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# The Effect of Income-Transfer Programs on Income Distribution

by

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## Introduction

Increased attention recently has been given to various income transfer schemes that would guarantee all U. S. citizens a minimum annual income. Christopher Green's book <u>Negative Taxes and the Poverty Problem</u>,[1/] provides an extensive discussion about the administrative and technical features of the major guaranteed annual income plans that have been proposed, such as negative income taxation, family allowances, and social dividends. The basic idea which such plans have in common is transferring revenue from people with incomes above some minimum level to people with incomes below that level.

Despite a growing amount of literature regarding negative income taxation and other plans as tools for reducing poverty, relatively little has been done with respect to the effects of such income transfer programs on the distribution of income in the U. S., beyond examining the immediate tax incidence of the transfer programs. While limiting an analysis to the immediate tax incidence does provide worthwhile information, it does

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1/ C. Green, <u>Negative Taxes and the Poverty Problem</u>, Washington, D. C., Brookings Institution, (June 1967).

not answer other important questions about the longer term economic effects of reducing income inequality. Such effects are important for several reasons. For example, the composition of aggregate consumption expenditures in the economy is likely to change due to the different propensities between sectors. Second, under certain conditions such as full employment, income transfers may have substantial effects on the level of money wages and general prices. Third, the absolute and relative shares of income for various sectors of the economy may be quite different when a new equilibrium is reached from what they were either before or immediately after the transfer program was initiated. Such obviously important questions would seem to warrant further examination.

In discussing the longer-term effects of income transfer programs we shall come upon an unexpected by-product. In particular, we shall find that the equilibrium value of the income of sectors which furnish the transfer (the "donor" groups) may be higher or lower than it was before the transfer program. This by itself is not too surprising, since we would expect the equilibrium values to depend on many factors, such as the value of the multiplier, the marginal propensity to consume, etc. The by-product will be that once the conditions are delineated which determine whether the income of donor groups increases or falls as a result of the program, then these same conditions may be used as guides in developing transfer programs that are Pareto-optimal. In other words, such conditions may be used to design transfer programs in such a way that no sector of the economy is made worse off, in terms of absolute real income, as a result of the program.

Designing transfer programs in a way that assures Paretian optimality can be an important tool for policymakers. Currently, there are extreme viewpoints on transfer programs, one extreme claiming any transfer program must make the donor group worse off, while others say transfer programs will make donor groups better off because of the generation of increased economic activity. Our tentative answer, which we are pursuing in more refined research, lies between these two extremes. It recognizes the possibility that some transfer programs may indeed reduce the real income of donor groups while other programs may increase the income of donor groups. Explicit recognition of such possibilities in the design of transfer programs hence could do much to gain more nearly universal acceptance of transfer programs as a means of reducing income inequality.

For purposes of discussion we are presenting a highly simplified type of model, which may be thought of as a first approximation to a more complex model the authors are working with in research on this subject. Our approach is comparative-static in nature, starting with an initial equilibrium condition, introducing an income transfer, determining the new equilibrium point and comparing the two. In so doing, we abstract from important questions concerning the time path of adjustments to the transfer program; questinns we hope to incorporate in the more detailed model.

We start first by introducing a two sector model of the economy, composed of wage-earners and non-wage-earners, and determine the effects on income shares of transferring income to wage-earners from non-wage-earners. In this two-sector model, the major result shows the conditions under which a transfer program will be Pareto optimal, but it does not give us the capability of designing programs to achieve Pareto optimality. We then extend the basic model to include a third sector, welfare recipients. The three sector model is first considered under conditions of under employment where changes in money and real income coincide, and next, under conditions of full employment where income changes are in money terms when potential real output is fixed. Likely changes in potential real output are incorporated into the model. The major result of the three sector model is that we are able to determine the conditions under which the real income of donor groups will increase, and that these conditions are such that they can be used to design programs that are Pareto optimal.

In discussing the implications of our results, we will note that when some widely accepted estimates of the magnitude of several of the important parameters are applied to the Paretian conditions, there is indeed the possibility that some transfer programs may reduce the income of donor groups. Such a possibility lends urgency to the tasks of both identifying and measuring the relevant parameters, and to incorporate the results of such investigations into actual policy formulation.

## The Two-Sector Model

Our basic two-sector model is derived from one presented by Arthur Smithies and attributed to J. Tinbergen.[2] The model consists of three behavioral relations and three accounting identities, as follows:

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2/ A Smithies, "The Behavior of Money National Income under Inflationary Conditions," in A. Smithies and J. K. Butters, eds., Readings in Fiscal Policy, Richard D. Irwin, Inc., Homewood, Illinois (1955) pp. 122-136.

1. Consumption function:

$$C = \lambda_1 L + \lambda_2 Z + \delta, \quad \text{where} \quad 0 < \lambda_2 < \lambda_1 \quad \text{and} \quad \delta > 0 \tag{1}$$

2. Demand for labor:

$$L = \gamma Y + \beta, \quad \text{where} \quad 0 \leq \gamma \leq 1, \; \beta \geq 0 \tag{2}$$

3. Investment function:

$$I = \alpha Y, \quad \text{where} \quad 0 \leq \alpha \leq 1 \tag{3}$$

4. Accounting identities:

$$Y = L + Z \tag{4}$$

$$Y = C + S \tag{5}$$

$$I = S \tag{6}$$

Consumption (C) is assumed to be a linear function of wage-eraners income, L, and non-wage-earners income, Z (see eq. 1). The wage-bill, or wage-earners income L, is assumed to be a linear functinn of national income Y (eq. 2), while net investment I, is taken to be proportional to national income. The marginal propensities to consume of wage-earners and non-wage-earners, $\lambda_1$, and $\lambda_2$, are necessarily positive and less than unity; we also assume $\lambda_1$ is greater than $\lambda_2$. Finally, national income is taken as the sum of wage and non-wage earners income; also as the sum of consumption plus savings (eq. 5). Savings, expost, are equal to investment.

Altogether, the basic model consists of six equations, which may be solved for the equilibrium values of the variables Y, C, I, L, Z, and S in terms of the parameters $\alpha$, $\gamma$, $\beta$, $\lambda_1$, $\lambda_2$ and $\delta$. In particular, the equations may be solved as presented to yield equilibrium values in the absence of a transfer program. Also, if a program is assumed which transfers a predetermined amount of income $\Delta\beta$ to wage-earners, equilibrium values with the transfer program can be obtained by adding $\Delta\beta$ to the right-hand side of equation 2 and solving the resulting system. These computations were made and results are presented in Table 1, where we compare equilibrium values (before and after the transfer program) for national income, the absolute levels of wage and non-wage-earners incomes, and the relative shares of the two sectors.[3/]

First, comparing national income with and without the transfer program, the difference is found to be:

$$\bar{Y}_1 - \bar{Y}_0 = \frac{\Delta\beta(\lambda_1 - \lambda_2)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha} \tag{7}$$

The denominator of equatinn (7) is one minus the slope of the aggregate demand function, hence is positive. Also $\Delta\beta$ is positive and $\lambda_1 > \lambda_2$. Therefore, national income will increase as a result of the program. Note that the amount of the increase in national income is the amount of the transfer multiplied by the difference in the marginal propensities to consume, all times the multiplier.

3/ The method of solution is found in the appendix, (part A).

Table 1. Equilibrium values of the two sector model with and without an income transfer program

| Variable | Equilibrium Values | |
|---|---|---|
| | Without a transfer program | With a transfer program |
| National Income (Y) | $\bar{Y}_0 = \dfrac{\beta(\lambda_1 - \lambda_2) + \delta}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ | $\bar{Y}_1 = \dfrac{(\beta + \Delta\beta)\ (\lambda_1 - \lambda_2) + \delta}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ |
| Wage-Earners Income (L) | $\bar{L}_0 = \dfrac{\beta(1 - \lambda_2 - \alpha) + \gamma\delta}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ | $\bar{L}_1 = \dfrac{(\beta + \Delta\beta)\ (1 - \lambda_2 - \alpha) + \gamma\delta}{1 - \lambda_1 - \lambda_2(1 - \gamma) - \alpha}$ |
| Wage-Earners Share (L/Y) | $\dfrac{\bar{L}_0}{\bar{Y}_0} = \dfrac{\beta(1 - \lambda_2 - \alpha) + \gamma\delta}{\beta(\lambda_1 - \lambda_2) + \delta}$ | $\dfrac{\bar{L}_1}{\bar{Y}_1} = \dfrac{(\beta + \Delta\beta)\ (1 - \lambda_2 - \alpha) + \gamma\delta}{(\beta + \Delta\beta)\ (\lambda_1 - \lambda_2) + \delta}$ |
| Non-Wage-Earners Income (Z) | $\bar{Z}_0 = \dfrac{-\beta(1 - \lambda_1 - \alpha) + \delta(1 - \gamma)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ | $\bar{Z}_1 = \dfrac{-\beta(\beta + \Delta\beta)\ (1 - \lambda_1 - \alpha) + \delta(1 - \gamma)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ |
| Non-Wage-Earners Share (Z/Y) | $\dfrac{\bar{Z}_0}{\bar{Y}_0} = \dfrac{-\beta(1 - \lambda_1 - \alpha) + \delta(1 - \gamma)}{\beta(\lambda_1 - \lambda_2) + \delta}$ | $\dfrac{\bar{Z}_1}{\bar{Y}_1} = \dfrac{-(\beta + \Delta\beta)\ (1 - \lambda_1 - \alpha) + \delta(1 - \gamma)}{(\beta + \Delta\beta)\ (\lambda_1 - \lambda_2) + \delta}$ |

Next, comparing wage earners income with and without the program, we find:

$$\bar{L}_1 - \bar{L}_0 = \frac{\Delta\beta(1 - \lambda_2 - \alpha)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha} . \qquad (8)$$

Note that the denominator of (8) and $\Delta\beta$ are positive. It also can be shown that $1 - \lambda_2 - \alpha$ is positive[4/]. Thus we reach the unsurprising result that wage-earners income increases as a result of the program. We also show in the appendix (part B) that the wage earners relative income share also increases as a result of the program. It follows immediately that the non-wage-earners relative share must decrease.

The key question, which we have left to last, is the change in the absolute level of the income of non-wage-earners, the donor group. This change will determine whether the program is Pareto optimal and will have substantial implications for the programs feasibility. Examining Table 1, we find

$$Z_1 - Z_0 = \frac{-\Delta\beta(1 - \lambda_1 - \alpha)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha} \qquad (9)$$

Note again that the denominator of (9) is positive, as is $\Delta\beta$. Hence the term $(1 - \lambda_1 - \alpha)$ is of crucial importance in determining whether $Z_1 - Z_0$ is positive or negative. But a priori, it is not known whether $1 - \lambda_1 - \alpha$ is positive or negative, hence the effect of the transfer program on non-wage-earners income depends on:

4/ Since $1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha > 0$; $1 - \lambda_1\gamma - \lambda_2 + \lambda_2\gamma - \alpha > 0$; hence $1 - \lambda_2 - \alpha > \gamma(\lambda_1 - \lambda_2) > 0$.

a. The propensity of businessmen to investment, $\alpha$. The larger the propensity to invest the more likely is $Z$ to increase.

b. The propensity of wage-earners to consume, $\lambda_1$. The larger is $\lambda_1$ the more likely is $Z$ to increase.

It should be noted that, with given values of $\delta, \gamma, \lambda_1, \lambda_2$ and $\alpha$, equation (9) determines whether $Z_1 \gtrless Z_0$, and the direction of change is independent of the magnitude of the transfer. In other words, in this two-sector model, equation 9 may not be used as a policy guide in formulating Pareto-optimal transfer programs, except insofar as programs are designed to influence the value of the parameters $\lambda_1$ and $\alpha$. However, equation 9 may be used to check whether a contemplated program will be Pareto optimal, given values of $\lambda_1$ and $\alpha$.

## The Three Sector Model

Our initial two sector model may be augmented by a third sector, welfare recipients, and the results of transfer programs from both wage and non-wage earners to welfare recipients analyzed. To accomplish this, we add $W$, the income of welfare recipients, to the right hand side of equations (1) and (4), and subtract the portion of the transfer coming from wage-earners, $\Delta\beta$, from the right hand side of (2). These adjustments in the system correspond to the following assumptions:

a. The marginal propensity to consume of welfare recipients is one.

b. Part of the transfer, $\Delta\beta$, comes from wage-earners while part comes from non-wage-earners.

The effect of the transfer program on equilibrium values may be determined by solving the expanded model and comparing results with those of the initial two sector model. This comparison is valid under the additional assumption that persons without income do not contribute to aggregate demand. These computations were made and results are presented in Table 2.

Examining the change in national income induced by the transfer program we find this to be given by:

$$\bar{Y}_2 - \bar{Y}_0 = \frac{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha} \qquad (10)$$

Note that both the numerator and denominator of (10) are positive, hence national income will increase as a result of a transfer program.[5/]

The income of welfare recipients obviously will increase, both in absolute amount and relative shares. We also show in the appendix (part C) that the relative share of both donor groups must decline. The key question again is what happens to the absolute income of the donor groups.

The change in wage-earners income is found to be:

$$\bar{L}_2 - \bar{L}_0 = \frac{-\Delta\beta(1 - \lambda_2 - \alpha) + \gamma W(1 - \lambda_2)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$$

We note again that the denominator of equation (11) is positive, which means the income of wage-earners will increase if and only if the numerator is positive, a condition which may be rewritten as:

5/ Note that $W(1 - \lambda_2) > \Delta\beta(\lambda_1 - \lambda_2)$, since $W \geq \Delta\beta$ and $1 - \lambda_2 > \lambda_1 - \lambda_2$, thus assuring the numerator is positive.

Table 2. Equilibrium values of National Income (Y). Wage-Earners Income (L). Non-Wage-Earners Income (Z) and their relative shares in the two and three sector models

| Variable | Two Sector Model<br>Without a transfer program | Three Sector Model<br>With a transfer program |
|---|---|---|
| Y | $\bar{Y}_0 = \dfrac{\beta(\lambda_1 - \lambda_2) + \delta}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ | $\bar{Y}_2 = \dfrac{(\beta - \Delta\beta)\ (\lambda_1 - \lambda_2) + \delta + W(1 - \lambda_2)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ |
| L | $\bar{L}_0 = \dfrac{\beta(1 - \lambda_2 - \alpha) + \gamma\delta}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ | $\bar{L}_2 = \dfrac{(\beta - \Delta\beta)\ (1 - \lambda_2 - \alpha) + \gamma\delta + \gamma W(1 - \lambda_2)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ |
| $\frac{L}{Y}$ | $\dfrac{\bar{L}_0}{\bar{Y}_0} = \dfrac{\beta(1 - \lambda_2 - \alpha) + \gamma\delta}{\beta(\lambda_1 - \lambda_2) + \delta}$ | $\dfrac{\bar{L}_2}{\bar{Y}_2} = \dfrac{(\beta - \Delta\beta)\ (1 - \lambda_2 - \alpha) + \gamma\delta + \gamma W(1 - \lambda_2)}{(\beta - \Delta\beta)\ (\lambda_1 - \lambda_2) + \delta + W(1 - \lambda_2)}$ |
| Z | $\bar{Z}_0 = \dfrac{-\beta(1 - \lambda_1 - \alpha) + \delta(1 - \gamma)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ | $\bar{Z}_2 = \dfrac{-(\beta - \Delta\beta)\ (1 - \lambda_1 - \alpha) - W(\gamma - \lambda_1\gamma - \alpha) + \delta(1 - \gamma)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$ |
| $\frac{Z}{Y}$ | $\dfrac{\bar{Z}_0}{\bar{Y}_0} = \dfrac{-\beta(1 - \lambda_1 - \alpha) + \delta(1 - \gamma)}{\beta(\lambda_1 - \lambda_2) + \delta}$ | $\dfrac{\bar{Z}_2}{\bar{Y}_2} = \dfrac{-(\beta - \Delta\beta)\ (1 - \lambda_1 - \alpha) - W(\gamma - \lambda_1\gamma - \alpha) + \delta(1 - \gamma)}{(\beta - \Delta\beta)\ (\lambda_1 - \lambda_2) + \delta + W(1 - \lambda_2)}$ |

$$\frac{\Delta\beta}{W} \leq \frac{\gamma(1 - \lambda_2)}{1 - \lambda_2 - \alpha} \qquad (12)$$

Thus, whether wage earners absolute income increases as a result of the program depends on: (a) the magnitude of the total transfer and the portion coming from wage-earners; (b) the difference between marginal propensities to consume of welfare recipients and non-wage-earners; (c) the slope of the demand for labor, and (d) the propensity to invest.

Now, for any given values of the parameters, equation (12) may be used for estimating the maximum proportion of any transfer which may come from wage earners and still maintain Paretian optimality insofar as wage-earners are concerned.

Turning to the change in the income of non-wage-earners, we find this change to be:

$$\bar{Z}_2 - \bar{Z}_0 = \frac{\Delta\beta(1 - \lambda_1 - \alpha) - W(\gamma - \lambda_1\gamma - \alpha)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha} \qquad (13)$$

Again, we note the denominator of (13) is positive. However, the numerator of (13) may be positive or negative.[6] Thus, as in the case of wage-earners, it is conditional whether the absolute income of non-wage-earners increases as a result of the transfer. Also, we again find the change depends critically on the propensity of businessmen to invest, the marginal propensity to consume of the wage-earners, and the portion of the transfer coming from the two groups.

6/ Since $W \geq \Delta\beta$ while $1 - \lambda_1 - \alpha \geq \gamma(1 - \lambda_1) - \alpha$.

One facet of these results is worthy of note. We have found that national income will increase as a result of the transfer program. Given this result, it follows from equation (12), that for any given total transfer, $W$, a ratio $\Delta\beta/W$ can always be found that will ensure Pareto optimality for wage-earners. A ratio $\Delta\beta/W$ that assures Pareto optimality for wage-earners, however, may not permit the same for non-wage-earners. Hence the key question for policy purposes is to find the range of ratios which assures Paretian optimality for both groups. With given values of the parameters, this range, if it exists, may be determined from equations (12) and (13).[7]

Summarizing the results of equations (11) and (13), we find that the effect of the program on wage and/or non-wage-earners income is dependent on:

(a) The relative shares of the transfer coming from each group.

(b) The ex ante propensity of businessmen to invest. The larger the propensity to invest, the more likely both $L$ and $Z$ are to increase.

(c) The slope $\gamma$ of the wage-earners income function. The larger the value of $\gamma$, the more likely is $L$ to increase and $Z$ to decrease.

(d) The relative marginal propensities to consume of the three groups.

For given values of the parameters, equations (11) and (13) provide a guide for estimating whether a transfer program may be Pareto optimal and

7/ Note that any range for which $0 < \frac{\gamma(1 - \lambda_1) - \alpha}{1 - \lambda_1 - \alpha} < \frac{\Delta\beta}{W} < \frac{\gamma(1 - \lambda_2)}{1 - \lambda_2 - \alpha}$ is a range that insures both $Z$ and $L$ will increase as a result of the transfer, provided $\gamma(1 - \lambda_1) - \alpha > 0$.

if so, the maximum relative share that can come from each group and still maintain Pareto optimality.

## Over-Full Employment

Up to this point, full multiplier effects on real income were allowed in the analysis, which implicitly assumed the economy was in less than full employment. If the possibility of over-full employment is considered, the results are altered significantly.

Assuming the economy is initially at the full employment equilibrium, any income transfer from wage-earners and non-wage-earners to welfare recipients will lead to an increase in aggregate demand, which in turn causes an inflationary gap. The inflationary gap is defined as the excess of aggregate demand over the maximum potential output measured at the full employment level of money income and the corresponding level of prices. Since the level of potential output is fixed, the resultant increase in money income causes the price level to rise.

In periods of under employment, changes in money and real income coincide. Under conditions of full employment when potential output is fixed, the situation is different. Money income of all sectors still may increase, but total real national income remains unchanged.

Changes in money income that result from a transfer program, under conditions of full employment are the same as the changes in both real and money income with less than full employment. Welfare recipients will have their absolute and relative money income shares increased, while the relative money income shares of wage-earners and non-wage-earners will decline. The absolute money income of the donor groups may increase or decrease, depending on the conditions outlined in equations (12) and (13). But since the relative share of money income of the donor groups decreases and real output is fixed, it follows that the real income of these groups decreases.

At this stage, it is obviously desirable to relax the assumption of fixed potential output in order to consider the implications of likely increases in potential output. Thus far, we have looked at investment only as a component of aggregate demand. But net investment also has a capacity-creating effect, thus expanding the nation's capital stock and the potential output an economy is capable of producing.

The potential output of an economy depends on the capacity-capital ratio, which for purposes of illustration may be assumed to be of the form $\frac{Y_p}{K} = \rho$, where $Y_p$ = capacity or potential output flow per year, $K$ = capital stock, and $\rho$ a constant ratio between the two. Thus given a capital stock $K$ the economy is capable of producing an annual product or income of $Y_p = \rho K$. From this it follows that some portion of net investment will cause a change in potential output or capacity.

$$\frac{dY_p}{dt} = \rho\frac{dK}{dt} = \rho I \qquad (14)$$

since net investment is defined as, $I \equiv \frac{dK}{dt}$.

Under our assumptions, investment is a function of national income. Hence, in the case of full employment, real investment equals $\alpha Y_{pi}$, where $Y_{pi}$ is defined as the initial potential output implying

$$\frac{dY_p}{dt} = \rho\alpha Y_{pi} \tag{15}$$

Should an increase in potential real income $Y_p$ equal the increase in money national income, $Y_m$, the results obtained in equation (11) and (13) would also hold in this case. The condition which would determine whether the change in $Y_p$ equals the change in $Y_m$, is derived in the appendix (part D) and given here as:

$$\rho = \frac{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)}{\left[\beta(\lambda_1 - \lambda_2) + \delta\right]\alpha} = \phi \tag{16}$$

Equation (16) may be useful, when quantitative estimates of the parameters are available, to determine whether $\Delta Y_p = \Delta Y_m$. In such an instance, implications concerning the changes in shares of income in real terms of both wage-earners and non-wage-earners may be determined from the conditions derived in the previous section, since real income and money income increased by the same amount. Similarly, should $\rho > \phi$, $\Delta Y_p > \Delta Y_m$, the results obtained under the condition of less than full employment would prevail. Also, given any value of $\rho$, it would be possible to determine from (16) how much of the transfer should come from wage-earners, $\Delta\beta$, to insure that $\Delta Y_p = \Delta Y_m$, if desirable. Another policy alternative also is implied by (16), namely other steps to increase $\alpha$.

When $\rho < \phi$, then $\Delta Y_p < \Delta Y_m$, which implies that the increase in potential output is less than the increase in national money income. In this case, the effects on absolute real income of donor groups will differ from previous results.

In the case of the wage-earners, the change in absolute real income, $\Delta L_R$, as a result of the income transfer may be expressed as

$$\Delta L_R = -\Delta\beta + \gamma\Delta Y_p \quad \text{or} \tag{17}$$

$$\Delta L_R = -\Delta\beta + \gamma\rho I$$

where $\gamma$ represents the proportion of the change in potential real income that will accrue to wage-earners. When $\gamma\rho I = \Delta\beta$ the absolute real income of wage-earners will remain unchanged after the income transfer. When $\gamma\rho I < \Delta\beta$ their absolute real income share will decline. Once $\gamma$ and the expected change in $Y_p$ are determined, the policy maker will know the amount of money income that can be transferred from wage-earners without affecting their absolute real income share.

Likewise, the change in non-wage-earners absolute real income, $\Delta Z_R$, is expressed as

$$\Delta Z_R = (1 - \gamma)\,\Delta Y_p + \Delta\beta - W \quad \text{or} \tag{18}$$

$$\Delta Z_R = (1 - \gamma)\,\rho I + \Delta\beta - W$$

where $(1 - \gamma)$ represents the share of the increase in potential output that will go to the non-wage-earners. When $(1 - \gamma)\rho I = W - \Delta\beta$, the absolute real income share of non-wage-earners, $Z_R$ will remain the same as the before transfer level. Should $(1 - \gamma)\,\rho I > W - \Delta\beta$, the absolute real income level of non-wage-earners would increase in spite of the income transfer.

It may be concluded that the absolute real income for both sectors will remain unchanged when the change in potential output is exactly equal to the total transfer to welfare recipients, or $\Delta Y_p = W$. Although wage-earners and non-wage-earners may be worse off in relative real income shares when $\Delta Y_p < \Delta Y_m$, their absolute real income is conditional on whether the proportion of the increase in potential output that goes to each sector is less than, equal to, or greater than, the share of the transfer coming from each group respectively.

## Implications

Summarizing the results of our simplified models, we find the following major points. First, the absolute income of recipient groups and the relative share of recipient groups both will increase as a result of an income transfer program. This holds in both real and money terms in the case of less than full employment, also in the case of full employment.

Second, the relative share of income of donor groups will decline as a result of the transfer, both in real and money terms. The key point is that the absolute money and real income of donor groups may increase with certain programs under certain conditions, and decrease under other programs and conditions. The conditions under which the absolute income of donor groups will increase may be used as a guide in determining the amount of the transfer and the portions to come from various donor groups when designing Pareto optimal program.

We note in passing that given some often quoted ranges of values of the relevant parameters, there may indeed be the possibility that the absolute income of donor groups may decrease with certain types of transfer programs. Thus, if the marginal propensities to consume of wage and non-wage-earners ($\lambda_1$ and $\lambda_2$) are of the order of .85 and .65, if the propensity to invest, $\alpha$, is of the order of .10 or less, and if the slope of the demand for labor is about .70, then the term $(1 - \lambda_1 - \alpha)$ in equation (11) indeed may be positive, as may be the term $(1 - \lambda_1 - \alpha)$ in equation (13). Such values of the relevant parameters thus raise the possibility that the absolute income of particular donor groups may decrease under particular types of transfer programs. This possibility, of course, is one which is desirable to avoid. Thus there is a need to incorporate analyses along the lines of those suggested here both to estimate the relevant parameters and to use the results as guides in policy formulation.

A keystone of Paretian welfare economics is that we can conclude the general welfare is promoted by some change only if no group is made worse off by the change, or if the group that is made worse off can be induced to voluntarily accept the change by redistributing some of the benefits received by other groups. At the practical policy level, it is a corollary that any change which makes all groups better off while harming no group is a change that should be made. Analyses along the lines indicated here thus provide the opportunity to formulate income transfer programs that will aid in achieving two very different goals--namely promoting equality of income distribution and maximizing the income of donor groups.

The models presented here obviously are oversimplified and much more work needs to be done in model construction and estimation before practical guides to the development of optimal transfer programs can be determined. In particular, the models presented do not conform as closely as one would desire to the format of national accounting statistics. Thus, for example, it would be desirable to incorporate refinements such as making consumption a function of disposable rather than total income, introducing additional sectors into the model, allowing for changes in imports and exports, etc. As noted previously, the authors are working on estimation of expanded models to include many factors. These expanded models are patterened after those of L. R. Klein in his work on econometric models of the United States at the Cowles Commission and the Brookings Institute. A sub-system of this more general model is presented in the appendix (part E) for the interested reader. The simplified models presented in this paper, however, serve the purpose of indicating the type of analysis involved and the benefits to be derived.

## Appendix

A. The derivation of equilibrium values in Table 1:

Given the system of equations (1) through (6), the equilibrium level of national income, $\overline{Y}_0$, is found by expressing both sides of the identity $I = S$, in terms of $Y$, so that $\alpha Y = Y - C$, and then substituting equations (1), (2) and (4) respectively, to yield the value for $\overline{Y}_0$ shown in Table 1.

The equilibrium value for $L$ then is found by substituting $\overline{Y}_0$ into equation (2). Similarly, the equilibrium value for $Z$ is found by substituting both $\overline{Y}_0$ and $\overline{L}_0$ into equation (4)

Having derived the equilibrium level of $\overline{Y}_0$, $\overline{L}_0$, and $\overline{Z}_0$, the relative shares for each sector follow directly.

Modifying the model by assuming a government transfer, changes the wage-earners income function from $L = Y + \beta$ to

$$\text{(2a)} \quad L = \gamma Y + \beta + \Delta\beta$$

The system of equations (1), (2a), (3), (4), (5) and (6) can similarly be solved for $\overline{Y}_1$, $\overline{L}_1$, $\overline{Z}_1$, $\overline{L}_1/\overline{Y}_1$, $\overline{Z}_1/\overline{Y}_1$ (the subscript 1 denotes equilibrium values of the two sector model into a government income transfer).

B. Changes in the relative income share of wage-earners in the two sector model:

Given: (i) $\overline{L}_0/\overline{Y}_0 = \dfrac{\beta(1 - \lambda_2 - \alpha) + \gamma\delta}{\beta(\lambda_1 - \lambda_2) + \delta}$

$$\text{(ii)} \quad \bar{L}_1/\bar{Y}_1 = \frac{\beta(1 - \lambda_2 - \alpha) + \gamma\delta + \Delta\beta\,(1 - \lambda_2 - \alpha)}{\beta(\lambda_1 - \lambda_2) + \delta + \Delta\beta(\lambda_1 - \lambda_2)}$$

Then $\bar{L}_1/\bar{Y}_1 > \bar{L}_0/\bar{Y}_0$, if and only if

$$\text{(iii)} \quad \frac{1 - \lambda_2 - \alpha}{\lambda_1 - \lambda_2} > \frac{\beta(1 - \lambda_2 - \alpha) + \gamma\delta}{\beta(\lambda_1 - \lambda_2) + \delta}$$

Letting $R = 1 - \lambda_2 - \alpha$

$S = \lambda_1 - \lambda_2$

the inequality (iii) can be rewritten as

$$\text{(iv)} \quad \frac{R}{S} > \frac{\beta R + \gamma\delta}{\beta S + \delta}$$

and since each term in (iv) is positive, the following operation is valid

$$R\beta S + R\delta > \beta RS + \gamma\delta S$$

which implies $\frac{R}{S} \gtrless \gamma$

Thus, $\bar{L}_1/\bar{Y}_1 - L_0/Y_0 > 0$ if and only if $\frac{1 - \lambda_2 - \alpha}{\lambda_1 - \lambda_2} > \gamma$

which we showed must hold in footnote number five on page 7.

C. To show the relative income share for wage-earners and non-wage-earners decline in the three sector model.

I. If $\frac{\Delta L}{\Delta Y} - \gamma < 0$ then $\frac{L_2}{Y_2} < \frac{L_0}{Y_0}$. But $\frac{\Delta L}{\Delta Y} - \gamma$ is necessarily negative.

To see this consider

$$-(1 - \lambda_2 - \alpha) + \gamma(\lambda_1 - \lambda_2) < 0$$

from footnote 5, page 7, hence

$$-\Delta\beta(1 - \lambda_2 - \alpha) + \Delta\beta\gamma(\lambda_1 - \lambda_2) < 0$$

$$-\Delta\beta(1 - \lambda_2 - \alpha) + \gamma W(1 - \lambda_2) + \Delta\beta\gamma(\lambda_1 - \lambda_2) - \gamma W(1 - \lambda_2) < 0$$

$$\frac{-\Delta\beta(1 - \lambda_2 - \alpha) + \gamma W(1 - \lambda_2)}{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)} + \frac{\gamma[\Delta\beta(\lambda_1 - \lambda_2) - W(1 - \lambda_2)]}{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)} < 0$$

$$\frac{-\Delta\beta(1 - \lambda_2 - \alpha) + \gamma W(1 - \lambda_2)}{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)} - \gamma < 0$$

which implies $\frac{\Delta L}{\Delta Y} - \gamma < 0$ and as a result of the transfer the relative share of wage-earners income will decline.

II. The relative income share of non-wage-earners will decrease when

$$\frac{\Delta Z}{\Delta Y} < 1 - \gamma \quad \text{or} \quad \frac{\Delta Z}{\Delta Y} + \gamma - 1 < 0$$

Given $\Delta Z = \dfrac{\Delta\beta(1 - \lambda_1 - \alpha) - W(\gamma - \gamma\lambda_1 - \alpha)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$

and $\Delta Y = \frac{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$

we could express the condition $\frac{\Delta Z}{\Delta Y} + \gamma - 1 < 0$ as

$$\frac{\Delta\beta(1 - \lambda_1 - \alpha) - W(\gamma - \gamma\lambda_1 - \alpha)}{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)} + \gamma - 1 < 0$$

Finding the common denominator and reducing, yields

$$\frac{(\Delta\beta - W)\,[(1 - \lambda_2 - \alpha) - \gamma(\lambda_1 - \lambda_2)]}{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)} < 0$$

It has been shown that the denominator is positive and $1 - \lambda_2 - \alpha > \gamma(\lambda_1 - \lambda_2)$. However, if part of the transfer is coming from wage-earners, such that $\Delta\beta < W$ the numerator is negative and $Z/Y$ must decrease whenever any amount is transferred from non-wage-earners to welfare recipients.

D. The condition which determines when the change in potential real output, $Y_p$ equals the change in money income, $Y_m$.

Given $\Delta Y_p = \rho\alpha Y_{pi}$

and $\Delta Y_m = \frac{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$

$\Delta Y_p = \Delta Y_m$ implies $\rho = \frac{\Delta Y_m}{\alpha Y_{pi}}$ or

$$\rho = \frac{\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)}{[1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha]\,\alpha Y_{pi}}$$

where $$Y_{pi} = \frac{\beta(\lambda_1 - \lambda_2) + \delta}{1 - \lambda_1\gamma - \lambda_2(1 - \gamma) - \alpha}$$

Thus the expression for $\rho$, reduces to

$$\rho = \frac{-\Delta\beta(\lambda_1 - \lambda_2) + W(1 - \lambda_2)}{[\beta(\lambda_1 - \lambda_2) + \delta]\,\alpha}$$

E. A subsystem of Klein's econometric model of the U.S.

a. Behavioral Relations

1. Consumption function

$$C_t = \lambda_0 + \lambda_1(L - T_L) + \lambda_2(Z - S_p - T_Z)_t + \lambda_3(A - T_A)_t$$
$$+ \lambda_4 C_{t-1} + U_{1t}$$

where:

$C$ = total consumer expenditures

$L$ = employee compensation (wage and salary)

$T_L$ = personal and payroll taxes less transfer associated with wage and salary income

$Z$ = non-wage non-farm income

$S_p$ = corporate savings

$T_Z$ = personal and corporate taxes less transfers associated with non-wage non-farm income

$A$ = farm income

$A - T_A$ = disposable farm income

$U_1$ = random distrubance

2. Investment function

$$I_t = \alpha_0 + \alpha_1(Z + A + D - T_Z - T_A)_{t-1} + \alpha_2 K_{t-1} + U_{2t}$$

where:

$I$ = gross private domestic investment

$D$ = capital consumption allowance

$K$ = end of year private capital stock

3. The Depreciation function

$$D_t = \varepsilon_0 + \varepsilon_1\left(\frac{K_t + K_{t-1}}{2}\right) + U_{3t}$$

4. Demand for labor

$$L_t = \gamma_0 + \gamma_1(Y + T + D)_t + \gamma_2(Y + T + D)_{t-1} + U_{4t}$$

where:

$Y + T + D$ = gross national product

5. Money wage rates

$$w_t - w_{t-1} = \Theta_0 + \Theta_1(N - N_L - N_Z - N_A)_t + \Theta_2(p_{t-1} - p_{t-2}) + \Theta_{3t} + U_{5t}$$

where:

$w$ = index of hourly wages

$N$ = number of persons in labor force

$N - N_L - N_Z - N_A$ = unemployed in number of persons

$p$ = general price index

$t$ = time trend in years

6. Agricultural income determination equation

$$A_t = \eta_0 + \eta_1[L + Z - S_p - T_L - T_Z]_t + \eta_2[L + Z - S_p - T_L - T_Z]_{t-1} + \eta_3(F_A)_t + U_{6t}$$

where:

$F_A$ = index of agricultural exports

7. Production function

$$Y + T + D = \beta_0 + \beta_1[h(N_L) + N_Z + N_A] + \beta_2\left(\frac{K_t + K_{t-1}}{2}\right) + \beta_{3t} + U_{7t}$$

where:

$h$ = index of hours worked per person per year

$N_L$ = number of wage and salary-earners

$N_Z$ = number of non-farm entrepreneurs

$N_A$ = number of farm operators

b. Accounting Identities

8. $Y_t + T_t + D_t = C_t + I_t + G_t + (F_E)_t - (F_I)_t$

where:

$G$ = government expenditures for goods and services

$F_E$ = exports of goods and services

$F_I$ = imports of goods and services

9. $Y_t = L_t + Z_t + A_t$

10. $K_t - K_{t-1} = I_t - D_t$

11. $h_t(\frac{w_t}{p_t})\ (N_L)_t = L$

The endogenous variables are:

$C_t$, $L_t$, $Z_t$, $A_t$, $I_t$, $D_t$, $K_t$, $Y_t$, $w_t$, $(N_L)_t$, $p_t$.