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131

Hamburgisches Welt-Wirtschafts-Archiv (HWWA)
Hamburg Institute of International Economics
2001
ISSN 1616-4814

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I am grateful to Th. Gries from the University of Paderborn for helpful comments on an earlier version. The subject of this paper is assigned to the HWWA's research programme "Internalization of Firms and Labor Markets".

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EUROPEAN INTEGRATION
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Abstract

How can long-term unemployment be reduced by policy measures of the government? In this paper a growth-matching-model is developed, in which the unemployment pool consists of heterogeneous unemployed workers, short-term and long-term unemployed, and with an endogenous skill-depreciation of the long-term unemployed emerging as technical progress accelerates. For innovation countries characterized by rapid technical progress we show that through subsidizing vacancy creation which causes a substitution and an income effect long-term unemployment can be reduced. Since the positive substitution effect implied by subsidizing vacancy creation outweighs the negative income effect induced by taxing the household's income, a positive employment effect results leading to additional job-matches and decreasing unemployment duration. Therefore, the introduction of subsidies will be favorable for achieving a reduction in long-term unemployment.

Zusammenfassung

Wie kann die Langzeitarbeitslosigkeit durch geeignete Politikmaßnahmen des Staates reduziert werden? In diesem Artikel wird ein Wachstums-Matching-Modell entwickelt, das durch heterogene Arbeitslose – Kurz- und Langzeitarbeitslose – und bei steigendem technischen Fortschritt durch die endogene Abwertung von Fähigkeiten und Fertigkeiten der Langzeitarbeitlosen charakerisiert ist. Für Innovationsländer, die mit akzelerierendem technischen Fortschritt konfrontiert sind, wird gezeigt, dass die Subventionierung von Vakanzen, bei der sowohl ein Substitutions- wie auch ein Einkommenseffekt entsteht, eine Reduzierung der Langzeitarbeitslosigkeit impliziert. Da der positive aus der Vakanzsubventionierung resultierende Substitutionseffekt den negativen Einkommenseffekt, der durch die Besteuerung der Einkommen verursacht wird, überwiegt, wird ein positiver Beschäftigungseffekt generiert. Dieser positive Beschäftigungseffekt führt zu zusätzlichen Job-Matchings und sinkender Arbeitslosigkeitsdauer. Somit kann der Staat mit Hilfe der Subventionierung von Vakanzen, eine Verringerung der Langzeitarbeitslosigkeit erreichen.

JEL Classification: E24; J41; O41

Keywords: long-term unemployment, growth, search, matching, subsidies

1 INTRODUCTION

Data on long-term unemployment[1] show a huge increase in the level and the growth rates of long-term unemployment in industrialized countries. A natural question then becomes, what policy measures should be introduced to reduce long-term unemployment?

Stylized Fact

Figure 1a shows a group of countries characterized by high shares of long-term unemployment. In 1975 Belgium displays 36 per cent long-term unemployment of total unemployment; this share increases until 1999 up to over 60 per cent. Italy and Ireland have nearly 67 respectively 57 per cent long-term unemployment in the end of the 90s. In this group the average growth rate of long-term unemployment is at about 2 per cent.

The countries shown in Figure 1b are characterized by medium levels and higher average growth rates of long-term unemployment. The share of long-term unemployment increases in Germany from 10 per cent in 1975 up to 50 per cent in 1999. France and the U.K. show nearly the same structure: their shares rise from 17 per cent in 1975 up to 40 per cent at the end of the last decade.

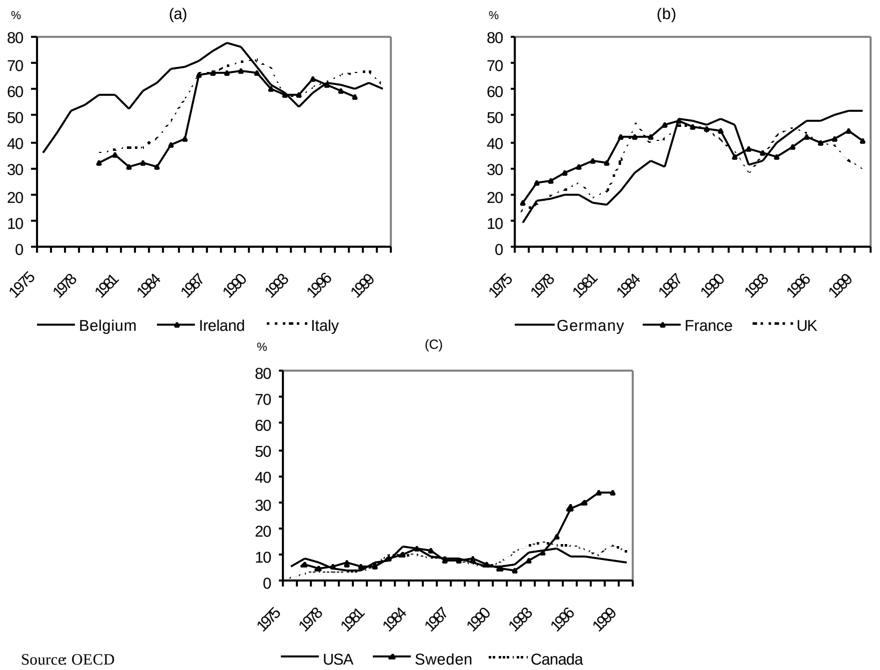
A third country group with relatively low levels but relatively high growth rates of long-term unemployment can be identified in Figure 1c. Canada starts with 1 per cent long-term unemployment and this increases up to nearly 11 per cent in 1999; Sweden starts with 6 per cent and ends up with 33 per cent. In the US the proportion of long-term unemployed workers is over the whole period almost constant at about 6 per cent and the average growth rate is constant as well. However, Sweden and Canada display annual average growth rates of 7 respectively 9 per cent.

As this stylized fact shows, there was a dramatic increase in long-term unemployment in nearly all industrialized countries.[2] Since industrialized economies are characterized by high levels of capital intensity and rapid technical progress, they are innovation rather

1 In the empirical definition long-term unemployed are defined as jobless workers being out of work for twelve or more months.
2 See also Layard, Nickell, Jackman (1991), Jones, Manning (1992), Ljungqvist, Sargent (1995, 1998).

than imitation countries. The latter economies are usually in the stage of catching up. The question for innovation countries arises, how this dramatic increase in long-term unemployment can be reduced and what kind of policy should be implemented to achieve this reduction.

Figure 1: Development of Long-Term Unemployment on Total Unemployment for Groups of Countries



Source: OECD

Policy options to reduce long-term unemployment are usually retraining and educational policies that enable workers to keep up with technological progress.[3] In this paper, however, it is suggested that subsidizing vacancy creation reduces long-term unemployment.[4] For innovation countries we show that the introduction of subsidies induces two effects for the labor market. The first one is a substitution effect which implies that, due to the granting of subsidies for the firm's search costs, additional vacancies are generated and, therefore, job-matching increases. The second one is an income effect which results from taxing the household's income to finance the subsidies. Because of this

3 See Acemoglu (1995), Boeri, Wörgötter (1998), Gora, Schmidt (1998), Mortensen, Pissarides (1999).

4 See also Coles, Masters (2000) who suggest that subsidizing vacancy creation is a better way than subsidizing retraining to reduce long-term unemployment.

effect, a reduction in capital accumulation and, therefore, less vacancies are implied. Since the substitution effect outweighs the income effect, a positive employment effect results leading to additional job-matches and the labor market becomes slack, i.e. for given vacancies unemployment reduces. Due to a heterogeneous unemployment pool, consisting of short-term and long-term unemployed workers, and due to a dependence of long-term unemployment on the duration of unemployment and on the rate of technical progress, the reduction in unemployment induces a reduction in the average duration of unemployment and, therefore, long-term unemployment will also decrease.[5] Hence, the introduction of subsidies for the firm's search costs arising from unfilled vacancies will be favorable for achieving a reduction in long-term unemployment.

These implications are derived in a growth-matching-model with the labor market characterized by matching-frictions and capital accumulation is described by a neoclassical growth process.[6] Matching-frictions represent the search process needed to fill vacancies.[7] Even in equilibrium, which is defined as a flow equilibrium, i.e. inflows are equal to outflows, the labor market is marked by search respectively matching-frictions. If no frictions were present, laid-off workers would find immediately new jobs and equilibrium unemployment would not exist. The existence of frictions implies further that outflows depend on the labor market tightness causing that the matching-probability is influenced by the levels of unemployment and vacancies. Therefore, each trading partner faces market externalities determined by the number of traders on each side of the market.[8]

Furthermore, due to matching-frictions, trading partners have some monopoly power and successful matching yields additional profits which are shared between firms and workers. The division of profits can be modeled by a Nash bargaining approach or simply by sharing the additional product with the sharing proportions determined by the bargaining power of the trading partners.[9] It is assumed that all job-workers pair are equally productive. Wages are then fixed by the sharing rule.

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5 For related discussions see Lockwood (1991), Pissarides (1992) and Blanchard, Diamond (1994).

6 The model is similar to that of Pissarides (1990), see also Postel-Vinay (1998), Merz (1995, 1999), Marimon, Zilibotti (1999).

7 See also Blanchard, Diamond (1994, 1989).

8 See Merz (1995) and Feve, Langot (1996).

9 See Gries, Jungblut, Meyer (1997 a, b)

The unemployment pool consists of heterogeneous unemployed workers and the fraction of long-term unemployment is determined by the duration of unemployment itself and by the rate of technical progress. The positive dependence between the average duration of unemployment and the fraction of long-term unemployed can be explained by the extreme skill-depreciation and by the motivation losses of the long-term unemployed[10] during their jobless time.[11] If the average, endogenously determined duration of unemployment increases, increased long-term unemployment is implied. Furthermore, increasing technical progress induces rising long-term unemployment, since long-term unemployed do not possess the know-how and the abilities to handle the latest production methods.

For attaining the steady-state solution and the determinants of the equilibrium level of long-term unemployment, an efficient factor allocation function and a balanced accumulation function are derived. The first mentioned function which is implied by intertemporal demand decisions of firms characterizes labor market structures and describes optimal factor allocation in the labor market. As long as the labor market has not reached the long-run equilibrium, structures will change permanently. This is reflected by differences in inflows and outflows and, therefore, by a permanently changing share of long-term unemployment. The second mentioned function represents the steady-state of the goods market and characterizes the growth process of the economy. In long-run equilibrium all relevant variables grow with the same rate and the vacancy level as well as the share of long-term unemployment have reached long-run positions.

To analyze the effect of subsidizing the firm's search costs on long-term unemployment, the government is introduced and the model is extended by the government's policy. Then the implications for long-term unemployment are discussed.

Therefore, the paper is organized as follows. In the next section the model is developed, the steady-state solution and the stability of the model is given in section 3. In section 4 the policy implications resulting from the introduction of subsidies are analyzed and section 5 concludes.

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10 See Layard (1997).
11 See Birk (2001).

2 THE ECONOMY

The Labor Market

The aggregate labor endowment of households is constant and denoted by $L = \overline{L}$. At any time labor is either employed or unemployed; the employed workers are denoted as E and the unemployed as U. Thus, the labor force is represented by

(1) $$\overline{L} = E + U.$$

The labor market is characterized by search frictions with firms looking for jobless workers filling vacancies and unemployed searching for a job. Both sides of the market have incomplete information about the opposite market side. The level of search activities is represented by the number of vacancies V, the number of unemployed U and the number of matches M formed at any point in time. Furthermore, since newly created vacancies depend on the latest technology, the rate of technological progress $\hat{l}$ determines also the number of matches and the growth rate of technological progress represents the diffusion of technological know-how. If an economy has a high rate of technological progress, only few unemployed workers can fill the vacancies and the number of matches will reduce, i.e. technological knowledge of the unemployed does not grow with the same rate as technological progress does. Therefore, the underlying matching technology is defined as

(2) $$M = m(U,V;\hat{l}) = V^{1-b} U^{b} \hat{l}^{-1}$$

with b as the search intensity of the unemployed and the matching function is assumed to be homogeneous of degree one. Furthermore, the indicator for labor market tightness is denoted by the ratio of vacancies to unemployed $q := V/U$ and

(3) $$p(q) := M/U = m(V/U,1;\hat{l}), \quad p_q > 0$$

is the matching-probability for the unemployed and

(4) $$q(q) := M/V = m(1,U/V;\hat{l}), \quad q_q < 0$$

is the probability of filling vacancies. Both probabilities depend on labor market tightness and reflect the externalities each trading partner faces. If the number of jobless workers increases, the matching-probability for the average unemployed will decrease and simultaneously the probability of filling vacancies will increase.

Due to constant returns of scale, the duration of unemployment is defined as

$$(5) \qquad r(q) := U/M, \quad r_q < 0$$

and it rises when the labor market becomes tighter which is characterized by increasing unemployment for given vacancies.

Furthermore, the unemployment pool is heterogeneous and two types of jobless workers are distinguished: short-term and long-term unemployed, U^S respectively U^L, and the heterogeneous unemployment pool is defined as

$$(6) \qquad U = U^S + U^L$$

$$(7) \qquad U = \left[1 - f(r;\hat{l})\right]U + f(r;\hat{l})\,U, \quad 0 < f < 1, \quad f_r, f_{\hat{l}} > 0$$

with $f(r;\hat{l})U$ as the long-term unemployed. The long-term jobless workers show significant different search behavior than short-term unemployed. They are looking for new jobs with less search intensity and, due to the long unemployment duration, they are demoralized and discouraged.[12] During their jobless time, their human capital is exposed to large depreciation losses and, since they are not trained and do not accumulate any additional knowledge, i.e. without allocating any resources to the long-term unemployed, they are not able to handle the latest production technologies. Therefore, the number of long-term jobless workers depends positively on the unemployment duration r and positively on the rate of technical progress $\hat{l}$.

If new job-matches are formed, each match generates additional revenues and, because both trading partners have monopoly power, unemployed workers and firms could bargain over the additional produced profits; or the profits are simply shared using a sharing rule. This sharing rule determines the profit proportion, the new workers get and, therefore, the wage results as a constant fraction of the marginal product

12 See also Layard, Nickell, Jackman (1991).

(8) $w = wF_E(k), \quad 0 < w < 1,$

with w denoting the sharing proportion and representing the monopoly power of unemployed workers.

The Goods Market

Each firm uses capital K, labor L and the current state of technological progress $l := l_0 e^{\hat{l}t}$ to produce a homogenous good X. Production is described by a Cobb-Douglas-function:

(9a) $X = F(K, lE) := K^a \left(lE\right)^{1-a}$

(9b) $\Leftrightarrow x = k^a$

with $x := X / lE$ and $k := K / lE$.

For the representative firm demand decisions concern changes in real capital and in employment. It is supposed that installation costs of $c_I I$ [with $0 < c_I < 1$] arise with c_I as the fraction of installation costs used for investments *I*.

The change in employment is determined by inflows in and outflows out of unemployment. The inflows are characterized by the separation of existing job-matches at any point in time and are described by the exogenously given separation rate n times the workers *E*. Thus, inflows characterize the number of unproductive jobs which generate layoffs.[13] On the other hand, the outflows are represented by the flow of newly formed job-matches and, therefore, by the matching-function $m(U,V;\hat{l})$. Firms create and offer new productive jobs in the labor market and they have to fill these vacancies by searching for suitable workers. At the aggregate level, the filling of vacancies depends on the number of unemployed, the number of offered vacancies, the search intensities of firms and unemployed and the rate of technical progress; all variables are expressed in the matching-function. Taking outflow and inflow together, the dynamics of employment result as the excess of outflows over inflows and can be expressed by

13 For an exogenous separation rate see also Pissarides (1990) and Postel-Vinay (1998) and for an endogenous rate see Mortensen/Pissarides (1994, 1998).

(10) $\dot{E} = m(U,V;\hat{l}) - nE.$

Each vacancy induces search costs in the amount of c_v with $c_v := c_{v0}e^{\hat{l}t}$. Since the newest jobs contain the latest technology, it is costly for the firm to find unemployed workers being able to handle latest technologies. Therefore, search costs grow with the rate of technical progress.

Taking these aspects into consideration, the representative firm faces the following intertemporal optimization problem with the current flow of profits as output minus factor payments minus search expenditures. Denoting *r* as the discount factor, the firms maximization can be written as

$$\max_{I,V} \int_0^\infty \{F(K, lE) - rK - wE - c_I I - c_v V\} e^{-rt} dt$$

$$\text{s.t. } \dot{E} = m(U,V;\hat{l}) - nE$$

$$\dot{K} = I$$

$$K(0), E(0), V(0), U(0) \quad \text{given.}$$

For solving the optimization problem, a present-value Hamiltonian function $\mathrm{H}(K,E,V,I,m_1,m_2)$ with costate variables m_i $[i = 1,2]$ is set up. Denoting F_j as the partial derivative of $F(\cdot)$ with respect to $j = K,E$, the Hamiltonian conditions are

(11) $\frac{\partial H}{\partial V} = 0 \quad \Leftrightarrow \quad -e^{-rt}c_v + m_1 \frac{\partial m}{\partial V} = 0$

(12) $-\dot{m}_1 = \frac{\partial H}{\partial E} \quad \Leftrightarrow \quad -\dot{m}_1 = e^{-rt}[F_E - w] - m_1 n$

(13) $\dot{E} = \frac{\partial H}{\partial m_1} \quad \Leftrightarrow \quad \dot{E} = M(U,V;\hat{l}) - nE$

(14) $\frac{\partial H}{\partial I} = 0 \quad \Leftrightarrow \quad -e^{-rt}c_I + m_2 = 0$

(15) $-\dot{m}_2 = \frac{\partial H}{\partial K} \quad \Leftrightarrow \quad -\dot{m}_2 = e^{-rt}[F_K - r]$

(16) $\dot{K} = \frac{\partial H}{\partial m_2} \quad \Leftrightarrow \quad \dot{K} = I$

with the transversality condition[14]

$$\lim_{t \to \infty} H(t) = 0.$$

The first order condition for capital respectively labor are given by[15]

(17) $F_K(k) = (1 + c_I)r$

(18) $F_E(k) = w + \frac{\hat{I} c_v}{1-b}\left[r - \hat{I} + b(\hat{U} - \hat{V}) + n\right] q^{b}$

with F_j $[j = K, E]$ as marginal products and the right hand sides are marginal costs of capital respectively labor.

After describing the intertemporal optimization problem of the representative firm, the model has to be closed by denoting aggregate income and the budget constraint. Factor income of the households Y is defined as the remuneration of production factors capital and labor

(19) $Y := rK + wE$

with the wage rate w.

The output is used for factor income Y, installation costs $c_I I$ and search costs $c_v V$ and is given by

(20) $X := Y + c_I I + c_v V.$

Both of the last terms represent the profit income of firms that is completely used for installation and search costs $(1 - w)F_E = c_I I + c_v V$.

14 See Michel (1982).
15 See appendix.

Equilibrium of the Goods Market

In the closed economy households consume and save a constant fraction of their income and the equilibrium for the goods market is characterized by

(21) $I = S = sY$

with S as savings and s as the saving rate.

3 STEADY-STATE SOLUTION

Analyzing the steady-state solution, the long-run equilibrium of the labor market and the steady-state of the goods market are derived separately and can be characterized by a efficient factor allocation function respectively a balanced accumulation function.

Steady-State of the Labor Market

The steady-state of the labor market is deduced using the flow condition for the labor market. This condition requires that inflows are identical to outflows and, therefore, the change in employment is zero:

(22) $\dot{E} = 0 \quad \Leftrightarrow \; V^{1-b} U^{b} \hat{l}^{-1} = nE.$

Furthermore, due to neglecting on-the-job-search, the flow of newly created vacancies is identical to the employment flow, i.e. $\dot{V} = \dot{E} = 0$, and because of a constant labor force, the employment and unemployment levels are constant in the long-run equilibrium, i.e. $\dot{E} = -\dot{U} = 0$. These conditions imply that steady-state labor market tightness is also constant, i.e. $\dot{q} = 0$, and that the steady-state growth rates of unemployment and vacancies are zero, i.e. $\hat{V} = \hat{U} = 0$.

If these conditions are used, the *efficient factor allocation function* for the stationary labor market can be derived:[16]

$$(23)\qquad q^{b} = \frac{l_0(1-a)(1-b)(1-w)}{c_{v0}\,\hat{l}\left[\left(\frac{a}{1+c_I}\right)k^{a-1} + n - \hat{l}\right]}\,k^{a} =: \Psi(k).$$

It shows all combinations of capital intensity and labor market tightness that reflect the long-run equilibrium of the labor market. The steady-state of the labor-market is influenced by several exogenous variables and it will change when the exogenous environment changes. In the (q^{b},k)-plane it has a positive concave shape.[17]

Furthermore, in the long-run labor market equilibrium the steady-state employment rate is given by[18]

$$(24)\qquad e(q) := \frac{E}{\bar{L}} = \frac{p(q)}{n + p(q)}, \quad e_q > 0.$$

Therefore, the employment probability depends positively on labor market tightness q and on the matching-probability $p(q)$ and negatively on the separation rate n. The higher the separation rate, the lower the steady-state employment rate. Furthermore, the steady-state unemployment rate is determined as well as

$$1 = e(q) + u(q),$$

where the steady-state unemployment rate $u(q)$ is defined as $u(q) := U/\bar{L}$.

Thus, the steady-state for the labor market is described by an efficient factor allocation function that defines all equilibrium combinations of labor market tightness and capital intensity.

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16 For the detailed derivation of the efficient factor allocation function see appendix.
17 See appendix.
18 See appendix.

Steady-State of the Goods Market

As common in neoclassical growth models, the long-run steady-state is characterized by a constant capital intensity, i.e.

(25) $\dot{k} = sy - (\hat{l} + \hat{E})k = 0$.

The steady-state of the goods market can be described by a *balanced capital accumulation function*:[19]

(26) $$q^b = \frac{l_0}{c_{v0}\hat{l}n}\left[k^a - \frac{(1+c_I s)\hat{l}}{s}k\right] =: \Phi(k) .$$

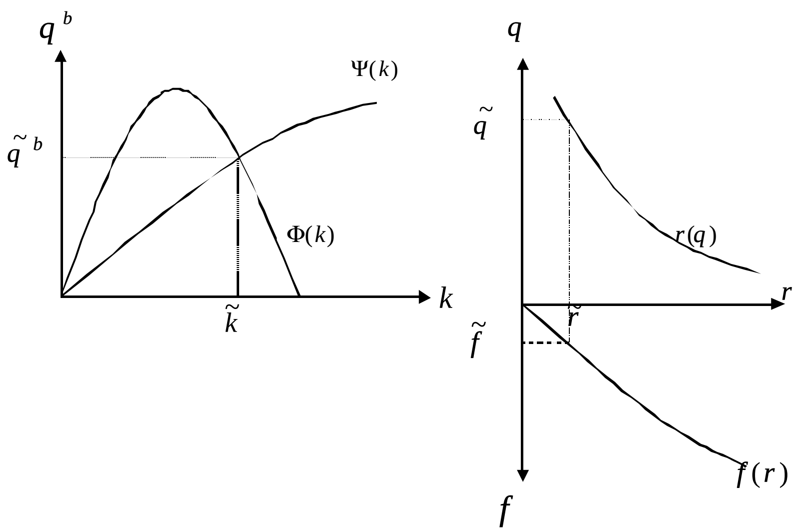
This function shows all combinations of labor market tightness and capital intensity characterizing the steady-state in the goods market.

Furthermore, in the (q^b, k)-plane the balanced accumulation function has – until the maximum is reached – a positive slope, in the maximum a slope of zero and behind the maximum a negative slope.[20]

After deriving the equilibrium conditions for the steady-state labor market respectively for the steady-state goods market separately, both determine together the overall steady-state, i.e. the efficient factor allocation function and the balance capital accumulation function simultaneously define the steady-state values for q^b and k. In Figure 2 the steady-state search equilibrium $(\tilde{q}^b, \tilde{k})$ is graphed at the intersection of both functions. Due to the shape of both functions, the steady-state exists and is unique.

19 See appendix.
20 See appendix.

Figure 2: Steady-State for the Innovation-Economy



Once the steady-state search equilibrium $(\tilde{q}^b, \tilde{k})$ for the innovation economy is determined, the steady-state values for the matching probability $\tilde{p}$, the steady-state employment rate $\tilde{e}$ and unemployment rate $\tilde{u}$ can be derived. The steady-state employment and unemployment levels are fixed as well: $\tilde{E} = e(\tilde{q})\bar{L}$ and $\tilde{U} = u(\tilde{q})\bar{L}$. Furthermore, steady-state labor market tightness determines equilibrium unemployment duration $\tilde{r}$ and the steady-state fraction of the long-term unemployed $\tilde{f}$ (see Figure 2).

Beside the determination of the steady-state labor market variables, the growth and accumulation process is fixed. In the long-run equilibrium the steady-state capital stock, the steady-state production and income level grow with the rate of technical progress, i.e. $\hat{X} = \hat{Y} = \hat{l}$.

Stability of the Steady-State

The transitional behavior of the labor market tightness is characterized by the dynamic factor allocation function[21]

21 See appendix.

$$q^{b} = \frac{(1-a)(1-b)(1-w)\,l_0}{c_{v0}\,\hat{l}\left[\left(\frac{a}{1+c_I}\right)k^{a-1} + b\left(\hat{U}-\hat{V}\right) + n - \hat{l}\right]} k^{a}\ .$$

Considering $\hat{U} = \hat{V} = -\hat{q}$, the function can be rewritten as

(27) $$\dot{q} = \frac{1}{b}\left[\frac{a}{1+c_I}k^{a-1} - \frac{(1-a)(1-b)(1-w)\,l_0}{c_{v0}\,\hat{l}} k^{a} q^{-b} + n - \hat{l}\right] q.$$

Equation (27) shows the transitional dynamics for q^{b}, i.e. labor market tightness increases if

$$\dot{q} > 0 \quad \Leftrightarrow \quad q^{b} > \frac{(1-a)(1-b)(1-w)\,l_0}{c_{v0}\,\hat{l}\left[n - \hat{l} + \frac{a}{1+c_I}k^{a-1}\right]} k^{a}.$$

Thus, labor market tightness increases, if the realized level of labor market tightness is greater than the equilibrium level and vice versa.

Furthermore, the following dynamic capital accumulation function can be derived as[22]

(28) $$\dot{k} = \frac{s}{1+c_I s}\left[k^{a} - \frac{c_{v0}\,\hat{l}\,(\hat{E}+n)}{l_0} q^{b}\right] - \left(\hat{l} + \hat{E}\right) k.$$

Equation (28) shows the transitional dynamics for the capital intensity; it increases if

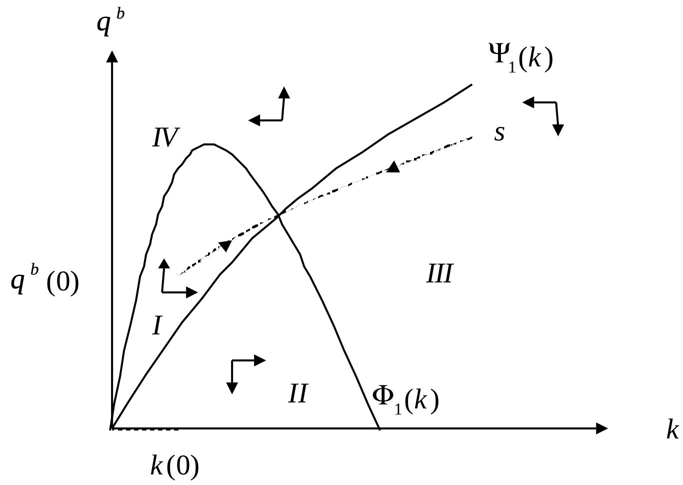
$$\dot{k} > 0 \quad \Leftrightarrow \quad q^{b} < \frac{l_0}{c_{v0}\,\hat{l}\,(\hat{E}+n)}\left\{k^{a} - \left(\frac{1+c_I s}{s}\right)(\hat{l} + \hat{E})k\right\}.$$

Thus, if a capital intensity is realized lying below the balanced capital accumulation function, this capital intensity is too small to generate the equilibrium capital accumulation in labor efficiency units. The realized capital intensity has to increase to reach the equilibrium capital intensity and vice versa.

22 For a detailed derivation see appendix.

Due to this analysis, the transitional dynamics shown in Figure 3 are implied. To achieve the long-run steady-state, the dynamic system not only has to be in the areas of *I* or *III*, it also has to be on the stable saddle path *s* and the starting variables $V(0), U(0), E(0)$ and $K(0)$ must have values which are already on the saddle path *s* in *t=0*.

Figure 3: Analysis of the Stability for the Steady-State



4 POLICY IMPLICATIONS: THE INTRODUCTION OF SUBSIDIES

For analyzing the implications of the introduction of subsidizing search cost for innovation economies, the model has to be extended for the actions of the government. It is supposed that the government is active only through redistributing subsidies to firms. The subsidies are given to the firms for their search costs arising with unfilled vacancies and are financed by taxing the factor income of the households. The taxes are completely spent for the subsidies.

In order to show the policy implications in innovation economies for long-term unemployment resulting from the introduction of subsidies, the implications for these economies have to be discussed. Therefore, we have to analyze, first, the influence of subsidies on the intertemporal demand decision of the representative firm and, second, the government has to be introduced. Third, the taxation effect at the consumer side has to be described.

Subsidizing search costs – the intertemporal demand decision of the firm and the substitution effect

The granting of subsidies for vacancies are reflected in the firm's intertemporal profit maximization. The government's subsidies are $dc_v V$ [with $0 \le d \le 1$] which arise for the firm's search to fill vacancies and the intertemporal profit maximization changes to

$$\max_{I,V} \int_0^\infty \{F(K, lE) - rK - wE - c_I I - (1-d)c_v V\} e^{-rt} dt$$

$$\text{s.t.} \quad \dot{E} = m(U,V;\hat{l}) - nE$$

$$\dot{K} = I$$

$$K(0), E(0), V(0), U(0) \quad \text{given.}$$

The first order condition for labor becomes

$$(29) \qquad F_E(k) = w + \frac{(1-d)\hat{l}c_v}{1-b}\left[r - \hat{l} + b\left(\hat{U} - \hat{V}\right) + n\right] q^b$$

and the efficient factor allocation function will be replaced by

$$q^b = \frac{l_0(1-a)(1-b)(1-w)}{c_{v0}(1-d)\hat{l}\left[\left(\frac{a}{1+c_I}\right)k^{a-1} + n - \hat{l}\right]} k^a =: \Psi_1(k).$$

If the effect of introducing subsidies into the labor market is evaluated, we get

$$\frac{\partial \Psi_1(k)}{\partial d} = \frac{l_0(1-a)(1-b)(1-w)}{c_{v0}(d-1)^2\,\hat{l}\left[\left(\frac{a}{1+c_I}\right)k^{a-1} + n - \hat{l}\right]} k^a > 0.$$

Therefore, the granting of subsidies to firms implies that the search costs for vacancies will decrease and additional vacancies are offered in the labor market which leads to an increase in the job-matching and in the employment level. Thus, via a positive substitution effect, the introduction of subsidies will induce additional labor demand and increasing employment is implied.

The government's budget-constraint

To describe the raising of taxes and the redistribution of subsidies, the government is introduced by supposing that the government is active only in collecting taxes which are completely spent for the subsidies of the firm's search costs. Since the tax revenue, defined as T, is completely spent for financing the search subsidies, the government has the following budget constraint

(30) $$T = dc_v V$$

with the right hand side representing the subsidies. The government controls the subsidy rate d such that the government's revenues are equal to its expenditures.

Financing the subsidies: the taxation of the factor income – the income effect

Since the tax revenues T are financed by taxing the household's income Y, it reduces by the tax amount and, therefore, the disposable income Y^v is given by

(31) $$Y^v = Y - T .$$

Regarding these extensions, the following balanced capital accumulation function is derived[23]

$$q^b = \Phi_1(k) := \frac{l_0}{c_{v0}\hat{l}n(1-d)}\left[k^a - \frac{(1+c_I s)\hat{l}}{s}k\right].$$

23 See appendix.

The effects resulting from the taxation of the household's income on the goods market will be obvious by taking the partial derivative with respect to the tax rate d

$$\frac{\partial \Phi_1(k)}{\partial d} = -\underbrace{\frac{l_0}{(1-d)^2 c_{v0} \hat{l} n}}_{>0} \underbrace{\left[k^a - \frac{(1+c_I s)\hat{l}}{s} k \right]}_{>0} < 0$$

This effect is interpreted as a negative income effect, since an increase in the subsidy rate decreases the disposable income and leads, via reduced savings and reduced investments, to decreasing capital accumulation. Therefore, less vacancies are offered and the labor market becomes depressed going together with increasing unemployment. Thus, the income effect induced by taxing the household's income is negative and steady-state employment is reduced.

Since we are analyzing the effects resulting from the subsidies for innovation economies, we have to discuss the implications for these economies characterized by high steady-state capital intensities. As we have seen, for such kind of economies, the introduction of subsidies induces a positive substitution and a negative income effect. Because the positive substitution effect is larger than the negative income effect, the overall impact on the labor market is positive and steady-state employment as well as steady-state labor market tightness increases for innovation economies (see Figure 4a).[24]

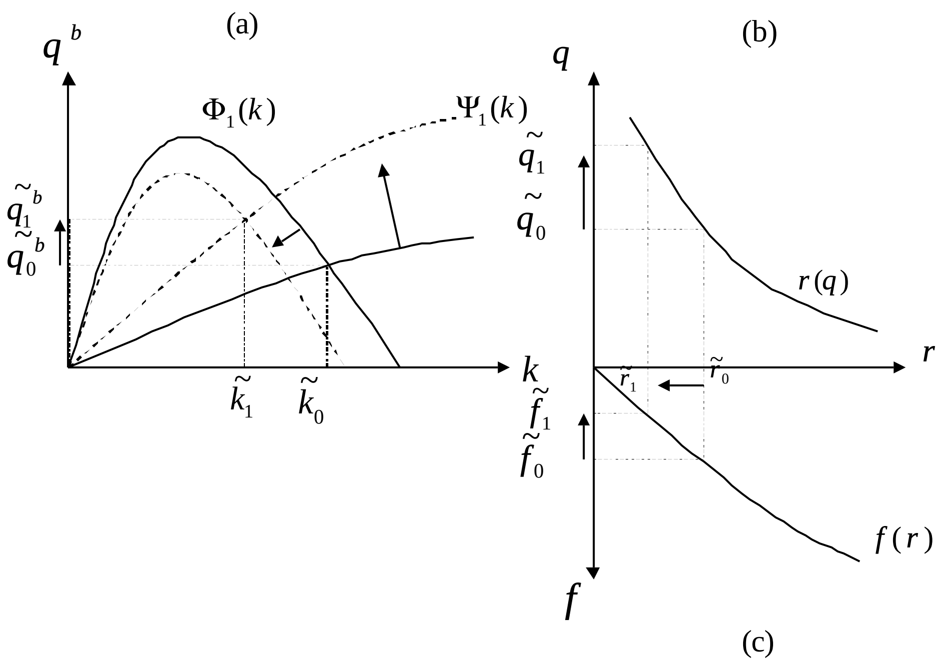
Impact on Long-term Unemployment

Furthermore, the increase in steady-state labor market tightness has implications for the duration of unemployment. Due to the negative relationship between labor market tightness and the duration of unemployment, i.e. the higher the unemployment, the longer it takes to leave unemployment and the higher the unemployment duration, increasing labor market tightness induces decreasing unemployment duration. This implication is obvious, since increasing labor market tightness implies shrinking unemployment. Therefore, subsidizing the search cost for vacancies causes a reduction in the duration of unemployment (from $\tilde{r}_0$ to $\tilde{r}_1$, see Figure 4b).

24 In Figure 5a the innovation economies are characterized by high capital intensities.

Considering the implications for the fraction of long-term unemployment, the reduction in the duration of unemployment induces a reduction in long-term unemployment as well. The share of steady-state long-term unemployment shrinks, simply because the average steady-state unemployment duration decreases (from $\tilde{f}_0$ to $\tilde{f}_1$, see Figure 4c).

Figure 4: Labor Market Effects of Subsidizing Search Costs



Hence, with increasing labor market tightness it becomes easier for an average unemployed worker to leave unemployment and to become matched with a vacancy. Therefore, the duration of unemployment decreases which induces a reduction in long-term unemployment. However, the effect on steady-state capital intensity is not clear; it can rise or shrink.

5 SUMMARY

The heterogeneity of the unemployment pool and the endogenous determination of the average duration of unemployment affect the level of long-term unemployment in innovation countries that produce with high capital intensities and that are characterized by inventing rapidly new technologies. In these economies, the introduction of subsidizing the firm's search costs induces a substitution and an income effect for the labor market. Since the positive substitution effect outweighs the negative latter one, the labor market becomes less depressed and firms offer more vacancies, i.e. labor market tightness increases. With increasing labor market tightness it becomes easier for an average unemployed worker to leave unemployment and to become matched with a vacancy. Therefore, the duration of unemployment decreases which leads to a reduction in long-term unemployment. Hence, the government can achieve a reduction in long-term unemployment by subsidizing the search costs of the firms which arise with unfilled vacancies.

6 APPENDIX

Lemma 1 *Using (14) and (15), then* $F_K(k) = (1+c_I)r$.

Proof. Differentiate (14) w.r.t. time and substitute it in (15), then $F_K(k) = (1+c_I)r$ is implied. ¦

Lemma 2 *Using (2), (11), (12),* $c_v = c_{v0}e^{\hat{l}t}$ *and* $l = l_0 e^{\hat{l}t}$*, then*

$$F_E(k) = w + \frac{\hat{l}c_v}{1-b}\left[r - \hat{l} + b\left(\hat{U} - \hat{V}\right) + n\right]q^b .$$

Proof. Differentiate (2) w.r.t. V and (11) w.r.t. time, use $c_v = c_{v0}e^{\hat{l}t}$ and $l = l_0 e^{\hat{l}t}$, then

$$e^{-rt}c_v(r-\hat{l}) + \frac{1-b}{\hat{l}}\left[\dot{m}_1 + m_1 b(\hat{U}-\hat{V})\right]q^{-b} = 0$$

$$\Leftrightarrow\ -\dot{m}_1 = \frac{\hat{l}}{1-b}e^{-rt}c_v(r-\hat{l})\, q^b + m_1 b(\hat{U}-\hat{V})$$

Substitute (12) for $-\dot{m}_1$, then

$$\frac{\hat{l}}{1-b}e^{-rt}c_v(r-\hat{l})q^b + m_1 b(\hat{U}-\hat{V}) = e^{-rt}[F_E - w] - m_1 n$$

and substitute (11) for m_1, then

$$c_v\frac{\hat{l}}{1-b}q^b\left[b(\hat{U}-\hat{V}) + n\right] = F_E - w - c_v\frac{\hat{l}}{1-b}q^b(r-\hat{l})$$

$$\Leftrightarrow\ F_E = w + c_v\frac{\hat{l}}{1-b}\left[r - \hat{l} + b(\hat{U}-\hat{V}) + n\right]q^b$$

Therefore, (18) is implied. ¦

Proposition 3 *Using (1), (8), (9a), (17), (18) and (22), the efficient factor allocation function* $\Psi(k) := \dfrac{l_0(1-a)(1-b)(1-w)}{c_{v0}\,\hat{l}[a/(1+c_I)k^{a-1} + n - \hat{l}]}k^a$ *follows.*

Proof. Differentiate (9a) w.r.t. *E*, substitute this and (8) in (18), then

$$q^b = \frac{l(1-a)(1-b)(1-w)}{c_v\,\hat{l}[r + b(\hat{U}-\hat{V}) + n - \hat{l}]}k^a .$$

Differentiate (9a) w.r.t. K and substitute this, (17), $l = l_0 e^{\hat{l}t}$ and $c_v = c_{v0}e^{\hat{l}t}$ in the above equation, then

$$\Psi(k) := \frac{l_0(1-a)(1-b)(1-w)}{c_{v0}\,\hat{l}[a/(1+c_I)k^{a-1} + b(\hat{U}-\hat{V}) + n - \hat{l}]}k^a$$

Furthermore, differentiate (1) w.r.t. time, then $\dot{E} = -\dot{U}$, use $q := V/U$, then $\dot{q} = \dot{V}/U - V\dot{U}/U^2$; use (1) and (22), then $\dot{E} = 0 = -\dot{U}$, $\dot{q} = 0$ and $\dot{V} = 0$ are implied and therefore $\hat{U} = \hat{V} = 0$. Substitute this in the above equation, the efficient factor allocation function $\Psi(k)$ follows. ¦

Proposition 4 *Suppose* $n - \hat{l} > 0$ *and* $k > (a_2 / a_3)^{1/1-a}$, $\Psi(k)$ *is an increasing concave function with*

$\Psi(0) = 0, \Psi(\infty) = \infty, \Psi'(0) = a_1 / a_2 < \infty, \Psi'(\infty) = 0, \Psi'(k) > 0, \Psi''(k) < 0$.

Proof. Equation (23) is equivalent to

$$\Psi(k) = \frac{a_1 k^a}{a_2 k^{a-1} + a_3}$$

$$\Leftrightarrow \Psi(k) = \frac{a_1 k}{a_2 + a_3 k^{1-a}}$$

with $a_1 := [l_0(1-a)(1-b)(1-w)/c_{v0}\hat{l}]$, $a_2 := a/(1+c_I)$ and $a_3 := n - \hat{l}$, then

$$\Psi(0) = 0,$$

$$\Psi(\infty) = \infty.$$

Using $n - \hat{l} > 0$, the properties of $\Psi'(k)$ follow directly from

$$\Psi'(k) = \frac{a_1[a_2 + a_3 k^{1-a}] - (1-a) a_1 a_3 k^{1-a}}{[a_2 + a_3 k^{1-a}]^2},$$

then

$$\Psi'(k) = \frac{a_1 a_2 + a a_1 a_3 k^{1-a}}{[a_2 + a_3 k^{1-a}]^2} > 0$$

$$\Psi'(k) = \frac{a_1}{a_2} < \infty,$$

$$\lim_{k\to\infty} \Psi'(k) = \lim_{k\to\infty} \frac{a a_1}{2[a_2 + a_3 k^{1-a}]} = 0.$$

Furthermore using $k > (a_2 / a_3)^{1/1-a}$, the properties of $\Psi''(k)$ follow directly from

$$\Psi''(k) = \underbrace{-2 a_1 a_2 [a_2 + a_3 k^{1-a}]^{-3} (1-a) a_3 k^{-a}}_{<0}$$

$$\underbrace{a a_1 a_3 k^{1-a} [a_2 + a_3 k^{1-a}]^{-2} (1-a)}_{>0} \underbrace{\left(1 - \frac{2 a_3 k^{1-a}}{a_2 + a_3 k^{1-a}}\right)}_{<0},$$

then

$$\Psi''(k) < 0$$

is implied. ¦

Proposition 5 *Using (1), (3) and (22), the steady-state employment rate* $e(q) = p(q)/[n + p(q)]$ *is implied.*

Proof. Equation (22) can be written as

$$\dot{E} = 0 \Leftrightarrow M = nE$$

and using (1) and (3), then

$$\frac{M}{U}U = nE$$

$$\Leftrightarrow \frac{M}{U}\bar{L} - \frac{M}{U}U = nE$$

$$\Leftrightarrow p(q)\bar{L} = (n + p(q))E.$$

Therefore, $e(q) := E/\bar{L} = p(q)/[n + p(q)]$ follows. ¦

Proposition 6 *Using (2), (9b), (10), (20), (21) and (25), the balanced accumulation function* $q^b = (l_0/c_{v0}\hat{l}n)\{k^a - [(1+c_I s)\hat{l}/s]k\} =: \Phi(k)$ *is implied.*

Proof. Using equations (20), (21) in efficiency units, then

(32) $x = (1+c_I s)y + c_v v$

Define $v := V/lE$ and use (2) and (10), then

(33) $v = \frac{(\hat{E}+n)}{l}q^b.$

Substituting (9b) and (33) in (32), then

$$y = \frac{1}{1+c_I s}\left[k^a - \frac{c_{v0}\hat{l}(\hat{E}+n)}{l_0}q^b\right].$$

Define $k := K/[lE]$, then $\dot{k} = \dot{K}/lE - (\hat{l}+\hat{E})k$. Use $\dot{K} = I = sY$, i.e. use (21), then

$$y = \frac{1}{s}\left[(\hat{l}+\hat{E})k + \dot{k}\right]$$

Equate the last equations and use (25) and $\dot{E} = \hat{E} = 0$, then

$$q^b = \frac{l_0}{c_{v0}\hat{l}n}\left[k^a - \frac{(1+c_I s)\hat{l}}{s}k\right] =: \Phi(k)$$

is implied. ¦

Proposition 7 *The balanced accumulation function is a concave function with* $\Phi(0) = 0, f(\infty) = -\infty, \Phi'(0) = \infty, \Phi'(\infty) = -a_5, \Phi'(k)\begin{cases}\geq 0\\ < 0\end{cases}$ for $ak^{a-1}\begin{cases}\geq a_5\\ < a_5\end{cases}$ *and* $\Phi''(k) < 0.$

Proof. Rewrite (26) as $\Phi(k) = a_4[k^a + a_5 k]$ with $a_4 := l_0/c_{v0}\hat{l}n$ and $a_5 := (1+c_I s)\hat{l}/s$, then $\Phi(0) = 0, f(\infty) = -\infty.$ Differentiate $f(k)$ w.r.t. *k*, then

$$\Phi'(k)\begin{cases}\geq 0\\ < 0\end{cases} \text{ for } ak^{a-1}\begin{cases}\geq a_5\\ < a_5\end{cases}$$

and, therefore,

$$\lim_{k\to 0}\Phi'(k) = \lim_{k\to 0}(ak^{a-1} - a_5) = \infty$$

$$\lim_{k\to\infty}\Phi'(k) = \lim_{k\to\infty}(ak^{a-1} - a_5) = -a_5.$$

Furthermore,

$$\Phi''(k) = (a-1)aa_4k^{a-2} < 0$$

is implied. ¦

Proposition 8 *The introduction of the government changes the balanced capital accumulation function to* $q^b = \frac{l_0}{c_{v0}\hat{l}n(1-d)}\left[k^a - \frac{(1+c_I s)\hat{l}}{s}k\right] =: \Phi_1(k)$.

Proof. Using (31) and (30), raising taxes, changes (21) to

(21') $S = I = sY^v = s(Y-T) = s(Y - dc_v V)$.

Therefore, the steady-state condition for the goods market (25) becomes

$$\dot{k} = sy^v - (\hat{l} + \hat{E})k = 0$$

(25') $\Leftrightarrow \dot{k} = s(y - dc_v v) - (\hat{l} + \hat{E})k = 0$.

Furthermore, raising taxes and distributing them to firms, changes (20) to

(20') $X := Y + c_I I + (1-d)c_v V + T$.

Using (30), (20') can be rewritten as $X := Y + c_I I + c_v V$ and in efficiency units as

(20'') $y = x - c_I i - c_v v$.

Substituting (9b), (21') and (33) in (20'') and rearrange it, then

(20''') $y = \frac{1}{1+c_I s}\left\{k^a - (1-c_I sd)\frac{c_{v0}\hat{l}n}{l_0}q^b\right\}$

follows.

Substituting $\dot{k} = 0$, $\hat{E} = 0$, (20'''), (21') and (33) in (25') and rearrange it, then

$$q^b = \frac{l_0}{c_{v0}\hat{l}n(1-d)}\left[k^a - \frac{(1+c_I s)\hat{l}}{s}k\right] =: \Phi_1(k)$$

is implied. ¦

References

Acemoglu, D. (1995)
Public Policy in a Model of Long-Term Unemployment, Economica, 62, 161-78.
Birk, A. (2001)
Qualification-Mismatch and Long-Term Unemployment in a Growth-Matching Model, HWWA Discussion Paper No. 128, 1-34.
Blanchard, O.; Diamond, P. (1989)
The Beveridge-Curve, Brookings Papers on Economic Activity, 1, 1-76.
Blanchard, O.; Diamond, P. (1994)
Ranking, Unemployment Duration and Wages, Review of Economic Studies, 61, 417-34.
Boeri, T.; Wörgötter, A. (1998)
Long-Term Unemployment and Social Assistance: Introduction, Empirical Economics, 23, 1-3.
Coles M.; Masters A. (2000)
Retraining and Long-Term Unemployment in a Model of Unlearning by Not Doing, European Economic Review, 44, 1801-22.
Feve, P.; Langot, F. (1996)
Unemployment and the Business Cycle in a Samll Open Economy: G.M.M. Estimation and Testing with French Data, Journal of Economic Dynamics and Control, 20, 1609-39.
Gora, M.; Schmidt, C. (1998)
Long-Term Unemployment, Unemployment Benefits and Social Assistance: The Polish Experience, Empirical Economics, 23, 55-85.
Gries, Th , Jungblut, S., Meyer, H. (1997a)
The Dynamics of Growth and Employment in a Two-Sector Model of Job-Matchings, in: Kischka, P.; Lorenz, H.; Derigs, U.; Domschke, W.; Kleinschmidt, P.; Mohring, R. (Hrsg.), Operations Research Proceedings 1997, Springer, Heidelberg, 241-6.
Gries, Th , Jungblut, S., Meyer, H. (1997b)
Job Matching, Structural Unemployment and Growth, Working Paper No. 9703, International Economics, University of Paderborn.
Jones, D.; Manning, D. (1992)
Long-term Unemployment, Hysteresis and the Unemployment-Vacancy Relationship: A Regional Analysis, Regional Studies, 26, 17-29.
Layard, R. (1997)
Preventing Long-Term Unemployment: An Economic Analysis, in: Snower, D., De La Dehasa, G. (Hrsg.), Unemployment Policy: Government Options for the Labour Market, Cambridge University Press.
Layard, R.; Nickell, S.; Jackman, R. (1991)
Unemployment: Macroeconomic Performance and the Labour Market, Oxford University Press, Oxford.
Ljungqvist, L.; Sargent, Th. (1995)
The Swedish Unemployment Experience, European Economic Review, 39, 1043-1070.

Ljungqvist, L.; Sargent, Th. (1998)
The European Unemployment Dilemma, Journal of Political Economy, 106, 31, 514-550.
Lockwood, B. (1991)
Information Externalities in the Labour Market and the Duration of Unemployment, Review of Economic Studies, 58, 733-753.
Marimon, R.; Zilibotti, F. (1999)
Unemployment versus Mismatch of Talents: Reconsidering Unemployment Benefits, Economic Journal, 109, 266-91.
Merz, M. (1995)
Search in the Labor Market and the Real Business Cycle, Journal of Monetary Economics, 36, 266-300.
Merz, M. (1999)
Heterogeneous Job-Matches and the Cyclical Behavior of Labor Turnover, Journal of Monetary Economics, 43, 91-124.
Michel, P. (1982)
On the Transversality Condition in Infinite Horizon Optimal Problems, Econometrica, 50, 4, 975-85.
Mortensen, D.; Pissarides, C. (1994)
Job Creation and Job Destruction in the Theory of Unemployment, Review of Economic Studies, 61, 397-415.
Mortensen, D.; Pissarides, C. (1998)
Technological Progress, Job Creation, and Job Destruction, Review of Economic Dynamics, 1, 733-53.
Mortensen, D.; Pissarides, C. (1999)
Unemployment Responses to `Skilled-Biased' Technology Shocks: The Role of Labor Market Policy, Economic Journal, 109, 242-65.
OECD (1983)
Employment Outlook, September, Table 24, 54, Paris.
OECD (1986)
Employment Outlook, September, Table L, 142, Paris.
OECD (1987)
Employment Outlook, September, Table Q, 332, Paris.
OECD (1991)
Employment Outlook, July, Table M, 259, Paris.
OECD (1995)
Employment Outlook, July, Table Q, 219, Paris.
OECD (1996)
Employment Outlook, July, Table Q, 202, Paris.
OECD (1998)
Employment Outlook, June, Table G, 208, Paris.
OECD (1999)
Employment Outlook, June, Table G, 242, Paris.
OECD (2000)
Employment Outlook, June,Table G, 220, Paris.
Pissarides, C. (1990)
Equilibrium Unemployment Theory, Basil Blackwell, Oxford.

Pissarides, C. (1992)
Loss of Skill During Unemployment and the Persistence of Unemployment Shocks, Quarterly Journal of Economics, 107, 1371-91.
Postel-Vinay, F. (1998)
Transitional Dynamics of the Search Model with Endogenous Growth, Journal of Economic Dynamics and Control, 22, 1091-1115.