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Filtering with search

Bart Taub

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Filtering with search

Bart Taub

This research is extremely preliminary. This paper subsumes and extends (8). The subject of this paper is assigned to HWWA's research program, "Internationalization of Firms and Labor Markets". I am grateful for comments from HWWA seminar participants.

HWWA DISCUSSION PAPER

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Abstract

A firm monopsonistically hires labor from a pool containing both skilled and unskilled workers. The marginal value of a worker depends on the match between the job and the worker’s skill level. Unskilled workers can have negative productivity if they are placed in a skilled job. The firm cannot distinguish the two types.

The workers are initially dispersed and search for the high wage jobs from the firm. The workers’ skill levels are correlated with their patience; equivalently, they obtain indirect benefits, such as non-firm-specific career capital, from jobs that use their skill appropriately. By judiciously choosing different wages for different types of jobs, the firm can partially filter the appropriate worker types and match them with the appropriate jobs. This mechanism works because the probability structure of the job offers changes as searchers accept jobs. This entails a delay in hiring workers who search, but the benefits from filtering that are requisite with the delay can exceed the benefits of hiring all workers immediately without filtering. The wage differentials assumed in standard search models are therefore motivated.

JEL classification: C73, D83, E24, J64

1. INTRODUCTION

This paper considers the situation in which there are classical job searchers, but the distribution of jobs changes as acceptances occur because new jobs do not replace the old jobs. As a result, workers are much more eager to accept low-wage jobs rather than wait for high-wage jobs, because the high-wage jobs are quickly depleted. In the extreme case where the number of jobs and workers are equal, all jobs are accepted immediately.

Such a construct can become a conscious strategy by an employer or supplier of search goods, such as airline seats. If searchers are of different types, and if the types are private information, the employer or firm wishes to identify types so as to appropriately match them to jobs or goods. By choosing wages judiciously, the types will partially sort themselves and match to appropriate jobs.

For example, a firm might have two types of job: unskilled (U) and skilled (S). The marginal product of a skilled worker will be higher in the skilled job, but the marginal product of an unskilled worker in the skilled job is actually lower than in the menial job. Yet the employer cannot identify types by direct observation. Moreover, he prefers not to "find out the hard way" by mis-assigning a worker to a job, as this entails adjustment costs.

I will simplify this story (which is surely exaggerated in the sense that some direct filtering is possible, and it is pretty easy to separate unskilled types from skilled types most of the time). I will assume the firm can never figure out a person's type directly, but it does know the payoff from the composition of types of its many employees. If it hires too many unskilled types in skilled jobs, it will observe the fraction of unskilled types in those jobs and productivity will shrink. This story is not completely fanciful in that in large firms there are many unobservable interactions between employees and an overall productivity associated with the group, not individuals.

If there is a correlation between productivity type and patience, then a way to separate the two types is to offer two separate wages that searchers encounter randomly. The assumption of the correlation of patience and skill is reasonable if skill is interpreted as a kind of capital; equivalently, workers obtain indirect benefits, such as non-firm-specific career capital, from jobs that use their skill appropriately. The patient, high-productivity types will wait for the high-wage jobs, while the low-productivity searchers will take anything, or at least be less patient. The firm can therefore sharpen its probability of the worker's type by looking at the wage accepted by the worker.

Some low-productivity workers will still end up in high-wage jobs. However, there will be fewer than if only a single wage were offered. As a result of this discrimination, the firm has a better, though still imperfect, mix between worker type and job type. There is a cost associated with this improved discrimination: the firm must wait for the high wage workers to accept jobs. In order to merit paying this cost, it must be the case that mis-matches are very costly: a skilled worker taking an unskilled job generates an opportunity cost from his failure to be matched to a high-productivity job, and a low-skill worker matched with a skilled job is very costly in terms of productivity.

A second milieu in which this mechanism could operate is airline seat pricing. Here the types are business versus casual travelers. Business travelers pay higher fares, but also cost more in the sense that they want to fly at peak times and demand to be pampered more. Putting a casual flyer in a business-traveler's seat is therefore costly in the sense of foregone revenue and higher cost—casual travelers don't respond to pampering with repeat business. One place where the analogy breaks down however, is that airline seats can be explicitly divided into peak and off-peak times. This enables a division of the market by means other than price; there are then effectively two markets.

The mechanism provides a rationale for the existence of wage disparities: they spring from the unobservability of types. The model has enough structure to have as an element of any empirical test this overall hypothesis. Markets in which there is wide disparity in pricing should also have more extensive private information, and vice versa. In addition, markets with this feature should also undergo an evolution of the prices of trades—that is, as the market gets depleted, there should be more higher-wage acceptances for example.

Another empirical attribute of markets with wage or price disparity is that there should be an excess of jobs over workers in markets where there is a lot of privacy of information about types. In that situation, the high wage types are more likely to wait, as the high-wage jobs don't get depleted as fast as they would if there were an equal number of workers and jobs. Thus there will be a surplus of jobs (or airline seats) at the end. In the case of airline seats this is a real cost that the firm must pay for its ability to discriminate.

The next section of the paper sets out the mechanics of a simple search model in which job matches result in the removal of jobs from the set of searchable vacancies. The dynamics of the model are characterized by the ratio of low-wage vacancies to all vacancies. If this ratio is low enough, search occurs that is like the standard steady-state search model. If the ratio crosses a threshold, the gains from search cease and all offers are taken. In subsequent sections this phenomenon is used to induce two types of workers to behave differently in response to wage offers.

2. SEARCH WITHOUT REPLACEMENT

In this section there is a single type of worker, each of whom initially searches for employment. There are different wages, but the impetus for the wage differentials is left unexplained for the moment; the analysis will center on the behavior of the searching workers. The simplest structure is assumed: if a worker accepts a job, he holds it permanently, and there is no further decision-making in the interaction of the worker and the firm once the job is accepted.

Searching workers behave according to McCall-type [3] search as presented in [4], but when searchers accept a job, the job is removed from the distribution of jobs as well as the searcher being removed. The basic intuition is that the best jobs get taken first, a kind of cream-skimming. This leaves a pool of worse jobs on subsequent rounds, adding to the pressure to accept lower-payoff jobs sooner rather than later. In its most extreme form, the number of initial jobs matches the number of workers and all jobs are accepted on the first round.

There is an accounting issue regarding the removal of matched workers: if there were finitely many of them, which seems at first like an assumption necessarily accompanying the no-replacement assumption, it would be necessary to account for the randomness in the matching process. That is, if there are just two searchers, one high-wage vacancy and two low-wage vacancies, then it is possible that only low-wage offers will be made to the two searchers in some periods, while a combination of one low-wage offer and one high-wage offer will be made in other periods. In order to avoid this "small sample" problem it will be assumed that there is a continuum of searchers and also of vacancies. The no-replacement concept is then modeled as a reduction of the measure of searchers and jobs, with the law of large numbers assumption ensuring that the ratio of matches reflects the underlying distribution of wage levels in the vacancies.

An initial problem. Starting in period 1, workers search for jobs. There is a fixed number of jobs. Initially, the jobs have distribution $F_1(\cdot)$. After the first period, if a worker accepts a job at wage w, that job disappears from the system. Also, that worker stays employed at that wage permanently and no longer searches.

Since jobs and their associated wages are removed from the system, after the first period the distribution of wage offers available in the second period, $F_2(\cdot)$, includes only wages spurned by workers in the first period. The search process then continues in a similar way in each subsequent period until all workers have accepted an offer.

All workers are identical in their preferences and opportunities. Any firm making an offer has exactly one worker who considers it, and every searcher gets an offer. It might be intuitively helpful to think of the distribution F as uniform with support $[0, w_1^*]$.

The Bellman equation of the searcher in an arbitrary period t is

$$v_t(w) = \max \left\{ \frac{w}{1-\beta}, \beta \int_0^{w_t^*} v_{t+1}(w') dF_{t+1}(w') \right\}$$

where w_t^* is the reservation wage for period t. The reservation wage for period t removes all wages above w_t^* from the distribution of available wages at $t+1$—hence the mixed t and $t+1$ subscripts on the right hand side.

The period-t distribution of wages F_{t+1} is related to the period-t distribution F_1 is

$$F_{t+1}(w) = \frac{F_t(w)}{F_t(w_t^{*,e})}$$

where $w_t^{*,e}$ is the equilibrium reservation wage—that is, in equilibrium $w_t^{*,e} = w_t^*$. This shows how the cream gets skimmed in the current period by individuals who have the top offers.

PROPOSITION 1: *In a Nash equilibrium every searcher finds a job within a finite amount of time, namely in the initial period.*

PROOF: Suppose there is a common reservation wage w_t^*. Due to the Nash equilibrium construct, each worker takes as given the behavior of the other workers, and each knows that in equilibrium all workers with offers above w^* take the offers, and those jobs are removed in the next round. Consider a worker with offer $w^* - \Delta$. If he waits then his probable offer in the next period is less than w^*, and this is discounted. Its value will then be discounted if he accepts it later, but this is a lower payoff than immediate acceptance. This contradicts the existence of a positive w^*. ■

If two searchers could cooperate and spurn their offers simultaneously, the same reasoning would apply to the searcher with the higher current offer. The Nash concept is therefore not masking other interesting equilibria.

PROPOSITION 2: *The Nash equilibrium is efficient.*

PROOF: If an individual waited, there would be an unfilled job paying no-one. On the other hand, if individuals can contract ex ante to share risk, it is not efficient—but risk sharing would not change the efficiency of everyone taking a job immediately. ■

It should be remarked that this result would not necessarily hold under search with replacement, because output can be increased by maximizing the number of workers in high-paying jobs.

More jobs than searchers. Now let the number of jobs exceed the number of workers. Search proceeds as in the standard search with replacement model: searchers with low draws turn jobs down and continue searching. Because searchers with high offers accept them, the pool shrinks on the next round. When enough jobs have been taken the previous logic reappears, and all searchers accept all offers.

In order to analyze this case, the model will be simplified so that there are only two wages, w_L and w_H, $w_L < w_H$. There are N searchers, and $M > N$ jobs; in keeping with the earlier remarks about accounting issues, N and M are sets on a continuum. In the initial period there are pM w_L jobs and $(1-p)M$ w_H jobs. When a worker accepts a job, that job is no longer available: in other words, there is sampling without replacement. All the workers have the same ability and the same discount factor β. What condition must hold for the searchers to take more than a single period to find jobs?

Denote the population of remaining searchers at time t by N_t, the total number of low wage jobs at the beginning of the period by M_{Lt}, and the number of high wage jobs at the beginning of the period by M_{Ht}. Thus, $N_1 = N$, $M_{L1} = pM$, and $M_{H1} = (1-p)M$. Let $\alpha_t = N_t/M_t$ denote the ratio of active searchers to the current number of job offers, and assume that searchers are evenly distributed across job offers. Also, the probabilities are indexed by time as well: $p_1 = p$, etc. It will be helpful in the analysis that follows to contemplate conditions that must hold in the second-to-last period, if there is one.

In the next-to-final period $T-1$, searchers will survive into the next period if

$$v_{T-1}(w_L) = \beta\left(p_T\frac{w_L}{1-\beta} + (1-p_T)\frac{w_H}{1-\beta}\right) > \frac{w_L}{1-\beta}$$

That is, search will continue as long as the probability of getting the high wage in the terminal period is high enough. Solving for p_T here, we have

$$p_T < \frac{\beta w_H - w_L}{\beta(w_H - w_L)} \equiv P^* < 1$$

the latter inequality being straightforward. Prior to period T, only the high wage is accepted, so all we need to do is add up the time it takes to violate this inequality.

Before T, those who accept jobs are removed from the system.

$$1 - p_t = \frac{M_{Ht}}{M_{Ht} + M_L}; \qquad M_{Ht} = M_{H,t-1} - \alpha_{t-1}M_{H,t-1}$$

where $\alpha_t \equiv N_t/M_t$. Then

$$1 - p_2 = \frac{M_{H2}}{M_L + M_{H2}} = \frac{(1-\alpha_1)M_{H1}}{M_L + (1-\alpha_1)M_{H1}} = \frac{1-\alpha_1}{\frac{p_1}{1-p_1} + 1 - \alpha_1}$$

or

$$p_2 = \frac{1}{1 + (1 - \alpha_1)\frac{1-p_1}{p_1}}$$

Thus it is only necessary to back up from the main inequality to get search to last more than one period. Thus if $T = 2$, we have

$$\frac{1}{1 + (1 - \alpha_1)\frac{1-p_1}{p_1}} = p_2 < \frac{\beta w_H - w_L}{\beta(w_H - w_L)} \equiv P^*$$

or

$$p_1 < \frac{1 - \alpha_1}{1 - \alpha_1 P^*}$$

Dynamics. The wage-offer probability can be iterated forward at arbitrary times t prior to the final period:

$$p_{t+1} = \frac{1}{1 + (1 - \alpha_t)\frac{1-p_t}{p_t}} \tag{2.1}$$

The dynamics of α are as follows:

$$\alpha_{t+1} = \frac{N_{t+1}}{M_{t+1}} = \frac{p_t N_t}{M_t - (1 - p_t)N_t} = \frac{p_t}{\frac{1}{\alpha_t} - (1 - p_t)} \tag{2.2}$$

That is, the next period ratio of workers to jobs consists of the number of workers who received low offers in the current period to the jobs remaining after the high offers were accepted.

Thus (p_t, α_t) form a nonlinear system. There is some intuition for the dynamics of this system. First, simulations show that α (the ratio of active searchers to remaining total job offers) shrinks over time. This seems counterintuitive, but the reason is that the relative shrinkage of searchers to jobs is always larger than the relative shrinkage of jobs. If the number of jobs initially far exceeds the number of workers, then α is negligible. In that case, it will tend to stay small, and the probability of a low offer p_t will persist near its with-replacement value. As the number of high wage jobs decreases, though, there is an increase of p_t, the probability of a low offer, and eventually the threshold is attained, with a precipitous "collapse".

Stationarity of the difference equation system requires $p = 1$, and $\alpha \leq 1$. It is only necessary to demonstrate stability of the stationary solution. The derivative of the right hand side of (1) is

$$\frac{1 - \alpha_t}{(1 - \alpha_t(1 - p_t))^2}$$

At $p_t = 1$ this is a fraction as long as $\alpha > 0$, and hence the difference equation (1) is stable at $p = 1$. However if α approaches 0, then we only have metastability. Indeed, at $\alpha = 0$ equation (1) becomes an identity. Thus if α shrinks fast, p stabilizes and P^* might not be attainable.

A benchmark with-replacement threshold. In the with-replacement model, continuation at the low wage occurs if

$$v(w_L) = \beta \left(pv(w_L) + (1-p)\frac{w_H}{1-\beta} \right) > \frac{w_L}{1-\beta}$$

Solving for $v(w)$ and then for the threshold p yields

$$P^* = \frac{\beta w_H - w_L}{\beta(w_H - w_L)}$$

which is identical to the no-replacement P^*! Thus if α is small, individuals wait for good jobs for a long time, because acceptance of the good jobs does not affect α very much.

This means that there are some settings where individuals will wait until they get a high offer and never reach the threshold. Intuitively, think of ten workers searching a million jobs with $p_1 = .5$. They will never reach the threshold and are effectively in a with-replacement world.

3. THE FIRM'S PROBLEM

It will now be assumed that there is a single monopsonistic firm offering jobs and controlling the wages. There is a finite number of workers. Unlike the model in the previous section, there are now two types of worker, unskilled (U) and skilled (S), and two corresponding types of job, unskilled and skilled. The firm cannot discern a worker's type, even after the worker is employed. Because the firm knows its technology, it can however measure the number of each type that it has hired.

The two types of searchers discount the future differently; their patience is correlated with their skill level. This assumption can be motivated in two ways. First, the skilled workers might obtain non-firm-specific human capital from matching to an appropriate job, the value of which exceeds the actual wage. A second motivation springs from the association of capital with skill: skilled workers can be viewed as embodying more human capital than the unskilled workers, so that the marginal product of their capital is lower, and concommitantly their patience is higher. [1]

There is an initial period in which the firm chooses the number of each type of job vacancy to post, and the wage to offer in each case. Each unfilled vacancy incurs a cost each period: one can think of this as advertising costs and administrative costs of interviewing candidates.

The two types of worker search for jobs, receiving one offer in each period if they are in search mode. If a worker accepts a job, he keeps it forever at the offered wage.

In this initial model it will be assumed that the firm chooses two wages, w_U and w_S, and that no revision of the wages is possible. The firm is faced with an initial pool of workers N_U and N_S of the unskilled and skilled types. The sum of the two types is N_0. In addition, the firm chooses M_U and M_S, the number of job vacancies of the two types.

[1] This latter motivation requires that the different marginal productivities of the human capital of the unskilled and skilled workers is somehow unarbitraged; no explanation is offered here.

The firm will have an optimization with an objective like the following:

$$
\begin{aligned}
h(\lambda_{ij}, w_U, w_S) = \max_{\{\lambda'_{ij}\}} \{ & (\phi_{UU} - w_U)\lambda'_{UU} + (\phi_{US} - w_U)\lambda'_{US} \\
& + (\phi_{SU} - w_S)\lambda'_{SU} + (\phi_{SS} - w_S)\lambda'_{SS} \\
-c(M_U - \lambda'_{UU} - \lambda'_{US} + M_S - \lambda'_{SU} - \lambda'_{SS}) & \\
& + \rho h(\lambda'_{ij}, w'_U, w'_S)\}
\end{aligned}
\tag{3.1}
$$

The symbols are as follows:

$h(\cdot)$: value function of the firm.

ϕ_{ij}: marginal product of the worker of type j who works in job i. Thus, ϕ_{SU} is the marginal product of an unskilled worker who works in a skilled job.

w_U, w_S: wages paid to in the unskilled and skilled jobs. The wages are the same as w_U and w_S.

λ_{ij}: the number of workers of type j in jobs of type i. If $i \neq j$, that is a mis-match, which is inefficient for the firm.

There is a cost c for unfilled vacancies.

The assumption that there are many workers is being used here; the firm's problem is deterministic by the law of large numbers. [2]

Interludes. Because of the search-without-replacement phenomenon, there will be three possible interludes in an equilibrium. In the first interlude, the U and S types both wait for high wage (w_S) jobs and spurn the low-wage jobs. In the second interlude, the low types, who are more impatient ($\beta_U < \beta_S$) take all offers. This interlude by construction lasts only one period: call this period t^*. The third and final interlude begins at $t^* + 1$. The low types are no longer present, having all accepted jobs at t^*; only the high types remain.

With only the high types remaining in the third interlude, we are back to the single-type, search-without-replacement setting. If there aren't very many jobs remaining, the high types will all accept jobs immediately; if there is a surplus of jobs they will wait for high jobs, at least for awhile.

If the firm is allowed to adjust wages in the final interlude, it might be tempted to lower the top wage since only the high types remain. If this happens, the S-types will anticipate it, and they will all accept jobs in the previous interlude, at t^*. This would be inefficient since the firm would have more mis-matches. In the remainder of this section, it will be assumed that the choice of wages is fixed after the initial period; the possibility of wage adjustment is considered at a more abstract level in a subsequent section.

In the third interlude the firm knows that all individuals accepting jobs are S-types. It should therefore match them all to high-skill jobs. If this is allowed, some will accept at the low wage if all offers are accepted and there will be pay disparity. If there are surplus jobs so that only w_S offers are accepted, then the appropriate matching takes place anyway.

The firm might therefore wish to create surplus jobs so as to better approximate the with-replacement model, thus more perfectly matching the S-types to S jobs. But if this

[2] The assumption of the law of large numbers is a means of avoiding small sample fluctuations. In that instance strategic behavior by the searchers would need to be accounted for.

is costly, the pressure would go the other way. Also, surplus jobs that lead to more patient searching also entail a delay cost for the firm—by failing to accept low offers, the S types are not working.

Control of interludes. The firm chooses the wages in the initial period, and possibly the number of jobs of each type. The firm can thereby control the timing of the interludes. The payoffs can then be calculated by adding up the payoffs that evolve within each interlude. In particular, the firm can completely eliminate the first interlude, in which the low types accept high offers only. This is a bad outcome for the firm since it matches U-types with S jobs. Intuitively, this is the worst type of match.

Technically, this means that it only needs to be checked that the low types will indeed all take jobs in the first period, and that the high types will accept only S-jobs in the first period.

Incentive constraints in the firm's objective. The analysis can now be taken a step further. Since the firm is getting two hidden types to reveal themselves via the search process, it seems worthwhile to model the problem more directly in terms of incentive constraints. Each type of worker has his behavior represented by a Bellman equation from the search model. Without an optimization, these Bellman equations are the "promise-keeping" equations (which can also be considered dynamic individual rationality constraints). With optimization, the Bellman equations also constitute incentive constraints.

In the standard dynamic contract model the incentive constraints can be modeled by first order conditions. Here, the first order conditions are represented by inequalities since the search Bellman equation is represented by a two-alternative optimization. Thus, the general Bellman equation for a searcher is

$$v(w,F) = \max\left\{\frac{w}{1-\beta_i}, \beta_i \int_0^{w^*(F)} v(w',F')dF'(w')\right\} \tag{3.2}$$

where F is the distribution of wages, and F evolves according to some rule

$$F'(w) = \phi[F, w^*(F)]$$

If there are two discrete wages, the equation looks like

$$v(w,\alpha,p) = \max\left\{\frac{w}{1-\beta_i}, \beta_i(pv(w_U,\alpha',p') + (1-p)v(w_S,\alpha',p'))\right\} \tag{3.3}$$

Now denote the high wage type by β_H. Then the incentive inequalities are

$$\frac{w_U}{1-\beta_H} \leq \beta_H(p'v(w_U,\alpha',p') + (1-p')v(w_S,\alpha',p')) \tag{3.4}$$

$$\frac{w_S}{1-\beta_H} \geq \beta_H(p'v(w_U,\alpha',p') + (1-p')v(w_S,\alpha',p')) \tag{3.5}$$

These equations can be combined with the Bellman equation recursion.

The low-types might also obey these equations in the early stages of the model, when high-wage jobs are still abundant. Thus, the dependence on the probability structure of the offers, here represented by p, must be explicit.

If the firm's problem is set up as a mechanism problem then the searcher's problem becomes a recursion in which the future values $v(w_U, \alpha', p')$ and $v(w_U, \alpha', p')$ are chosen directly:

$$v = \max\left\{\frac{w}{1-\beta_i}, \beta_i(p\underline{v} + (1-p)\overline{v})\right\} \tag{3.3$'$}$$

In addition, the evolution equation for the probabilities and α are constraints for the firm:

$$p' = \frac{1}{1 + (1-\alpha)\frac{1-p}{p}} \tag{3.6}$$

$$\alpha' = \frac{p}{\frac{1}{\alpha} - (1-p)} \tag{3.7}$$

Characteristics of the equilibrium. The equilibrium results in three stages. In the first stage (stage I) the low types (i.e., those with the lowest β, β_L) wait for high-wage jobs. Since some of them find such jobs, they diminish in frequency, and the fraction $1-p$ (the probability of a high-wage offer) shrinks. At the threshold value of p, the second stage occurs (stage II) the low types β_L accept all offers, leaving only the high β_H types. In stage III only the high types are left and they wait for high offers if there are enough of them left. It is possible that they also accept all offers though if the ratio of searchers to jobs (α) is high. If α is low enough, the high types wait for high offers. In order for this to occur there must be surplus jobs, so that search has a high payoff.

The model results in full or partial separation of the two types of searchers. The low types accept all offers in stage II, while the high types do not. Thus any worker who accepts in stage III is known to be a high type, while the probability of a low type accepting a high wage offer in stage I or II is reduced, simply because of the removal of the high types due to their patience. This is a kind of adverse selection, because low types will get high-wage jobs, which is a mismatch. The matching is improved, however, because of the filtering of the high types to the end of the search process.

Unlike standard adverse selection models, there is not a sharp distinction between pooling and separating equilibria. The low types do get selected adversely, but the probability of identifying the high types is increased by the search filter. Adding surplus jobs, which can be costly (or flying an aircraft with empty seats) adds cost for the firm but results in better filtering of the types. This can be viewed as a cost of information.

The staged dynamic programming problem. The three stages of the dynamic programming problem can now be stated. The final stage (III) is simply the basic search without replacement problem (SWR), in which all the searchers are the high type. The firm has an initial state of matched workers. If its payoffs are linear, as has been assumed so far, that state is irrelevant for the continuation. The firm's value is then just the discounted value of the acceptance process as the searchers accept high-wage jobs—or low wage jobs if the high-wage jobs get too depleted. Suppressing the vacancy cost term, the value is thus

$$h_{III}(\{\lambda_{ij}\}, w_U, w_S, \alpha_S, p_S) = \max\{(\phi_{US} - w_U)\lambda_{US} + (\phi_{SS} - w_S)\lambda_{SS} + \rho h_{III}(\{\lambda_{ij}\}', w_U', w_S') + h_{II}^*\}$$

where h_{II}^* is the terminal value of the stage II process—the workers hired then stay on without influencing the stage III problem directly. The value implicitly allows the number of employees to remain static after the initial level.

Since the assumption so far is that the wages are not adjusted at each stage, the firm's maximization is not dynamic: the searchers will simply search until they find an appropriate wage.

As an accounting matter, the number of searchers is the net after the number of employees $\lambda_{US} + \lambda_{SS}$:

$$\alpha_S' = \frac{N_S - (\lambda_{US}' + \lambda_{SS}')}{(M_U + M_S) - (\lambda_{US}' + \lambda_{SS}' + \lambda_{UU}' + \lambda_{SU}')} \quad (3.8)$$

that is, the ratio of active skilled searchers to remaining jobs of both types nets out the jobs accepted. Similarly, the probability of a high wage draw on the current round is

$$1 - p' = \frac{M_S - (\lambda_{US}' + \lambda_{SS}')}{(M_U + M_S) - (\lambda_{US}' + \lambda_{SS}' + \lambda_{UU}' + \lambda_{SU}')} \quad (3.9)$$

Note that in stage II no skilled workers accept the low wage, so $\lambda_{US} = 0$. Also note the timing: if λ_{ij} is the jobs currently filled, then the probability of a high wage offer is affected on the next round.

These equations can be solved for λ_{US} and λ_{SS}, since the unskilled employees λ_{UU} and λ_{SU} are fixed when stage III begins. The evolution of α_S and p are already known from equations (3.6-7), so λ_{US} and λ_{SS} can be found as well.

Some additional relations can be immediately noted. First, in stage III all the unskilled searchers have accepted jobs, so $N_U = \lambda_{UU} + \lambda_{SU}$. Because of this, the above relations can be written

$$\alpha_S = \frac{N_S - (\lambda_{US} + \lambda_{SS})}{(M_U - N_U + M_S) - (\lambda_{US} + \lambda_{SS})}$$

that is, the ratio of active skilled searchers to remaining jobs of both types nets out the jobs accepted. Similarly,

$$1 - p = \frac{M_S - (\lambda_{US} + \lambda_{SS})}{(M_U - N_U + M_S) - (\lambda_{US} + \lambda_{SS})}$$

The second relation is that the skilled workers will not accept unskilled wages unless they reach the threshold P_S^*—call this stage IV. At that point they all accept all offers. Thus, in stage III λ_{US} is fixed and is zero. In stage IV, $\lambda_{US} + \lambda_{UU} = N_U$, and $\lambda_{SU} + \lambda_{SS} = N_S$.

The value in stage IV can be directly calculated as a present value. The value of stage III can be calculated since stage IV will be reached in a fixed and predictable time T_{IV}. The present value of stage III can also be calculated as well.

The remaining issue is how to calculate α_U, α_S, and p in the most complicated stage, stage II. (Stage I is ruled out a priori since it allows unskilled searchers to accept only skilled jobs, thus failing to filter out the unskilled types.) Stage II can be constructed to last only one period, however, reducing the complication. The present value of stage II can then be directly calculated.

In stage III the skilled workers still face some low-wage offers: those are the low-wage offers that skilled workers drew in stage II and rejected. Thus, the search problem can still be nontrivial for the skilled workers, in the sense that they will not accept low offers in stage III, so stage III can last more than one period. If the firm gets a highly negative payoff from skilled workers doing unskilled jobs (they become union agitators) it will "seed" its job pool with excess high-wage jobs to ensure that the high-wage workers take only high-wage jobs. If it did not seed the market in this way, stage IV would occur too soon and there would be too many skilled workers in unskilled jobs.

If the firm does not lose much by having skilled workers in low-wage jobs, it can go immediately to stage IV from stage III, accepting the mixing of skills with jobs in both states.

Computation. The model so far is rather "busy": the parameters include three discount factors and four marginal productivity factors; the firm must initially choose the number of jobs and ongoing wages, and the α and p measures must be dynamically updated. Computation to demonstrate the existence of interesting behavior will now be developed.

The most interesting case results in the U-types being filtered out, with the S-types being gradually filtered out in subsequent periods. It will also be interesting to demonstrate that excess U jobs must be created, and that different parametric conditions can result in degenerate outcomes, such as immediate acceptance of all jobs.

In the interesting case, the problem of the two searcher types can be characterized by their values in each stage. These outcomes can be imposed as constraints on the principal in order to characterize overall behavior.

Translating the value recursion into employment. The firm adds workers deterministically from the continuum. In the "interesting case" of case II followed by several periods of case III, the behavior of the searchers translates into number of workers hired in each category. By imposing this structure (in the case of fixed wages) the firm's value iteration can be calculated, and then the wages that satisfy the assumed searcher value recursions can be found. That is, in stage II the constraints for the skilled searchers are

$$\frac{w_U}{1-\beta_S} \leq \beta_S(p' v_S(w_U, \alpha', p') + (1-p') v_S(w_S, \alpha', p')) \tag{3.10}$$

$$\frac{w_S}{1-\beta_S} \geq \beta_S(p' v_S(w_U, \alpha', p') + (1-p') v_S(w_S, \alpha', p')) \tag{3.11}$$

and for the unskilled (and impatient) searchers,

$$\frac{w_U}{1-\beta_U} \geq \beta_U(p' v_U(w_U, \alpha', p') + (1-p') v_U(w_S, \alpha', p')) \tag{3.12}$$

Since the unskilled workers take all offers in stage II, it follows directly that

$$\underline{v}_U = \frac{w_U}{1-\beta_U}, \qquad \overline{v}_U = \frac{w_S}{1-\beta_U}$$

but to satisfy the incentive constraint they must not be tempted to wait: therefore it must be the case that their discounting puts them above P^*:

$$p'_{II} > P^*_U = \frac{\beta_U w_S - w_U}{\beta_U(w_S - w_U)}$$

On the other hand, the skilled (and patient) workers accept only the high offers, so the reverse inequality must be satisfied

$$p'_{II} \leq P^*_S = \frac{\beta_S w_S - w_U}{\beta_S(w_S - w_U)}$$

In stage III this same inequality must still be satisfied; in stage IV it reverses.

Given the probabilities, the employment dynamics can be calculated.

$$\lambda'_{UU} = p'(N_U - \lambda_{UU} - \lambda_{SU}) + \lambda_{UU}$$

$$\lambda'_{SU} = (1-p')(N_U - \lambda_{UU} - \lambda_{SU}) + \lambda_{SU}$$

In the simplest case the firm starts with zero employees in stage II, and so the equations are simpler:

$$\lambda'_{UU} = p' N_U$$

$$\lambda'_{US} = (1-p')N_U$$

The dynamics of the skilled workers reflect their non-acceptance of the low wage in stages II and III

$$\lambda'_{US} = \lambda_{US}$$

$$\lambda'_{SS} = (1-p')(N_S - \lambda_{US} - \lambda_{SS}) + \lambda_{SS}$$

Finally in stage IV,

$$\lambda'_{US} = p'(N_S - \lambda_{US} - \lambda_{SS}) + \lambda_{US}$$

$$\lambda'_{SS} = (1-p')(N_S - \lambda_{US} - \lambda_{SS}) + \lambda_{SS}$$

Observe that adding these two constraints together accounts for all the skilled searchers.

The firm's problem in stage III can now be stated (again suppressing the vacancy cost terms):

$$\begin{aligned} h_{III}(\lambda_{ij}, \{w_U, w_S\}, p) = \max_{\lambda'_{US}, \lambda'_{SS}} \{ \; & (\phi_{UU} - w_U)\lambda'_{UU} \\ & + (\phi_{US} - w_U)\lambda'_{US} \\ & + (\phi_{SU} - w_S)\lambda'_{SU} \\ & + (\phi_{SS} - w_S)\lambda'_{SS} \\ & + \rho h_{III \text{ or } IV}(\lambda'_{ij}, p')\} \end{aligned} \tag{3.13}$$

subject to

$$\lambda'_{US} = \lambda_{US} \tag{3.14}$$

$$\lambda'_{SS} = (1-p')(N_S - \lambda_{US} - \lambda_{SS}) + \lambda_{SS} \tag{3.15}$$

$$1-p' = \frac{M_S - (\lambda'_{US} + \lambda'_{SS})}{(M_U - N_U + M_S) - (\lambda'_{US} + \lambda'_{SS})} \tag{3.16}$$

$$\frac{\beta_U w_S - w_U}{\beta_U(w_S - w_U)} \leq p' \leq \frac{\beta_S w_S - w_U}{\beta_S(w_S - w_U)} \tag{3.17}$$

The first inequality is needed even at this stage because the unskilled types must not be tempted to "wait it out" in stage II.

The terminal period in stage IV can be accounted for by the requirement

$$\begin{aligned} &h_{IV}(\lambda'_{ij}, p' > P^*) \\ &\quad = \frac{(\phi_{UU} - w_U)\lambda'_{UU} + (\phi_{US} - w_U)\lambda'_{US} + (\phi_{SU} - w_S)\lambda'_{SU} + (\phi_{SS} - w_S)\lambda'_{SS}}{1-\rho} \end{aligned} \tag{3.18}$$

subject to

$$\lambda'_{US} = p'(N_U - \lambda_{US} - \lambda_{SS}) + \lambda_{US}$$

$$\lambda'_{SS} = (1-p')(N_U - \lambda_{US} - \lambda_{SS}) + \lambda_{SS}$$

These are the same equations that applied to the unskilled workers in stage II.

The stage II problem is includes the evolution of the unskilled types. The number of each type of job M_i and the wages are chosen at this stage:

$$\begin{aligned} h_{II}(\{\lambda_{ij}\}, p) = \max_{\{\lambda'_{UU}, \lambda'_{SU}, \lambda'_{US}, \lambda'_{SS}\}, M_U, M_S, \{w_U, w_S\}} \{ \quad & (\phi_{UU} - w_U)\lambda'_{UU} \\ & + (\phi_{US} - w_U)\lambda'_{US} \\ & + (\phi_{SU} - w_S)\lambda'_{SU} \\ & + (\phi_{SS} - w_S)\lambda'_{SS} \\ & + \rho h_{III \text{ or } IV}(\{\lambda'_{ij}\}, p')\} \end{aligned} \tag{3.19}$$

subject to

$$\lambda'_{UU} = p'(N_U - \lambda_{UU} - \lambda_{SU}) + \lambda_{UU} \tag{3.20}$$

$$\lambda'_{SU} = (1-p')(N_U - \lambda_{UU} - \lambda_{SU}) + \lambda_{SU} \tag{3.21}$$

$$\lambda'_{US} = \lambda_{US} \tag{3.22}$$

$$\lambda'_{SS} = (1-p')(N_S - \lambda_{US} - \lambda_{SS}) + \lambda_{SS} \tag{3.23}$$

$$1-p' = \frac{M_S - (\lambda'_{SU} + \lambda'_{SS})}{(M_U + M_S) - (\lambda'_{UU} + \lambda'_{SU} + \lambda'_{US} + \lambda'_{SS})} \tag{3.24}$$

$$\frac{\beta_U w_S - w_U}{\beta_U(w_S - w_U)} \leq p' \leq \frac{\beta_S w_S - w_U}{\beta_S(w_S - w_U)} \tag{3.25}$$

In the next section some computational examples are presented.

4. THE GAIN FROM FILTERING

The firm could obtain workers immediately by simply creating the same number of jobs as workers, then letting the workers fill all the jobs immediately, taking whatever was offered. The proportion of each type in each job is then proportional to the number of job types. As a result, many workers will be mismatched with their job type. Call this strategy *random employment.*

With some parameterizations, the search filter outperforms random employment. This is so even when it takes many periods to fill the skilled jobs, because the productivity gains from a correct match are so high. For example here is a set of simulated values in which the firm can choose the number of skilled jobs M_S, but takes as fixed the number of unskilled jobs and also takes the wages as fixed:

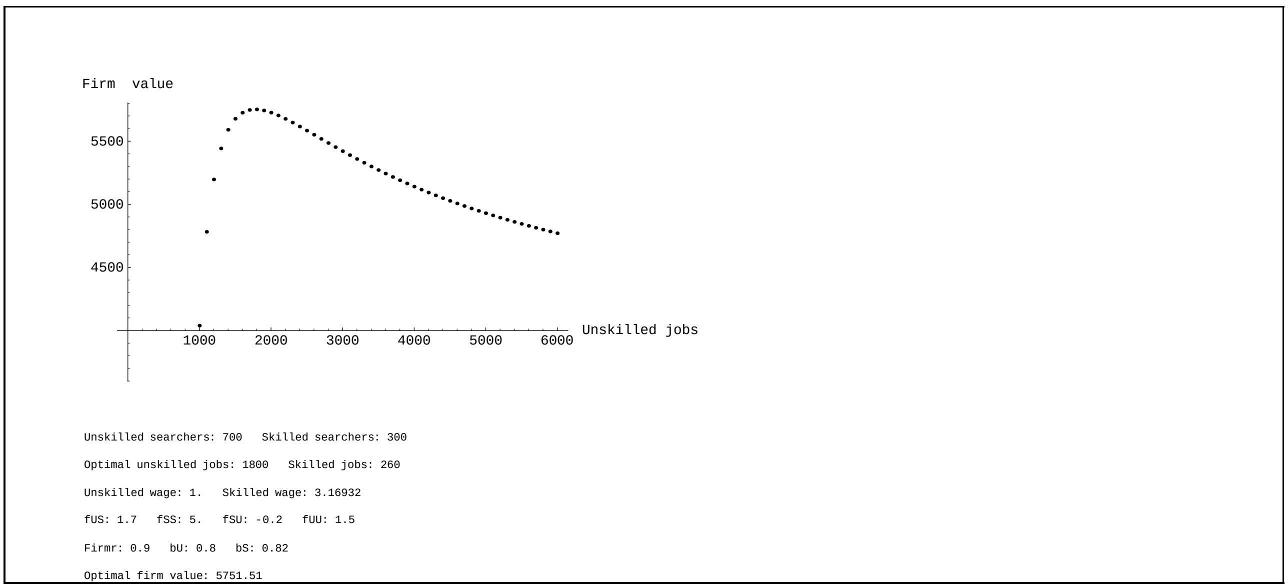
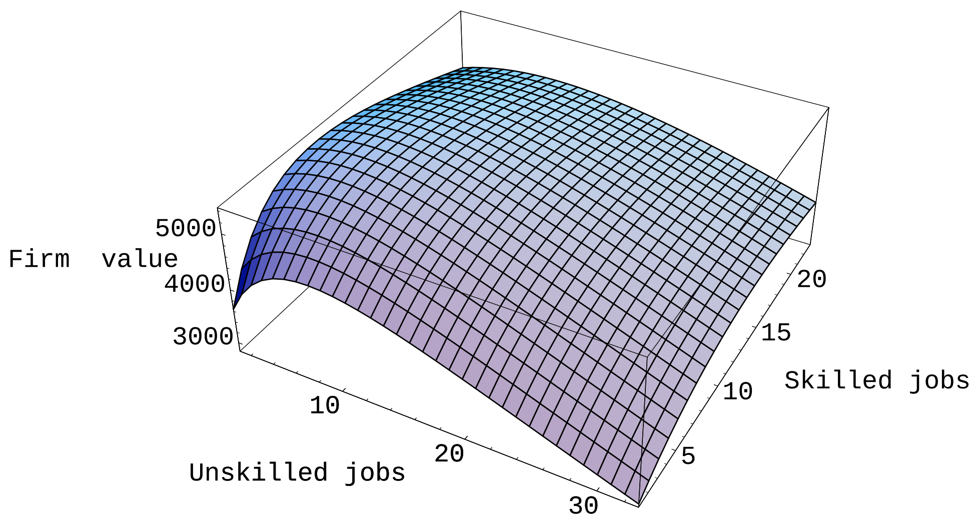


FIGURE 4.1

The duration of the search by the skilled types is four periods—there is a nontrivial amount of waiting on the part of the firm. One can see how the gain arises: the search filter puts more workers in skilled positions. With random employment and an initial number of searchers 1000 of whom 300 are skilled, optimal random employment maximizes the number of unskilled jobs; this is because matching an unskilled worker with a skilled job has such a negative effect, that it is better to ensure that the unskilled workers end up in unskilled posititions.

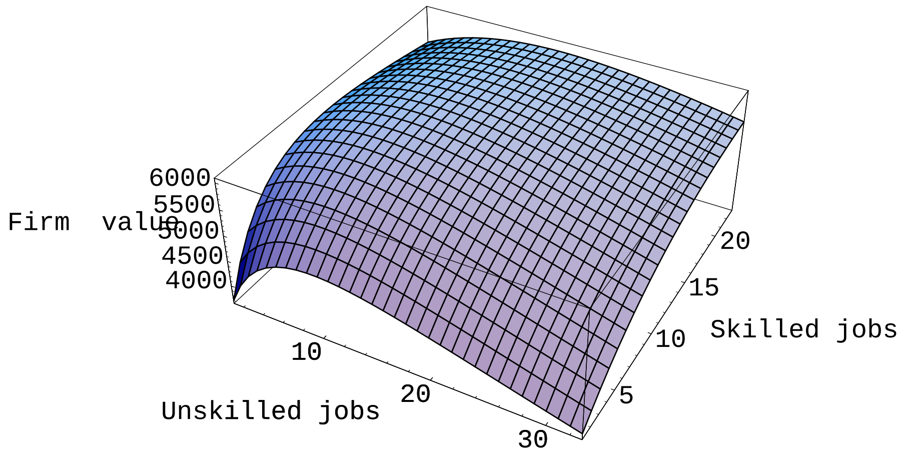
By making unskilled workers have better productivity in skilled positions, the optimal strategy reverses, and it is desirable to have mostly skilled jobs if there is random employment, and search filtering is unproductive.

Optimizing the number of jobs and the wage. If the firm can choose the number of U-type jobs as well as the number of S-type jobs, but must pay a penalty for unfilled vacancies (think of this as an advertising cost) then there will be an optimal mix of jobs. The simulations below show two examples. In each example the unskilled wage is required to equal 1.0. The firm chooses the skilled wage, and also the number of vacancies of each type.




```
Unskilled searchers: 700   Skilled searchers: 300
Optimal unskilled jobs: 1250
Optimal skilled jobs: 1550
Unskilled wage: 1.   Skilled wage: 1.3425
Marginal Productivity: fij, i is job, j is worker type:  fUS: 0.2   fSS: 4.   fSU: -0.8   fUU: 2.5
UVacancyCost: 0.05   SVacancyCost: 0.07
Firmr: 0.9   bU: 0.8   bS: 0.85
Optimal firm value: 5388.77
Check threshhold for U-searchers:  PStar for U-types =
 0.270083 < 0.446429 necessary for single-period cull of U-types
Search duration for high types after t=1:
PStar: 0.484765   LowProbability: 0.660819   DurationCount: 1
813.53 Second, Null<
Final job counts:
   lUS: 30.9917   lSS: 269.008    lSU: 387.5   lUU: 312.5
Random matching outcome with no search:
   Unskilled workers: 700.   Skilled workers: 300.   Unskilled jobs: 446.429   Skilled jobs: 553.571
   Employment value under random assignment: -258.929
  lUS: 133.929    Skilled workers in skilled jobs HlSSL: 166.071  lSU: 387.5  lUU: 312.5
Random matching outcome with no search:
   Unskilled workers: 700.   Skilled workers: 300.   Unskilled jobs: 700.   Skilled jobs: 300.
   Employment value under random assignment: 3570.
  lUS: 210.    Skilled workers in skilled jobs HlSSL: 90.  lSU: 210.  lUU: 490.
```

FIGURE 4.2




```
Unskilled searchers: 700   Skilled searchers: 300
Optimal unskilled jobs: 1750
Optimal skilled jobs: 2050
Unskilled wage: 1.   Skilled wage: 1.34326
Marginal Productivity: fij, i is job, j is worker type:  fUS: 0.2   fSS: 4.   fSU: -0.8   fUU: 2.5
UVacancyCost: 0.025   SVacancyCost: 0.027
Firmr: 0.9   bU: 0.8   bS: 0.85
Optimal firm value: 6031.54
Check threshhold for U-searchers:  PStar for U-types =
 0.271682 < 0.460526 necessary for single-period cull of U-types
Search duration for high types after t=1:
PStar: 0.485893   LowProbability: 0.588944   DurationCount: 1
813.43 Second, Null<
Final job counts:
   lUS: 32.6087   lSS: 267.391    lSU: 377.632   lUU: 322.368
Random matching outcome with no search:
   Unskilled workers: 700.   Skilled workers: 300.   Unskilled jobs: 460.526   Skilled jobs: 539.474
   Employment value under random assignment: -46.0526
  lUS: 138.158    Skilled workers in skilled jobs HlSSL: 161.842  lSU: 377.632  lUU: 322.368
Random matching outcome with no search:
   Unskilled workers: 700.   Skilled workers: 300.   Unskilled jobs: 700.   Skilled jobs: 300.
   Employment value under random assignment: 3570.
  lUS: 210.    Skilled workers in skilled jobs HlSSL: 90.  lSU: 210.  lUU: 490.
```

FIGURE 4.3

The firm would like to maximize the number of jobs and to choose a ratio of job types, so as to emulate an ideal search with replacement environment. By adding the cost of unfilled jobs, the firm chooses to reduce the total number of jobs.

The number of jobs filled by each type is calculated for the optimum job combination. In addition, at the bottom of each plot the firm's payoffs are calculated under the assumption that it conducts a single-period match without search on the part of the workers. In the first of those simulations, the ratio of job types is chosen simply to match the searcher types. In the second case, the ratio of job types is chosen to match the ratio found by the optimization in the search model. It is apparent in both cases that the number of matches between skilled workers and skilled jobs is significantly raised by using the search filter, and that the firm's value is radically increased by using the search filter.

The simulations show that there is a differential wage, but the differential is relatively small so that the firm gets a significant amount of surplus from matching skilled workers with skilled jobs. The firm chooses a large number of skilled-job vacancies relative to the ratio of skilled workers in the population. The search process results in many skilled jobs being filled by unskilled workers, but the primary result is that most of the skilled workers are matched with skilled jobs.

The firm's strategy includes not only a wage differential, but increasing the ratio of skilled vacancies above the ratio of skilled workers to unskilled workers. This increases the probability of a skilled worker getting appropriately matched with a skilled job. This increase in the skilled job ratio stops short of inducing the unskilled workers to wait for a skilled offer. The strategy results in more unskilled workers getting skilled offers in the first period, but is compensated by the improved matching of skilled workers that takes place subsequently.

5. A MORE FORMAL MECHANISM

In this section the firm is allowed to update the wage. The firm explicitly treats the searchers' value functions as state variables updated by the search Bellman equations, which are now constraints. The main technical challenge is translating the search Bellman equations into evolution equations for the low-wage probability p and for the ratios of searcher populations to jobs.

Searchers know the path of prices—since there is a continuum of searchers and no common-value problem, they have perfect foresight about the price sequence. Their optimization will take this into account.

Let the sequence of current and future wages (or prices) be denoted $\tilde{w}$. Then $\tilde{w}_t = (\{w_{it}\}, \{w_{i,t+1}\}, \ldots, \{w_{i,T}\})$, where T is the terminal period—which could be infinity— and $\{w_{it}\}$ is the collection of wages available in the period. If there are just two types of job then $\{w_{it}\} = \{w_{1t}, w_{2t}\}$. Then $\tilde{w}' = (w_{t+1}, w_{t+2}, \ldots, w_T)$. The searcher's problem is now

$$v(w, \tilde{w}, F) = \max\left\{\frac{w}{1-\beta_i}, \beta_i \int_0^{w^*(F)} v(w', \tilde{w}', F')dF'(w')\right\} \tag{5.1}$$

The distribution function F could be made to subsume the dynamics of the wage set, but it is better to keep it separate because the dynamics of F are not controlled directly by the firm, as the decisions of searchers do matter.

The firm will have the distribution of value states of searchers as state variables. Those values then evolve according to (5.1) as in a standard contracting model. The second key constraints for the firm consist of the analogues of the evolution equations when wages change dynamically.

If there are two discrete wages, the searcher's optimization problem looks like

$$v(w, \{w_U, w_S\}, \alpha, p) = \max\left\{\frac{w}{1-\beta_i}, \beta_i(p'v(w'_U, \{w'_U, w'_S\}, \alpha', p') + (1-p')v(w'_S, \{w'_U, w'_S\}, \alpha', p'))\right\} \tag{5.2}$$

Because there are only two wages, there are only two value states for searchers. The searchers' Bellman equations now become value-continuation equations:

$$v_{Ui} = \max\left\{\frac{w_U}{1-\beta_i}, \beta_i(p'v'_{Ui} + (1-p')v'_{Si})\right\} \tag{5.3}$$

$$v_{Si} = \max\left\{\frac{w_S}{1-\beta_i}, \beta_i(p'v'_{Ui} + (1-p')v'_{Si})\right\} \tag{5.4}$$

where i is the searcher's type: v_{UU} is the value of an unskilled searcher with a low value for example.

A detail. If the firm changes the wage across periods, a modeling decision must be made: does the wage change affect already-employed workers, or only new hires? From a dynamic programming point of view it is easiest to assume that accepted offers are stuck with the offered wage, but in that case the firm must keep track of the fraction of employees by the date of their hire and the wage offered. If wages decrease over time, for example, older workers will have a higher wage. (This mimics what happened to pilot salaries when the airlines were deregulated.)

On the other hand, if wages are adjusted for those already employed as well as for new offers, this must be accounted for in the value recursion of the searchers, and the value of accepting an offer will no longer be the present value of the wage; instead, it will be a more complicated present value and will require prescience about the future wage.

Using the first scheme, each of the present-period payoff terms in the principal's objective will take the more complicated form

$$\sum_{k-1}^{K}(\phi_{ij} - w_{ik})\mu_{ijk} \tag{5.5}$$

where μ_{ijk} indexes the workers of type j who have type i jobs and who work for wage k, and where

$$\lambda_{ij} = \sum_{k=1}^{K}\mu_{ijk} \tag{5.6}$$

The firm's problem is now the more complicated expression

$$
\begin{aligned}
h(\{\mu_{ijk}\},\{w_U,w_S\},\{v_{ij}\},p) = \sup_{\{M_U,M_S\}} \max_{\{\mu'_{ij}\},\{w'_U,w'_S\},\{v'_{ij}\}} \{ & \sum_{i\in\{U,S\},j\in\{U,S\}} \sum_{k=1}^{K} (\phi_{ij} - w_{Uk})\mu_{ijk} \\
& + (\phi_{UU} - w'_U)\mu'_{UU} \\
& + (\phi_{US} - w'_U)\mu'_{US} \\
& + (\phi_{SU} - w'_S)\mu'_{SU} \\
& + (\phi_{SS} - w'_S)\mu'_{SS} \\
& - (M_U + M_S - \sum_{i,j} \mu'_{ij})c) \\
& + \rho h(\{\mu_{ijk}\}', \{w'_U, w'_S\}, \{v'_{ij}\}, p')\}
\end{aligned}
\tag{5.7}
$$

subject to the searchers' value recursions (5.3-4), keeping in mind the accounting relation (5.6), and where the sup is taken only in the initial period. The final term in the contemporaneous payoff is the penalty cost of unfilled jobs c. Including this penalty guarantees a finite number of vacancies will be created by the firm. The realizations of the λ_{ij} are determined by the value recursion equations:

$$
\begin{aligned}
\lambda'_{UU} &= \begin{cases} \lambda_{UU} + p(N_U - \lambda_{UU} - \lambda_{SU}) & v_{UU} = \frac{w_U}{1-\beta_U} \\ \lambda_{UU} & v_{UU} = \beta_U(p' v'_{UU} + (1-p')v'_{SU}) \end{cases} \\
\lambda'_{SU} &= \lambda_{SU} + (1-p)(N_U - \lambda_{UU} - \lambda_{SU})
\end{aligned}
\tag{5.8}
$$

$$
\begin{aligned}
\lambda'_{US} &= \begin{cases} \lambda_{US} + p(N_S - \lambda_{US} - \lambda_{SS}) & v_{US} = \frac{w_U}{1-\beta_S} \\ \lambda_{US} & v_{US} = \beta_S(p' v'_{US} + (1-p')v'_{SS}) \end{cases} \\
\lambda'_{SS} &= \lambda_{SS} + (1-p)(N_S - \lambda_{US} - \lambda_{SS})
\end{aligned}
\tag{5.9}
$$

These equations could equivalently be expressed in terms of μ'_{ij}, using equation (5.6). These constraints express the fact that employment increases in the low wage category only if searchers are impatient to accept those jobs. An implicit set of constraints is

$$
v'_{UU} \geq \frac{w'_U}{1-\beta_U} \qquad v'_{US} \geq \frac{w'_U}{1-\beta_S} \tag{5.10}
$$

and

$$
v'_{SU} = \frac{w'_S}{1-\beta_U} \qquad v'_{SS} = \frac{w'_S}{1-\beta_S} \tag{5.11}
$$

Finally, the evolution of the probability when both types are potentially searching is

$$
p' = \frac{M_U - \lambda'_{UU} - \lambda'_{US}}{M_U + M_S - (\lambda'_{UU} + \lambda'_{US} + \lambda'_{US} + \lambda'_{SS})} \tag{5.12}
$$

There are also initial conditions. In the initial period, there are no employees, so $\lambda_{ij} = 0$. In addition, there must be initial values for the searchers, v_{ij}, and these must be consistent with the initial wages w_U and w_S.

This setup leads to the following proposition:

PROPOSITION 1: *There is a unique solution to the principal's problem (5.7-12).*

The proof is standard and is contained in the appendix.

6. DISCUSSION

One could view this model as a cousin of the standard model of price discrimination. In the standard model there are two types of demanders, and their types are observable. The price is used to extract rent from each type. In this model, the types are not observable, and the price is used to reveal the types. Because the price (the wage here) does double duty, there is some inefficiency.

The model takes some inspiration from the matching literature, which studies algorithms for efficiently matching men with women, students with colleges, and similar problems. In addition, the search literature itself has studied matching when types are unknown, with the model of Jovanovic [2] the primary reference. In that model, firms and workers learn simultaneously about the quality of a match after it has occurred, with the option of termination for matches that are discovered to be suboptimal. That literature lacks a feature that is present here: prices. Prices, along with discounting frictions, get searchers to reveal their types, and this can improve the allocation. This suggests one direction in which to expand the model, namely to study classic matching situations but allow prices. These models differ from the ones here in the sense that there is typically an assumption of symmetry between two populations of types that are seeking matches, such as men and women.

The model also has some similarities with the model of Shi and Cao [6]. In that model there is more than one firm, and each firm can create a single job and its wage, and post the wage to searchers. As with the model here, there is a cost of maintaining a vacancy, and there is also a searcher-to-job ratio that affects the probability structure. The thrust of the model is that workers can collide if multiple workers apply for a single job, resulting in wasted search time and it jobs that go unfilled due to lack of applicants. Types of workers and of jobs are identical, centering the focus of the model on firms' wage-setting strategies. The searchers see all wages at once however, and their strategies have to do with trading off the probability of successful application versus the wage, due to the collision probability. Another similar model is that of [5], in which there are different jobs and worker types, and with a desirable coordination between worker types and job types as here.

Aside from possible applications such as pricing and wage setting, the model extends existing theories of information. Those theories rest primarily on the revelation principle. This principle seems excessively strict in its effects: nonconvexity, discontinuity and nonexistence of equilibrium are generic problems with that approach. The search filter seems to avoid these problems, at least at this primitive level. Of key interest is differentiability: the behavior of the optimum can be perturbed at least at interior points in a continuous way. This might pave the way for a theory of the comparative statics of information.

A paper that is similar in spirit in this regard—that is, it successfully uses price as a conduit of information—is [1]. In that model, learning about product quality takes place through experimentation by consumers; as quality is revealed through this process, price adjusts. The central result in [1] is that firms—who initially are as ignorant of demand as consumers—initially charge low prices in order to subsidize experimentation by consumers. The model here is in this spirit, in that the firms in essence pay for the information possessed by the searchers by paying a higher wage to searchers who accept skilled jobs.

Numerous extensions suggest themselves. The number of jobs and searcher types could be increased, or extended to a continuum in continuous time. The firm could be modeled as selling slots, such as airline seats, to customers, instead of filling jobs with workers. Once hired, worker types could become slowly manifested, resulting in reshuffling or wage adjustment or termination, in the manner of Jovanovic's model. The dynamics of wages or prices when revision is allowed can be explored, as can the effect of multiple firms on the efficiency of the filtering process.

APPENDIX

This appendix provides an outline of the proof of existence and uniqueness for the firm's flexible-wage problem. The iteration constraints for the employment fractions μ_{ijk} and for the probability p are straightforward. The probability lies on the probability simplex which is closed and bounded; the employments are bounded by the initial number of searchers.

The searcher values are bounded by the upper bound on wages, which in turn can be fixed as the upper bound on the marginal productivities; for example if ϕ_{SS} is the maximum of the marginal productivities, then $v'_{ij} \leq \frac{\phi_{SS}}{1-\beta_S}$ will bound the values of the searchers. The value states are therefore elements of a compact set.

The number of jobs M_i is not a-priori bounded. However, if M is increased without bound, the with-replacement model is replicated, as are the corresponding values. This generates an upper bound on the firm's value. The supremum therefore exists.

The firm's value recursion is a mapping from the space B of bounded functions on $[0, N_U + N_S]^4 \times [0, \frac{\phi_{SS}}{1-\beta_S}]^4 \times [0, \phi_{SS}]^2 \times [0, \infty]^2$, where the elements of the space are of the form $(\{\mu_{ij\cdot}\}, \{v_{ij}\}, (w_U, w_S), (M_U, M_S))$. into the space of similar functions. (Not only is the space B the space of bounded functions; there is a bound that can be stated.) The discounting and monotonicity properties needed for sufficiency in a contraction mapping argument as in [7, p. 54] are obvious, so there is a fixed point.

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