizing the exchange rate during the interim period—on interest rate volatility. Evidently, a modest intensification of intervention activities from $\eta_1 = 1.0$ to $\eta_2 = 1.5$ would, *ceteris paribus*, entail an increase in interest rate volatility in the short run while in the middle and in the long run volatility is reduced. On the other hand, the increase of intervention activities on a larger scale (e.g. from $\eta_1 = 1.0$ to $\eta_3 = 3.0$) leads to a volatility decrease for the rest of the interim period.

4 Concluding remarks

Based on a monetary flex-price exchange rate model this paper derives closed-form solutions of the term structure of interest rate differentials when the economies under consideration plan to form a monetary union. Two alternative systems of floating exchange rates prior to entrance into the union are considered, namely a free-float and a managed-float pre-switch regime. The respective term structures of interest rate differentials are compared with respect to volatility properties. The economic significance in exploring differential volatility stems from the fact that for domestic investors of a small open domestic economy, which—under perfect capital mobility—faces an exogenous term structure of world interest rates, the volatility of interest rate differentials may serve as an important prediction of the volatility of domestic interest rates.

Under a free-float pre-switch regime there exists a clear-cut relationship between the volatility of interest rate differentials and the term. At the beginning of the interim period lower-term differentials exhibit less volatility than longer-term differentials while the reverse is true on an exactly specified time interval at the end of the interim period. Under a managed-float this volatility ranking no longer holds in general. This lack is due to the intervention parameter η.

Evidently, the possibility of central-bank intervention—aimed at stabilizing the exchange rate prior to the fixing—introduces strong parameter non-linearities into the dynamics of interest rates prior to the switch. These non-linearities give rise to a wide spectrum of possible volatility processes prior to the introduction of the common currency and therefore may have significant impacts on financial applications. The problem may aggravate if the central banks decide—for whatever reasons—to change the intensity of their intervention policy. Clearly, frequent changes in the parameter η introduce further complexities into the evolution of interest rate volatilities.

Another insight concerns the singular role of instantaneous interest rates whose importance for the theory of bond and option pricing are well documented in the literature (see e.g. Cochrane 2000, Part III). In contrast to all strictly positive-term interest rate differentials, instantaneous differentials can remain excessively volatile under an exchange rate arrangement which is characterized by extremely high leaning-against-the-wind interventions.

Finally, the paper discusses econometric issues. Two challenging tasks for future research consist of developing new as well as gathering experience with existing econometric techniques for estimating and drawing statistical inference from interest rate volatility processes during this transitional period between the exchange rate systems.

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