Comparative Study of Short-Term Time Series Models: Use of Mobile Telecommunication Services in CR Regions

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Abstract

In the area of time series analysis and prediction there are comparatively many methods available, out of which the extrapolation method has been practically applied most often. Currently, combined models have been serving more and more in experimentation. This study is aimed at construction of adequate models of the indicators observed development tendencies, assessment of selected individual models and subsequent aggregation of these into combined models, including a comparative analysis of both types of models. To find suitable candidates for predicting within the time series analysed, SAS system has been used. Outcomes of the empirical study have shown promising results in the use of combined models for time series processing. The techniques presented in the paper are illustrated with examples of short-term time series on monthly and quarterly basis, in the field of mobile telecommunication services, their consumption and use in various regions of the Czech Republic. A description of the current state of use of selected services in separate CR regions, including urban and rural areas, is a natural part of the paper. ICT including mobile phones and use of mobile services, especially in rural areas, is still widely discussed topic.

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Keywords

Time series models, Box-Jenkins Methodology, mobile telecommunications, regions in CR, combined forecasting models, regional policies.

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Introduction

The issue of regional support is one of the most debated topics worldwide. Regional policies are focused at all regions and cities in the European Union. The principal target of those is the support of jobs creation, competitiveness of companies, growth of economics, sustainable development and improvement of the EU citizens´ life quality (European Commission, 2016). Special attention is paid by the regional policies to special development problems of cities and country regions, too.

Regional policies are the principal EU investment policies and are materialized by means of three main funds: European Fund for Regional Development (ERDF), Cohesion Fund (CF), and European Social Fund (ESF). Thereby the necessary scope for investment aimed at the Europe 2020 strategy for the intelligent and sustainable growth targets reaching, supporting inclusion in the EU, is provided. Financing within the scope of cohesion policy for the 2014-2020 period is concentrated at eleven topical targets, out of which the following four are ERDF key priorities: research and innovations, information and communication technologies, competitiveness of small and medium enterprises and low-carbon economics (European Commission, 2016).

This paper is concerned with one of the priorities from above, namely, the information and communication technologies (ICT). Many authors have already dealt with ICT, with mobile telecom and availability of mobile services in separate regions or country areas particularly. In the Czech Republic the issue of high-speed connection to internet and its application in country regions, inclusive of programme equipment analysis of the companies working in such regions, was studied by Vaněk et al. (2010), within the scope of an extensive survey of ICT development in CR regions.

The authors have continued immediately next year, besides studying the high-speed internet connection issue as such, by analysing acquirement of it by agricultural enterprises in the country (Vaněk et al., 2011). Next, Reigadas et al. (2015) analysed the issue of small country areas in Spain, having bad, or none at all, approach to mobile telecommunication services and chance of wireless infrastructures exploitation, what could, according to the study outcomes, help in solving this problem to a degree. Baro and Endouware (2013) studied the problem of mobile services use in country areas of Nigeria, affecting population´s socio-economic life within the areas. Core of the study by Sundquist and Markendahl (2015) is situated in cost analysis of the regulation measures accessible, that could help improving coverage of country areas by the mobile operators´ signals. Use of mobile phones by Cyprus farmers has been analysed by Adamides and Stylianou (2013). They examined first of all methods of information sharing in agriculture in their study, and they reached a conclusion that, in 98 % of the cases, mobile phone has been exploited for it. Lim et al. (2012) studied the possibility of 3G mobile subscription prediction in separate Chinese provinces, with regard to regional differences.

This paper is aimed at offering a short synoptical study of mobile telecommunications services use in a broader sense - not just in the country but in the whole Czech Republic at the level of individual CR regions. Area of regional development as well as the development of rural areas is still highly topical. This problematic is well known to mobile operators too. Each of mobile operators goes for the greatest possible coverage. They are aimed at coverage of every piece of Czech territory, especially country areas. This effort will bring them greater satisfaction of existing customers and also opportunity to gain new customers. At present is the attention focused on increasing the availability of broadband services, but this is not the only issue. For people living in the country is very important to use all mobile services provided to them by their mobile operators – especially voice services as well as SMS and MMS. Unfortunately problems with coverage in the country still persists and therefore it is not enabled to everyone to make a call or send SMS.

The object of analysis are the services, let us say, revenues from the services, offered by the youngest mobile operator acting on the Czech telecom market - the Vodafone Czech Republic, a.s. (VF) - to their final customers. VF acts as direct provider (SMS services, voice and data services subject to VAT - called TELCO services in the following), as well as an intermediary covering the Audiotex service analysed here (payments to third parties not subject to VAT), inclusive of the numbers of customers generating the revenues. In this study, the indicator of customer numbers is limited on natural persons only (non-company customers) not using the prepaid cards, rather paying for the services after their use only, once a month, by means of invoice (so called consumer postpaid customers). Adequate development trend models construction for time series of the indicators studied, especially the quality assessment of time series models offered by the SAS system, inclusive of comparative analysis of individual and combined models, is going to be the object of this study. Meade and Islam (2015) were engaged in their work with various techniques of time series forecasting, but primarily analyzed the importance of ICT in the field of forecasting and conversely the importance of forecasting for ICT.

Time series analysis, inclusive of their future behaviour prediction, became a very dynamically developing discipline recently. A lot of new efficient and untraditional procedures and methods of time series modelling arose. The principal reason of growing importance of this discipline is the fact of its success in description of the frequently encountered dynamic systems. Modern methodology of statistical procedures has recommended the following sequence of stages when processing time series analyses and forecasting (SAS/ETS User´s Guide, Version 6, 1993):

- data collection
- data examination and possibly cleaning of those
- data preparation for modelling
- preliminary data analysis
- design of a suitable candidate for prognosticating
- obtaining a sample of variables (as far as a multivariate series is being processed
- processing of preliminary modelling
- accuracy and stability of the model detection
- selection and adjustment of the final model

- implementation of the final model.

This research has been mainly focused at one of the stages of the sequence presented above, namely at the design of appropriate candidates for the purpose of making extrapolation forecasts and assessment of these based on selected assessment criteria, inclusive of the mentioned comparison of single models and combined models aggregating forecasts obtained from various single models (Evans, 2002).

The approach to solution of univariate models is possible either using classical trend models (Hindls et al., 2000), or by means of the Box-Jenkins methodology. Interpretation of the Box-Jenkins methodology outcomes is relatively complex. In spite of it, this methodology is considered universal framework of modelling techniques and time series prognosticating. This methodology serves primarily as a starting point for modelling of non-stationary time series and seasonal time series with a complex stochastic structure. Models based on this methodology are considerably flexible and capable of fast adaptation to changes of the modelled process nature. They can describe such time series, too, where the classical analysis models fail (Cipra, 1986). In order to efficiently exploit these methods usually time series long enough are needed, covering about 50 observations. One of the simplest special models, employed by this methodology, is the so-called moving averages model, MA model. Another important special case of Box-Jenkins models are the autoregressive models, AR models. By means of combination, the mixed so-called ARMA model arises. The mixed model is based on an assumption of stationarity of the time series given (Martin et al., 2013). Most of the real economic time series do not satisfy this assumption and in order to apply ARMA models they have to be stationarized somehow. A suitable device to reach this end is to obtain differences of neighbouring time series values. An ARMA model applied on a time series modified this way is denoted the integrated model - ARIMA model. Models of this type are widely applied in various fields. E.g., in the area of hydrology, Dastorani et al. (2016) were dealing with individual cases of the models in their work aimed at examining utility of separate ARIMA model structures for monthly rainfall forecasting in Iran. Success of ARIMA models has been confirmed by Mishra and Singh (2013) in their study dealing with monthly forecasting of peanut prices in Delhi, just applying ARIMA and ANN models. According to all the indicators of predictive model quality assessment selected, the Box-Jenkins methodology, i.e., the ARIMA models, presented itself as more appropriate for prices forecasting of a farm commodity chosen.

Materials and methods

Time series analysis is becoming one of the most important areas in the development of statistics. It finds application in all fields of human activity, such a for example social sciences, medicine engineering. Area of ICT is not the exception. Development of ICT is one of the four main investment priorities for regional policy that targets all areas, urban or rural. For ERDF will be a priority to expand the deployment of broadband connectivity, as well as the development of ICT products and services and foster the use of ICT in these areas (European Commission, 2016).

For the empirical analysis, total of 70 short-term time series provided by Vodafone Czech Republic, Ltd. have been used, out of which 42 with monthly data collection frequency and 28 with quarterly frequency. In all the cases these were time series from the area of mobile telecommunication services in the Czech Republic. The time series concerning indicators of receipts originally were collected on monthly frequency and for the purpose of this analysis they were aggregated into quarterly time series. Length of the reference period ranged from 12 to 36 data.

The current positions of separate CR regions from the viewpoint of the indicators studied have been presented by means of basic statistical measures of location. For the description of development of the indicators studied index analysis has been employed.

Searching for an optimal time series analysis model can sometimes be very lengthy. In order to accelerate this process the model selection regime offered by the Time Series Forecasting System (TSFS) module can be employed. This module brings a chance to diagnose the time series from the viewpoint of trend or seasonality presence, or a necessity of logarithmic transformation of data. The model selection regime then is based on these diagnostic tests when looking for the most appropriate model. The generated models already take into account all the necessary adjustments in the data analysed, e.g. seasonal adjustment in case of detection of significant seasonal component in the time series analysed.

Three best models from the wide supply

of the adaptive, classical analytical and ARIMA models have been chosen for each of the time series analysed, based on diagnostic tests. Single models obtained this way have been then subjected to experimentation concerning their aggregation, i.e., forming of combined models offering forecasts as certain combinations of the forecasts supplied by separate individual models. The SAS System offers a chance of combining separate forecasts by way of a simple or weighted arithmetic average with varying system of weights. In this particular case weighted arithmetic average with the so-called regression weights offered has been chosen. The weights somehow penalize forecasts burdened by higher prediction errors.

In order to compare quality of the models estimated for all time series analyzed, be it the individual or the combined ones, the Mean Absolute Percent Error (MAPE) has been used, defined as

$$M.A.P.E. = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{y_t - y_t'}{y_t}\right| \qquad (1)$$

where y_t , or y'_t (t = 1, 2, ..., n) are the actual or predicted values of the time series given and n is the number of the time series observations. MAPE is the most common measure of forecast error. This statistics is widely used by many organizations when assessing forecast accuracy. MAPE is easy to interpret because thinking in percentage terms is very comfortable for most people. In order to support appropriateness of the models chosen the determination coefficient R^2 has been used, defined as

$$R^2 = \frac{\sum_{t=1}^{n}(y_t - y_t')^2}{\sum_{t=1}^{n}(y_t - \bar{y})^2} \qquad (2)$$

R^2 is often used by many authors around the world. It has ben applied, together with the MAPE measure, by Papagera et al. (2014), too, in their research dealing with water balance analysis and water supply changes in the area of Lake Koronia and with forecasting of the indicators studied by means of Artificial Neural Networks (ANN). Another criterion for ANN models quality assessment has been the Root Mean Square Error (RMSE).

The coefficient of determination has been applied, too, by Dastorani et al. (2016) in their work where they analysed and assessed appropriateness of forecasting models for monthly forecasts of rainfall based on time data collected from nine stations in the north-east of Iran, together with analysis of autocorrelation and partial autocorrelation functions of the models, which was employed as a further quality assessment criterion of the models constructed, in this paper, too. Applicability of most time series analytical models, namely, is dependent on satisfaction of the assumption, that, a lot of the residues has a nature of the so-called white noise, which is an uncorrelated random quantity with zero mean and constant variance (Seger and Hindls, 1993). Autocorrelation function (ACF) and partial autocorrelation function (PACF) graphs are one of the devices for the time series models quality assessment (Ranjit et al., 2015).

Autocorrelation function of the residues serves in estimation of autocorrelations between residues with varying time shift. The outcome of autocorrelation function looks like a column where there are the coefficients of correlation presented, between the Y_t series analysed and the Y_{t+k} series, where t = 2, 3, 4, …,T. Besides these correlation coefficients the SAS system offers 95% confidence intervals for the coefficients, in graphical form, too. If the autocorrelation function value exceeds the limit of such interval, it is statistically significant. It means that, the residues need not be of the white noise type, and it is giving a signal to the processing body to introduce a modification of the data, such as, e.g., an adequate transformation of these.

Autocorrelation function supplies an information on the level of relationship between the Y_t and Y_{t+k} quantities, regardless of the impact of other quantities situated between them. In some cases this information is not sufficient. The function expressing level of relationship between the Y and Y_{t+k}, where t = 2, 3, 4, …,T and it is also taking into question correlation coefficients obtained for all the lower time shifts, is the so-called partial autocorrelation function (PACF). The partial autocorrelation function outcomes are interpreted same as with the autocorrelation function, graphically it can be presented by means of correlogram. PACF is a very useful device when establishing the order of autoregression, if autoregression models (SAS/ETS User´s Guide, Version 6, 1993) are being constructed.

Results and discussion

Current state and development analysis of the indicators studied

A total of three indicators were analysed in separate regions of the Czech Republic: number of consumer postpaid customers, total revenues paid by these customers from TELCO services and revenues from Auditex service. Table 1 presents average shares of separate CR regions on the total number of customers and total volume of revenues from the given operator services studied over the period July 2013 - June 2016. The results obtained demonstrate that, the strongest region, i.e., region with the highest share both of the number of customers and of the total volume of revenues from the services by the given operator is Prague Region. The second and third place are occupied by Central Bohemia Region and South Moravian Region. At the last place of the fictitious ladder then is Karlovy Vary Region.

For description of development of the indicators studied of customer numbers and volumes of revenues from TELCO services, value of the basic index from the last period under study can be used, since its values support the total development tendency of the indicators over the period given. Values of the index numbers are summarized in Table 2. The customer numbers in separate regions same as the revenues from TELCO services sold do not present any significant deviations - increase or decrease on the contrary. The customer numbers indicator shows a gradual slight downward trend in most of the CR regions, except for Prague Region, Central Bohemia Region, Liberec Region, Karlovy Vary Region and Ústí nad Labem Region. The revenues from TELCO services sold indicator shows in almost all the regions a slight upward trend, in the regions Hradec Králové Region and Zlín Region there is a constant trend, what is confirmed by the time series diagnostic tests results in the following part of the paper, too. In 6 cases of monthly time series - two of these have been represented just by this indicator - no trend was identified in the tests (sometimes a time series with constant trend may be denoted as trendless series). What concerns the indicator of Audiotex services sold revenues volumes, this indicator shows a very irregular course in all the regions and the base index would have a zero predictive value when analysed from the development viewpoint. Therefore, it is not included in the table 2.

Diagnostic test results of the time series studied

The Table 3 presents results of diagnostic tests of the time series analysed, in an overview. The results are presented not only in absolute terms but in percentages, too.

The table contains absolute numbers of time series studied, showing a trend, seasonality or a need of logarithmic transformation considering

Region	avg share of the total customer base	avg share in total sales of TELCO svc	avg share in total sales of ATX svc
South Bohemia Region	5%	5%	5%
South Moravian Region	12%	12%	12%
Karlovy Vary Region	3%	3%	3%
Hradec Králové Region	5%	5%	5%
Liberec Region	4%	4%	4%
Moravian-Silesian Region	9%	8%	8%
Olomouc Region	6%	5%	6%
Pardubice Region	5%	5%	5%
Plzeň Region	5%	5%	4%
Prague Region	17%	20%	16%
Central Bohemia Region	12%	13%	13%
Ústí nad Labem Region	7%	8%	8%
Vysočina Region	4%	3%	5%
Zlín Region	6%	5%	5%
TOTAL	**100%**	**100%**	**100%**

Source: Own processing based on data provided by Vodafone Czech Republic, Ltd.

Table 1: Average shares of separate regions on total values of the indicators studied.

Region	number of customers - base index in %	revenue from TELCO svc - base index in %	revenue from ATX svc - base index in %
South Bohemia Region	93	102	63
South Moravian Region	96	104	64
Karlovy Vary Region	102	112	75
Hradec Králové Region	91	100	50
Liberec Region	105	115	75
Moravian-Silesian Region	91	107	58
Olomouc Region	90	101	76
Pardubice Region	90	98	58
Plzeň Region	98	109	68
Prague Region	105	110	91
Central Bohemia Region	104	112	64
Ústí nad Labem Region	103	114	100
Vysočina Region	91	102	105
Zlín Region	91	100	68

Source: Own processing based on data provided by Vodafone Czech Republic, Ltd.

Table 2: The base index values of customer numbers and TELCO services revenues.

	Type of time series	monthly	monthly in %	quarterly	quarterly in %	Grand Total	Grand Total
Trend	**no**	6	14%		0%	6	9%
	yes	36	86%	28	100%	64	91%
	Grand Total	42	100%	28	100%	70	100%
Seasonality	**maybe**		0%	28	100%	28	40%
	no	30	71%		0%	30	43%
	yes	12	29%		0%	12	17%
	Grand Total	42	100%	28	100%	70	100%
Log transformation	**maybe**	28	67%	28	100%	56	80%
	no	14	33%		0%	14	20%
	Grand Total	42	100%	28	100%	70	100%
ACF/PACF	**no**	8	19%	3	11%	11	16%
	yes	34	81%	25	89%	59	84%
	Grand Total	42	100%	28	100%	70	100%

Source: Own processing

Table 3: Absolute and percentage results of diagnostic tests.

periodicity of the series given, further on their relative shares on the total number of indicators studied, also in distinction into monthly and quarterly time series. Need of logarithmic transformation has not been proven by the diagnostic test at all, only a possibility to transform the original time series this way has been found. This concerns 56 time series analysed, out of these exactly one half being those quarterly and one half monthly. Presence of the trend component has been identified in most series, only in 6 cases of those monthly has not.

Seasonal component has been proven in monthly time series only, in 29 % cases of the total number of series with monthly frequency. In all cases of quarterly time series there was a chance of seasonal element detected, but this was neither confirmed nor excluded, either.

Assessment of individual models

In Table 4 the models are presented proven as those most appropriate for analysis and prognostication of the time series processed, as based on the diagnostic tests mentioned above and on the assessment criterion chosen. Besides the most appropriate models also the share of these, let us say, their percentage on separate time series under study groups, classified according to data collection periodicity, inclusive of the absolute number of models with the MAPE measure within a certain interval.

	monthly time series		quarterly time series		summary	
Overview of selected individual models	**Models proportion**	**Number of models**	**Models proportion**	**Number of models**	**Models proportion**	**Number of models**
Linear (HOLT) Exponential Smoothing	11%	13	0%		6%	13
MAPE - 5 and smaller	9%	11	0%		5%	11
MAPE - 20 and higher	2%	2	0%		1%	2
Damped Trend Exponential Smoothing	14%	17	0%		8%	17
MAPE - 5 and smaller	9%	11	0%		5%	11
MAPE - between 15 and 20	1%	1	0%		0%	1
MAPE - 20 and higher	4%	5	0%		2%	5
Random Walk with Drift	3%	4	0%		2%	4
MAPE - 5 and smaller	3%	4	0%		2%	4
Log Winters Method Additive	6%	7	25%	21	14%	28
MAPE - between 10 and 15	1%	1	2%	2	1%	3
MAPE - 5 and smaller	5%	6	11%	9	7%	15
MAPE - between 15 and 20	0%		6%	5	2%	5
MAPE - 20 and higher	0%		6%	5	2%	5
Winters Method Additive	10%	12	23%	19	15%	31
MAPE - between 10 and 15	1%	1	1%	1	1%	2
MAPE - 5 and smaller	9%	11	14%	12	11%	23
MAPE - between 15 and 20	0%		4%	3	1%	3
MAPE - 20 and higher	0%		4%	3	1%	3
Log Damped Trend Exponential Smoothing	12%	14	0%		7%	14
MAPE - 5 and smaller	6%	7	0%		3%	7
MAPE - between 15 and 20	1%	1	0%		0%	1
MAPE - 20 and higher	5%	6	0%		3%	6
Winters Method Multiplicative	6%	7	10%	8	7%	15
MAPE - between 10 and 15	0%		1%	1	0%	1
MAPE - 5 and smaller	6%	7	4%	3	5%	10
MAPE - between 15 and 20	0%		2%	2	1%	2
MAPE - 20 and higher	0%		2%	2	1%	2
Linear Trend	1%	1	0%		0%	1
MAPE - 5 and smaller	1%	1	0%		0%	1
Log Linear Trend	1%	1	0%		0%	1
MAPE - 5 and smaller	1%	1	0%		0%	1
Log Linear (HOLT) Exponential Smoothing	11%	13	0%		6%	13
MAPE - 5 and smaller	4%	5	0%		2%	5
MAPE - between 15 and 20	1%	1	0%		0%	1
MAPE - 20 and higher	6%	7	0%		3%	7
Double (Brown) Exponential Smoothing	5%	6	0%		3%	6
MAPE - 5 and smaller	5%	6	0%		3%	6
Log Double (Brown) Exponential Smoothing	2%	2	1%	1	1%	3
MAPE - 5 and smaller	2%	2	0%		1%	2
MAPE - 20 and higher	0%		1%	1	0%	1
Simple Exponential Smoothing	5%	6	0%		3%	6
MAPE - 5 and smaller	1%	1	0%		0%	1
MAPE - 20 and higher	4%	5	0%		2%	5

Source: Own processing

Table 4: Percentages of selected models in the time series groups classified by periodicity.

	monthly time series		quarterly time series		summary	
Overview of selected individual models	**Models proportion**	**Number of models**	**Models proportion**	**Number of models**	**Models proportion**	**Number of models**
Log Simple Exponential Smoothing	4%	5	0%		2%	5
MAPE - 5 and smaller	1%	1	0%		0%	1
MAPE - 20 and higher	3%	4	0%		2%	4
Log Random Walk with Drift	1%	1	0%		0%	1
MAPE - 20 and higher	1%	1	0%		0%	1
Log Winters Method Multiplicative	4%	5	7%	6	5%	11
MAPE - between 10 and 15	1%	1	1%	1	1%	2
MAPE - 5 and smaller	3%	4	1%	1	2%	5
MAPE - between 15 and 20	0%		2%	2	1%	2
MAPE - 20 and higher	0%		2%	2	1%	2
Linera Trend with Autoregressive Errors	0%		4%	3	1%	3
MAPE - between 15 and 20	0%		2%	2	1%	2
MAPE - 20 and higher	0%		1%	1	0%	1
Log ARIMA (0,1,1,)(1,0,0,)s NOINT	0%		4%	3	1%	3
MAPE - between 15 and 20	0%		1%	1	0%	1
MAPE - 20 and higher	0%		2%	2	1%	2
ARIMA (0,1,1,)(1,0,0,)s NOINT	0%		1%	1	0%	1
MAPE - 20 and higher	0%		1%	1	0%	1
Linear Trend with Seasonal Terms	3%	4	12%	10	7%	14
MAPE - between 10 and 15	0%		1%	1	0%	1
MAPE - 5 and smaller	3%	4	11%	9	6%	13
Log Linear Trend with Seasonal Terms	0%		12%	10	5%	10
MAPE - 5 and smaller	0%		10%	8	4%	8
MAPE - between 15 and 20	0%		1%	1	0%	1
MAPE - 20 and higher	0%		1%	1	0%	1
Log Linear Trend with Autoregressive Errors	0%		1%	1	0%	1
MAPE - between 15 and 20	0%		1%	1	0%	1
Linear Trend with Autoregressive Errors	1%	1	1%	1	1%	2
MAPE - 5 and smaller	1%	1	0%		0%	1
MAPE - between 15 and 20	0%		1%	1	0%	1
Summary	**100.00%**	**119**	**100%**	**84**	**100%**	**203**

Source: Own processing

Table 4: Percentages of selected models in the time series groups classified by periodicity (continuation).

It is seen from the outcomes obtained that, in most cases one of the exponential smoothing models and its possible modifications have been selected. As the most appropriate models for analysis and forecasting of short-term time series with monthly periodicity the following models have been found of use:

- Linear (Holt) Exponential Smoothing and its logarithmic transformation
- Damped Trend Exponential Smoothing and its log transformation
- Winters Method (both additive and multiplicative) and its log transformation.

In the quarterly time series the following models have been found of use primarily:

- Winters Method Additive and Log Winters Method Additive
- Winters Method Multiplicative and Log Winters Method Multiplicative
- Linear Trend with Seasonal Term and log Linear Trend with Seasonal Term.

The Box-Jenkins ARIMA models have, in case of looking for the most appropriate model, almost in all the cases where they have been offered for time series modelling by SAS system, placed themselves at the third to fifth place.

The *MAPE* value (its amount) fluctuates according to situation given. No universally accepted limit has been established for it. Practically, it is possible to meet a situation when a value at 15 % is required, or 5 % otherwise. Anyway, a model showing the *MAPE* value at about 10 % is acceptable. The outcomes obtained show that, in 63 % of time series analysed, models with *MAPE* below 15% have been set up.

The R^2 coefficient of determination has been employed in order to confirm appropriateness of the time series models chosen. Table 5 below demonstrates the results of the two measures used, (*MAPE* and R^2). It is possible to regard these separately, or in aggregation as well. It is seen from Table 5 that, were the two measures of appropriateness assessment considered together, the desired R^2 value would be above 80 % and the *MAPE* value at the same time up to 15 %. Then 81 models out of the total of 203 would present themselves as those very appropriate for modelling of the time series studied. In such a case always 3 best models would be selected for 64 time series as to the *MAPE* value, 2 models would be offered by SAS system to 5 time series and one series would be processed by one only model.

Furthermore, analysis of autocorrelation functions and partial autocorrelation functions has been applied as the next assessment criterion of the models employed quality. In 84 % cases of time series analysed the selected models have satisfied this criterion.

Comparative analysis and combined models assessment

In most individual models chosen the outcomes have been obtained that only slightly differed as to MAPE levels. These were the reasons, too, why construction of aggregated prediction models has been tried. Combined predictions were obtained for separate time series with MAPE values as given in Table 6. For 47 time series the combined models have been loaded by a lower MAPE prediction error - a not very significant one, mostly, anyway - than the single models originally selected. Out of the total of 70 time series analysed the combined models presented themselves as more appropriate for analysis and forecasting of the indicators analysed, as to the MAPE viewpoint, in 55 % of the time series with monthly frequency and in 86 % with quarterly frequency.

In the most successful combinations the models of simple exponential smoothing, exponential smoothing with muffled trend and the Winters´model of exponential smoothing have been represented most often. As for the number of single models of which the combined models have been constructed, the combination of three single models was the most frequented, in five cases only two models have been involved.

Time series in the area of telecommunications, especially in the domain of third parties services supply (Audiotex, here) frequently record a complex structure, their development is affected by many factors and their modelling can present a difficult problem. Outcomes of the studies dealing with time series modelling and prognosticating using Box-Jenkins methodology show that, this problem could be solvable just through its application. However, the time series studied in this paper do not satisfy the B-J methodology condition concerning the extent of data files. A further research could aim at an empirical analysis of still more extensive set of time series, experimentation with various lengths of the reference period and collection of longer short-term time series but those long-term ones, too.

type of series/R-square	MAPE - 20 and higher	MAPE - between 10 and 15	MAPE - between 15 and 20	MAPE - 5 and smaller	Total number of models
monthly	30	3	3	83	119
more than 80%				55	55
between 30% and 80%	9	3	3	21	36
less than 30%	21			7	28
quarterly	18	6	18	42	84
more than 80%				26	26
between 30% and 80%	8	4	14	16	42
less than 30%	10	2	4		16
Total number of models	48	9	21	125	203

Source: Own processing

Table 5: Overview of the *MAPE* and R^2 results.

INDICATOR	type of series	Type of combination	MAPE
Karlovy Vary Region - ATX revenue	monthly	Combination of 2 best models	29.4698
Hradec Králové Region - ATX revenue	monthly	Combination of 2 best models	**28.4011**
Moravian-Silesian Region - ATX revenue	monthly	Combination of 2 best models	**26.5839**
Zlín Region - ATX revenue	monthly	Combination of 2 best models	**24.0700**
Hradec Králové Region - revenue from TELCO services	monthly	Combination of 2 best models	21.8686
Prague Region - number of costumers (CP)	monthly	Combination of 3 best models	0.5813
South Bohemia Region - number of costumers (CP)	monthly	Combination of 3 best models	0.7892
South Moravian Region - number of costumers (CP)	monthly	Combination of 3 best models	**0.6547**
Karlovy Vary Region - number of costumers (CP)	monthly	Combination of 3 best models	0.7291
Hradec Králové Region - number of costumers (CP)	monthly	Combination of 3 best models	0.7721
Liberec Region - number of costumers (CP)	monthly	Combination of 3 best models	0.7084
Moravian-Silesian Region - number of costumers (CP)	monthly	Combination of 3 best models	0.8596
Olomouc Region - number of costumers (CP)	monthly	Combination of 3 best models	0.7963
Pardubice Region - number of costumers (CP)	monthly	Combination of 3 best models	0.7399
Plzeň Region - number of costumers (CP)	monthly	Combination of 3 best models	0.6618
Central Bohemia Region - number of costumers (CP)	monthly	Combination of 3 best models	0.5739
Ústí nad Labem Region - number of costumers (CP)	monthly	Combination of 3 best models	0.7315
Vysočina Region - number of costumers (CP)	monthly	Combination of 3 best models	0.8099
Zlín Region - number of costumers (CP)	monthly	Combination of 3 best models	0.7259
Prague Region - ATX revenue	monthly	Combination of 3 best models	**17.2691**
South Bohemia Region - ATX revenue	monthly	Combination of 3 best models	**22.3199**
Liberec Region - ATX revenue	monthly	Combination of 3 best models	27.4538
Olomouc Region - ATX revenue	monthly	Combination of 3 best models	**30.2340**
Pardubice Region - ATX revenue	monthly	Combination of 3 best models	**20.8409**
Plzeň Region - ATX revenue	monthly	Combination of 3 best models	**28.1354**
Central Bohemia Region - ATX revenue	monthly	Combination of 3 best models	**20.3211**
Ústí nad Labem Region - ATX revenue	monthly	Combination of 3 best models	**20.4871**
Vysočina Region - ATX revenue	monthly	Combination of 3 best models	**11.6700**
Prague Region - ATX revenue	quarterly	Combination of 3 best models	**15.1758**
South Bohemia Region - ATX revenue	quarterly	Combination of 3 best models	19.0482
South Moravian Region - ATX revenue	quarterly	Combination of 3 best models	**12.0788**
Karlovy Vary Region - ATX revenue	quarterly	Combination of 3 best models	**17.2793**
Hradec Králové Region - ATX revenue	quarterly	Combination of 3 best models	**19.7755**
Liberec Region - ATX revenue	quarterly	Combination of 3 best models	**14.2904**
Moravian-Silesian Region - ATX revenue	quarterly	Combination of 3 best models	23.6456
Olomouc Region - ATX revenue	quarterly	Combination of 3 best models	28.0381
Pardubice Region - ATX revenue	quarterly	Combination of 3 best models	**19.1458**
Plzeň Region - ATX revenue	quarterly	Combination of 3 best models	**14.3764**
Central Bohemia Region - ATX revenue	quarterly	Combination of 3 best models	**23.2126**
Ústí nad Labem Region - ATX revenue	quarterly	Combination of 3 best models	**17.5013**
Vysočina Region - ATX revenue	quarterly	Combination of 3 best models	**13.1043**
Zlín Region - ATX revenue	quarterly	Combination of 3 best models	17.1540
Prague Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.6276**
South Bohemia Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.6532**
South Moravian Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.5062**
Karlovy Vary Region - revenue from TELCO services	monthly	Combination of 3 best models	14.7589
Liberec Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.7244**
Moravian-Silesian Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.5628**

Source: Own processing

Table 6: The *MAPE* values of combined models for all the indicators studied.

INDICATOR	type of series	Type of combination	MAPE
Olomouc Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.4959**
Pardubice Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.6477**
Plzeň Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.5230**
Central Bohemia Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.4832**
Ústí nad Labem Region - revenue from TELCO services	monthly	Combination of 3 best models	14.5551
Vysočina Region - revenue from TELCO services	monthly	Combination of 3 best models	0.6289
Zlín Region - revenue from TELCO services	monthly	Combination of 3 best models	**0.5389**
Prague Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.4933**
South Bohemia Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.7126**
South Moravian Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.4981**
Karlovy Vary Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.6053**
Hradec Králové Region - revenue from TELCO services	quarterly	Combination of 3 best models	**10.4817**
Liberec Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.7252**
Moravian-Silesian Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.6778**
Olomouc Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.6251**
Pardubice Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.6157**
Plzeň Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.5067**
Central Bohemia Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.5666**
Ústí nad Labem Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.6812**
Vysočina Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.4489**
Zlín Region - revenue from TELCO services	quarterly	Combination of 3 best models	**0.6593**
South Moravian Region - ATX revenue	monthly	No combintion	

Source: Own processing

Table 6: The *MAPE* values of combined models for all the indicators studied (continuation).

Conclusion

Mobile telecommunications services are primarily a source of information and enable to improve the quality of life, particularly in rural areas. The importance of mobile services lies especially in the expanding information base, which results in greater availability of various services (internet banking, reservation systems - the purchase of tickets, self-learning etc.). In contrast, mobile services also allow to inform the public about life in the village, local products offer, community events or accommodation options, which are a source of income (tourism = Rural Development).

Consumption of selected mobile services in the Czech Republic has been found as one quite evenly distributed from the CR regional breakdown viewpoint. The separate regions, let us say, the consumer postpaid customers from separate regions, have been using given services of Vodafone Czech Republic Ltd., with almost equal shares as to their numbers in all the regions, as it is shown in the Table 1. Separate regions include urban areas as well as rural areas. Rural and the sparsely populated areas are still disadvantaged because of weakening telecommunication systems, although access to high-capacity telecommunications is essential for the social, cultural and economic life of each of us. ICT development and access to all mobile services is very important for everyone of us whether we live in the city or in the countryside.

In spite of a very irregular development in some of the time series analysed, the models generated for their development description and possible forecasts of their future behaviour have been loaded by comparatively low prediction errors, be it for the single models, or for the aggregation of these. Outcomes of this study show that, the SAS System generated models can be successfully employed for extrapolation forecasting in time series under study. In order to assess whether it is recommendable to apply this process of forecasting, it will be needed to provide empirical analyses in larger extent than the one described in this paper.

It is obvious, too, from the empirical research outcomes that, in order to obtain a more exact forecast, construction of combined models is definitely recommendable. This has been confirmed, e.g., in the research results by Oliveira and Ludermir (2016), the core of which has been laid in quality assessment of the forecasts obtained through aggregation, let us say, combination of single models. A more detailed exploration of combined forecasts has to be provided applying experimental ways.

Models of exponential smoothing and logarithmic transformations of these have primarily pushed through and participated in the combined models construction, out of all the models examined. The Box-Jenkins ARIMA models ranked almost always at the third to fifth place in the sequence of models when looking for the most appropriate one, as seen from the MAPE assessment measure viewpoint. ARIMA model estimation is a very complex matter and it is processed over several stages (identification, estimation, verification) that assume experience and perfect knowledge of the methodology. SAS System facilitates the procedure to the analyst by means of the TSFS module, since it processes the stages mentioned automatically. It is needed to underline, anyway, that this procedure need not bring an optimal, let us say, satisfactory result.

Zahan and Kenett (2013) applied TSFS in forecasting energy consumption in selected countries of Asia, inclusive of the development analysis of the indicator. Quality of the prognostic models here was assessed by means of MAPE.

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