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## Transportation Research Forum

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# Investigating Mixed Logit Analysis of Critical Headways at a Single-Lane Instrumented Roundabout

by Alex Hainen

*This paper examines 29,403 entering vehicles that rejected two or more headways for a total of 69,123 rejected headways. A detailed series of temporal parameters was established and used to estimate a mixed binary logit model and understand rejection/acceptance decisions. This technique allows for the parameter estimates to vary across the population and across the set of decisions that drivers made and suggests that drivers may modify their critical headway as they wait at the yield bar. The results from this paper indicate that future consideration of capacity using a dynamic critical headway could be useful in modeling and capacity estimation.*

## INTRODUCTION

Roundabouts have gained much popularity and usage in the United States over the past decade. As designers and planners start to consider them as an alternative, much effort has been spent in capacity analysis during the design phase. One popular analysis technique is to use microsimulation. Microsimulation involves running a virtual model of the intersection or facility under user defined conditions and recording the observed performance. The critical headway is a very important setting for microsimulation. Critical headway is the minimum amount of time between circulating vehicles that a driver entering the roundabout will choose to proceed.

Critical headway at roundabouts has been studied for decades. Some of the earliest work on the subject was conducted in the 1970s in the UK and has evolved over time (Kimber 1980). Other work over time has included studies by Troutbeck (1992), Wu (2012), Raff and Hart (1950), Sieglöch (1973), Polus et al. (2005), Pimentel et al. (2013), and Gazzari et al. (2012). One of the popular methods that emerged was the logit analysis, which predicts the probability of accepting a headway as  $P_a = \exp(U_a) / (1 + \exp(U_a))$  where  $U_a$  is a utility function based on the circulating headway and the waiting time at the yield bar (Hewitt 1983). These equations can be calibrated in the field and the critical headway is then identified as the headway that is acceptable to half the drivers. In other words, the critical gap is identified when  $P_a = P_r$  (where  $P_r$  is the probability of rejecting a headway, or  $P_r = 1 - P_a$ ). This model can also be extended with a more robust mixing formulation discussed further in the methodology section.

Another popular critical headway estimation technique is the maximum likelihood approach (Tian, et al. 1999). This method identifies each driver's critical headway by comparing the largest rejected headway and the accepted headway by assuming a probabilistic distribution. Troutbeck (1992), Brilon et al. (1999), and Weinert (2000) have studied the impact of different distributions in their works. Another analysis by Wu (known as equilibrium of probabilities) is also used to estimate the critical headway. The equilibrium of probabilities uses a macroscopic model that doesn't need an *a priori* assumption about the distribution as required in the maximum likelihood method. Each of these techniques have numerous papers by their authors.

The fundamental challenge with the critical headway is that it cannot be directly measured and is, instead, a latent value that must be estimated using various techniques. The critical headway has been traditionally estimated as a fixed value and does not accommodate change by driver or by headway sequence. In other words, the critical headway for drivers does not change as drivers wait

at the yield bar to enter the roundabout even though the drivers are likely assembling short-term observations to calibrate their own critical headway. However, drivers waiting at the yield bar may be subject to several sequential headways where they will learn and change their decision making on accepting or rejecting a headway. To accommodate this phenomenon, mixed logit analysis is used in this paper to understand additional factors and reveal information about the driver decision-making process.

### OBJECTIVE

The objective of this paper is to understand how the critical headway may be changing as drivers wait to enter a roundabout and variations across the sample population. As drivers wait at the yield line, each sequential headway is an opportunity for drivers to calibrate themselves to current traffic flow. This paper examines how drivers are using observations and other pieces of information to adjust their critical gap. Technological advances allowed a single-lane roundabout to be instrumented and observe a large sample of nearly 100,000 acceptance and rejection decisions of headways over six weeks. Mixed binary logit analysis is used to further support the notion that critical headways change as drivers wait at the yield bar. This analysis will help shape future estimation of critical headways using contemporary modeling techniques. The characteristics identified in the model can be considered by researchers within a simulation environment to enhance microsimulation analysis at roundabouts.

### DATA COLLECTION

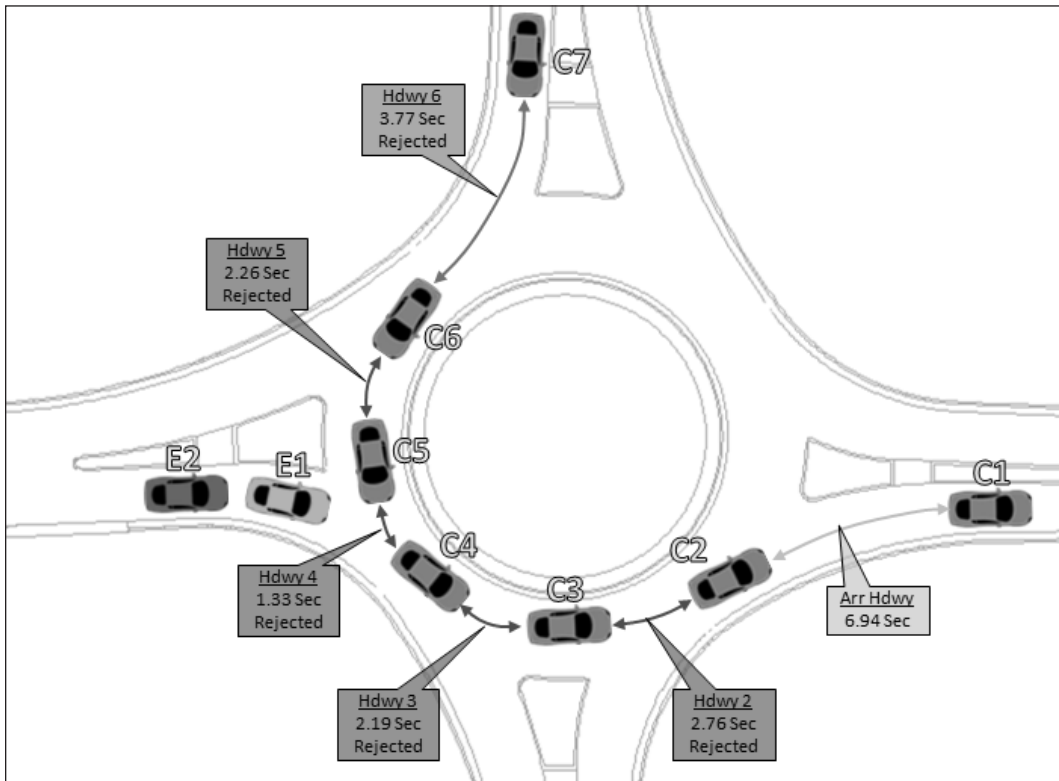
The single-lane roundabout at W 106<sup>th</sup> St, Spring Mill Rd in Carmel, IN, was instrumented with 12 wireless magnetometers to provide vehicle detection at the entrance, exit, and circulating path of each of the four approaches (Hainen, et al. 2013). This roundabout has been in operation for over 10 years and the driver population is considered experienced with roundabouts (Carmel, IN, is a community with over 70 roundabouts). The sensor layout is shown in Figure 1, where the two-letter label indicates (1) the approach and (2) the sensor position (for example, “We” indicates the west approach entering sensor). Sensors were field-located and installed between the wheel tracks of the vehicle paths (redundant sensors were placed wide outside of the wheel tracks, but not necessarily based upon matching data with the primary sensors more than 99.5% of the time). Detection records were recorded over six weeks from mid- July to late August, 2012. The roundabout is in a residential area and video data for the first two weeks were analyzed to confirm minimal truck traffic (much less than 1%). Due to the expensive equipment and complex installation, this was the only roundabout instrumented for this study.

The wireless magnetometers work similarly in logical operation to a traditional inductive loop detector. When a vehicle occupies the detector, an “on” state is noted and logged to the nearest millisecond. When the vehicle leaves the detector, an “off” state is noted and also logged to the nearest millisecond. (There are a few other detector diagnostic statuses in the data, but the “on” and “off” records are all that are required for the analysis in this study.) Figure 2 shows an actual field-documented and recorded example where vehicle E1 is waiting to enter the roundabout. Vehicle E1 rejected five headways (including the arrival headway) and accepted on the sixth headway. This indicates that a headway of 3.77 seconds was larger than the critical headway for driver E1 where the driver determined there was enough room to enter the roundabout. Figure 3 shows this particular example as synthesized with video data. It is important to note that the video was not used for data reduction as in many past studies and that the video is only used to confirm the detector data. The sample sensor data are shown in Table 1. These data are then reduced to a series of headways and decisions.

**Figure 1: Sensor Layout at the Roundabout at Spring Mill Rd @ W 106th St**



**Figure 2: Example of a Vehicle Waiting to Enter the Roundabout**



One important note for the examples in Figure 2 and Figure 3 is that the arrival time was also likely larger than the critical headway. For this particular case, the driver arrived at the yield bar at nearly the same time as the first circulating vehicle (C1 in Figure 2) arrived and thus the entering vehicle E1 had to yield. This dynamic of arriving vehicles and circulating vehicles is dependent on many parameters. Since the aim of this paper is to evaluate how critical headway is changing over time, the final reduced set of data used in the models only considers vehicles that, at a minimum, rejected headway #2. This ensures that each entering vehicle came to a stop and that drivers assessed a minimum of two headways before accepting. Since the first rejected headway upon arrival is unbounded, it was not used in the data set. The second rejected headways for these vehicles were used along with subsequent rejected headways to build the final data set.

**Figure 3: Video Observation of Example Vehicle Waiting to Enter the Roundabout**



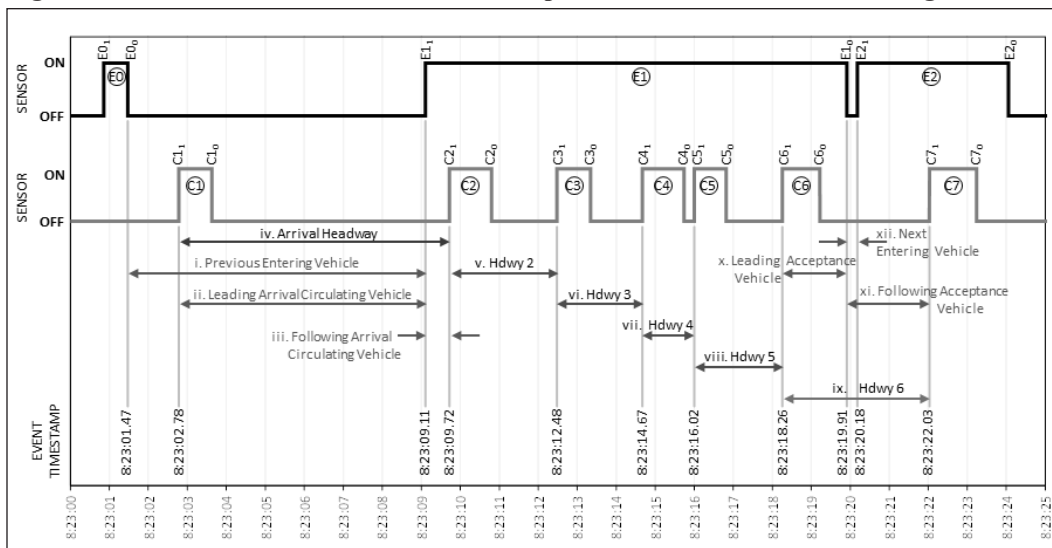
**Table 1: Sample Sensor Data from Example Vehicle Waiting to Enter the Roundabout**

SENSOR	STATUS	VEH-ID	CODE	TSTAMP
Entering	ON	0	E01	7/12/2012 08:23:00.86
Entering	OFF	0	E00	7/12/2012 08:23:01.47
Circulating	ON	1	C11	7/12/2012 08:23:02.78
Circulating	OFF	1	C10	7/12/2012 08:23:03.63
Entering	ON	1	E11	7/12/2012 08:23:09.11
Circulating	ON	2	C21	7/12/2012 08:23:09.72
Circulating	OFF	2	C20	7/12/2012 08:23:10.80
Circulating	ON	3	C31	7/12/2012 08:23:12.48
Circulating	OFF	3	C30	7/12/2012 08:23:13.34
Circulating	ON	4	C41	7/12/2012 08:23:14.67
Circulating	OFF	4	C40	7/12/2012 08:23:15.74
Circulating	ON	5	C51	7/12/2012 08:23:16.02
Circulating	OFF	5	C50	7/12/2012 08:23:16.82
Circulating	ON	6	C61	7/12/2012 08:23:18.26
Circulating	OFF	6	C60	7/12/2012 08:23:19.22
Entering	OFF	1	E10	7/12/2012 08:23:19.91
Entering	ON	2	E21	7/12/2012 08:23:20.18
Circulating	ON	7	C71	7/12/2012 08:23:22.03
Circulating	OFF	7	C70	7/12/2012 08:23:23.24
Entering	OFF	2	E20	7/12/2012 08:23:24.06

### Assembly of Records for Modeling

The raw sensor data were turned into a series of variables that could be used for modeling. An example of the reduced data for vehicle E1 is shown in Figure 4. The upper square wave shows the ON/OFF status for the entering sensor and the lower square wave shows the ON/OFF status for the circulating sensor. By referencing key times, a series of temporal variables is established to generate a series of records for each rejected/accepted headway. Headway was calculated as the time from the “on” event of vehicle  $n$  to the “on” event of vehicle  $n+1$  for the circulating sensors (items “iv” to “ix” in Figure 4). The delay of the entering vehicle was calculated from the “on” time to the “off” time of the entering sensor for each entering vehicle. By pairing both the entering and circulating records, the number of headways each entering vehicle rejected was observed along with the magnitude of each headway. From this set, each headway could be used to build a cumulative average, minimum, and maximum rejected headways. These are important variables that summarize the decisions that a driver made while waiting.



**Figure 4: Visualization of Sensor Data and Temporal Variables Used for Modeling**

Other key temporal variables include the position of the entering vehicle in time relative to both of the *circulating* vehicles for the arrival headway (items “ii” and “iii” in Figure 4) and both of the *circulating* vehicles for the acceptance headway (items “x” and “xi” in Figure 4). Lastly, temporal variables describing the entering vehicle E1 position relative to leading and following *entering* vehicles were compiled. Item “i” in Figure 4 shows that substantial time had passed between the previous entering lead vehicles E0 and E1. This indicates that vehicle E1 was not waiting in a queue (this is important since queued vehicles may be pre-calibrated by observing headways of the leading entering vehicle as they wait in the queue). Also, item “xii” in Figure 4 shows the time between vehicle E1 entering the roundabout and the next entering vehicle arriving at the yield bar. This information indicates that, in this example, E1 had vehicle(s) queued behind waiting. This was hypothesized to add to driver distraction (realizing that vehicles were pulling up behind) and also driver pressure as they felt more urgent to accept a headway on behalf of entering for queued vehicles. This move up time is discussed in detail in NCHRP Report 572 (2001).

The final record set is summarized in Table 2. Each headway is a record and includes temporal information from some of the other headways experienced by the driver for a set. These data were also combined with entering, exiting, and circulating volumetric information, which is also pertinent information that drivers will leverage while making rejection/acceptance decisions.



**Table 2: Assembly of Data Records for Example Vehicle Waiting to Enter the Roundabout**

Observation Vehicle	Headway Number	Headway (Seconds)	Accepted headway indicator (1 = true, 0 = otherwise)	i. Time between previous entering vehicle (seconds)	ii. Time between leading arrival circulating vehicle (seconds)	iii. Time between the following arriving circulating vehicle (seconds)	iv. Arrival Headway (Seconds)	Cumulative average rejected headway from headway 2 to headway i-1 (seconds)	Cumulative maximum rejected headway from headway 2 to headway i-1 (seconds)	Cumulative minimum rejected headway from headway 2 to headway i-1 (seconds)	Cumulative delay (seconds)	ix. Accepted headway (seconds)	x. Time between the following the leading acceptance headway vehicle and entering the roundabout (seconds)	xi. Time between entering the roundabout and the following acceptance headway vehicle (seconds)	xii. Time between entering the roundabout and the next entering vehicle arrival at the yield bar (seconds)
E1	2	2.76	0	7.64	6.32	0.62	6.94	--	--	--	3.37	3.77	0.92	2.85	0.27
E1	3	2.19	0	7.64	6.32	0.62	6.94	2.76	2.76	2.76	5.56	3.77	0.92	2.85	0.27
E1	4	1.35	0	7.64	6.32	0.62	6.94	2.47	2.76	2.19	6.89	3.77	0.92	2.85	0.27
E1	5	2.24	0	7.64	6.32	0.62	6.94	2.10	2.76	1.35	9.15	3.77	0.92	2.85	0.27
E1	6	3.77	1	7.64	6.32	0.62	6.94	2.14	2.76	1.35	10.06	3.77	0.92	2.85	0.27

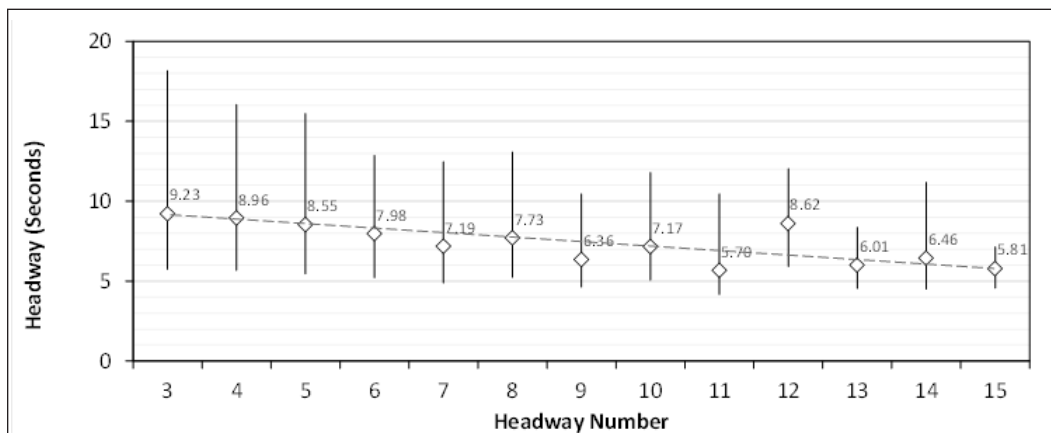
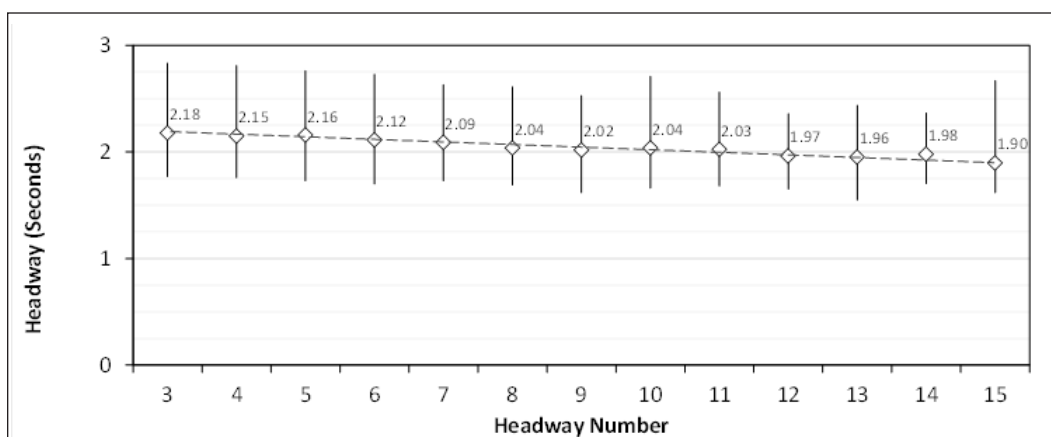
\* Small Roman numerals (i., ii., ...) correspond to figures. Not all variables available for modeling.

## ANALYSIS AND RESULTS

Starting with empirical observations, stock plots based on the field-measured data for both the accepted and rejected headways are shown by headway sequence number (Figure 5). Each vertical bar represents the spread of the 25<sup>th</sup> and the 75<sup>th</sup> percentile headways, and the diamond marker represents the median headway. These figures provide some very intuitive evidence that the headways, by sequence, decrease as drivers wait. Since headway #2 was only considered for vehicles that rejected a minimum of two headways, the headways sequence starts on the third headway.

Decreasing acceptance headways (Figure 5a) means that drivers are willing to lower their critical headways a bit after rejecting several headways. This is also dependent on prevailing conditions at the roundabout where, under lighter conditions, drivers are able to accept larger headways in the earlier sequences, whereas drivers will feel forced to accept a much smaller headway during busy periods as they wait (this was clearly observed in the raw detector data and thus reflected in the model estimation).

With regards to the rejected headway decisions, the average rejected headway as drivers wait through a sequence of headways also decreases as sequence number increases (Figure 5b). This means that drivers are more discerning when they adjust their critical headways and only the tightest headways (headways that are now known to the driver to be extremely close to the critical headway) are rejected later on. This first-order magnitude, empirical analysis demonstrates that both the accepted and rejected headways are decreasing as the headway sequence increases.

**Figure 5: Empirical Stock Chart Distributions (Quartiles 1, 2, and 3) of Headways by Headway Sequence (with Frequency Table)****a) Decreasing Mean and Interquartile Range of Accepted Headways****b) Decreasing Mean and Interquartile Range of Rejected Headways**

Headway	2	3	4	5	6	7	8	9	10	11	12	13	14	>=15
Reject	15,152	7,568	4,016	2,267	1,376	886	608	424	313	230	178	136	102	77
Accept	N/A	7,584	3,237	1,586	809	439	252	156	104	71	49	37	31	88

**c) Count of Rejected and Accepted Headways in the Final Dataset**

As previously mentioned, the critical headway is recognized to be a latent value that cannot be directly observed. Turning again to Figure 5, the difference in the median accepted headway for headway sequence number 3 (9.23 seconds in Figure 5a) and the median rejected headway for headway sequence number 3 (2.18 seconds in Figure 5b) encompasses the actual critical headway. As the headway sequence increases, the difference between the median accepted headway and median rejected headway decreases. For example, the median accepted headway for headway sequence number 15 (5.81 seconds in Figure 5a) and the median rejected headway for headway sequence number 15 (1.90 seconds in Figure 5b) start to converge around the traditionally estimated critical headway value of 3.5 to 4.5 seconds. Traditional estimation techniques from Troutbeck (1992) and Wu (2012) may be used to estimate the critical headway, but this dataset lends itself to further analysis for understanding the driver decision-making process.

## Mixed Binary Logit Analysis

In past estimation approaches by other researchers, binary logit analysis was an early technique used to estimate the critical headway. A binary logit model uses parameters to predict the probability of a driver making a discrete choice to either accept or reject a headway. Again, a utility function can be defined as:

$$(1) \quad U_n = \beta_i X_{in} + \varepsilon_n$$

where  $X_{in}$  is a vector of data that characterizes the circumstances of a particular headway decision making instance.  $\beta_i$  is a vector of estimable parameters and  $\varepsilon_n$  is an error term. The estimation of  $\beta_i$  is done using maximum likelihood (Washington, Karlaftis, and Mannering 2011). The multinomial logit model (generalized formulation of the binary logit model) is based on McFadden's assumption that the error term  $\varepsilon_n$  in the utility function is distributed as a type 1 generalized extreme value, sometimes referred to as the Gumbel distribution (McFadden and Train 2000). The formula for a generalized multinomial logit then becomes:

$$(2) \quad P_{in} = \frac{e^{\beta_i X_{in}}}{\sum_{\forall I} e^{\beta_i X_{in}}}$$

For the reduced binary decision case where only two choices are available (to accept headway or reject a headway), the model can be reduced to the binary logit form shown below. The equation for the binary logit shown below also includes a mixing function:

$$(3) \quad P_{in} = \int \frac{1}{1 + e^{\beta_i X_{in}}} f(\beta|\varphi) d\beta$$

Where  $P_{in}$  is the mixed logit probability, which is a weighted average about the density function  $f(\beta|\varphi)d\beta$  over varying parameter estimates (McFadden and Train 2000). For the mixing function, the  $\beta$  is the mean and the  $\varphi$  is the standard deviation of the parameter distribution. This mixing function allows the parameter estimates to vary over the sample data set instead of being fixed for all samples. A normal distribution was used for estimation in this analysis, but a variety of distributions could be used as the analyst determines is appropriate. Model estimation based on maximum likelihood was conducted using Halton draws or quasi-random selection for generating search space efficiently (Halton 1960).

## Mixed Binary Logit Results

The mixed binary logit model was estimated using simulation-based maximum likelihood, and the results are shown in Table 3. Statistically significant variables were added based on the results of models that used different subsets of the variables. While models were estimated using the full 6-week dataset, a reduced dataset of 47,975 records was used to estimate the distributions of the random parameters due to software limitations (the difference in the distribution of variables was statistically insignificant, so consistent parameter estimates hold true). Estimates for fixed parameters are shown along with their  $t$ -statistics. It should be noted that large  $t$ -statistics are a function of large sample size (which is also the reason that all parameter estimates are significant at the  $\alpha=1\%$  level). For the random parameters, the means and standard deviations of the mixing distributions are included. The elasticities (and pseudo-elasticities for indicator variables) are shown in Table 4. These indicate the change in the probability of accepting a headway for a 1%

change (or one-unit change for indicator variables) in the independent variables (these are average values and will vary for the random parameters across the population). Variables were added and removed in a forward-selection fashion.

**Table 3: Mixed Binary Logit Estimation Results for Headway Acceptance**

Variable	Parameter estimate (Standard Deviation)	t-Statistic
Constant	-19.44	-47.84
Headway (seconds)	3.16 (1.61)	53.43 (52.12)
Headway sequence number ( <i>number of rejected headways – 1</i> )	0.35	23.13
Cumulative yield bar delay greater than 10-seconds ( <i>1=true, 0=otherwise</i> )	-2.77	-27.45
Previous one-minute circulating volume in front of the approach	0.035	4.82
Previous one-minute entering volume for the entire roundabout	0.029	5.83
Previous one-minute circulating volume at the upstream approach	0.039	4.82
Cumulative average rejected headway ( <i>from headway 2 to headway<sub>n-1</sub></i> )	1.59	43.97
Time (in seconds) between arriving at the yield bar and the previous entering vehicle leaving the yield bar	-0.028	-11.63
Time (in seconds) between leaving the yield bar and the next entering vehicle occupying the yield bar	0.020	6.34
Time (in seconds) between the entering vehicle arriving at the yield bar and the first circulating vehicle passing in front of the approach	-0.021 (0.36)	-8.01 (15.19)
PM peak-hour indicator ( <i>1=true, 0=otherwise</i> )	0.64	7.80
Weekday (Monday-Friday) indicator ( <i>1=true, 0=otherwise</i> )	0.38 (0.18)	2.10 (4.31)
Sample size, <i>n</i> ( <i>reduced set for distribution estimation</i> )		47,975
Log-likelihood		-7990.06

Turning to variable analysis, a positive parameter estimate indicates that drivers are less likely to reject a headway, and a negative sign suggests that a driver is more likely to reject a headway. The most pertinent information drivers use during the accept/reject decision-making process is (1) the size of the headway under consideration, (2) how many headways have been rejected, and (3) how long the driver has been waiting. The parameter for headway (in seconds) intuitively indicates that larger headways are more likely to be accepted. The random parameter aspect suggests that there are many other factors that will change the way a given headway looks to drivers depending on how long they've been waiting, how many headways they've rejected, and many other factors further discussed in the model. The fact that this variable has a distribution strongly indicates that a dynamic process of adjusting a driver's critical headway is evident, and the additional model variables help to identify some of these mechanisms. For item (2), as the number of rejected headways increases, the probability of accepting a headway increases. This is intuitive as drivers perceive each rejection as a unit that cumulatively increases the probability of accepting a headway. Also, drivers will be able to leverage the information from each rejected headway to better calibrate themselves where they'll be more likely to accept a headway. Finally, for item (3), sensitivity analysis was used to identify that a binary indicator variable of waiting more than 10 seconds at the yield bar was found to be highly significant for drivers where they will be less likely

to accept a given headway. This may be due to conservative drivers who are willing to wait longer for an acceptable headway, or perhaps drivers who may have had sufficient time to identify a more acceptable headway further upstream where they desire to wait for a desirable headway.

**Table 4: Elasticities for Mixed Binary Logit Model of Headway Acceptance**

Variable	Elasticity
Headway (seconds)	0.0387
Headway sequence number ( <i>number of rejected headways – 1</i> )	0.00433
Cumulative yield bar delay greater than 10-seconds ( <i>1=true, 0=otherwise</i> )	-0.0339
Previous one-minute circulating volume in front of the approach	0.00043
Previous one-minute entering volume for the entire roundabout	0.00036
Previous one-minute circulating volume at the upstream approach	0.00048
Cumulative average rejected headway ( <i>from headway 2 to headway<sub>n-1</sub></i> )	0.0195
Time (in seconds) between arriving at the yield bar and the previous entering vehicle leaving the yield bar	-0.00034
Time (in seconds) between leaving the yield bar and the next entering vehicle occupying the yield bar	0.00024
Time (in seconds) between the entering vehicle arriving at the yield bar and the first circulating vehicle passing in front of the approach	-0.00262
PM peak-hour indicator ( <i>1=true, 0=otherwise</i> )	0.00783
Weekday (Monday-Friday) indicator ( <i>1=true, 0=otherwise</i> )	0.00467

Looking at volumes, when the previous one-minute circulating volume in front of the approach is higher, drivers are more likely accept a headway. During heavy traffic conditions, drivers likely feel added stress about how busy the roundabout is, and increased traffic causes drivers to be more observant and discerning where they will be more likely to accept a headway that could be questionable close to their critical headway.

The previous one-minute entering volume for the entire roundabout is another variable that contributes to the probability of a driver accepting a headway. The higher the previous one-minute total entering volume, the more likely a driver will accept a headway. There are a few mechanisms driving this. First, a higher previous entering volume at the roundabout indicates an increased ability of vehicles in general to enter the roundabout. This is somewhat tricky because more entering vehicles at other approaches can also become circulating vehicles in front of the approach under consideration. However, relatively higher upstream circulating volumes can be useful to help drivers calibrate their perception of headways as discussed in the previous paragraph and in the next paragraph.

The variable for previous one-minute circulating volume at the upstream approach suggests that higher upstream circulating volume increases the probability of accepting a headway. (This is in contrast to higher upstream entering volume, which was not significant). The difference is that drivers waiting to enter the roundabout are able to better observe the circulating headways further out and have a longer time to observe the headway as they are presented with the option to accept. Also, it may be easier to gauge an exiting decision of a circulating vehicle.

Another very important finding is that a higher cumulative average rejected headway increases the probability of accepting a headway. This suggests that if drivers have accumulated a relatively large cumulative average rejected headway, they are more likely to recognize headways where they could have entered and thus will be more prone to accept the current headway under consideration. This makes sense from a driver perspective where rejecting a few large (and possibly acceptable)

headways will cause the driver to be more likely to accept a headway as they calibrate and lower their critical headway.

The variables for the time between two entering vehicles at a given approach are also important. The more time (in seconds) between arriving at the yield bar and the previous entering vehicle leaving the yield bar, the less likely the probability of accepting a headway. This shows that drivers waiting in a queue tend to observe the headways of the car they're waiting behind. This is an opportunity for driver's to calibrate their headway observations before they're waiting at the yield bar. If the time is large (indicating that the driver wasn't in a queue), then they won't have information ahead of time (this was the case shown in Figure 4 with vehicle E0 preceding vehicle E1 with a relatively large amount of time, where E1 is unlikely to have spent time waiting in queue behind E0).

A second variable relating sequential entering vehicles is the time (in seconds) between leaving the yield bar and the next entering vehicle occupying the yield bar (or move-up time according to NCHRP 572). The more time between the next following entering car, the more likely that a driver will accept a headway. This information shouldn't be used directly for analysis since this can't be known at a given headway  $n$ , but it can be used as a proxy for the probability that the entering vehicle was in front of a queue and had entering vehicles queued behind (an example is shown in Figure 4 with E2 following very closely behind E1). Such a queue could cause the driver to feel more pressure and distraction than if there was no queue.

Another variable that is an important spatial relationship is the time (in seconds) between the entering vehicle arriving at the yield bar and the first circulating vehicle passing in front of the approach (item "iii" in Figure 4). If there is more time between arrival at the yield bar and the first circulating vehicle passing in front of the approach, this will be seen by the driver as an instance where they might have been able to enter. This is really their first frame of reference for observing time and headways at a roundabout, so it's important for the initial adjustment of a critical headway, and the magnitude will depend on each vehicle's relative positioning.

Finally, two time of day/week variables were found to be significant. An indicator for the PM peak-hour was estimated and shows that drivers are more likely to accept a given headway during these conditions. While at first this may seem only attributable to heavier volumes, most of the decision-making component related to volumes is captured in other variables and this PM peak-hour may be capturing other driver behavior during a stressful period. Also, a binary indicator variable for weekdays was found to be significant and, on average, increased the probability of accepting a headway. This is likely due to heavier traffic and a higher value of time during the work week (Monday-Friday) period where drivers are less likely to accept a headway during the weekend period (or rather stated that drivers tend to have a higher critical headway).

## CONCLUSIONS

This paper provides very important insight about the critical headway and the decision making process drivers face when accepting or rejecting headways at a single lane roundabout. The data collection process, a major advancement over past studies, used video collection over a limited time frame. This data collection process observed 29,403 vehicles 24-hours a day over six weeks.

1. Based on empirical observations and traditional critical headway estimation techniques, there is evidence that the critical headway changes across drivers and headways as a driver waits at the yield bar. This is important information that can be used to enhance existing models.
2. The median accepted headway shows a consistent trend of decreasing over time. This suggests that drivers' critical headway value is changing based on each additional rejected headway they sit through. The median rejected headways confirms that drivers eventually reduce their critical gap and are less likely to reject longer headways as they wait.

3. Mixed binary logit analysis was used to assess different variables that affect drivers' decisions. Each of these variables could be included in microsimulation to better predict the acceptance/rejection decision for headways.
4. By using a dynamic critical headway, microsimulation and other modeling approaches could cause capacity analysis to be improved. Using a more accurate estimate of critical headway over time spent waiting at the yield bar has implications of additional capacity at roundabouts. Drivers could be modeled as accepting a headway earlier in the circulating stream sequence based on lowered critical headways.

The discussions and conclusions identified by the model are important findings for traffic engineers. Future work will include incorporating these results into calibration of a microsimulation model and also exploring other yield situations where similar technology and methodology can be used. In particular, multilane roundabouts should be examined but will require additional data reduction techniques from the sensors.

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