DISCUSSION PAPER

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Abstract

The aim of the paper is to develop a theoretical framework for analyzing regional adjustment processes after idiosyncratic shocks. The model builds on the macroeconomic approach with monopolistic competition on goods and labor markets. It is shown how interregional factor mobility affects wage-setting behavior and labor supply. The dynamics of this extended model are explicitly taken into account. For a special case we can derive the characteristics of the adjustment process analytically. Under certain conditions the model exhibits hysteresis, i.e. temporary shocks on the price level or the unemployment rate can have a permanent effect on production and potential labor supply. It is argued that a suitable method for investigating regional adjustment processes empirically would be a panel VAR approach with integrated and co-integrated variables.

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1 INTRODUCTION

Adjustment processes in face of asymmetric regional shocks are a central topic on the economic policy agenda. The coherence within an economic and monetary union (e.g. within the EMU) crucially depends on effective mechanisms that lead back to equilibrium if regions were affected by idiosyncratic disturbances.[1] The more specialized regions within a large union are, the more likely seems a divergence in general business conditions: while some regions suffer from deep structural recessions, others are booming. This is important because there are good reasons to expect that increasing integration of markets will significantly lower the diversification in the regions' product mix. Empirical evidence based on comparisons to the U.S. suggests that the degree of specialization in Europe is still quite low.[2] Hence for many observers a large potential exists for intensifying the interregional division of labor in Europe. One the one hand this would lead to gains in efficiency, but on the other it makes regions more vulnerable. Increasing the sensitivity of a system of regions with respect to shocks might be especially a problem within a monetary union where the instrument of a compensating monetary policy is no longer available and fiscal interventions are quite limited. In such a situation the burden of adjustment has to be borne by other mechanisms, i.e. price reactions on goods and factor markets and the mobility of factors of production, especially of labor. However, given the labor market inflexibility typical European countries are blamed for, a marked response of relative prices and/or strong migration processes are hardly realistic at least in the short und medium run. Under these conditions the capability of the system to dampen possible shocks will be quite limited. As a consequence, since quantities adjust instead of prices and labor is immobile, excess unemployment may persist over longer periods of time.

Despite of its relevance for economic policy, the economics of regional adjustments appears to be not very well developed in new regional economics yet. One branch of the literature is closely related to the theory of optimal currency areas.[3] Several empirical papers investigate the synchronization of shocks occurring in countries or regions (cf.

1 The problem of asymmetric shocks for the European Monetary Union is investigated by several authors (see, for instance, *Krugman* (1993), *von Hagen*, *Hammond* (1998)).

2 The degree and development of specialization in Europe and the U.S. is discussed in *Fujita et al.* (1999), *Krugman*, *Venables* (1996), *Amiti* (1999), *Brülhart* (1998), *Brülhart, Torstensson,* (1996), *De Nardis et al.* (1996), *Kim* (1995), *Molle, Boeckhout* (1995), *Molle* (1997) and *Möller, Tassinopoulos* (2000).

3 *Mundell* (1961)

Bayoumi, *Eichengreen* (1993), *Obstfeld*, *Peri* (1998)). The degree of synchronization is taken as an important criterium for answering the question whether different economies should form an monetary union or not. Another branch of literature follows the pioneer work of *Blanchard, Katz* (1992). The central aim of their study is to identify the specific mechanisms that are responsible for economic adjustments after regions were hit by idiosyncratic shocks. Under different institutional settings, it can be expected that countries or regions show quite different characteristics in this respect. In contrast to the approach chosen by *Blanchard, Katz* (1992), applications to the situation in typical European countries have to take into account that labor markets are far from being perfectly competitive. Therefore the papers by *Decressin*, *Fatas* (1995) and *Möller* (1995), both designed for applications in the European context, include the possibility of persistent unemployment. Although there are some similarities in the chosen model and the empirical strategy, the cited studies come to different views about the role of migration as an adjustment process in typical European countries. On the one hand, *Decressin*, *Fatas* (1995) conclude that migration is of minor importance in this respect. Instead of workers' mobility it is especially participation behavior that reacts in response to an economic shock. This result is in sharp contrast to the U.S. where – according to the results of *Blanchard*, *Katz* (1992) – migration almost bears the full burden of adjustment. On the other hand, based on panel VAR estimates, *Möller* (1995) provides evidence that at least within Western Germany participation (as well as wage) responses are weak and sometimes in the wrong direction while migration gives rise to long-run adjustment. Contrary to the *Blanchard*, *Katz* results, however, migration responses are extremely sluggish in the German context. While in the U.S. the lion's share of the necessary processes is already completed within a time span of one year, the migration stream in Germany sets in only two or three years after the occurrence of the shock. This result fits well to the casual observation that it took decades (not years) to overcome deep crises in some West German regions which were caused by adjustment problems of specific sectors concentrated in these regions (for example coal, steel and shipbuilding). Compared to this evidence, regional adjustment forces in the U.S. appear to be much stronger.

There are also severe deficits in our theoretical understanding of regional adjustment processes, especially in the presence of imperfect labor markets. Only few systematic attempts have been made to extent the modern theory of unemployment (see *Layard et* *al.* (1991, 1994), *Phelps* (1994)) to the regional dimension.[4] Most existing approaches are rather partial in nature. Typically they deal with only an isolated aspect of the complex processes interacting for regional adjustment. An example is the work on the wage curve initiated by *Blanchflower* and *Oswald* (1990, 1994). For different countries and mircro-data sets they and several other authors find strong evidence for a negative convex relationship between the level of regional wages and regional unemployment. A number of independent studies in the mid-nineties come up with the conclusion that the elasticity of regional wages with respect to unemployment is in the order of magnitude of minus 0.1. As a puzzling fact, this elasticity appears to be rather independent of the institutional settings on the labor markets. Although recent studies[5] have challenged this "quasi law", the responsiveness of worker's remuneration to the state of the regional labor market is supported in general. However, this is only one facet of regional adjustments. This is also conceded by the pioneers of the wage curve. In recent contributions *Oswald* (1996, 1998), for instance, stresses a different aspect. He points to the fact, that labor mobility interrelated with house ownership might play an important role for understanding how labor markets digest a regional crisis.

The aim of the present paper is to develop a comprehensive theoretical framework for analyzing regional adjustment dynamics. In section 2, we first give a non-formal overview of various mechanisms that make up regional shock absorption. Then we present a theoretical analysis where the focus is on the interaction of migration with wage and price setting, while other elements of the theoretical model like the participation decision or capital mobility are sketched only briefly. Using a log linear approximation, the adjustment processes for the various equations are modeled in section 3 and integrated into a linear differential equation system. In section 4, we investigate the dynamic characteristics of this system. It is of special interest, whether or not temporary shocks have a permanent effect on some regional variables. This phenomenon has become known under the notion of "regional hysteresis".[6] For a simplified variant of the model it is possible to analyze the conditions under which regional hysteresis occurs. The paper ends with some conclusions in section 5.

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4 There are, however, important contributions to the development of a comprehensive theory of regional evolutions integrating insights from new growth theory (*cf. Fujita et al.* (1999) and *Walz* (1997, 1999)).

5 For a theoretical extension and innovative empirical study on this relationship using panel econometrics see *Nickell et al.* (2000).

6 See, for instance, *Krugman* (1991a,b,c), (1993), *Möller* (1994), *Venables* (1999).

2 ELEMENTS OF A THEORETICAL MODEL

2.1 Adjustment processes: An informal overview

Let us first describe different facets of regional adjustment processes in a non-formal way. Consider a region that was hit by a severe adverse shock. As a consequence, production will be depressed and unemployment rises. The recession of regional economy triggers several adjustment mechanisms. The most important ones concern the following aspects:

- mobility of labor: a part of the labor potential will leave the region temporarily (commuters) or permanently (migrants); since excess supply on the labor market is reduced, this behavior will foster the process of regaining pre-shock employment rates;

- participation: theoretically, effects in both directions are possible depending on the relative magnitude of the income and substitution effect; hence the response of the internal labor supply to a regional crisis might be negative or positive; in the former case the "discouraged worker effect" dominates, while in the latter the "additive worker effect" is more powerful; empirically there exists strong evidence that participation declines as a response to bad economic conditions; in this case participation would serve as a buffer;

- (real) wages and product prices: increasing unemployment lowers the outside option of workers and most likely undermines the bargaining power of unions; hence unemployment functions as a worker's disciplinary device (as *Shapiro*, *Stiglitz* (1984) have put it); since wage pressure is reduced, the profitability of firms improves; firms are able to lower prices; this would lead to a recovery of demand and, eventually, to new employment;

- price of immobile goods and services: the economic contraction leads to a decline in the prices of non-tradables; lower costs of living render the necessary downward adjustment of real (product) wages less critical to workers;

- capital flows: the excess supply of (qualified) workers together with shrinking wages and the lower regional price level for non-tradables could help to entice capital from outside the region; if capital inflows were strong enough, the structural crises would quickly be overcome by the creation of new working places; however, whether or not a region is attractive to investors also depends on expectations about the economic prospects of the region; these expectations might be negatively affected by the initial crisis; if nobody likes to invest in a "looser" region, the economic and labor market problems would be aggravated;

- effects on productivity and effort: during an economic crisis workers are more concerned with the stability of their working places; since the outside possibilities have deteriorated, the economic loss in case of lay-off would be higher; hence it is rational to show more effort in order to avoid being caught with shirking.

The complexity of regional adjustment after a regional shock is evident. Moreover, all what we know about regional development points to the fact that catch-up processes after deep regional crises are slow.[7] Since the duration of these amendment periods are better measured in decades than in years, it seems worthwhile to analyze the adjustment dynamics explicitly.

In the remainder of this section elements of a formal model are developed in order to capture different aspects of regional adjustment in a more stringent way. We first present a model of production, price setting and labor demand before including elements of migration and labor supply theory. Then special emphasis is laid on the analysis of wage setting, where an approach is chosen that extends the standard model to the regional dimension.[8]

2.2 Production, price setting and labor demand

Consider an economy which can be divided into a large number of regions so that a single region is small compared to the aggregate. Let production in region *r* be described by a CES-function with labor and capital input:

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7 From a growth-theoretic perspective the hypothesis of sluggish regional adjustment processes is also corroborated. The literature on regional convergence typically reveals low rates of convergence. For the general convergence debate see *Baumol* (1986) and *Abramovitz* (1986). The discussion in the nineties was strongly influenced by the empirical approach of *Barro, Sala-i-Martin* (1991, 1992). In this context see also *Neven, Gouyette* (1995), *Stahl* (1997), *Lopez-Bazo* (1999).

8 The standard approach of wage and price setting originates from *Layard, Nickell, Jackman* (1991). For a survey of the theory and recent extensions see *Möller, Beißinger* (1999).

$$Y_r = A_r\left(E_r^n N_r^{(\sigma-1)/\sigma} + E_r^k K_r^{(\sigma-1)/\sigma}\right)^{h\sigma/(\sigma-1)} \quad . \tag{1}$$

N_r are the number of workers in region r, h is an economies-of-scale parameter and σ the elasticity of substitution. A_r represents neutral increases in efficiency and E_r^n and E_r^k stand for efficiency or distribution parameters, respectively.

According to the standard Dixit-Stiglitz model of monopolistic competition, demand on the product market in region *r* is

$$Y_r^d = D_r P_r^{-\eta_r}, \tag{2}$$

where D_r is an exogenous variable reflecting aggregate income and the size of the region. P_r denotes the regional price level relative to the aggregate and $\eta > 1$ is the price elasticity of product demand in that region in absolute terms. The marginal condition derived from profit maximization leads to

$$N_r^* = \left(E_r^n X\right)^{\sigma} A_r^{\sigma-\mu} Y_r^{\mu_r} \left(\frac{W_r}{P_r}\right)^{-\sigma} \tag{3}$$

and

$$K_r^* = \left(E_r^k X\right)^{\sigma} A_r^{\sigma-\mu} Y_r^{\mu_r} \left(\frac{R}{P_r}\right)^{-\sigma}, \tag{4}$$

where $\mu := \sigma - \frac{1}{h}(\sigma - 1)$ and $X := \left(1-\eta^{-1}\right)h$. Note that under constant returns to scale, the parameters h and μ are both equal to unity.

In the context of models of monopolistic competition, eq.(3) is known as a *price-setting equation* (see *Layard et al.* 1991). In logs this equation can be written as

$$p_r = w_r - \tfrac{\sigma-1}{h\sigma} a_r - \left(1 + \tfrac{1-\sigma}{h\sigma}\right) y_r + \tfrac{1}{\sigma} n_r + \bar{x}_r^p \, . \tag{5}$$

with $\bar{x}_r^p := -\left(x + e_r^n\right)$. Note that $\bar{x}_r^p$ decreases when the efficiency of labor or the degree of competition rises. The benchmark case with perfect competition in the product market $(\eta_r \to \infty)$ and constant returns to scale $(h=1)$ is covered as a special case. Then the price-setting equation reduces to a standard labor demand equation because the product price is exogenous to the firm in this case:

$$n_r = (\sigma - 1) a_r + y_r - \sigma \left(w_r - p_r - e_r^n \right) \quad . \tag{6}$$

2.3 Migration

The classical economic theory of migration in the *Harris*, *Todaro* (1970) tradition[9] derives that migration depends on the expected real income differential adjusted for the costs of migration. In a slightly more general approach the basic idea can be extended to differences in utility levels determining the decision to migrate.[10]

Let F_r and F describe the expected utility for living in- or outside of region *r*, respectively:

$$\begin{aligned} F_r &= (1-u_r)\, q_r\, \Phi\left(\frac{W_r}{P_r^c}\right) + u_r\, q_r\, \Phi\left(\frac{B_r}{P_r^c}\right) + (1-q_r)\, \Phi\left(\frac{\bar{B}_r}{P_r^c}\right) \quad \text{and} \\ F &= (1-u)\, q\, \Phi\left(\frac{W}{P^c}\right) + u\, q\, \Phi\left(\frac{B}{P^c}\right) + (1-q)\, \Phi\left(\frac{\bar{B}_r}{P_r^c}\right), \end{aligned} \tag{7}$$

where the $\Phi(\cdot)$ is a utility function with the usual properties. The expected utility in both cases stems from the weighted average of three components, real wage income $W_r^r := W_r / P_r^c$, real income in case of unemployment B_r / P_r^c and the real value of social assistance in case of being out of the labor force $\bar{B}_r / P_r^c$. The weights are determined by the participation and unemployment rate q_r and u_r, respectively.[11]

In what follows, regional deviations from the aggregate are marked by a tilde, for instance $\tilde{u}_r := u_r - u$. Using a linear approximation to the utility function (see appendix 1), the difference in expected utility between the home and the target region in case of risk neutral agents can be written as

9 *Harris, Todaro* (1970)

10 Since the costs of migration are partly sunk, they create a "zone of indeterminancy" for individuals. By aggregation, however, the migration rates will be smoothed. So we can neglect the problem here.

11 The variables without subscript denote the corresponding aggregate variables.

$$\tilde{F}_r = (1-u_r)\, q_r\, \tilde{W}_r^r + u_r\, q_r \tilde{B}_r^r + (1-q_r)\, \tilde{\bar{B}}_r^r - q\left[W^r - B^r\right]\tilde{u}_r + \\ + \left[(1-u_r)W^r + u_r\, B^r - \bar{B}^r\right]\tilde{q}_r. \qquad (8)$$

Eq. (8) has a plausible interpretation. The expected utility differential depends positively on real wage differentials, differences in social security standards and participation and inversely on differences in the unemployment rate. The weighting factors for the real wage, unemployment insurance and social assistance differential are determined by the employment, unemployment and non-participation probabilities, respectively. The unemployment rate is weighted by the difference in the utility levels of receiving wage income or unemployment insurance payments. Finally, the difference in participation is weighted by the participation/ non-participation income differential.

Let ω_r and ω denote the rates of out-migration and immigration, respectively. The probability of leaving the region depends negatively on the expected net utility differential between the home and the target region as well as the monetary equivalent for the costs of out-migration C_r^m.[12] Vice versa the same holds for immigration. Hence

$$\omega_r = \underset{(-)}{\Omega_r}\left(\tilde{F}_r + C_r^m\right) \quad \text{and} \quad \omega = \underset{(+)}{\Omega}\left(\tilde{F}_r - C^m\right) , \qquad (9)$$

where $\Omega_r(\cdot)$ and $\Omega(\cdot)$ are functions that guarantee a non-negative function value. Moreover it is reasonable to assume $\Omega_r(\cdot) < 1$ and $\Omega_r\left(\tilde{F}_r + C_r^m\right) = 0$ if the argument is positive.

2.4 Wage formation

In a first variant we assume that a monopoly union in region *r* maximizes the expected utility of a representative agent taking into account the option to migrate. It is clear that introducing an option to migrate to another region cannot reduce the expected utility because it extends the opportunity space. In other words: the value of the option to migrate cannot be negative. At the limit, - if conditions in the home region are clearly

12 Note that the costs of out-migration can be negative if the target region is attractive because of non-wage characteristics.

preferable to the conditions elsewhere, or, the costs of migration are prohibitively high - the option to migrate becomes worthless.

According to these considerations, the expected utility of a representative worker in region *r* is

$$Z_r = (1-\omega_r)F_r + \omega_r(F - C_r^m) = F_r - \omega_r(\tilde{F}_r + C_r^m) = F_r - \Omega_r\left(\tilde{F}_r + C_r^m\right)\cdot\left(\tilde{F}_r + C_r^m\right), \quad (10)$$

where the migration rate ω_r becomes zero if $\tilde{F}_r + C_r^m \geq 0$.

Assuming that the (regional) union maximizes the expected utility of a representative agent in that region one obtains the first order condition

$$\frac{\partial Z_r}{\partial W_r} = F_r'\left[1 - \Omega_r' \cdot \left(\tilde{F}_r + C_r^m\right) - \Omega_r\right] \overset{!}{=} 0, \quad (11)$$

where $F_r' = \partial F_r / \partial W_r$ and Ω' denotes the first derivative of the function with respect to the argument.[13] It can be shown that for Z_r becoming a maximum it is required that $F_r' = 0$.[14] At first glance there seems to be no difference in the behavior of a regional monopoly union whether acting in an isolated region or in a multi-regional context. Note, however, that the function F_r' is not identical in the two cases.

In the appendix 2 it is shown that in case of risk-neutral agents the optimal real wage in consumption units (or *consumption wage*) a monopoly union would set is:

$$W_r^c = \left(1 - \frac{1}{\varepsilon_N + \varepsilon_q + \varepsilon_M}\right)^{-1} B_r^r \quad \text{with } \varepsilon_N + \varepsilon_q + \varepsilon_M > 1, \quad (12)$$

where ε_N, ε_q and ε_M are the real wage elasticities of labor demand, participation and migration, respectively. Eq. (12) indicates that the optimal wage claim depends positively on the replacement income B_r^r in case if unemployment. The mark-up depends

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13 Note that $\tilde{F}_r' = F_r'$ if the home region is small as compared to the target region which is assumed here.

14 The term in brackets is positive as long as the elasticity of the out-migration with respect to the argument does not exceed unity which has to assumed also from stability reasons.

negatively on the real wage elasticities of labor demand, participation and migration ε_N, ε_q and ε_M. In the sense that interregional migration increases the responsiveness of labor supply with respect to the real wage, it dampens the wage aspirations of workers for a given institutional setup.

In a second variant of modeling wage setting, the theory of wage bargaining is used.[15] Following the standard assumption the utility function of the union depends on employment and the difference between wages and the value of the outside option

$$\Theta_r = N^{\gamma}\left[W_r^r - Z_r^r\right], \tag{13}$$

where γ describes a parameter for the weight the union attaches to employment and Z^r is the outside option which here comprises the possibility to leave the labor force or to migrate.[16] Note that Z^r is a function of the wage rate and unemployment in the home region among others.

Let the profit of a representative firm in region *r* be denoted by $\Pi\left(W_r^r, R\right)$, where R is the exogenous interest rate. Maximizing the Nash product yields (see appendix 3)

$$W^r = \left[1 - \frac{1 - Z^{r\,\prime}}{\zeta}\right]^{-1} Z^r \tag{14}$$

with

$$\zeta := \gamma\varepsilon_N + \frac{(1-\theta)(\bar{\kappa}+\alpha)}{\theta(1-\alpha-\beta)},$$

where $\bar{\kappa} := (\eta-1)^{-1}(\varepsilon_N\alpha + \varepsilon_K\beta)$ is an indicator for market power of firms on the goods markets.

As in the monopoly union model, the bargained real wage is a mark-up over the outside option. This mark-up exceeds unity since $Z^{r\,\prime} < 1$. As expected, wage pressure is reduced if the bargaining power of firms relative to that of workers increases and the more weight the union attaches to employment. Ceteris paribus the bargained real wage will

15 See for a discussion of this approach, for instance, *Manning* (1993), (1995).
16 A similar approach is used by *Fuest, Thum* (2000), who discuss the welfare effects of migration using a dual labor market model.

be dampened by a higher wage elasticity of labor demand ε_N. Finally, rising market power of firms on the goods market will increase the bargained real wage.

The derivations from above can be interpreted as description of the behavior of an atomistic wage bargain at the firm level or as a region-wide bargain. In what follows we consider the first case in which the outside option is not affected by the behavior of the individual wage-setting agents and hence $Z^{r\,\prime}=0$. Then the optimal real wage can be derived from eq. (14) as an implicit function

$$W_r^c = \frac{\zeta}{\zeta-1}\left\{F_r\left(W_r^c,\cdot\right)-\Omega_r\left[\tilde{F}_r\left(W_r^c,\cdot\right)+C_r^m\right]\tilde{F}_r\left(W_r^c,\cdot\right)\right\}. \tag{15}$$

To obtain the impact of the different variables on the wage rate, note that the derivative of Z_r with respect to a regional (X_r)or aggregate variable (X) is

$$\frac{\partial Z_r}{\partial X_r} = F_r{}'(1-\Omega_r'\tilde{F}_r-\Omega_r) = F_r{}'\left[1-\Omega_r\left(1-\varphi\right)\right] \tag{16}$$

and

$$\frac{\partial Z_r}{\partial X} = F'\left(\Omega_r'\tilde{F}_r+\Omega_r\right) = F'\Omega_r\left(1-\varphi\right), \tag{17}$$

respectively, where the prime indicates the derivative of the corresponding function with respect to the argument and φ is the elasticity of the migration rate with respect to differences in expected utility

$$\varphi = \frac{\Omega_r{}'}{\Omega_r}\tilde{F}_r > 0 \quad \text{if} \quad \tilde{F}_r < 0$$
$$\varphi = 0 \quad \text{if} \quad \tilde{F}_r \leq 0.$$

It is reasonable to assume that the reaction of the migration rate to changes in the expected utility differential is inelastic ($0 \leq \varphi < 1$), hence

$$\operatorname{sign}\left(\partial Z_r/\partial X_r\right) = \operatorname{sign}\left(F_r{}'\right) \text{ and } \operatorname{sign}\left(\partial Z_r/\partial X\right) = \operatorname{sign}\left(F'\right).$$

Then the partial derivative of the outside option with respect to the regional wage rate is unambiguously positive, while it is unambiguously negative for the regional unemployment rate:

$$Z^r_{r,w_r} = q_r\left(1-u_r\right)\left[1-\left(1-\varphi\right)\Omega_r\right] \qquad \text{with} \quad 1 > Z^r_{r,w_r} > 0$$
$$Z^r_{r,u_r} = -q_r(W_r{}^c - B_r{}^r)\left[1-\left(1-\varphi\right)\Omega_r\right] < 0,$$

where a subscript indicates the derivative with respect to the corresponding variable. It can also be excluded that the partial derivative of the aggregate wage level is negative and that of the aggregate unemployment rate is positive:

$$Z^r_{r,w} = q(1-u)\left(1-\varphi\right)\Omega_r \geq 0$$
$$Z^r_{r,u} = -q(W^c - B^r)\left(1-\varphi\right)\Omega_r \leq 0,$$

while the partial derivatives with respect to the participation rate are not determined in sign.

The derivative of equilibrium wage setting in region *r* with respect to the unemployment rate is of crucial importance. From the implicit function

$$W_r^c - \frac{\zeta}{\zeta-1} Z_r^r\left(W_r^c, u_r, \cdot\right) = W_r^c - \mu Z_r^r\left(W_r^c, u_r, \cdot\right) = 0 \qquad (18)$$

one obtains the partial derivative as

$$\frac{\partial W_r^c}{\partial u_r} = \frac{\mu Z^r_{r,u_r}}{1-\mu Z^r_{r,w_r}}, \qquad (19)$$

A higher unemployment rate unambiguously lowers the outside option ($Z^r_{r,u_r} < 0$), and hence the nominator is negative. Because the elasticity of the outside option with respect to changes in the regional wage is smaller than unity, the sign of the derivative in eq. (19) must be negative. In other words, the regional wage-setting curve is falling in unemployment/wage-space. Analogously one can derive that all other variables driving the value of the outside option upwards increase wage pressure. Other things being equal, this is the case for both variants of social security payments. The upshot of these considerations is

$$W_r^c = W_r^c\Big(\underset{(-)}{u_r}, \underset{(-)}{u}, \; \underset{(+)}{W^c}, \underset{(?)}{q_r}, \; \underset{(?)}{q}, \underset{(+)}{B_r^r}, \underset{(+)}{\overline{B}_r^r}\Big). \tag{20}$$

Note that eq. (20) gives wages-setting in a static economy. If *per capita* income in the economy grows at the rate of growth of technical knowledge, it can reasonably be assumed that all variables relevant for wage formation grow at this rate in the long run. Since from growth-theoretic considerations this rate must also be equal to the rate of productivity growth, it is standard to introduce the level of productivity in the wage equation.[17] In logarithmic form one then obtains

$$w_r^{c*} = a_r + H\Big(\underset{(-)}{u_r}, \underset{(?)}{q_r}, \underset{(+)}{\overline{x}_r^w}\Big), \tag{21}$$

where $\overline{x}_r^w$ collects institutional and other exogenous variables.

3 MODELING REGIONAL ADJUSTMENT PROCESSES

In this section the basic dynamic relationships important for analyzing regional adjustment processes are described. To keep the notation simple, the time index is omitted. We start with participation and labor supply. Then the dynamics of regional unemployment and the wage-price spiral as well as inter-regional capital flows are considered.

3.1 Participation and labor supply

By definition, labor supply can be written in logs as $\ell_r = q_r + m_r$. Denoting first differences by a dot over the corresponding variable yields for the growth rate of labor supply[18]

17 See, for instance, *Nickell et al.* (2000).

18 Throughout the paper growth rates are approximated by log differences.

$$\dot{\ell}_r = \dot{q}_r + \dot{m}_r \,. \tag{22}$$

With no natural population growth log changes of the potential labor supply $\dot{m}_r$ are equal to minus the net (out-)migration rate which can be defined from eq. (9) as

$$\dot{m}_r = -\left[\Omega_r(\cdot) - \Omega(\cdot)\right] := -\tilde{\Omega}_r\Big(\underset{(-)}{\tilde{F}_r}, \underset{(-)}{C_r^m}, \underset{(+)}{C^m}\Big). \tag{23}$$

Assuming that the substitution effect dominates the income effect, the desired participation rate depends positively on the expected income on the labor market relative to the "non-work" social security level $\bar{B}_r$:

$$q_r^* = q_r^*\Big(\underset{(+)}{w_r^c}, \underset{(-)}{u_r}, \underset{(+)}{b_r^r}, \underset{(-)}{\bar{b}_r^r}\Big). \tag{24}$$

The corresponding dynamic adjustment process for the log participation rate is

$$\dot{q}_r = \xi\left[q_r^*(\cdot) - q_r\right]. \tag{25}$$

A log-linear approximation of eq. (24) together with eq. (25) gives

$$\dot{q}_r = -\xi_1 q_r + \xi_2 w_r^c - \xi_3 u_r + x_r^q \,, \tag{26}$$

where x_r^q collects the influence of exogenous structural variables affecting the participation decision like social and unemployment assistance.[19] To derive an equation that fits into the other elements of the model one has to consider the wedge between the consumption wage being relevant for wage setting, participation and migration and the product wage being relevant for price setting and labor demand. Let the log wedge be described by

$$g_r := w_r^c - w_r^r = p_r^c - p_r^r \,. \tag{27}$$

Using this equation the dynamic participation relationship reads

19 The parameters are defined to be positive.

$$\dot{q}_r = -\xi_1 q_r + \xi_2 \left(w_r - p_r \right) + \xi_2 g_r - \xi_3 u_r + x_r^q . \qquad \text{(dynamics of participation)} \quad (26)'$$

Inserting eq. (23) and eq. (25) into eq. (22) yields an expression for log changes in labor supply

$$\dot{\ell}_r = \xi \left[q_r^*(\cdot) - q_r \right] - \tilde{\Omega}_r (\tilde{F}_r, \cdot) \quad \text{with} \quad \frac{\partial \tilde{\Omega}_r}{\partial \tilde{F}_r} < 0 . \qquad (28)$$

Considering the variables that determine the difference in expected utility one obtains:

$$\tilde{\Omega}_r = \tilde{\Omega}_r \left(\underset{(+)}{u_r}, \underset{(-)}{W_r^c}, \underset{(-)}{u}, \underset{(+)}{W^c}, \underset{(-)}{\tilde{B}_r^r}, \underset{(-)}{\tilde{\tilde{B}}_r^r} \right) \qquad (29)$$

After collecting structural variables as well as those being determined outside the region in a new variable x_r^m and using a log-linear approximation, eq. (29) simplifies to

$$1\text{n}\tilde{\Omega}_r = \omega_1 u_r - \omega_2 w_r^c + x_r^m . \qquad (30)$$

Since the effect of unemployment and wages from participation and out-migration goes into the same direction, log changes in labor supply can be written as

$$\dot{\ell}_r = -(\xi_3 + \omega_1) u_r + (\xi_2 + \omega_2) w_r^c - \xi_1 q_r + x_r^q - x_r^m , \qquad (31)$$

or,

$$\dot{\ell}_r = -\lambda_1 u_r + \lambda_2 \left(w_r - p_r \right) + \lambda_2 g_r - \xi_1 q_r + x_r^\ell . \qquad \text{(dynamics of labor supply)} \quad (31)'$$

Hence changes in labor supply of region *r* depend negatively on the level of the unemployment and participation rate, and positively on the level of the real wage in that region. A closer inspection of the exogenous variables shows that a higher unemployment rate as well as lower wages outside the region exert a positive effect on regional labor supply. Lower social standards outside the region, whether related to wage replacement payments or social assistance, increase the regional labor supply as well. The effect of social assistance in the home region, however, is ambiguous. On the one hand it clearly lowers participation, but on the other hand it helps to dampen out-migration.[20]

20 It should be noted that the model here does not take into account selectivity effects that come into play if persons with special characteristics migrate because of differences in the social security levels. This issue is subject of a number of papers, see, for instance, *Krueger* (2000), *Bertola* (2000).

3.2 Unemployment

The standard log-linear approximation for the unemployment rate[21] can be written in dynamic form as

$$\dot{u}_r = \dot{\ell}_r - \dot{n}_r \,. \tag{32}$$

From the price-setting equation (5) one obtains

$$n_r^* = -\sigma(w_r - p_r) - \tfrac{1-\sigma}{h} a_r + \left(\sigma + \tfrac{1-\sigma}{h}\right) y_r + \sigma\left(x + e_r^n\right). \tag{33}$$

With re-defined parameters the dynamic adjustment process can be written as

$$\dot{n}_r = \nu\left(n_r^* - n_r\right) = -\nu_1 n_r + \nu_2 y_r - \nu_3 (w_r - p_r) - (\nu_2 - \nu_3) a_r + x_r^n \,,$$

where $x_r^n := -\nu_1 \sigma \overline{x}_r^p$, or, after substituting for the employment variable

$$\dot{n}_r = \nu\left(n_r^* - n_r\right) = -\nu_1 (\ell_r - u_r) + \nu_2 y_r - \nu_3 (w_r - p_r) - (\nu_2 - \nu_3) a_r + x_r^n \tag{34}$$

with $\nu_1 < 1$, $\nu_2 := \nu_1\left(\sigma + \frac{1-\sigma}{h}\right)$ and $\nu_3 := \nu_1 \sigma$. Hence $\nu_2 > \nu_3$ as long as $\sigma < 1$. Note that under constant returns to scale $(h = 1)$ the parameters ν_1 and ν_2 are equal.

Inserting eq. (31) and eq. (34) into eq. (32) and considering the wedge between the product and consumption wage then yields

$$\begin{aligned} \dot{u}_r = {} & -(\lambda_1 + \nu_1) u_r + (\lambda_2 + \nu_3)\left(w_r - p_r\right) + \nu_1 \ell_r - \nu_2 y_r - \xi_1 q_r + (\nu_2 - \nu_3) a_r + \\ & + \lambda_2 g_r + x_r^u \qquad \text{(dynamics of unemployment)} \end{aligned} \tag{35}$$

with $x_r^u := x_r^\ell - x_r^n$.

It turns out that the dynamic development of unemployment depends positively on the level of real wages and the active population, and negatively on participation and pro-

21 Note that, in contrast to the other variables of the model, the unemployment rate is not measured in natural logarithms.

duction. Note that the unemployment generating effect of real wages stems from two sources: While labor demand shrinks, higher real wages lead to positive impulses on labor supply via participation or migration.

3.3 The wage-price spiral and aggregate demand

As outlined in the previous section, wage setting is determined by consumer price inflation, the general productivity trend and specific factors that are captured by the function $H(\cdot)$. If the consumption wage $w_r - p_r^c = w_r - p_r - g_r$ falls below the target value given by $a_r + H(\cdot)$, wage pressure in region r starts to rise, i.e. $\dot{w}_r > 0$, and vice versa.[22] Hence

$$\dot{w}_r = \eta\Big(a_r + \underset{(+)}{H(u_r, q_r, \cdot)} - \big(w_r - p_r - g_r\big)\Big), \tag{36}$$

or, in a log-linear version

$$\dot{w}_r = -\eta_1 (w_r - p_r) + \eta_1 (a_r + g_r) - \eta_2 u_r - \eta_3 q_r + x_r^w, \qquad \text{(dynamic wage setting)} \quad (36)'$$

An analogous reasoning holds for price setting. Define a function

$$G(a_r, y_r, n_r, \cdot) := -\tfrac{1-\sigma}{h\sigma} a_r + \Big(1 + \tfrac{1-\sigma}{h\sigma}\Big) y_r - \tfrac{1}{\sigma} n_r + x + e_r^n,$$

such that the desired price level can be written as

$$p_r^* = w_r - G(a_r, y_r, n_r, \cdot) \tag{37}$$

If $G(\cdot)$ falls short of the regional product wage $w_r - p_r$, the regional price level will tend to increase:

$$\dot{p}_r = \gamma\Big((w_r - p_r) - \underset{(+)}{G(a_r, y_r, n_r, \cdot)}\Big). \tag{38}$$

Using $u_r = \ell_r - n_r$, one obtains in a linear specification

22 Here and in what follows, a dot over a variable stands for the first derivative with respect to time.

$$\dot{p}_r = \gamma_1(w_r - p_r) - \gamma_2 y_r + \gamma_3(\ell_r - u_r) + (\gamma_2 - \gamma_1) a_r + x_r^p \text{ (dynamic price setting)} \qquad (38)$$

with $\gamma_1 < 1, \gamma_2 := \gamma_1\left(1 + \frac{1-\sigma}{h\sigma}\right), \gamma_3 := \frac{\gamma_1}{\sigma}$ and $x_r^p := \gamma_1\left(\overline{x}_r^p + \varepsilon_r^p\right)$, where ε_r^p stands for any other exogenous influence on price setting. Note that $\gamma_2 > \gamma_1$ if $\sigma < 1$ and that under constant returns to scale $\gamma_2 = \gamma_3$.

It has to be stressed that in the model with monopolistic competition, - once prices and wages have been settled, - production is determined by demand. As outlined in section 2.2, the demand in region *r* depends on the relative prices and the size of the region (or the number of firms) among others. It is reasonable to assume that the number of firms is proportional to the size of the potential workforce in region under consideration.

The dynamics of regional production as driven by aggregate demand can then be derived from eq. (2) as

$$\dot{y}_r = \vartheta\Big(d_r - \underset{(+)}{\eta_r} p_r - (y_r - \ell_r)\Big), \qquad (39)$$

or,

$$\dot{y}_r = -\delta_1(y_r - \ell_r) - \delta_2 p_r + x_r^d . \qquad \text{(dynamics of product demand) (39)'}$$

Changes in the regional consumption price index are partly determined by the index of tradables, and partly by changes in the price index of immobile goods and services. The latter is a function of excess demand on the local markets for these immobile goods and services. It can be assumed that at least partly the supply of these local goods is inelastic. Hence the corresponding prices are mainly determined from the demand side. Without having a more elaborated theory it can be assumed that the regional *per capita* income is the dominating factor. Neglecting inter-regional net income streams, the *per capita* income can be approximated by *per capita* production. Hence we write the dynamics of the regional price wedge as

$$\dot{g}_r = -\zeta_1 g_r + \zeta_2(y_r - \ell_r) + x_r^g . \qquad \text{(dynamics of the price wedge) (40)}$$

3.4 Interregional capital flows and total factor productivity

The model outlined so far has neglected interregional capital flows. Capital flows, however, play an important role for regional adjustment processes. Without deriving the corresponding relation more rigorously, we try here to sketch the necessary extension of the theoretical framework to incorporate the factor capital.

Capital inflow into the region depends on differences in the marginal return to capital. For a CES production function the marginal product of capital in units of regional products is

$$\frac{\partial Y_r}{\partial K_r} = \left(1-\eta_r{}^{-1}\right) E_r^k A_r{}^{-\frac{1-\sigma}{\sigma}} \left(\frac{Y_r}{K_r}\right)^{\frac{1}{\sigma}} . \tag{41}$$

In a multi-regional context the real returns to capital have to be adjusted for interregional price differences. If capital owners partly live outside the region then it is the aggregate price level that counts. Moreover, the installation costs in case of investments depend on the price of immobile factors such as land which is also captured in regional consumption prices. Therefore it is reasonable to approximate capital inflows in log-linear form as[23]

$$\dot{k}_r = \kappa_1 (y_r - k_r) - (\kappa_1 - 1) a_r + \kappa_2 p_r - \kappa_3 g_r + x_r^k \quad . \qquad \text{(flow of capital)} \tag{42}$$

Furthermore, an important side-effect of investment should be considered. Capital inflows into a region are typically related to new technology. New growth theory provides arguments that investments exert a positive externality fostering total factor productivity. This can be modeled here as an influence of investment on the dynamics of the variable A_r :

$$\dot{a}_r = -\tau_1 a_r + \tau_2 \dot{k}_r + x_r^a . \qquad \text{(dynamics of total factor productivity)} \tag{43}$$

23 In their famous paper *Feldstein, Horioka* (1980) have found that regional investment also depends on the regional saving rate. This result is in a certain contrast to the rationality of capital owners and free mobility to capital. We do not consider this aspect in the context here.

3.5 The complete dynamic model

In the previous subsections the dynamic model was completely described. The nine endogenous variables can be grouped into three classes:

- **employment related variables**:
  potential labor supply ℓ_r, participation q_r, the unemployment rate u_r;

- **wages and prices**:
  the nominal wage w_r, the price index for tradable production goods p_r and the price gap between the production and the consumption wage g_r; these variables implicitly define the product and consumption wage: $w_r - p_r$ and $w_r - p_r^c = w_r - p_r - g_r$, respectively;

- **capital, production and total factor productivity**:
  capital k_r, production y_r, total factor productivity a_r.

In order to present an overview of the model all relevant dynamic equations are repeated here:

$$\dot{w}_r = -\eta_1 (w_r - p_r) + \eta_1 (a_r + g_r) - \eta_2 u_r - \eta_3 q_r + x_r^w \qquad \text{(dynamic wage setting)} \; (36)'$$

$$\dot{p}_r = \gamma_1 (w_r - p_r) - \gamma_2 y_r + \gamma_3 (\ell_r - u_r) + (\gamma_2 - \gamma_1) a_r + x_r^p \qquad \text{(dynamic price setting)} \; (38)'$$

$$\dot{q}_r = -\xi_1 q_r + \xi_2 \left(w_r - p_r \right) + \xi_2 g_r - \xi_3 u_r + x_r^q \qquad \text{(participation)} \; (26)'$$

$$\dot{\ell}_r = -\lambda_1 u_r + \lambda_2 \left(w_r - p_r \right) + \lambda_2 g_r - \xi_1 q_r + x_r^\ell \qquad \text{(labor supply)} \; (31)'$$

$$\begin{aligned} \dot{u}_r = -(\lambda_1 + \nu_1) u_r + (\lambda_2 + \nu_3) \left(w_r - p_r \right) + \nu_1 \ell_r - \nu_2 y_r - \xi_1 q_r + (\nu_2 - \nu_3) a_r + \\ + \lambda_2 g_r + x_r^u, \qquad \text{(dynamics of unemployment)} \end{aligned} \qquad (35)$$

$$\dot{y}_r = -\delta_1 (y_r - \ell_r) - \delta_2 p_r + x_r^d \qquad \text{(product demand)} \; (39)'$$

$$\dot{g}_r = -\zeta_1 g_r + \zeta_2 (y_r - \ell_r) + x_r^g \qquad \text{(dynamics of the price wedge)} \; (40)$$

$$\dot{k}_r = \kappa_1(y_r - k_r) - (\kappa_1 - 1)\,a_r + \kappa_2 p_r - \kappa_3 g_r + x_r^k \qquad \text{(flow of capital)} \quad (42)$$

$$\dot{a}_r = -\tau_1 a_r + \tau_2 \dot{k}_r + x_r^a \qquad \text{(dynamics of total factor productivity)} \quad (43)$$

As has been pointed out, regional adjustment even under some "heroic simplifying assumptions" is a highly interdependent and complicated process.

4 SOLUTION OF THE MODEL

4.1 Long-run equilibrium and hysteresis

We now turn to the long-run equilibrium of the model. In long-run equilibrium wage-setting and price-setting are mutually consistent, there is an equilibrium in the "battle of the markups". Hence the regional unemployment rate is on its quasi-equilibrium or "natural" level u_r^*. Moreover, there will be neither migration nor capital flows and participation is constant. An important question is whether the steady state is unique or not.

The dynamic model outlined in section 3 can be written in general form as

$$\dot{\mathbf{x}}_t = \mathbf{C} \;\; \mathbf{x_t} + \mathbf{z}_t \,. \qquad (44)$$

If the transition matrix $\mathbf{C}$ has full rank, then an unique equilibrium exists:

$$\mathbf{x}_r^* = -\mathbf{C}^{-1}\mathbf{z}_r \qquad (45)$$

Otherwise the model shows hysteresis implications so that temporary shocks have permanent effects. What could be the sources of regional hysteresis? An important element is related to migration. The migration decision involves sunk costs. Once a person has left the home region, the costs involved through migration are not taken into account in further decisions. If migration was triggered by a severe adverse regional shock, the effect of the shock remains, even if the shock fades out. There are other possible sources of regional hysteresis. For example the wage process can be based on insider decisions. Once an insider has become an outsider, his weight in the wage-setting process might be reduced substantially. Insofar as a severe recession increases the number of outsiders in this sense, a temporary shock can have permanent effects. Insider dominated wage decisions, however, are not modeled here.

Of course, the possibilities for analyzing a complex dynamic adjustment process analytically are quite limited. We therefore argue for the use of a suitable empirical methodology to determine the speed of adjustment for the various endogenous variables and other parameters of interest. The basic idea in this context is to approximate the linear differential equation system (44) by a system of difference equations. This can be shown as follows: A VAR approach with exogenous variables can be written as

$$\mathbf{A(L)}\,\mathbf{x_t} := \left(\mathbf{I} - \mathbf{A}_1\mathbf{L} - \mathbf{A}_2\mathbf{L}^2 - \ldots - \mathbf{A}_p\mathbf{L}^p\right)\mathbf{x_t} = \boldsymbol{\varepsilon}_\mathbf{t}\,, \tag{46}$$

where $\mathbf{A(L)}$ is a suitable matrix lag polynomial of order p and $\boldsymbol{\varepsilon}_t$ is a vector of disturbances (representing exogenous influences here). In case that $\mathbf{A(L)}$ can be inverted
A standard re-formulation of eq. (46) gives

$$\Delta\mathbf{x}_t = -\mathbf{A}(1)\,\mathbf{x}_{\mathbf{t}-1} - \sum_{i=1}^{p-1}\mathbf{G}_i\Delta\mathbf{x}_{t-i} + \boldsymbol{\varepsilon}_t \tag{47}$$

with

$$\mathbf{G}_i := \sum_{j=i+1}^{p}\mathbf{A}_j\,,$$

where $\mathbf{A}_j$ is the coefficient matrix of lag j in $\mathbf{A(L)}$.[24] Such a system can suitably estimated by a the method of vector autoregressions (VAR). To take account of the possibility of a rank deficit in the transition matrix, a VAR approach for integrated and cointegrated variables should be used.[25] To check the number of non-zero eigenvalues of the transition matrix $-\mathbf{A}(1)$, it is possible, in principle, to apply the well-known rank test developed in the cointegration literature.[26] For a single data set, this technique is sufficiently elaborated. The use of regional panel data sets, however, still poses some methodological problems. Recent developments in this field give hope that the remaining

24 Note that for p=1 the second term on the right-hand side of eq. (47) vanishes. In this case eq. (47) gives the direct discrete analogon to the linear system of first-order differential equations of eq. (44). The reason to choose the more general formulation of eq. (47) is that it allows more flexibility for modeling the adjustment process.

25 To reduce the number of integrated variables one can use the regional variables as deviations from the corresponding aggregate ones.

26 See for a in-depth description of this test and the corresponding methodology *Lütkepohl* (1991).

problems will be solved in the near future. Then a powerful instrument would be available to study regional adjustment dynamics empirically on a sound econometric basis. The analytical results obtained in the present paper could then be checked quantitatively.

4.2 A special case

In order to get an analytically tractable model we consider a special case. Let us neglect the fact that participation is endogenous. The difference between consumption and product prices will be disregarded and no capital flows are allowed. This yields the following simplified model:

$$\dot{w}_r = -\eta_1 (w_r - p_r) - \eta_2 u_r + x_r^w \qquad \text{(dynamic wage setting)} \qquad (36)''$$

$$\dot{p}_r = \gamma_1 (w_r - p_r) - \gamma_2 y_r + \gamma_3 (\ell_r - u_r) + x_r^p \qquad \text{(dynamic price-setting)} \qquad (38)''$$

$$\dot{y}_r = -\delta_1 (y_r - \ell_r) - \delta_2 p_r + x_r^d \qquad \text{(demand for goods)} \qquad (39)$$

$$\dot{u}_r = -(\lambda_1 + \nu_1) u_r + (\lambda_2 + \nu_3)(w_r - p_r) + \nu_1 \ell_r - \nu_2 y_r + x_r^u \qquad \text{(unemployment)} \qquad (35)'$$

$$\dot{\ell}_r = -\lambda_1 u_r + \lambda_2 (w_r - p_r) + x_r^\ell , \qquad \text{(potential labor supply)} \qquad (31)''$$

where $x_r^p := -\gamma_1 \left(x + e_r^n \right)$ and $x_r^u := x_r^\ell + \frac{\nu_1}{\gamma_1} \sigma x_r^p$

In matrix form one obtains

$$\dot{\mathbf{x}}_{r,t} = \mathbf{C}\, \mathbf{x}_{r,t} + \mathbf{z}_{r,t} ,$$

or,

$$\begin{pmatrix} \dot{w}_r \\ \dot{p}_r \\ \dot{y}_r \\ \dot{u}_r \\ \dot{\ell}_r \end{pmatrix} = \begin{pmatrix} -\eta_1 & \eta_1 & 0 & -\eta_2 & 0 \\ \gamma_1 & -\gamma_1 & -\gamma_2 & -\gamma_3 & \gamma_3 \\ 0 & -\delta_2 & -\delta_1 & 0 & \delta_1 \\ \lambda_2 + \nu_3 & -(\lambda_2 + \nu_3) & -\nu_2 & -(\lambda_1 + \nu_1) & \nu_1 \\ \lambda_2 & -\lambda_2 & 0 & -\lambda_1 & 0 \end{pmatrix} \begin{pmatrix} w_r \\ p_r \\ y_r \\ u_r \\ \ell_r \end{pmatrix} + \begin{pmatrix} x_r^w \\ x_r^p \\ x_r^y \\ x_r^u \\ x_r^\ell \end{pmatrix} \qquad (48)$$

The determinant of the transition matrix is

$$|\mathbf{C}| = \gamma_1\delta_1(\eta_2\lambda_1 + \eta_1\lambda_2)(\nu_2 - \nu_1)\,. \tag{49}$$

With all parameters being positive and $h \neq 1$ (implying $\nu_1 \neq \nu_2$ and $\gamma_2 \neq \gamma_3$) it follows that eq. (48) has a unique steady solution according to eq. (45). Hence there exists a well-defined relationship between the set of exogenous variables and the equilibrium dependent variables. After a temporary shock in one of the exogenous variables, the system re-adjusts to the initial steady state. Depending on the eigenvalues of the transition matrix $\mathbf{C}$, however, this could rather be a time-consuming process.

The situation is qualitatively different under constant returns to scale ($h = 1$). In this important special case, the transition matrix shows a rank deficit. Hence the solution of the system is path-dependent. This means that the steady-state solution of at least one of the endogenous variables depends on the initial shock which implies that a temporary shock can have a permanent impact. Let us consider the situation in more detail. Inspection of $\mathbf{C}$ reveals a linear dependency between the third and fifth column. Applying the solution technique outlined in appendix 4, it turns out that the equilibrium for the regional wage and price variables as well as for the unemployment rate is *not* affected by the initial conditions.

Define $\hat{x}_r^* := \overset{*}{\mathbf{x}}_{r,1} - \overset{*}{\mathbf{x}}_{r,0}$, where the two vectors on the right-hand side denote the new and the initial steady-state. The solution shows that

$$\hat{p}_r^* = \hat{w}_r^* = \hat{u}_r^* = 0, \quad \text{whereas} \quad \hat{y}_r^* \neq 0 \quad \text{and} \quad \hat{\ell}_r^* \neq 0\,. \tag{50}$$

Hence regional production or income as well as the potential labor force are “hysteresis variables”. Moreover it turns out that in equilibrium the deviation from the initial steady-state solution for these variables, - expressed by the corresponding elasticities, - is identical. After some simple but tedious algebra one obtains more specifically:

$$\frac{\partial \hat{y}_r^*}{\partial w_{r,0}} = \frac{\partial \hat{\ell}_r^*}{\partial w_{r,0}} = 0 \tag{51}$$

$$\frac{\partial \hat{y}_r^*}{\partial p_{r,0}} = \frac{\partial \hat{\ell}_r^*}{\partial p_{r,0}} = \sigma\frac{\nu_1}{\gamma_1} > 0 \tag{52}$$

$$\frac{\partial \hat{y}_r^*}{\partial y_{r,0}} = \frac{\partial \hat{\ell}_r^*}{\partial y_{r,0}} = 0 \qquad (53)$$

$$\frac{\partial \hat{y}_r^*}{\partial u_{r,0}} = \frac{\partial \hat{\ell}_r^*}{\partial u_{r,0}} = -1 \qquad (54)$$

$$\frac{\partial \hat{y}_r^*}{\partial \ell_{r,0}} = \frac{\partial \hat{\ell}_r^*}{\partial \ell_{r,0}} = 1\,. \qquad (55)$$

Several points have to be stressed: First, the result implies that per capita income $y_r - \ell_r$ and productivity $y_r - n_r$ are *not* influenced by temporary shocks. The results are thus compatible with conditional β-convergence.[27] Second, temporary shocks in production (demand) have no long-run impact on any of the endogenous variables. Hence in this respect the model shares the neutrality property of the “structuralist” labor market model.[28] Third, the variables indicating the economic power of the region (overall workforce and production) are subject to hysteresis effects. It must be pointed out, however, that no external effects like, for instance, the possible disutility from a higher population density are modeled here. Fourth, temporary wage-shocks have no long-run effects. By contrast, an initial shock in the price variable causes such an effect on production and labor supply. According to the results here, the possibility to impose higher prices for a certain period of time fosters the economic development of the region. The magnitude of the long-run impact on the region’s domestic product and workforce increases with the elasticity of substitution. A higher price level reduces the real wage in production units. This triggers employment growth especially if the elasticity of substitution is high. The overall effect is the stronger, the slower prices adjust to the new equilibrium level (γ_1 low) and the more flexible are changes in employment (ν_1 high). Fifth, the elasticity of the workforce in long-run equilibrium with respect to an initial shock in unemployment and labor supply is minus and plus unity, respectively. Consider, for instance, a one per cent increase in unemployment caused by exogenous reasons. With all adjustments being completed, unemployment is back to its original level but the workforce has declined by one per cent. Since in the simplified model participation is constant, this implies that the full burden of adjustment is borne by (out)migration. As a consequence production is reduced by the same relative amount.

27 See *Barro, Sala-i-Martin* (1991), (1992).
28 For a recent survey see *Beißinger, Möller* (2000).

This is mirrored by a situation where the labor supply increases by one per cent. In this case the adjustment is through the creation of new employment and increasing demand.

5 CONCLUSIONS

The starting point of the analysis here is the observation that under the typical institutional environment of European economies, regional adjustment processes in response to idiosyncratic shocks are sluggish. Given that the success of the European Economic and Monetary Union in the medium and long-run critically depends on the capability of regional economies to absorb changes in their specific economic environment, it is required to investigate the corresponding adjustment dynamics more in detail.

The response of a regional economy to shocks has numerous facets. Even under some heroic simplifications the model shows a considerable degree of complexity. The theoretical framework used here is that of monopolistic competition. To adopt this leading macroeconomic approach to the regional context, several modifications and extensions are necessary. In our view this gives some new insights into the functioning of a highly interactive intra- and interregional system. Among others, it is shown that the *outside option* as an important determinant of wage-setting depends on the possibility to migrate. This creates an interdependency with variables and institutional conditions in other regions that are important for the migration decision. Through its influence on labor supply and unemployment, migration feeds back to regional wage setting.

The analysis of regional adjustment dynamics reveals several other specific features. A crucial aspect is whether or not the process of regional evolution involves hysteresis. In a technical sense hysteresis or path dependence occurs if the transition matrix in the dynamic system describing regional adjustments exhibits a rank deficit. In this case temporary shocks have an impact on the long-run equilibrium of at least one of the endogenous variables. This is not the case, if the transition matrix can be inverted.

The specific dynamic system can be derived from the theoretical framework and log-linear adjustment hypotheses. For a special case we analyze the dynamic implications of the model more closely. It turns out that under the assumption of constant returns to scale, the constellation of parameters in the transition matrix gives rise to hysteresis. From the inspection of the steady-state solution it follows that the equilibrium values of wages, prices and unemployment are not affected by the initial conditions. Hence the exogenous variables and model parameters uniquely determine the equilibrium values of these endogenous variables. Regional production and the potential labor supply, however, are influenced by the initial conditions and, therefore, by temporary shocks. In other words, these variables are path-dependent under the given assumptions. More specifically, it can be shown that temporary shocks in the price level, in labor supply and unemployment can affect the economic power of the region in the long run, while temporary wage and production (demand) shocks are irrelevant in this respect. A further important result concerns the long-run elasticities of production and labor supply with respect to a temporary labor supply or unemployment shock. The numerical values for these elasticities are plus and minus unity, respectively. This means that the burden of adjustment in case of an unemployment shock entirely relies on migration, while in case of a labor supply shock, the adjustment works through the creation of new employment and production. It seems economically plausible that the long-run impact of a temporary price shock on production and the regional workforce depends on the elasticity of substitution and the relative magnitude of inertia in price-setting and labor demand.

Of course, the possibilities of analyzing a complex dynamic adjustment process analytically, are quite limited. We therefore argue for the use of a suitable empirical methodology to determine the speed of adjustment for the various endogenous variables and other parameters of interest. The basic idea in this context is to approximate the linear differential equation system by a system of difference equations. Such a system can suitably estimated by a the method of vector autoregressions (VAR) with integrated and co-integrated variables.

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APPENDIX

Appendix 1: The difference in expected utility between two regions

With a superscript r indicating real values of the corresponding variables one obtains for the difference in expected utility for living in- or outside of region r :

$$\tilde{F}_r := F_r - F = (1-u_r)\, q_r\, \Phi\left(W_r^r\right) - (1-u)\, q\, \Phi\left(W^r\right) + u_r\, q_r\, \Phi\left(B_r^r\right) - u\, q\, \Phi\left(B^r\right) + (1-q_r)\, \Phi\left(\bar{B}_r^r\right) - (1-q)\, \Phi\left(\bar{B}^r\right). \quad \text{(A-1)}$$

Define $\tilde{u}_r := u_r - u$ or $u = u_r - \tilde{u}_r$ and the same for $\tilde{W}_r^r$, $\tilde{B}_r^r$, $\tilde{\bar{B}}_r^r$ and $\tilde{q}_r$.

Substituting in eq. (A-1) yields

$$\tilde{F}_r = (1-u_r)\, q_r\, \Phi\left(W_r^r\right) - (1-u_r+\tilde{u}_r)\,(q_r-\tilde{q}_r)\, \Phi\left(W^r\right) + u_r\, q_r\, \Phi\left(B_r^r\right) - (u_r-\tilde{u}_r)\,(q_r-\tilde{q}_r)\, \Phi\left(B^r\right) + (1-q_r)\Phi\left(\bar{B}_r^r\right) - (1-q_r+\tilde{q}_r)\Phi\left(\bar{B}^r\right),$$

or,

$$\tilde{F}_r = (1-u_r)\, q_r\left[\Phi\left(W_r^r\right) - \Phi\left(W^r\right)\right] + u_r\, q_r\left[\Phi\left(B_r^r\right) - \Phi\left(B^r\right)\right] + (1-q_r)\times \times\left[\Phi\left(\bar{B}_r^r\right) - \Phi\left(\bar{B}^r\right)\right] - q\left[\Phi\left(W^r\right) - \Phi\left(B^r\right)\right]\tilde{u}_r + +\left[(1-u_r)\Phi\left(W^r\right) + u_r\,\Phi\left(B^r\right) - \Phi\left(\bar{B}^r\right)\right]\tilde{q}_r. \quad \text{(A-2)}$$

Linearization of the first term in brackets gives

$$\Phi\left(W_r^r\right) - \Phi\left(W^r\right) = \Phi'\left(\breve{W}_r^r\right)\cdot \tilde{W}_r^r \quad \text{(A-3)}$$

where $\breve{W}_r^r := 0.5\left(W_r^r + W^r\right)$. Using an analogous transformation for the second and third term in brackets leads to

$$\tilde{F}_r = (1-u_r)\, q_r\, \Phi'\left(\breve{W}_r^r\right)\tilde{W}_r^r + u_r\, q_r \Phi'\left(\breve{B}_r^r\right)\tilde{B}_r^r + (1-q_r)\,\bar{\Phi}'\left(\breve{\bar{B}}_r^r\right)\tilde{\bar{B}}_r^r - q\left[\Phi\left(W^r\right) - \Phi\left(B^r\right)\right]\tilde{u}_r + \left[(1-u_r)\Phi\left(W^r\right) + u_r\,\Phi\left(B^r\right) - \bar{\Phi}\left(\bar{B}^r\right)\right]\tilde{q}_r \quad \text{(A-4)}$$

or, in case of risk neutral agents

$$\tilde{F}_r = (1-u_r)\, q_r\, \tilde{W}_r^r + u_r\, q_r \tilde{B}_r^r + (1-q_r)\,\tilde{\bar{B}}_r^r - q\left[W^r - B^r\right]\tilde{u}_r + \left[(1-u_r)W^r + u_r\, B^r - \bar{B}^r\right]\tilde{q}_r \quad \text{(A-5)}$$

where $\bar{B}^r$ comprises the monetary equivalent of the value of leisure.

Appendix 2: Utility Maximization of the Monopoly Union

The expected utility of a representative worker is

$$Z_r := F_r - \omega_r \tilde{F}_r = F_r - \Omega\left(\tilde{F}_r, C_r^m\right)\tilde{F}_r \,, \tag{A-6}$$

where

$$F_r = (1-u_r)\, q_r\, \Phi\left(W_r^r\right) + u_r\, q_r\, \Phi\left(B_r^r\right) + (1-q_r)\,\Phi\left(\bar{B}_r^r\right) \quad \text{and}$$
$$\tilde{F} := F_r - F \quad \text{with} \quad F = (1-u)\, q\, \Phi\left(W^r\right) + u\, q\, \Phi\left(B^r\right) + (1-q)\,\Phi\left(\bar{B}^r\right).$$

Let the variables $\bar{B}^r$ and $\bar{B}_r^r$ comprise the monetary equivalent of the advantages of not participating (i.e. the value of extra leisure, the absence of disutility of work or being available to the labor market etc.). For the first order condition to be met, it is sufficient to set the derivative of expected utility in region *r* with respect to the nominal wage equal to zero. Assume that the utility maximizing participation decision of households conditional to the wage rate has already been taken. Then by applying the envelope theorem the aforementioned derivative is

$$\frac{\partial F_r}{\partial W_r^r} = q_r (1-u_r)\,\Phi'\left(W_r^r\right) - q_r u_r' \left[\Phi\left(W_r^r\right) - \Phi\left(B_r^r\right)\right].$$

Using the elasticity $\varepsilon_u := u_r' \dfrac{W_r^r}{u_r}$ yields

$$\frac{\partial F_r}{\partial W_r^r} = q_r \left\{ (1-u_r)\,\Phi'\left(W_r^r\right) - u_r \varepsilon_u \left(\frac{\Phi\left(W_r^r\right) - \Phi\left(B_r^r\right)}{W_r^r} \right) \right\}. \tag{A-7}$$

From the definition of the unemployment rate

$$u_r := 1 - \frac{N_r (W_r / P_r)}{L_r (W_r / P_r^c)} \tag{A-8}$$

one obtains

$$\frac{\partial u_r}{\partial W_r} = -\frac{L_r N_r' - N_r L_r'}{L_r^2} = \frac{\dfrac{L_r'}{L_r} - \dfrac{N_r'}{N_r}}{\dfrac{L_r}{N_r}}. \tag{A-9}$$

By introducing the real wage elasticities of labor demand and supply

$$\varepsilon_N := -\frac{\partial N_r}{\partial W_r^r}\frac{W_r^r}{N_r} \quad \text{and} \quad \varepsilon_L := \frac{\partial L_r}{\partial W_r^r}\frac{W_r^r}{L_r}$$

the real wage elasticity of unemployment can be written as

$$\varepsilon_u = \frac{\partial u_r}{\partial W_r^r}\frac{W_r^r}{u_r} = \frac{1-u_r}{u_r}(\varepsilon_N + \varepsilon_L) = \frac{1-u_r}{u_r}\left(\varepsilon_N + \varepsilon_q + \varepsilon_M\right) \qquad \text{(A-10)}$$

and hence

$$u_r\varepsilon_u = (1-u_r)\left(\varepsilon_N + \varepsilon_q + \varepsilon_M\right).$$

Substitution into eq. (A-7) yields

$$\frac{\partial F_r}{\partial W_r^r} = q_r(1-u_r)\left\{\Phi'\left(W_r^r\right) - \left(\varepsilon_N + \varepsilon_q + \varepsilon_M\right)\left(\frac{\Phi\left(W_r^r\right) - \Phi\left(B_r^r\right)}{W_r^r}\right)\right\}. \qquad \text{(A-11)}$$

Using the first order condition gives

$$W_r^r\,\Phi'\left(W_r^r\right) - \left(\varepsilon_N + \varepsilon_q + \varepsilon_M\right)\left(\Phi\left(W_r^r\right) - \Phi\left(B_r^r\right)\right) = 0.$$

For the optimal real wage one obtains with $\tau := \Phi' W_r^r \big/ \Phi\left(W_r^r\right)$:

$$W_r^r = \Phi^{-1}\left\{\frac{1}{1-\tau\left(\varepsilon_N + \varepsilon_q + \varepsilon_M\right)^{-1}}\Phi\left(B_r^r\right)\right\}. \qquad \text{(A-12)}$$

In the special case of risk neutral agents eq. (A-12) can be simplified to

$$W_r^r = \frac{1}{1-\left(\varepsilon_N + \varepsilon_q + \varepsilon_M\right)^{-1}}B_r^r. \qquad \text{(A-13)}$$

Appendix 3: Maximizing the Nash-Product

The outcome of the bargaining process at the firm level can be described by maximizing the Nash product (leaving aside the index for the region):

$$\max_{W^r} = \theta \ln \Theta\left(W^r, Z^r\right) + (1-\theta)\ln \Pi\left(W^r, R\right) \quad \text{(A-14)}$$

As a first order condition one obtains

$$\theta\frac{\Theta'}{\Theta} + (1-\theta)\frac{\Pi'}{\Pi} = 0 \quad \text{(A-15)}$$

Let $\varepsilon_\Theta := \frac{\Theta'}{\Theta W^r}$ and $\varepsilon_\Pi := -\frac{\Pi'}{\Pi W^r}$ be the elasticities of union's utility and firm's profits with respect to the wage. Then it follows from eq. (A-15)

$$\theta\varepsilon_\Theta + (1-\theta)\varepsilon_\Pi = 0 \quad \text{or} \quad \frac{\varepsilon_\Theta}{\varepsilon_\Pi} = \frac{1-\theta}{\theta}. \quad \text{(A-16)}$$

Hence in the optimum the wage elasticities relevant to unions and firms are inversely related to their bargaining power.

As defined in the main text, the utility function of a representative union is

$$\Theta = N^\gamma\left[W^r - Z^r\right]. \quad \text{(A-17)}$$

The derivative with respect to the wage rate is

$$\Theta' = \gamma N^\gamma \frac{N'}{N}\left(W^r - Z^r\right) + N^\gamma\left(1 - Z^{r\,\prime}\right). \quad \text{(A-18)}$$

Hence

$$\frac{\Theta'}{\Theta} = \gamma\frac{N'}{N} + \frac{1 - Z^{r\,\prime}}{W^r - Z^r},$$

or,

$$\varepsilon_\Theta := \frac{\Theta' W^r}{\Theta} = -\gamma\varepsilon_N + \frac{\left(1 - Z^{r\,\prime}\right)W^r}{W^r - Z^r}. \quad \text{(A-19)}$$

Correspondingly it follows from $\Pi := Y - WN - RK$ that

$$\frac{\Pi'}{\Pi} = \frac{Y' - N - W^r N' - R^r K'}{Y - W^r N - R^r K}$$
$$= -\frac{\left(Y_N / W^r - 1\right)\varepsilon_N \alpha + \left(Y_K / R^r - 1\right)\varepsilon_K \beta + \alpha}{W^r (1-\alpha-\beta)}. \qquad \text{(A-20)}$$

Define $\kappa := (\eta - 1)/\eta$ as an indicator of competition on the goods market, where η is the elasticity of demand. Optimal factor demand implies $W^r = \kappa Y_N$ and $R^r = \kappa Y_K$. Together with

$$\frac{Y_N}{W^r} = \frac{Y_K}{R^r} - 1 = \kappa^{-1} - 1 = \frac{1}{\eta - 1} \qquad \text{(A-21)}$$

the wage elasticity of profits can be derived as

$$\varepsilon_{\Pi, W^r} := -\frac{\Pi' W^r}{\Pi} = \frac{(\eta-1)^{-1}(\varepsilon_N \alpha + \varepsilon_K \beta) + \alpha}{1 - \alpha - \beta} \qquad \text{(A-22)}$$

In the special case of perfect competition $(\eta \to \infty)$ on goods markets one has

$$\varepsilon_{\Pi, W^r} := -\frac{\Pi' W^r}{\Pi} = \frac{\alpha}{1 - \alpha - \beta} \qquad \text{(A-23)}$$

Note that profits are more sensitive to wages if the degree of competition on goods markets decreases.

Let $\gamma = 1$, assume perfect competition of goods market, a Cobb-Douglas technology and $Z^{r\,'} = 0$. Then as a benchmark case

$$W^r = \frac{\alpha + \theta(1-\alpha-\beta)}{\alpha} Z^r = \left[1 + \frac{\theta(1-\alpha-\beta)}{\alpha}\right] Z^r \qquad \text{(A-24)}$$

For the general case one obtains

$$W^r = \left[1 - \frac{1 - Z^{r\,\prime}}{\gamma\varepsilon_N + \dfrac{(1-\theta)(\overline{\kappa}+\alpha)}{\theta(1-\alpha-\beta)}}\right]^{-1} Z^r = \left[1 - \frac{1 - Z^{r\,'}}{\zeta}\right]^{-1} Z^r = \frac{\zeta}{\zeta - \left(1 - Z^{r\,'}\right)} Z^r = \mu\ Z^r, \qquad \text{(A-25)}$$

where $\overline{\kappa} := (\eta - 1)^{-1}(\varepsilon_N \alpha + \varepsilon_K \beta)$ is an indicator for market power of firms on the goods markets and

$$\zeta := \gamma\varepsilon_N + \frac{(1-\theta)(\overline{\kappa}+\alpha)}{\theta(1-\alpha-\beta)}.$$

Appendix 4: Solving a dynamic system with a rank deficit of the transition matrix

Consider a dynamic system of the form

$$\dot{\mathbf{x}}_t = \mathbf{C} \quad \mathbf{x_t} \, . \tag{A-26}$$

It is assumed that the matrix **C** exhibits a rank deficit. Let the variables collected in **x** be sorted so that in the partitioned system

$$\begin{pmatrix} \dot{\mathbf{x}}_{\mathbf{1t}} \\ \dot{\mathbf{x}}_{\mathbf{2t}} \end{pmatrix} = \begin{pmatrix} \mathbf{C_{11}} & \mathbf{C_{12}} \\ \mathbf{C_{21}} & \mathbf{C_{22}} \end{pmatrix} \begin{pmatrix} \mathbf{x_{1t}} \\ \mathbf{x_{2t}} \end{pmatrix} \tag{A-27}$$

the matrix $\mathbf{C_{11}}$ has full rank, while the columns of $\mathbf{C}_{12}$ und $\mathbf{C}_{22}$ can be represented with a suitable chosen matrix **F** as

$$\mathbf{C_{12}} = \mathbf{C_{11}F} \quad \text{and} \quad \mathbf{C_{22}} = \mathbf{C_{21}F} \tag{A-28}$$

From

$$\mathbf{x}_{1\mathbf{t}} = \mathbf{C}_{\mathbf{11}}^{-1}\left(\dot{\mathbf{x}}_{1\mathbf{t}} - \mathbf{C_{12}}\mathbf{x}_{2\mathbf{t}}\right) \tag{A-29}$$

and

$$\mathbf{C_{22}} - \mathbf{C_{21}}\mathbf{C}_{\mathbf{11}}^{-1}\mathbf{C_{12}} = \mathbf{0} \tag{A-30}$$

follows that

$$\dot{\mathbf{x}}_{\mathbf{2t}} = \mathbf{D} \quad \dot{\mathbf{x}}_{1\mathbf{t}}, \quad \text{where} \quad \mathbf{D} := \mathbf{C_{21}}\mathbf{C}_{\mathbf{11}}^{-1} \quad . \tag{A-31}$$

Integration of eq. (A-31) yields

$$\mathbf{x}_{2\mathbf{t}} - x_{2\mathbf{0}} = \mathbf{D} \quad \left(\mathbf{x}_{1\mathbf{t}} - \mathbf{x}_{1\mathbf{0}}\right) \tag{A-32}$$

In the steady state it must hold that

$$\mathbf{x}_1^* = -\mathbf{C}_{\mathbf{11}}^{-1}\mathbf{C_{12}}\mathbf{x}_2^* \tag{A-33}$$

and

$$\mathbf{x}_2^* = \mathbf{G}^{-1}\mathbf{x}_{2\mathbf{0}} - \mathbf{G}^{-1}\mathbf{D} \quad \mathbf{x}_{1\mathbf{0}}, \tag{A-34}$$

where $\mathbf{G} := \mathbf{I} + \mathbf{D}\mathbf{C}_{\mathbf{11}}^{-1}\mathbf{C_{12}}$.

Using eq. (A-33) also the equilibrium solution for $\mathbf{x}_1^*$ can be easily calculated.