THE MEASUREMENT OF BIASED EFFICIENCY GAINS IN U. S. AND JAPANESE AGRICULTURE TO TEST THE INDUCED INNOVATION HYPOTHESIS

By

HANS PETER BINSWANGER

1 9 7 3

NORTH CAROLINA STATE UNIVERSITY
AT RALEIGH

ABSTRACT

BINSWANGER, HANS PETER. The Measurement of Biased Efficiency Gains in U. S. and Japanese Agriculture to Test the Induced Innovation Hypothesis. (Under the direction of RICHARD ADAMS KING.)

The basic purpose of this study was to test the induced innovation hypothesis at a very basic level. The hypothesis states that biases in efficiency gains arising from technical change and other sources are endogenously determined by economic forces rather than exogenously by the physical, chemical and biological laws of nature.

The key idea of the test is as follows: If biases are determined exogenously, two countries with different factor endowments and differences in other economic variables would experience the same patterns of biases over a prolonged period of time. If the biases differ, there is a strong presumption that these differences have been determined by endogenous economic forces.

Biases were measured for the agricultural sectors of the United States and Japan. Estimation equations for the biases in the case of a many-factor production process were developed using a Transcendental Logarithmic cost function in factor-augmenting form. Using these equations it is possible to divide observed share changes into a component due to efficiency gains and a component due to price changes. The components due to efficiency gains were then used to construct indices of biases.

The application of this method required that one first estimate the parameters of the Translog cost function. This was done with cross-section state data for the United States using generalized least squares

techniques. The estimated parameters also provided estimates of a set of elasticities and cross elasticities of factor demand and a set of elasticities of substitution.

The resulting indices of bias indicate marked differences between the United States and the Japanese experience. The conclusion reached is that the basic premise of the induced innovation hypothesis is correct. Biases are determined by economic forces. However, little evidence as to the precise inducement mechanisms was found.

THE MEASUREMENT OF BIASED EFFICIENCY GAINS IN U. S. AND JAPANESE AGRICULTURE TO TEST THE INDUCED INNOVATION HYPOTHESIS

by

HANS PETER BINSWANGER

A thesis submitted to the Graduate Faculty of
North Carolina State University at Raleigh
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

DEPARTMENT OF ECONOMICS

RALEIGH

1 9 7 3

APPROVED BY:

Chairman of Advisory Committee

BIOGRAPHY

The author was born June 9, 1943 in Kreuzlingen, Switzerland. He received his secondary education in Frauenfeld, graduating from the Thurgauische Kantonsschule in 1963.

He received a certificate of Political Science from the University of Paris in 1964 and his Ingenieur Agronom diploma from the Eidgenössische Technische Hochschule in Zürich in 1969. He held a graduate research assistantship in the Department of Economics at North Carolina State University from September 1969 to September 1972. Since that time, he has served as a research associate at the University of Minnesota.

The author is married to the former Monique Annie Soubes, and they have a daughter named Ingrid.

ACKNOWLEDGMENTS

The author wishes to express sincere appreciation to the many persons who have contributed to his graduate study and research. He is especially grateful to Dr. Richard King, Chairman of the Graduate Committee, and to the other members of the Graduate Committee. They have provided a unique research environment with an exceptional extent of freedom, generous help and constructive criticism during all phases of the study.

Gratitude is expressed to the Schweizerische Schulrat and the Department of Economics of North Carolina State University for financial assistance which made it possible for the author to come to North Carolina State University and devote his full attention to his education and this study.

This study was partially financed by the Agency for International Development under a research contract with North Carolina State University entitled "Agricultural Diversification and Trade in Latin America," and represents a contribution towards forming the theoretical guidelines for further research on that contract.

The author wishes to thank Mrs. Ann Dees for her very competent help in editing and typing the manuscript. Special appreciation is expressed to the staff of the Statistical Laboratory who provided assistance in computer programming and computation.

Finally, the author expresses his thanks to his wife Monique, for her strong encouragement to undertake this program and for her sacrifice during its entire duration.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

Page

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDIX TABLES

Page

Appendix C

Appendix D

CHAPTER 1
INTRODUCTION

The formal analysis of economic growth has started as an analysis of the accumulation of factors of production, in particular of capital. The simple Harrod-Domar models regarded increases in the capital-labor ratio as the only source of increases in per capita income. Therefore, increases in investment into physical capital (through increases in the savings rate) were considered the single most important policy goal for countries trying to achieve growth.

However, with the work of Solow (1957) and others, it soon became apparent that increases in the physical capital-labor ratio could explain only a very small part of the increases in per capita incomes. What the capital-labor ratio could not explain was termed technical change, although the more neutral term "efficiency increases" might have generated less controversy or misunderstanding. Since then, Denison (1969), Jorgenson and Griliches (1967) and others have made great efforts to allocate "technical change" to various elements of efficiency increases or quality changes of traditional factors--increases in education, quality changes of capital equipment and land, changes in utilization rate of capital and economies of scale, etc.

The ultimate source of these changes is always some sort of investment, although not the traditional investment of the Harrod-Domar model into new units of already-developed physical capital. Schultz (1964) therefore stresses that less developed countries will not be able to obtain growth by investing more in capital goods of traditional form. Instead they will have to create new institutions capable of providing improved inputs into production such as a better educated labor force, better intermediate inputs (e.g., seeds) and new capital equipment adapted to the local conditions. Some institutions (extension, information) and the proper market incentives for the diffusion of the new inputs will also be required. Government activity in the production and diffusion of new techniques is necessary because private firms will be unable to capture all the benefits of their investments. Schultz argues that only if the less developed countries are successful in this endeavor will they obtain growth.

Suppose a country is successful in obtaining efficiency growth. The rate of growth of labor income and employment (not necessarily the wage rates) will depend not only on the rate of efficiency growth but also on whether the ensuing efficiency growth will be biased, i.e., labor-saving or labor-using. If the countries simply import techniques from the developed countries without adapting them to their own factor endowments, their efficiency growth will be labor-saving and labor incomes and employment will not rise very fast or may even decline. On the other hand, if they could develop their own techniques or adapt advance techniques such that for a given increase in total factor efficiency (or productivity) they would use substantially lower capital-labor ratios than the techniques of the developed countries, labor incomes and employment would rise more quickly. The induced innovation hypothesis maintains that this is possible and will occur if the necessary institutions exist and factor prices reflect the true opportunity cost of factors.

The basic idea of the induced innovation hypothesis is that biases are not determined outside of the economic system but depend on the conditions prevailing within each economy. In particular, it states that biases in efficiency gains will depend on relative factor supplies. As the relative factor supplies change, the efficiency gains will be biased to save the factor which has become more scarce. The hypothesis is sort of an investment theory of biased efficiency gains.

In different existing forms of the hypothesis, the biases are induced in different ways--either by factor prices, changes in factor prices or by factor shares. Only empirical evidence can decide which inducement mechanism is the correct one.

The terms factor-saving and factor-using biases are unfortunate because efficiency gains will most often reduce the absolute amount used per unit of output of all factors. However, the terms factor-saving and factor-using do not refer to the absolute requirements but to the relative speed with which the requirements are reduced. Efficiency gains are said to be saving the factor which has its input requirements reduced in the highest proportion at constant factor prices. Absolute changes of factor productivity are not considered.

The most important empirical evidence needed to take induced innova-ation out of the area of speculation is a test of the endogeneity of biases of technical change. At the most basic level this can be done as follows: measure the biases of efficiency gains in two countries in which relative factor supplies have historically moved in different directions or at differing rates. If the countries experienced the same pattern of biases during the same time periods, the biases are exogenous because they have not been influenced by the conditions within each economy. Other-wise they are endogenous. The objective of this thesis is to perform this test. If biases were found to be exogenously determined, the induced innovation hypothesis could be discarded because nothing could be done to change the characteristics of new technology. However, the finding of this thesis is that the biases are endogenous.

A more detailed discussion of the induced innovation hypothesis and its empirical relevance is presented in the last chapter. It is possible to measure biases of efficiency gains without knowing the sources of the biases. Therefore, Chapters 2 to 5 deal with the measurement problem alone. In particular, in Chapter 2 we present definitions of biases, discuss the key problems of measurement and review Solow's and Sato's work in this area. Chapter 3 is devoted to the derivation of a method to measure biases for production processes with more than two factors. This method is then applied to the agricultural sectors of Japan from 1883 to 1962 and the United States from 1912 to 1968 in Chapters 4 and 5.

The agricultural sector was chosen because there was little change in its product, because sufficient historical data were available and because a many-factor case may provide more information on the causes of biases than a two-factor case.

CHAPTER 2
DEFINITIONS OF BIASES, FACTOR AUGMENTATION AND THE MEASUREMENT OF BIASES IN THE TWO-FACTOR CES CASE

This chapter first discusses the definition of Hicks neutrality and of biases in efficiency gains and the general problem of measuring biases. Then Solow's and Sato's approach to measuring biases with a slightly different definition of neutrality is reviewed. A discussion of the factor-augmenting hypothesis is included in that review.

The definitions of biases have been derived to deal with technical change problems. The presently available methods of measuring the biases, however, do not allow one to distinguish whether the biases have actually been due to technical change or to efficiency improvements caused by education, soil improvements, etc.

Technical change in some capital good may affect the efficiency of all cooperating factors and the same is true of a more educated labor force. No attempt is therefore made to attribute biases to any particular source of factor quality improvement and the term efficiency gain rather than technical change will be used.

Efficiency gains simply mean that the unit isoquant of a production process shifts closer to the origin. To characterize these shifts as to biases, a particular point on the isoquant has to be considered. Farrell (1957) has introduced the following useful distinction between economic and technical efficiency (see Figure 1).

Any point on the unit isoquant such as B or A is technically efficient, while C is not. For a given set of prices only one point is economically efficient; unit costs are only minimized at A. The theory of biased efficiency gains in the Hicksian sense asks how the economically

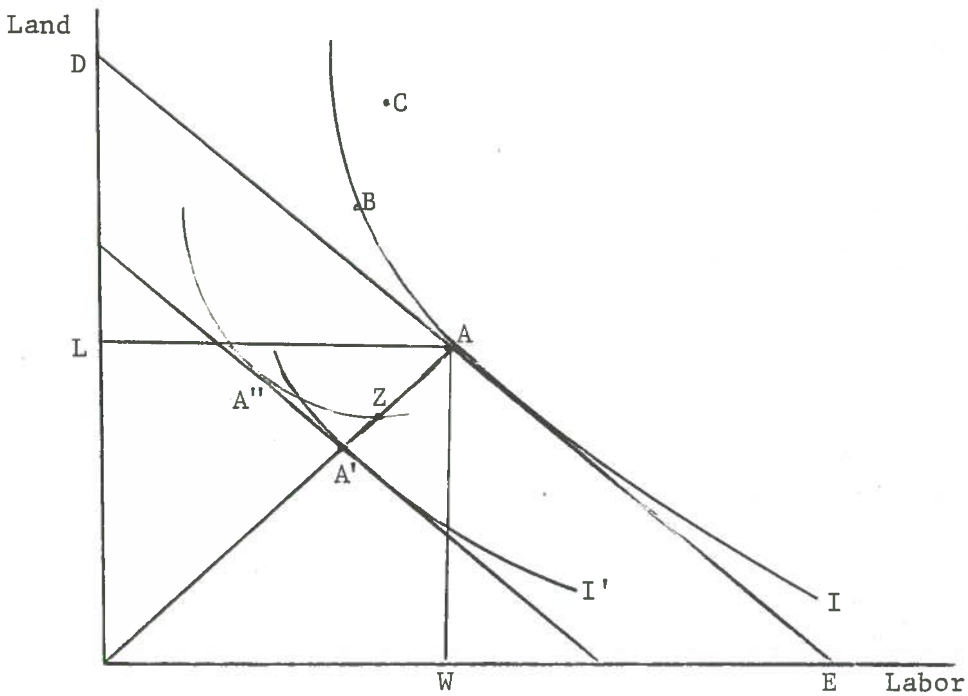

Figure 1. Technical and economic efficiency

efficient point moves inwards over time at constant factor price ratio. If it moves inwards along the ray OA to A', the efficiency gain is said to be Hicks neutral. If it moves to A", the change has been labor-saving. More specifically, efficiency gains are said to be lavor-saving, labor-neutral or labor-using depending on whether, at constant factor prices, the labor-land ratio decreases, stays constant or increases. This definition can be immediately transformed into a definition in terms of factor shares at constant factor prices.

Efficiency gains are labor-saving, labor-neutral or labor-using according to whether the labor share decreases, stays constant or increases at constant factor prices. This definition generalizes easily to the many-factor case and will lead to one single measure of the biases for each factor. If a definition in terms of the factor ratios were used it would be necessary to consider n - 1 factor ratios for each factor to determine biases. Therefore, the definition in terms of shares is used in the following chapters. The rate of the factor i bias is measured as:

$$B_i\Big|_{\text{relative factor prices}} = \frac{d\alpha_i^*}{dt} \cdot \frac{1}{\alpha_i} \lesseqgtr 0 \quad \text{Hicks} \begin{cases} \text{i-saving} \\ \text{i-neutral} \\ \text{i-using} \end{cases} \tag{1}$$

where α_i is the share of factor i in total costs.

To estimate biases it is, however, not possible to look simply at historic factor share changes. The observed share changes have come about through biased technical change and through ordinary factor substitution after changes in the prices of the factors. The basic problem is, therefore, to sort out to what extent the share changes have been due to biased technical change and to what extent to price changes. This can only be done, in a graphic sense, if the curvature of the isoquant is known. The substitution parameters of the production process have to be estimated before any biases can be measured. In the following chapters this will be done for the agricultural sector using a cost function to characterize the production process.

Solow and Sato have used a different definition of Hicks neutrality to derive measures of biases for the two-factor CES case. The basic problem is the same of splitting factor ratio changes into a part due to biased technical change and a part due to price changes. The remainder of the chapter discusses Solow's and Sato's approach and includes a discussion of the factor-augmenting hypothesis. Solow uses another definition of Hicks neutrality which does not differ, however, in substance from the earlier one.[1]

---

[1]For a review of different definitions of biases see Nadiri (1970).

Efficiency gains are Hicks neutral if, at constant factor ratio, the marginal rate of substitution remains unchanged. In Figure 1, I" is a labor-saving technical change because at the point Z (constant factor ratio) the marginal rate of substitution between land and labor has increased as compared to point A. The definition differs from the previous one not in substance but in what it holds constant. Using capital (K) and labor (L) instead of land and labor, Solow's definition can be written as follows:

$$\frac{d}{dt} \text{MRS}\Big|_{\frac{K}{L}} = \frac{d}{dt}\left(\frac{f_K}{f_L}\right) = -\frac{d}{dt}\left(\frac{dL}{dK}\right) \gtreqless 0 \longrightarrow \text{Hicks} \begin{cases} \text{labor-saving} \\ \text{neutral} \\ \text{labor-using} \end{cases} \qquad (2)$$

where f_K and f_L stand for the marginal products. This can be transformed into a measure of the bias (Solow, 1967) which is the proportional rate of change in the marginal rate of substitution.

$$Q\Big|_{\frac{K}{L}} = \frac{d}{dt} \log \frac{\partial F/\partial K}{\partial F/\partial L} = \frac{d}{dt} \left(\frac{f_K}{f_L}\right) \Big/ \left(\frac{f_K}{f_L}\right) \gtreqless 0 \longrightarrow \text{Hicks} \begin{cases} \text{labor-saving} \\ \text{neutral} \\ \text{labor-using} \end{cases} \qquad (3)$$

This definition has the advantage that there is one single measure in the two-factor case for both factors, while using (1) there will be two measures which will be different for the two factors.[2]

To measure Q, it is necessary to assume that technical change is factor-augmenting and to choose a particular form of a production function. A general production function can be specialized as follows:

---

[2]For the two-factor case one could write a single measure involving shares as follows:

$$D\Big|_{\frac{W}{T}} = \frac{d}{dt} \log \frac{\alpha_L}{\alpha_K} \lesseqgtr 0 \longrightarrow \text{Hicks} \begin{cases} \text{labor-saving} \\ \text{neutral} \\ \text{labor-using} \end{cases}$$

$$Y = f(X_1, X_2, \ldots, X_n, T) = \emptyset\ (A_1X_1, A_2X_2, \ldots, A_nX_n), \quad (4)$$

where X_i are factor flows per unit of time and T is a technology variable related to time. The As are augmentation or efficiency coefficients of the factors. Equiproportional increases of A_i and X_i have identical effects on output. The right-hand side of (4) is a specialization of the left-hand side. In it, efficiency changes may affect neither the form nor the parameters of the production function. Efficiency changes are treated as if they were simple changes in the "effective" quantity of a factor. If the true production function is Cobb-Douglas, it means that the exponents are constant over time and space. If it is CES, then the elasticity of substitution remains constant. If it is a more general production function in which the elasticity of substitution is variable, the parameters of the function (which also determine how the elasticity of substitution varies with factor input) must be constant. If in empirical investigation one is certain of the functional form used, a test of the constancy over time of the parameters of that particular function can be used to test the factor-augmenting hypothesis. This is also done in Chapter 4.

Invariance of the other parameters of the production function, except for the As, does not mean that countries are on the same production function, as is sometimes assumed in trade theory. The difference in the As is sufficient to place the unit isoquants of two countries in different positions in the factor space.

Although the effects on output of a change in A_i and in X_i are the same, there is an important difference in these variables. The As are functions of time (or technical change occurring over time). At a given moment of time their level cannot be altered by the entrepreneur. Hence, at a particular moment they are not influenced by factor prices [3] or output levels. The Xs can be observed over time and therefore have a time dimension, but they are the decision variables of the entrepreneur and depend on the ruling factor prices and the output level. In making the production decions, the entrepreneur need not know the As. What he observes are the marginal products of the Xs which reflect the efficiency of the inputs in natural units. He also observes the prices of the factors in natural units. This is all the information he needs for cost minimization at a particular moment of time. For him, the question of measurability of the As is immaterial. As the As change over time, this changes the marginal products of the factors in natural units, which forces him to adjust the factor quantities. Objections to factor-augmenting technical change therefore cannot be based on the fact that entrepreneurs do not know the levels of the As.

It is important to note that a particular efficiency change, say a new seed variety, affects not only the A level of the seeds but also the A levels of all cooperating factors. If it is possible to measure changes in the As for each factor, this does not mean that these changes can be identified with particular investments into the efficiency of the factor considered. This is the reason that, as of now, it is not possible to distinguish between biases due to technical change and biases due to other investments into factor efficiency.

---

[3] However, they have an influence on the current factor prices. Also, under the IIH they are influenced by the past history of the factor prices (or shares).

To measure biases, Solow, Drandakis and Phelps, and Sato assume a two-factor CES production function in augmenting form. As Drandakis and Phelps (1966) show, Q of (3) can then be written as follows:

$$Q \Big|_{\frac{K}{L}} = \frac{1-\sigma}{\sigma} \left(\dot{a}_L - \dot{a}_K\right), \tag{5}$$

where a = log A and the dot denotes the time derivative. Therefore, $\dot{a}$ is the proportional instantaneous rate of change over time in the level of A.

This equation shows the important fact that relatively labor-augmenting technical change need not be labor-saving. Three cases exist:

Cases: (1) $\sigma = 1$ Technical change is always neutral;

(2) $\sigma < 1$ Technical change is labor-saving if $\dot{a}_L > \dot{a}_K$, it is capital-saving if $\dot{a}_L < \dot{a}_K$; (6)

(3) $\sigma > 1$ Technical change is labor-saving if $\dot{a}_L < \dot{a}_K$, it is capital-saving if $\dot{a}_L > \dot{a}_K$.

That relatively labor-augmenting technical change ($\dot{a}_L > \dot{a}_K$) is labor-using for $\sigma > 1$ is explained as follows. The increase in efficiency of labor allows entrepreneurs to reduce the amount used. But with higher marginal product at a constant price, there is now an incentive to substitute labor for capital. The elasticity of substitution is sufficiently large that the incentive to use more labor due to its efficiency increase overrides the initial saving made possible by the efficiency increase.

To use (5) for measurement of the bias, $\dot{a}_L$ and $\dot{a}_K$ must either be known or mathematical expressions for them in terms of known variables have to be substituted into (5) Sato (1970) has derived such expressions and measured time series of augmentation coefficients for the U. S. manufacturing sector:

$$\dot{a}_i = \frac{\sigma \dot{w}_i - (\dot{y} - \dot{x}_i)}{\sigma - 1} \qquad \begin{array}{l} \sigma \neq 1 \\ i = 1, 2 \end{array} \tag{7}$$

where lower case letters with dots are logarithmic time derivatives and

W_i = wage of factor i,

Y = output, and

X_i = quantity of factor i.

Substituting into (5) leads to

$$Q\Big|_x = \frac{\dot{x}_K - \dot{x}_L}{\sigma} + (\dot{w}_K - \dot{w}_L) \gtreqless 0 \longrightarrow \text{Hicks} \begin{cases} \text{labor-saving} \\ \text{neutral} \\ \text{capital-saving} \end{cases} \tag{8}$$

This is the sum of the proportional change in relative factor prices and the proportional change in the factor ratio weighted by the elasticity of substitution.

Data for all the variables in (8) can be found, so that the biases can be measured, provided σ is known. While $Q|_x$ is measured in time series, σ should be obtained from cross-sectional studies because it is a measure of the extent of substitutability at a particular moment of time.

While Chapter 4 uses a many-factor framework and a different definition of neutrality, the basic approach to derive expressions of the biases will be the same. Again, cross-sectional measures of the cost function parameters have to be found before any bias can be measured.

CHAPTER 3
MEASUREMENT OF BIASED EFFICIENCY GAINS IN THE N-FACTOR CASE: THEORY

Any measurement of bias requires that one find out either what would have happened to factor shares or factor ratios, given constant factor prices, or how marginal rates of substitution would have changed at constant factor ratios. This involves the derivation of expressions for factor-augmenting series. In Appendix A, a theory is derived to do that for the general n-factor case and an arbitrary twice differentiable production function. Actual application of this theory nevertheless requires the choice of specific production or cost function.

The following functions were considered: CES, constant ratio of elasticity of substitution (Hanoch, 1970), generalized Leontief (Diewert, 1971) and the Transcendental Logarithmic (Translog) production and cost functions. The CES function was rejected because it implies that all Allen partial elasticities of substitution must be equal for all factor pairs, which is far too restrictive. A similar restrictiveness ruled out the constant ratio of elasticities of substitution function. Due to its quadratic form in square roots, the generalized Leontief function is extremely difficult to handle with factor augmentation. The functions which were both general enough and straightforward enough to apply were the Transcendental Logarithmic (Translog) production and cost functions (Christensen, Jorgenson and Lau).[4]

---

[4]L. R. Christensen, D. W. Jorgenson, and J. L. Lau. Conjugate duality and the transcendental logarithmic production function. Unpublished paper presented at the Second World Congress of the Econometric Society, Cambridge, England, September 1970.

The mathematics of the cost function are simpler than those of the production function. Its estimation equations satisfy some econometric assumptions better and the actual estimation of it was more successful than the production function estimation. The theory is therefore derived here in terms of the Translog cost function.

The Translog Case: Model A

Every production function has a minimum cost function as its dual. This function, which may not be expressible in closed form even though the production function is, relates factor prices to the cost of the output. Therefore, the cost function contains all the information about the production process which the production function contains.

A minimum per unit cost function with technical change can be specialized into factor-augmenting form analogous to (4)

$$U = f(W_1, W_2, \ldots, W_n, T) = \phi\left(\frac{W_1}{A_1}, \frac{W_2}{A_2}, \ldots, \frac{W_n}{A_n}\right), \tag{9}$$

where U is per unit cost, and W_i are the factor prices. The As are the same augmentation parameters as the ones of the dual production function. Equi-proportional changes in A_i and X_i have effects of equal magnitude but opposite sign on unit cost. As the MP of factor i is uniformly increased, more ot it is substituted for others in exactly the same way as if the price of it had fallen.

Let $R_i = \frac{W_i}{A_i}$ be the factor price in the augmented unit space. The Translog unit cost function can be written as

$$U = \nu_o \left[\prod_{i=1}^{n} R_i^{\nu_i}\right] \left[\prod_{i=1}^{n} R_i^{1/2 \sum_{j=1}^{n} \gamma_{ij} \ln R_j}\right]. \tag{10}$$

The part within the first brackets is a Cobb-Douglas function. If the cost function were Cobb-Douglas, the production function would be also (for proof, see Hanoch, 1970). We therefore can think of the terms in the second bracket as amendments to the Cobb-Douglas function which change the elasticities of substitution away from one. The function allows arbitrary and variable elasticities of substitution among factors.

The function is linear in logarithms:

$$\ln U = \ln \nu_o + \sum_i \nu_i \ln R_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln R_i \ln R_j \ . \qquad (11)$$

The function can be considered a functional form in its own right or regarded as a logarithmic Taylor series expansion to the second term around input prices of 1 or an arbitrary twice differentiable cost function (Christensen, _et al_.)[5] With the proper set of constraints on its parameters it can therefore be used as an approximation to any one of the known costs and production functions. As the simplest example, the constraints for Cobb-Douglas simply set all γ_{ij} equal to zero. The constraints for an approximation to the CES are more complicated according to Christensen, _et al_.

Since γ_{ij} are the values of the cross derivatives at input levels of one, they have to be symmetric or

$$\gamma_{ij} = \gamma_{ji} \ . \qquad (12)$$

Also, since it is a cost function, it has to satisfy the economic constraint of linear homogeneity, _i.e._, unit costs double when all factor prices double. This implies

---

[5] See footnote 4.

$$\sum_{i=1}^{n} \nu_i = 1 ,$$

$$\sum_{i=1}^{n} \gamma_{ij} = 0 , \tag{13}$$

$$\sum_{j=1}^{n} \gamma_{ij} = 0 .$$

The certainty of linear homogeneity in prices is a major advantage of using the cost function.

The Shepard Duality Theorem (Hanoch, 1970) gives one of the fundamental relationships between the cost and production function:

$$\frac{\partial U}{\partial W_i} = X_i . \tag{14}$$

The other duality relationship is the familiar

$$\frac{\partial Y}{\partial X_i} = \frac{W_i}{P} , \tag{15}$$

where P is the product price. In augmented units, (14) becomes

$$\frac{\partial U}{\partial R_i} = \frac{\partial U}{\partial W_i} \frac{dW_i}{dR_i} = A_i X_i = Z_i , \tag{16}$$

where the Z_i are the factor quantities in the augmented unit space. The first derivatives of the Translog function, with respect to the log of the factor prices, are equal to the shares

$$\frac{\partial \ln U}{\partial \ln R_i} = \frac{\partial U}{\partial R_i} \frac{R_i}{U} = \frac{Z_i R_i}{U} = \frac{A_i X_i (W_i/A_i)}{U} = \frac{W_i X_i}{U} = \alpha_i . \tag{17}$$

Taking these derivatives, we have

$$\alpha_i = \nu_i + \sum_j \gamma_{ij} \ln R_j \qquad i = 1, \ldots, n . \tag{18}$$

Differentiating (18) totally, we have

$$d\alpha_i = \sum_{j=1}^{n} \gamma_{ij} \, d\ln R_j \qquad i = 1, \ldots, n . \tag{19}$$

The proportional (log) change of a ratio is the difference of the proportional changes of its numerator and denominator.

$$d\alpha_i = \sum_{j=1}^{n} \gamma_{ij} \, (d\ln W_j - d\ln A_j) \qquad i = 1, \ldots, n. \tag{20}$$

Separating terms and using matrices

$$\begin{bmatrix} d\alpha_1 \\ \vdots \\ d\alpha_n \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nn} \end{bmatrix} \begin{bmatrix} d\ln W_1 \\ \vdots \\ d\ln W_n \end{bmatrix} - \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nn} \end{bmatrix} \begin{bmatrix} d\ln A_1 \\ \vdots \\ d\ln A_n \end{bmatrix} ,$$

$$\text{or } d\alpha = \gamma d\ln W - \gamma d \ln A. \tag{21}$$

γ is not of full rank due to the homogeneity constraint. But calling an arbitrary factor the n'th factor

$$\gamma_{in} = -\sum_{i=1}^{n-1} \gamma_{ij} . \tag{22}$$

Using (22) to remove γ_{in} from (21), we have

$$d\alpha_i = \sum_{j=1}^{n-1} \gamma_{ij} dw_j - \sum_{j=1}^{n-1} \gamma_{ij} \, da_j , \tag{23}$$

where $dw_j = d\ln W_j - d\ln W_n = d\ln \left(\frac{W_j}{W_n}\right)$

and $da_j = d\ln A_j - d\ln A_n = d\ln \left(\frac{A_j}{A_n}\right)$.

The notation is chosen so as to leave apparent the analogy with Appendix A. Let Γ be the truncated (n - 1) x (n - 1) matrix of the γ_{ij} which is of full rank. Then

$$d\alpha_{(n-1)x1} = \Gamma\ dw - \Gamma\ da\ , \tag{24}$$

which gives us the solution for the changes in the A ratios

$$da = dw - \Gamma^{-1}\ d\alpha\ . \tag{25}$$

With the discrete time equivalent of (25), time series of the augmentation series can be estimated provided reliable estimates of the Γ^{-1} matrix are available. Going one step further, the share changes which would have occurred had factor prices remained constant, can be estimated directly. They are the share changes needed to estimate the biases according to Equation (1). Call these changes $d\alpha^*$, which can be obtained from system (24) by setting dw = 0. Then

$$d\alpha^* = -\ \Gamma da. \tag{26}$$

And substituting da from (25),

$$d\alpha^* = d\alpha - \Gamma dw\ . \tag{27}$$

According to (1), we can immediately judge the nature of technical change for factors i-1, ..., n-1. Also, since

$$\sum_{i=1}^{n} d\alpha_i^* = 0\ ,$$

we have the solution for the n'th factor

$$d\alpha_n^* = - \sum_{i=1}^{n-1} d\alpha_i^* . \tag{28}$$

Equation (27) has a nice simplicity to it. To find out what the factor share changes would have been had factor prices remained constant, simply subtrace from the observed factor share changes that part which was caused by changing factor price ratios. The matrix contains the information by how much the changes in factor price ratios alone could have altered shares. A completely hueristic derivation of (27) is also possible:

Writing (20) without technical change, i.e., with all dln A = 0, we have:

$$d\alpha_i = \sum_j \gamma_{ij} \text{ dln } W_j \qquad i = 1, \ldots, n. \tag{29}$$

This is the share's change due to factor price changes alone. Now suppose that technical change over time alters the share by $d\alpha_i^*$, defined earlier as the change which would have occurred without factor price changes.

Then we could rewrite (29) for technical change.

$$d\alpha_i = \sum_j \gamma_{ij} \text{ dln } W_j + d\alpha_i^* , \tag{30}$$

where $d\alpha_i$ is the observed change in shares and dln W_j are the observed factor price changes. Converting to full rank as above, we then have

$$d\alpha = \Gamma dw + d\alpha^*$$

or $$d\alpha^* = d\alpha - \Gamma dw ,$$

which is the same as (27). This derivation depends also on the factor-augmenting hypothesis because the Γ matrix must be constant.

Before (27) can be used with time series data, the coefficients of the Γ matrix have to be estimated. This must be done with cross-section data where, ideally, all units are on exactly the same production function. We can then assume that all A_i are equal to one for all units and rewrite Equation (18):

$$\alpha_i = \nu_i + \sum_j \gamma_{ij} \ln W_j + \varepsilon_i \qquad i = 1, \ldots, n \qquad (31)$$

and use this system of equations to estimate the γ_{ij} coefficients. Of course, one will never find a cross section where all units are on exactly the same production function. Ways to deal with this problem are discussed in Chapter 4.

The Translog Case: Model B

Model A assumes that the rate of biases is not constant over time. This, of course, is the proper assumption if induced innovation is to be investigated over longer periods of time. For shorter time periods it is, however, possible to assume that the biases are constant. If this is done, biased technical change at constant exogenous rates can be introduced in the Translog cost function in a way similar to the way Christensen *et al.*[6] introduced it into the corresponding production function

$$\ln U = \ln \nu_o + \sum_i \nu_i \ln W_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln W_i \ln W_j$$
$$+ \nu_t \ln t + \sum_i \omega_i \ln W_i \ln t + \omega_t (\ln t)^2 , \qquad (32)$$

where t stands for time.

---

[6] See footnote 4.

Upon differentiation, the share equations become:

$$\frac{\partial \ln U}{\partial \ln W_i} = \alpha_i = \nu_i + \sum_j \gamma_{ij} \ln W_j + \omega_i \ln t \ . \qquad (33)$$

This is the estimation Equation (31) with time entering as a variable. ω_i is the constant exogenous rate of factor i bias. If (33) is used as a regression equation with a time series or a combination of cross section and time series, the introduction of time in this way will ensure that biased technical change at constant rates will not bias the estimates of the γ_{ij}. Furthermore, the coefficients $\hat{\omega}_i$ can be used to derive another set of price-corrected shares series, say $d\alpha_i^{**}$, which can be used with Equation (1) to estimate the biases for the particular period:

$$d\alpha_i^{**} = \hat{\omega}_i \, d\ln t \qquad i = 1, \ldots, n \ . \qquad (34)$$

Of course, this model cannot be used to extrapolate outside of the short regression period because then the assumption of a constant exogenous rate of bias is tenuous. However, the consistency of the $d\alpha_i^{**}$ series for the period in which both models can be used will be of great value to assess the quality of the series derived.

Factor Demand Elasticities and Elasticities of Substitution in the Translog Cost Function

The following relationships exist between the parameters of the Translog cost function, the factor demand elasticities, and the Allen elasticities of substitution (see Appendix Equation (A.23)) of the implied production function:

$$\sigma_{ij} = \frac{1}{\alpha_i \alpha_j} \gamma_{ij} + 1 \qquad \text{for all } i \neq j \, , \tag{35}$$

$$\sigma_{ii} = \frac{1}{\alpha_i^2} (\gamma_{ii} + \alpha_i^2 - \alpha_i) \qquad \text{for all } i \, , \tag{36}$$

$$\eta_{ij} = \frac{\gamma_{ij}}{\alpha_i} + \alpha_j \qquad \text{for all } i \neq j \, , \tag{37}$$

$$\eta_{ii} = \frac{\gamma_{ii}}{\alpha_i} + \alpha_i - 1 \qquad \text{for all } i \, . \tag{38}$$

These relationships are needed to judge the estimated coefficients of the Translog production function since $\gamma_{ij} = \frac{\partial^2 \ln U}{\partial \ln R_i \, \partial \ln R_j}$ has no direct economic meaning. They show that the γ_{ij} act as modifiers of the elasticity of substitution away from one. In the Cobb-Douglas case, all γ_{ij} are zero. The formulae also show elasticities of substitution and factor demand elasticities as functions of the factor shares. Proof of (35) and (36):

$$\gamma_{ij} = \frac{\partial^2 \ln U}{\partial \ln W_i \, \partial \ln W_j} = \frac{\partial}{\partial \ln W_j} \left(\frac{\partial U}{\partial W_i} \cdot \frac{W_i}{U} \right)$$

$$= W_j \frac{\partial}{\partial W_j} \left(\frac{\partial U}{\partial W_i} \cdot \frac{W_i}{U} \right)$$

$$= W_j \left(\frac{\partial^2 U}{\partial W_i \partial W_j} \cdot \frac{W_i}{U} - \frac{W_i}{U^2} \frac{\partial U}{\partial W_j} \frac{\partial U}{\partial W_i} \right) . \tag{39}$$

By (14) therefore,

$$\gamma_{ij} = \frac{W_i W_j}{U} \frac{\partial^2 U}{\partial W_i \partial W_j} - \frac{W_i W_j}{U^2} X_i X_j$$

$$\frac{\partial^2 U}{\partial W_i \partial W_j} = \frac{U}{W_i W_j} (\gamma_{ij} + \alpha_i \alpha_j) \quad .$$

Again by (14),

$$\frac{\partial^2 U}{\partial W_i \partial W_j} = \frac{\partial X_i}{\partial W_j} \quad ,$$

and the factor demand elasticity therefore is:

$$\eta_{ij} = \frac{\partial X_i}{\partial W_j} \cdot \frac{W_j}{X_i} = \frac{1}{\alpha_i} (\gamma_{ij} + \alpha_i \alpha_j) \quad .$$

This proves (37). By (A.27),

$$\eta_{ij} = \alpha_j \sigma_{ij} \tag{40}$$

so that

$$\sigma_{ij} = \frac{1}{\alpha_i \alpha_j} \gamma_{ij} + 1 \quad . \qquad \text{Q.E.D.}$$

The proof for σ_{ii} is similar except that in Equation (39) an additional term is introduced because $\frac{\partial P_i}{\partial P_i} = 1$. This accounts for the $- \alpha_i$ in (36).

Both the elasticities of substitution and the elasticities of factor demand are variables and depend on factor shares. They will be computed from the cross-sectional estimates using average factor shares.

CHAPTER 4
CROSS-SECTIONAL ESTIMATION OF THE PARAMETERS OF THE COST FUNCTION

Data

The cross-sectional estimation of the cost function was done with state data from the United States. Japanese cross-section data were not gathered. If the factor-augmenting hypothesis holds, then the γ_{ij} parameters are the same for the United States and Japan and the U. S. γ_{ij} parameters also hold for Japan.

Four sets of cross-section data were obtained for 39 states or groups of states. The cross sections were derived from census and other agricultural statistics for the years 1949, 1954, 1959, and 1964. The combination of cross sections over time poses problems which are discussed in the section on error specification.

The time series component is important because it allows a test of the factor-augmenting hypothesis (constancy of coefficients over time) and the estimation of biases by method B, by measuring the ω_i coefficients. Discussion of these two aspects is done towards the end of this chapter.

The data are discussed in detail in Appendix D. In general, Griliches' (1964) definitions of factors were used. He distinguishes the following five factors: land (D), labor (L), machinery (M), fertilizer (F) and all others (O). Intermediate inputs are included in this list and the function fitted corresponds to a gross output function rather than a value added function.[7]

---

[7]The conditions for fitting a value added function rather than an output function are rather restrictive. Ethier (1971) has shown that

Most of the data come from published USDA sources. Expenditures on factors usually are actual expenditures and, where applicable, imputed expenditures for wages of family members, interest charges, depreciation and taxes. Quantity data are derived as price weighted indices of physical units (land and fertilizer) or the sum of individually deflated expenditures (all other) or a combination of these methods (machinery and labor). The quantity data were already computed in Fishelson (1968), who used Griliches' (1964) data with slight changes. Expenditure and quantity data are consistent with each other. The price data were obtained by dividing the expenditure data by the quantities.

Any proportionality errors in the data will affect the estimates of the ν_i but not of the γ_{ij} in Equation (31). No attention will be given to the ν_i estimates because such proportionality errors exist at least in the wage rate statistics which seem to have a downward bias in recent periods (personal communication, USDA). Other types of data errors are neglected.

Neutral Efficiency Differences among States

Cross-sectional efficiency differences among the states would lead to biased results. This problem is discussed as "management bias" by Griliches (1957). Timmer (1970), among others, discusses a method to handle this in the Cobb-Douglas case. A separate intercept is specified for each firm or state. Applied to the Translog function this means:

"value added" and intermediate inputs must be combined in fixed proportion to obtain correct results with a value added function. This condition is not met in agriculture.

$$\ln U = \ln \nu_0 + \ln \delta_\ell + \sum_i \nu_i \ln W_{i\ell} + \sum_i \sum_j \gamma_{ij} \ln W_{i\ell} \ln W_{j\ell} , \qquad (41)$$

where ℓ refers to the state number ℓ. Of course, this equation is econometrically underidentified and cannot be estimated by least squares methods. But if we take first derivatives, all intercepts drop out and we again obtain the Equations (29).

$$\alpha_i = \nu_i + \sum_j \gamma_{ij} \ln W_{j\ell} \qquad i = 1, \ldots, n,$$

which are all fully identified. In the same way, we can introduce different time intercepts into (41) which will leave (31) unaffected.

The fact that Translog cost functions have the same estimation equations, (31), even if their intercepts differ is an advantage. It takes care of the problem of "management bias" which occurs in production or cost function fitting when variables such as research and development or education are left out of the model but affect the efficiency of all included factors equally or neutrally. The left out factors then act as shifters of the production function or cost function and can be expressed as differences in the intercept of the production or cost function.

Treatment of Non-neutral Efficiency Differences among Groups of States

Adding an intercept term for each state in each of the Equations (31) would, of course, take care of any non-neutral efficiency differences. But then, the model is, of course, underidentified.

It was therefore decided to add dummies for groups of states into (31) if this would improve the specification of the model (as discussed in the section on specification). The cost function corresponding to (31) with regional dummies is obtained by integration.

$$\ln U = \ln \nu_o + \sum_i \nu_i \ln W_i + \sum_i \sum_j \gamma_{ij} \ln W_i \ln W_j + \sum_i \sum_k \delta_{ik} d_k \ln W_k \qquad (42)$$

where d_k is the dummy corresponding to the group of states k.

Non-neutral efficiency differences among groups of states might arise because of (a) the non-neutral effect of educational differences, (b) non-neutral climatic and soil effects, or (c) apparent non-neutral effects arising from differences in product groups. Five groups of states were distinguished (for a specific list, see Appendix D).

Treatment of Non-neutral Efficiency Gains Occurring over Time

If the γ_{ij} are estimated in a time series using Equation (31), these estimates will be biased if biased efficiency gains occur at the same time. If the period is short and the biases in efficiency gains occur at constant rates, one can include time variables in the estimation equations as in Model B and use the following estimation equations:

$$\alpha_{it} = \nu_i + \sum_j \gamma_{ij} \ln W_{jt} + \omega_i \ln t + \varepsilon_i . \qquad (43)$$

This will at the same time remove the source of bias in the $\hat{\gamma}_{ij}$ and provide regression estimated $\hat{\omega}_i$ for each factor during the period of the data used to fit the equation.

The final estimation of the γ_{ij} is done using all four sets of cross-section data combined. Therefore, time will be included as a variable.

These discussions show that problems of management and education variables and of technical change can be handled in a simple way with the

Translog cost function. All this would apply as well to the Translog production function.

Error Specification

Within each of the four cross sections the error terms of the estimation equations for the five shares are interdependent because, for each state, the same variables which might influence the factor shares in addition to the prices are left out of all share equations. This violates the condition of error independence which would have to hold if ordinary least squares (OLS) methods were to be used for each equation. The error structure for the shares i and m and the states k and ℓ within each cross section is as follows (neglecting all dummies):

$$\alpha_{ik} = \nu_i + \sum_j \gamma_{ij} \ln W_{jk} + \varepsilon_{ik}$$

$$\alpha_{m\ell} = \nu_m + \sum_j \gamma_{mj} \ln W_{j\ell} + \varepsilon_{m\ell} \tag{44}$$

$$E(\varepsilon_{ik}) = E(\varepsilon_{m\ell}) = 0$$

$$E(\varepsilon_{ik}\varepsilon_{m\ell}) = \begin{cases} \sigma^2 & \text{if } j{=}m \text{ and } k{=}\ell \\ 0 & \text{if } j{=}m \text{ and } k{\neq}\ell \\ \omega_{im} & \text{if } j{\neq}m \text{ and } k{=}\ell \\ 0 & \text{if } j{\neq}m \text{ and } k{\neq}\ell \end{cases} \tag{45}$$

However, the explanatory variables in (44) are the same variables with the same values for both equations. As long as no constraints across equations are imposed on (44), OLS and GLS estimators are the same. This is no longer true as soon as restrictions across equations are imposed. GLS estimators are more efficient than OLS estimators (Theil, 1971, re consumer allocation problem). Therefore, restricted generalized least squares estimates were used (Zellner, 1962, 1963).[8]

If more than one cross section is used, there is an additional error interdependence over time for the errors of the same shares equation. Taking share i in two time periods, say t and s, with the states again denoted by k and ℓ, the error structure is as follows (neglecting all dummies and the time variable):

$$\alpha_{itk} = \nu_0 + \sum_j \gamma_{ij} \ln W_{jtk} + \varepsilon_{itk}$$
$$\alpha_{is\ell} = \nu_0 + \sum_j \gamma_{ij} \ln W_{js\ell} + \varepsilon_{is\ell} \qquad (46)$$

then

$$E(\varepsilon_{itk}) = 0$$
$$E(\varepsilon_{itk}\varepsilon_{is\ell}) = \begin{cases} 0 & \text{if } k \neq \ell \\ \sigma^2 & \text{if } k=\ell \text{ and } s=t \\ \sigma^2 \rho^{(s-t)} & \text{if } k=\ell \text{ and } s \neq t \end{cases} \qquad (47)$$

where ρ is the autocorrelation coefficient and (s-t) the number of years between the two cross sections.

The correct way of handling both the interdependence within a cross section and the interdependence over time when cross sections are pooled would be to use a GLS model and specify a separate equation for each

---

[8]The computer program used was: Triangle Universities Computing Center: Two and Three Stage Least Squares, Research Triangle Park, North Carolina, 1972. This program (henceforth called TTLS) only computes F-statistics for constraints of the form $R\beta = 0$. Since the constant returns for scale constraint is of the form $R\beta = \delta$, it could not be tested. Also, it would involve the five factor shares when equations for only four shares could be estimated due to singularity problems.

share in each year; then test and impose the symmetry and homogeneity constraints and the constraints that the γ_{ij} parameters are constant over time. This would have led to a 16-equation model with over 70 constraints This by far exceeded the capacity of the TTLS program. The correct procedure would also have required that one could impose constraints of equality of the autocorrelation coefficients over time on the estimated variance covariance matrix (see Appendix B), which was not possible with TTLS.

The following procedure was therefore adopted: (1) Hypotheses not involving time concerning the γ_{ij} of the same or different shares equations were tested in models containing four shares equations using data from a single cross section only. These tests were also used to decide in which equation dummies should be included. (2) The testing of the constancy of the γ_{ij} parameters over time was done in models containing a separate equation for the same share for each cross section included. As an example, to test whether the γ_{ij} parameters of the labor share equation were constant over time, a two-equation GLS model containing a labor share equation for the 1949 data set and a labor share equation for the 1959 data set was estimated and a test performed to ascertain whether the regression coefficients in the two equations were the same. (3) The actual estimation of the γ_{ij} coefficients was done with a data set which contained all four cross sections using a constrained GLS model containing four shares equations. This procedure took into account error interdependence among share equations but not error interdependence over time. While the resulting estimates are unbiased, they will not be most efficient ones. No tests are performed with this regression and the t-ratios of the estimates are probably overstated.

It was decided to neglect the problem of possible heteroscedastic disturbances for each share within each cross section. This could arise if the data to which the equations were fitted were averages of series with different numbers of observations in each state. Since the data come from various sources with different estimation procedures, where for each variable different numbers of observations may be involved, the data cannot be adjusted by a unique number.

Choice of Specification to Be Used for Estimating Purposes

The system to be estimated contains equations for five expenditure shares. But only four of these equations are linearly independent. So one equation has to be dropped. In a statistical sample the estimated coefficients will differ depending upon which equation is dropped. Another question to be decided is in which equations are regional dummies to be included; *i.e.*, for which factor should one allow biased regional efficiency differences. Theoretical reasoning does not help much to make the above choices. Neither should the estimates of the coefficients be used as guides. Significance tests were therefore used despite the fact that using such tests as criteria might lead to sequential estimation problems (Wallace and Ashar, 1972). In the absence of theoretical justification, they are the only information available. A specification was sought which would best satisfy the symmetry and homogeneity constraints of the cost function; *i.e.*, lead to the smallest relative increase in error sums of squares when imposed. The following specifications were considered (Table 1).

Table 1. List of different specifications tried

Specification	Share equations included simultaneously	Regional dummies in following equations
I	Land, labor, machinery, fertilizer	In no one
II	Land, labor, machinery, other inputs	In all
III	Land, labor, machinery, fertilizer	In fertilizer only
IV	Land, labor, machinery, fertilizer	In all

An initial idea of the influence of regional dummies, which differentiate the specifications, and of the homogeneity constraint can be seen by looking at the R^2 of single equations fitted individually by OLS in the cross section 1949 and 1964 (Table 2). The R^2s are not very high and increase when dummies are added. They do not decrease strongly when the homogeneity constraint is imposed. Very high R^2s were not expected.

Table 2. Single-equation R^2 of the cost function estimation equations

Equation	Land	Labor	Machinery	Fertilizer	Other
1949 data					
No regional dummies	.52	.57	.61	.66	.52
With regional dummies	.71	.64	.70	.88	.61
With regional dummies and homogeneity constraint	.71	.63	.66	.87	-[a]
1964 data					
No regional dummies	.41	.57	.30	-	-
With regional dummies	.56	.64	.51	.69	-
With regional dummies and homogeneity constraint	.54	.64	.51	.65	-

[a]Equation not fitted.

The increase of the R^2 is large enough to indicate that non-neutral efficiency differences are present with respect to all factors which is a first indication that either Specification II or IV might be preferable. Of course, the R^2 declines when the homogeneity constraints are imposed, but the decline is small.

Multicollinearity is no problem in any of the equations. The correlation coefficients among the independent variables are quite low, as one would expect for factor prices. The absence of multicollinearity is an additional advantage of cost function fitting over production function fitting.

To show the extent of interdependence between the errors of different shares equations within each cross section, Table 3 reproduces the estimated correlation coefficients of residuals for Specification IV

Table 3. Contemporaneous residual correlation coefficients, Specification IV

Equation	Labor	Land	Fertilizer	Other
1949 data with regional dummies in all equations				
Land	-.2854	.3095	-.1887	-.3463
Labor		.0449	-.0073	-.7242
Machinery			-.2598	-.5032
Fertilizer				n.a.[a]
1964 data with regional dummies in all equations				
Land	-.2051	-.0045	.1054	n.a.
Labor		-.0090	-.0570	n.a.
Machinery			-.1795	n.a.
Fertilizer				n.a.

[a] n.a. = not available, models not fitted.

in the 1949 and 1964 data sets. The correlation coefficients are the ones which are used for estimating the variance-covariance matrix in the GLS procedure. They are not as high as expected. For the other three specifications and other years they were of about the same magnitude, but they were not very stable for different cross sections.

On the other hand, error correlation over time is quite important. Correlation coefficients of residuals of the same factor share equation fitted in two different cross sections are given in Table 4. If the residual correlation came from an autocorrelation model, these correlation coefficients would imply one-year error correlation coefficients of about .98.

Table 4. Intertemporal residual correlation coefficients, Specification IV

Equation	Land	Labor	Machinery	Fertilizer
1949 and 1964 data sets with regional dummies in all equations	.87	.78	.62	.75

As explained above, the final decision on specification was made by considering the relative increases in the error sums of squares when symmetry and homogeneity constraints are imposed. The higher the relative increase in SSE, the higher the F-levels of these constraints. Table 5 shows the results.

At first these tests were only done in the 1949 cross section. Since in the 1949 data set the imposition of the homogeneity constraint and the symmetry constraint increased the weighted error sums of squares, the least in Specification III (smallest F-values), this specification was

Table 5. Test results for choice of specification

Specification	Equations included simultaneously	Regional dummies	Data set	Symmetry constraint	Homogeneity constraint	Cobb-Douglas constraint
	Specification description			F-values[a]		
I	Land, labor, machinery, fertilizer	None	1949	1.42[b]	1.82[c]	8.76[d]
II	Land, labor, machinery, other	In all	1949	1.98[e]	-	3.60
III	Land, labor, machinery, fertilizer	In fertilizer only	1949	.92[g]	.68[h]	7.17[i]
			1954	1.85	.81	-
			1959	4.47	1.34	-
			1964	8.68	2.21	-
IV	Land, labor machinery, fertilizer	In all	1949	1.38[e]	1.58	3.60[f]
			1954	1.14	.46	-
			1959	1.67	1.89	-
			1964	4.19	1.49	-

[a]The significance level of the numerator is taken as 120 for all cases since the F-value is not available for the other numbers. Small changes in the numerator at this number of degrees of freedom have almost no influence on the F-values.

[b]6 and 132 degrees of freedom; .05 significance level ≈ 2.17; .01 significance level ≈ 2.96.

[c]4 and 132 degrees of freedom; .05 significance level ≈ 2.45; .01 significance level ≈ 3.48.

[d]20 and 132 degrees of freedom; .05 significance level ≈ 1.66; .01 significance level ≈ 2.03.

[e]6 and 116 degrees of freedom; .05 significance level ≈ 2.17; .01 significance level ≈ 2.96.

Table 5 (continued)

[f]20 and 116 degrees of freedom; .05 significance level ≈ 1.66; .01 significance level ≈ 2.03.

[g]6 and 128 degrees of freedom; .05 significance level ≈ 2.17; .01 significance level ≈ 2.96.

[h]4 and 128 degrees of freedom; .05 significance level ≈ 2.45; .01 significance level ≈ 3.48.

[i]20 and 128 degrees of freedom; .05 significance level ≈ 1.66; .01 significance level ≈ 2.03.

applied to all data sets. Then the symmetry constraint was, however, rejected in the data sets 1959 and 1964.

Considering all data sets, Specification IV appears to be superior. On average, the homogeneity constraint increases the weighted error sums of squares the least in Specification IV. This is also the case for the symmetry constraint although it is still rejected in the 1964 data set. This specification was therefore selected for estimation purposes.

It should be noted that acceptance of the homogeneity constraint for the cost function does not imply that the production function is homogeneous of degree 1. Even for a nonhomogeneous production function, the unit cost function must be homogeneous of degree 1 in factor prices. If the production function exhibits returns to scale, the unit cost function should properly include Y as an argument. As long as Y enters just as an additive term in logarithms in the unit cost function (Equation (22)), it will, however, drop out when taking derivatives so that it will leave all estimation equations unaffected. Non-neutral scale effects are not considered.

Before reporting on the estimates, the other test results are examined. The Cobb-Douglas constraint was only tested with the 1949 data set. It is clearly rejected in each model tried. This, of course, could be expected from the R^2 of the single equations because the Cobb-Douglas constraint simply sets all coefficients equal to zero. It could only be accepted if the equations had no explanatory power at all. Since the cost function is not Cobb-Douglas, neither is the production function (see also p. 14).

The Estimates of the γ_{ij} Coefficients

For estimation purposes the four cross sections were combined as explained previously. Therefore, no account was taken of error interdependence over time. The t-ratios reported are therefore overstated to some extent and can at best give indications of the relative magnitude of the true t-ratios.

Specification IV is used for estimation pruposes. The four equations are estimated jointly using restricted generalized least squares, with the homogeneity constraint and the symmetry constraint imposed (see Appendix B for details). Regional dummies are added in each factor share equation. Time is included as a variable in natural logs with 1948 as year 1 and 1964 as year 16 (see Equation (45)).

Table 6 reports the regression results. A nonzero γ_{ij} implies that the corresponding partial elasticity of substitution is not 1. Therefore, a $\gamma_{ij} = 0$ is not a "bad" result and implies substantial price influence on factor use. If γ_{ij} is less than zero, the elasticity of substitution is less than 1, and vice versa. A better judgment of the estimates of the γ_{ij} is possible from the implied elasticities of

Table 6. Restricted estimates of the coefficients of the Translog cost function and t-ratios[a]

Equation	Variable										
	Land	Labor	Machinery	Ferti-lizer	Ln year	Inter-cept	MN	GR	SE	GS[c]	Other[b]
Land	.07747 (6.02)	-.03613 (3.25)	.00478 (.47)	.01066 (2.14)	.00847 (1.47)	.2603 (9.96)	-.1021 (10.2)	-.0394 (4.1)	-.1073 (8.9)	-.0577 (4.7)	-.05678
Labor		-.06367 (3.67)	-.00661 (.59)	-.02805 (4.97)	-.05482 (9.08)	.5218 (14.91)	.0194 (1.63)	-.0016 (.15)	.0169 (1.09)	.0246 (1.63)	.13446
Machinery	(Symmetric)		-.03485 (1.31)	-.00877 (.97)	.02498 (4.66)	.0926 (3.46)	-.0033 (.41)	-.0369 (5.08)	-.0186 (1.86)	.0072 (.73)	.04545
Fertilizer				.00068 (.12)	.00178 (.63)	.0745 (5.6)	.0104 (2.5)	-.0041 (1.10)	.0370 (7.24)	-.00247 (.49)	.02548
Other[b]											-.14861

[a]Critical values with 578 degrees of freedom are t.05 = 1.96 and t.01 = 1.65. T-ratios may be overstated due to error interdependence over time.

[b]Compiled using the homogeneity constraint, not estimated.

[c]MN, GR, SE, GS are dummies for mixed northern agriculture, grain farming states, Southeast, and Gulf states, respectively. The intercept stands for Western states and the coefficients of MN, GR, SE, GS are deviations from this intercept. For the states included in each group, see Appendix D Table 1.

factor demand and elasticities of substitution (Tables 7 and 8). They are computed according to Equations (35), (36), (37) and (38) using the unweighted average factor shares of the 39 states in the period 1949-1964. The elasticities of input demand are most useful to judge the diagonal elements of the γ matrix while the off-diagonal elements are better judged from the elasticities of substitution.

All own demand elasticities have the correct sign. The demand for land appears very inelastic. The demand elasticities for machinery and other inputs are larger than 1, a fact to keep in mind for the time series analysis since it implies that a rise in the corresponding prices will, other things equal, lead to a fall in the factor share. The lower part of Table 7 shows the values of the demand elasticities if the function was Cobb-Douglas and the actual factor shares were used as estimates of its coefficients. Negative elasticities of substitution imply that the two factors are complements.[9]

The closest substitute of land is fertilizer, as one might expect. Machinery also appears to be a good substitute for land, while labor is not. Other inputs appear to combine with land in almost fixed proportions. Labor's best substitute seems to be other inputs, and not machinery, as initially expected. Considering that other inputs contain all intermdeiate inputs and outside services, the strong substitutability becomes more plausible, as intermediate inputs substitute for inputs produced with the use of labor on the farm itself. (Note that

---

[9]Own elasticities of substitution have little economic meaning. They are simply transforms of the factor demand elasticities, which explains why they cannot be infinitely large. They obey the following adding up constraint: $\sum_j \alpha_j \sigma_{ij} = 0$ (Allen, 1938).

Table 7. Factor demand and cross-demand elasticities[a] implied in the estimated γ_{ij} and the standard errors around their value in the Cobb-Douglas case[b]

Equation	Land	Labor	Machinery	Fertilizer	Other
Estimated Translog values[c,d]					
Land	-.3356 (.09)	.0613 (.07)	.1792 (.07)	.1062 (.03)	-.0112
Labor	.0308 (.04)	-.9109 (.06)	.1256 (.04)	-.0577 (.02)	.8122
Machinery	.1833 (.07)	.2560 (.08)	-1.0886 (.18)	-.0239 (.06)	.6733
Fertilizer	.4506 (.10)	-.4878 (.20)	-.0991 (.30)	-.9452 (.16)	1.0815
Other	-.0046	.6690	.2720	.1053	-1.0417
Cobb-Douglas values for comparison[e]					
Land	-.8491	.3008	.1475	.0356	.3652
Labor	.1509	-.6992	.1475	.0356	.3652
Machinery	.1509	.3008	-.8525	.0356	.3652
Fertilizer	.1509	.3008	.1475	-.9644	.3652
Other	.1509	.3008	.1475	.0356	-.6348

[a]Each element in the table is the elasticity of demand for the input in the row after a price change of the input in the column. These elasticities are not symmetric.

[b]The shares used are the same as the Cobb-Douglas η_{ij}.

[c]$SE(\eta_{ij}) = \frac{SE(\gamma_{ij})}{\alpha_i}$.

[d]$\eta_{ij} = \frac{\gamma_{ij}}{\alpha_i} + \alpha_j$, $\eta_{ii} = \frac{\gamma_{ii}}{\alpha_i} + \alpha_i - 1$.

[e]$\eta_{ij} = \alpha_j$, $\eta_{ii} = \alpha_i - 1$.

Table 8. Estimates of the partial elasticities of substitution and standard errors around 1.0[a]

Factor	Land	Labor	Machinery	Ferti-lizer	Other
Land	-2.225	.204	1.215	2.987	-.031
Labor		-3.028	.851	-1.622	2.224
Machinery			-7.379	-.672	1.844
Fertilizer	(Symmetric)			-26.573	2.961
Other					-2.852

[a] $\sigma_{ij} = \frac{\gamma_{ij}}{\alpha_i \alpha_j} + 1$, $\sigma_{ii} = \frac{1}{\alpha_i^2} (\gamma_{ii} + \alpha_i^2 - \alpha_i)$. The elasticities of substitution are symmetric.

intermediate inputs produced and consumed on the same farm are included in neither input nor output statistics.) Also the substitutability between labor and machinery is still quite high.

The complementary relationships of fertilizer with labor and machinery is a little surprising, as is the strong substitutability between fertilizer and other inputs. Griliches (1964) tested whether the agricultural production function was Cobb-Douglas by testing whether the elasticity of substitution between labor and all other groups of capital inputs taken together was close to 1. He accepted the hypothesis. It is interesting to note that his result is not necessarily inconsistent with the above estimates since the average of all ten off-diagonal elements of the σ_{ij} matrix in Table is .992. But for technical change questions, it makes an enormous difference whether the average of the elasticities of substitution is 1, with some being larger and some smaller, or each individual elasticity of substitution is equal to 1. All factor-augmenting technical change is neutral in the latter, but not in the former case.

Overall, the γ_{ij} estimates seem to be adequate. No absurd results were obtained. To see whether error interdependence over time has a large influence on the estimates, restricted estimates with the same model were also obtained separately for each of the four data sets. The estimates from the pooled data were compared individually with the average estimates for the four data sets. The estimates were very close. In particular both sets imply complementarity for the same factor pairs except the machinery-fertilizer pair. The own demand elasticities were very similar except that other inputs had an elasticity of less than 1 for the average estimates of the four data sets. Some of the cross elasticities were not very stable over the four sets while the stability of the own elasticities was quite good. Again, in none of the sets did any absurd results occur.

Returning to Table 6, the time coefficients ($\hat{\omega}_i$ in Equation (45)) imply that technical change between 1949 and 1964 has been strongly labor-saving and machinery-using. The t-ratios are so high that it is quite safe to assume that they would come out significant in a model which would take into account error interdependence over time. The land- and fertilizer-using biases are small and nonsignificant.

The coefficients imply changes of the factor shares due to technical change alone as shown in Table 9, assuming factor prices constant between 1948 and 1964. These share changes will be checked against the changes $d\hat{\alpha}^*$ for the same period estimated with the U. S. time series data. These

Table 9. Regression estimates of biased technical change, Model B, 1948-1964

Input	1948 value of observed share	Changes which would have occurred in the absence of factor price changes[a]
	(percent)	(percent)
Land	9.4	+2.3
Labor	37.7	-15.1
Machinery	12.2	+6.9
Fertilizer	2.4	-.5

[a]Computed as $(\hat{\omega}_i \cdot \Delta \ln t)100$.

time series estimates (Equation (36)) use the $\hat{\gamma}_{ij}$ estimated in the same equations as the $\hat{\omega}$.

The comparison of the two sets of estimates will provide a check of the internal consistency of the approach. Exact numerical correspondence is not expected since the cross-section approach implies constant rates of bias while the time series approach does not. But direction and relative magnitude should be the same.

Test of Constancy of Coefficients over Time (the Factor-Augmenting Hypothesis)

This test is done separately for each equation. A two-equation GLS model is fitted for each share with the 1949 data used for the first equation and the 1959 data for the second equation. Specification IV is used with the homogeneity constraint imposed on the data, assuming this to be the true specification. Table 10 shows the resulting F-statistics. The hypothesis is never rejected at the .01 level of significance although it is rejected in two equations at the .05 level of significance. The tests can therefore be interpreted as support of the factor-augmenting

Table 10. Results of test for constancy of coefficients over time

Equation	F-statistic	df	Critical F
Land	.36	4/60	
Labor	3.57	4/60	F .05 = 2.52
Machinery	3.14	4/60	F .01 = 3.65
Fertilizer	.69	4/60	

hypothesis. Certainly the results would not suggest abandoning of the hypothesis for further work.

For several reasons these tests have to be viewed with caution. Apart from being asymptotic tests, the problem of sequential testing is quite important because the particular equation specification in which to run the test was chosen using previous test results on the same sample. If the test is run with Specification III and the data sets 1949 and 1964, the F-levels exceed the .01 level of significance by a wide margin for the labor and machinery equation. But these test results are not reported in detail and not taken into account because Specification III was rejected on the basis that it did not satisfy the symmetry constraints. A test in a wrong specification has no meaning.

CHAPTER 5
THE EMPIRICAL MEASURES OF BIASES IN EFFICIENCY GAINS

This chapter presents the derived series of biases for the United States and Japan using Model A. It also presents the series of actual factor shares, relative factor prices and input quantities per unit of output. The series will be interpreted with respect to the induced innovation hypothesis only in the next chapter. This chapter is descriptive and attempts to evaluate critically the quality of the derived series.

The basic estimation equations for the biases come from equation system (27)

$$d\hat{\alpha}_i^* = d\alpha_i - \sum_{j=1}^{n-1} \hat{\gamma}_{ij} \, d\ln w_j \,, \tag{47}$$

where the $d\hat{\alpha}_i^*$ is the change in the share of factor i which would occur in the absence of ordinary factor substitution due to price changes, $d\alpha_i$ is the actual total change in share i which includes this effect of the price changes, and $d\ln w_i$ is the proportional change of the ratio of the price of factor i to the price of other inputs (a choice which is arbitrary). For actual estimation purposes, series of three-year moving averages of the shares and the factor prices were constructed (see Appendix D). Then discrete differences of these moving averages at four-year intervals were taken and used in the discrete change equivalent of (27). The $\hat{\gamma}_{ij}$ were the ones estimated in the U. S. cross-section regressions. It was assumed they were the same for the whole period and for Japan.

Equation (27) can be converted into a standardized measurement of the bias for each share by dividing the share changes through by the levels of the actual shares in a base period. This leads to the discrete change equivalent of Equation (1) which measures the rate of the biases.

$$\hat{B}_i = \frac{\Delta\hat{\alpha}_i^*}{\alpha_i^o} \lesseqgtr 0 \longrightarrow \text{Hicks} \begin{cases} \text{i-saving} \\ \text{i-neutral} \\ \text{i-using} \end{cases} . \qquad (48)$$

Addind the B_i for all four-year intervals (with B_i of the base period equal to 1) gives cumulative standardized series of α_i^* as a fraction of the base period.

An approach is only as good as its assumptions. The key assumptions here are simple cost minimization and the constancy of the γ_{ij} coefficients over time and space. The former assumption is no problem because it implies neither profit maximization nor nonintervention by the government in goods and factor markets. Only if the government regulates both prices and quantities of factors of production is there a big problem. Quantity controls alone will be reflected in corresponding price changes and vice versa and, therefore, will not disturb the measurements. They may, of course, have induced biases.

The constancy over time and space is more troublesome. When tested, it was not supported as well as one might wish. But there is no way around the assumption.

Even the constancy over time assumption is not as restrictive as it sounds. It does not preclude variable elasticity of substitution and of factor demand. Furthermore, these elasticities may have arbitrary values. The approach allows for neutral and non-neutral efficiency differences between the regions considered. In short, it does not require countries to be on the same production function. Only the γ_{ij} parameters of the functions have to be the same.[10]

This approach is also less restrictive than other approaches used up to now in that it allows the production function to be nonhomogeneous and allows economies of scale, provided they affect all factors neutrally. It also uses five factors rather than two, which means that it does not impose a separability constraint between capital and labor on the one hand and intermediate inputs on the other. From the point of view of its assumptions, the approach should, therefore, be superior to other known approaches.

All resulting series are presented graphically. The corresponding numerical values are tabulated in Appendix C. Figures 2 and 6 show cumulative bias series in percent of their 1912 and 1893 values using a semi-logarithmic scale. The slope of each of the series is the B_i coefficient (Equation (1)) which measures the rate of the bias. The series themselves show cumulative effects. As an example, the fertilizer line in Figure 2 indicates that, given the biases which occurred, the fertilizer share would have quadrupled between 1912 and 1962 had all the factor prices remained constant; _i.e._, had no factor substitution along a given production function occurred. The rather constant slope of the line indicates that the rate of the bias remained fairly constant throughout the period.

---

[10]If estimates of the γ_{ij} coefficients were available for different time periods and countries, it would be possible to relax the assumption of constancy of coefficients over time and space. But the justification of Equation (27) then depends on the heuristic derivation and can no longer be based on a factor-augmenting framework. In that case, technical change cannot be considered to be factor-augmenting since it alters the parameters of the cost function.

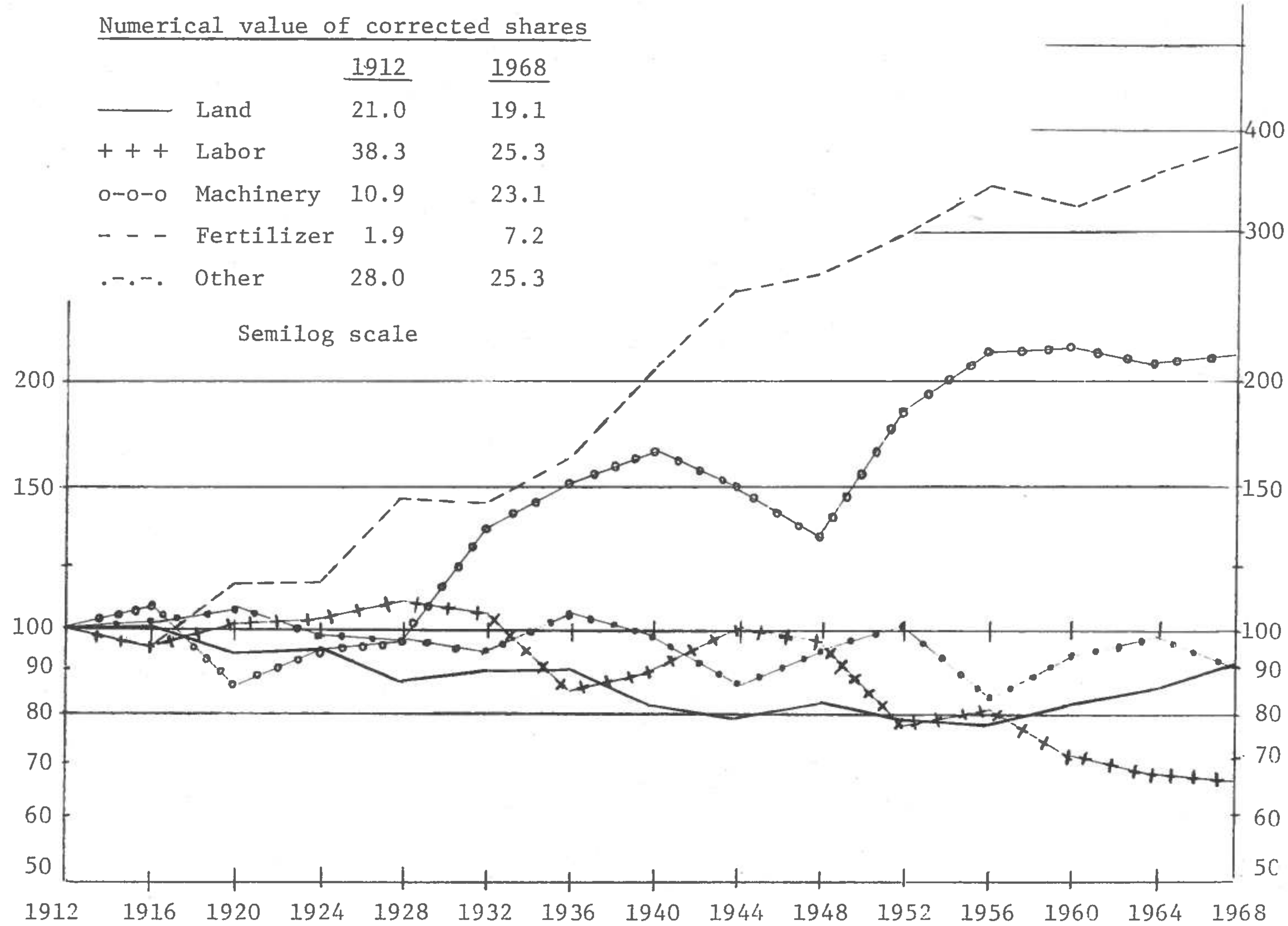

Figure 2. U. S. indices of biases in technical change: Model A estimates of α^*, 1912 = 100

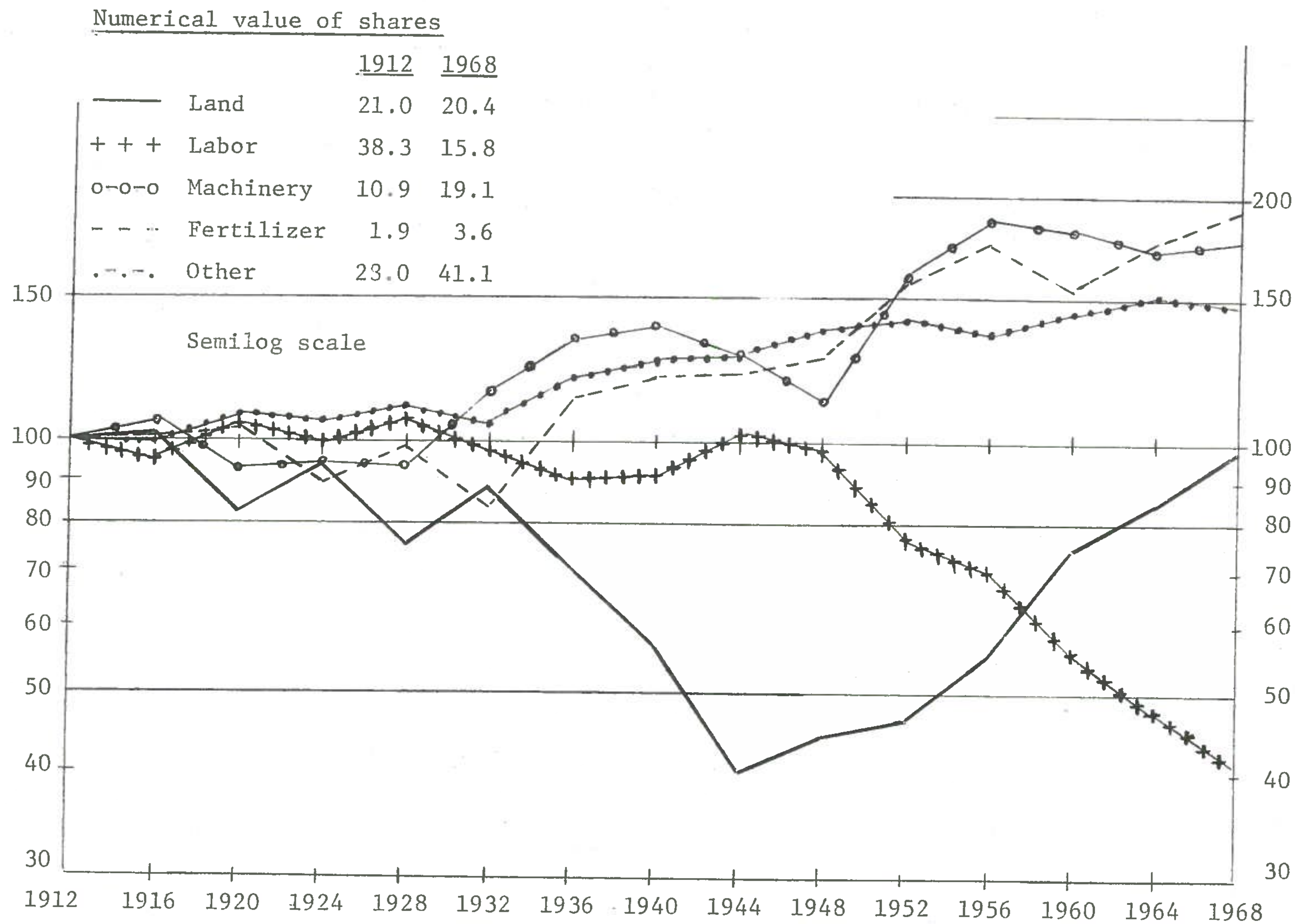

Figure 3. U. S. actual development of the factor shares, in percent of their 1912 value

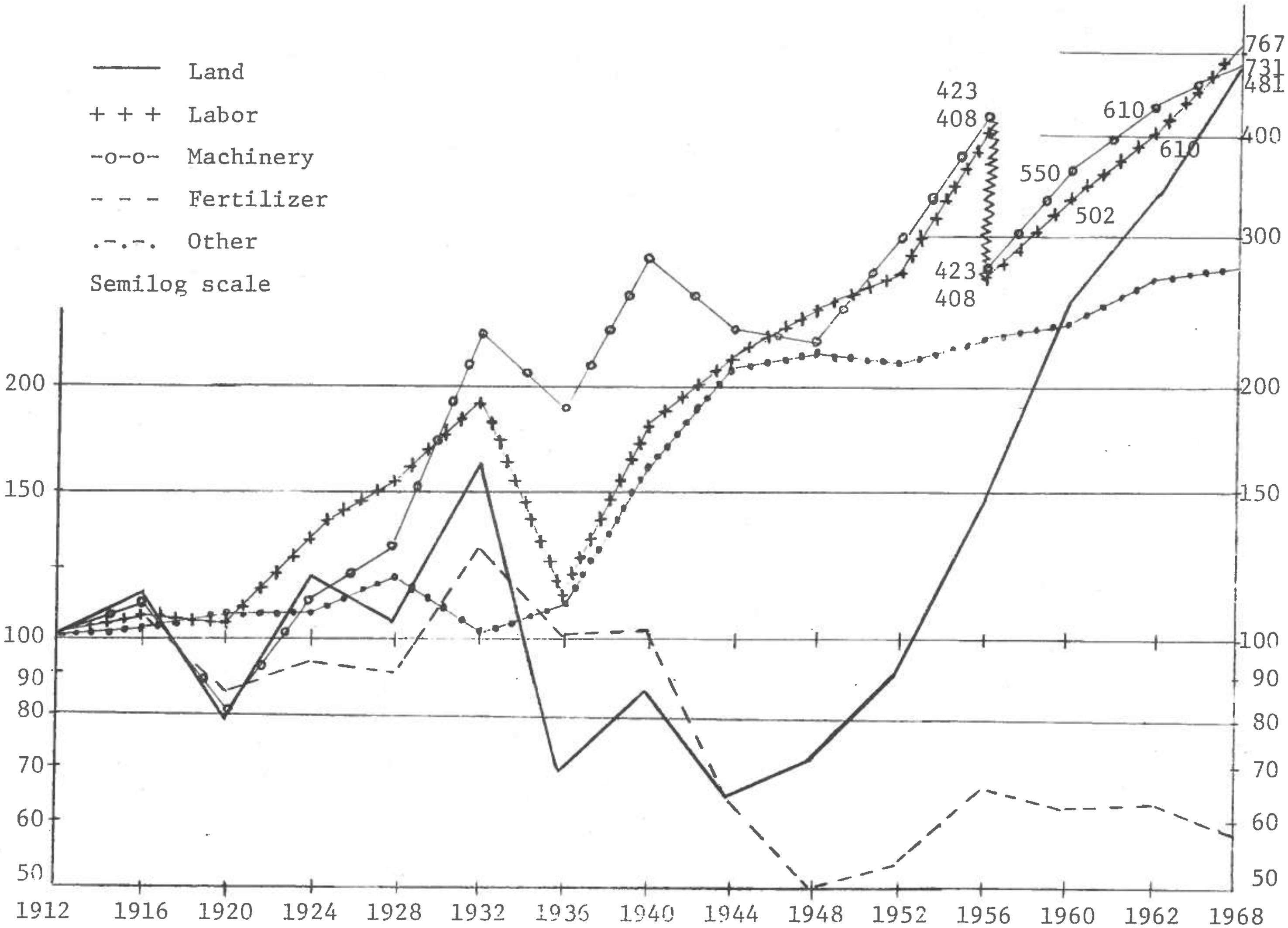

Figure 4. U. S. indices of input prices relative to aggregate output prices, 1912 = 100

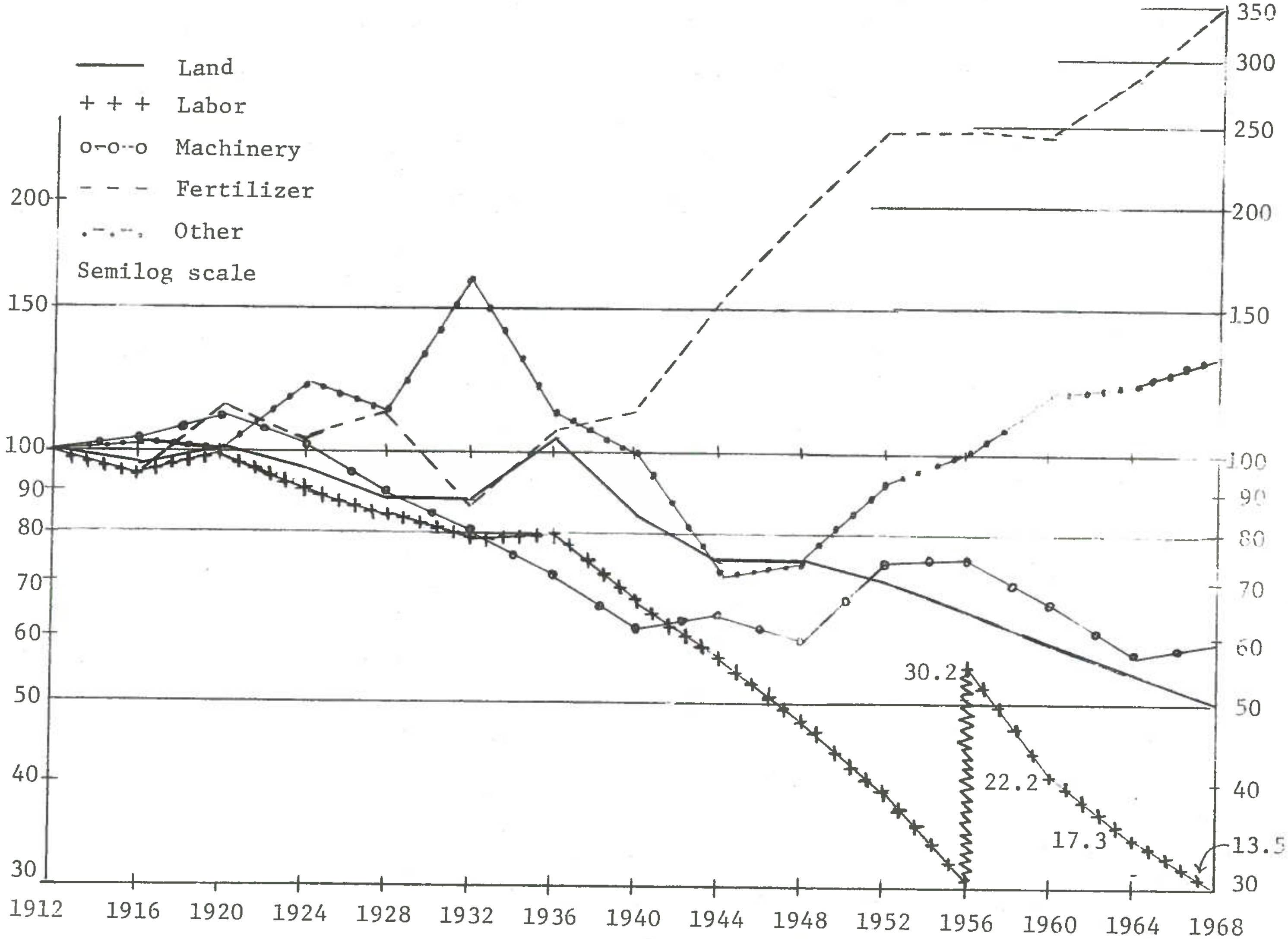

Figure 5. U. S. quantity indices of inputs per unit of output, 1912 = 100

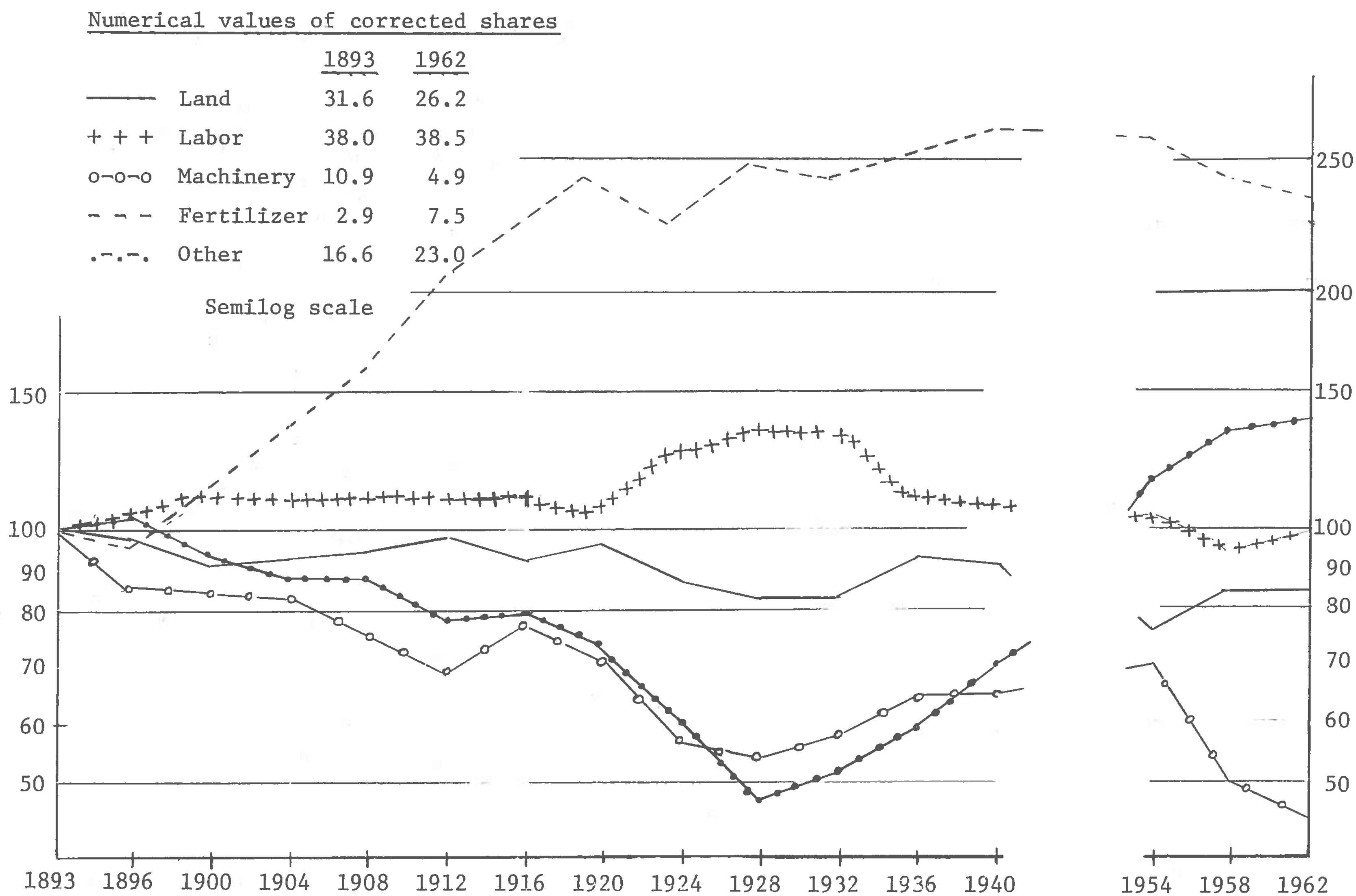

Figure 6. Japanese indices of biased technical change: Model A estimates of α^*, 1893 = 100

Figures 3 and 7 show the actual share movement for the United States and Japan in the same semilogarithmic scale and as a percent of the 1912 value of the actual shares. The actual share changes reflect the influence of both bias and ordinary factor substitution. Figures 4 and 8 represent indices of the prices of the factors relative to the output price for the two countries.[11]

Indices of quantities of inputs per unit of output are shown in Figures 5 and 9. These graphs clearly indicate that when speaking of factor-using efficiency gains, the amount of inputs per unit of output will not necessarily increase. The output indices used for the series are the total value of sales, home consumption, inventory change (and government payments for the United States) divided by the aggregate output price index. This relatively crude measure seemed adequate for the illustrative purposes for which it was used. The output index is never used in the derivation of the α^* series.

According to Figure 2, efficiency gains in the United States have been strongly fertilizer-using and machinery-using. At first they were labor-neutral and then substantially labor-saving. Land was first saved and then used while other inputs experienced neutral efficiency gains over the whole period.

Japanese efficiency gains were fertilizer-using in a much earlier period than in the United States. After 1920 they were fertilizer-neutral. Machinery had a negative overall bias, which is in strong contrast to the

---

[11] The price series actually used in computation were series of the prices of each factor relative to the price of other inputs. The graphs are intended to show the price movements of the factors relative to each other and not the absolute price level changes.

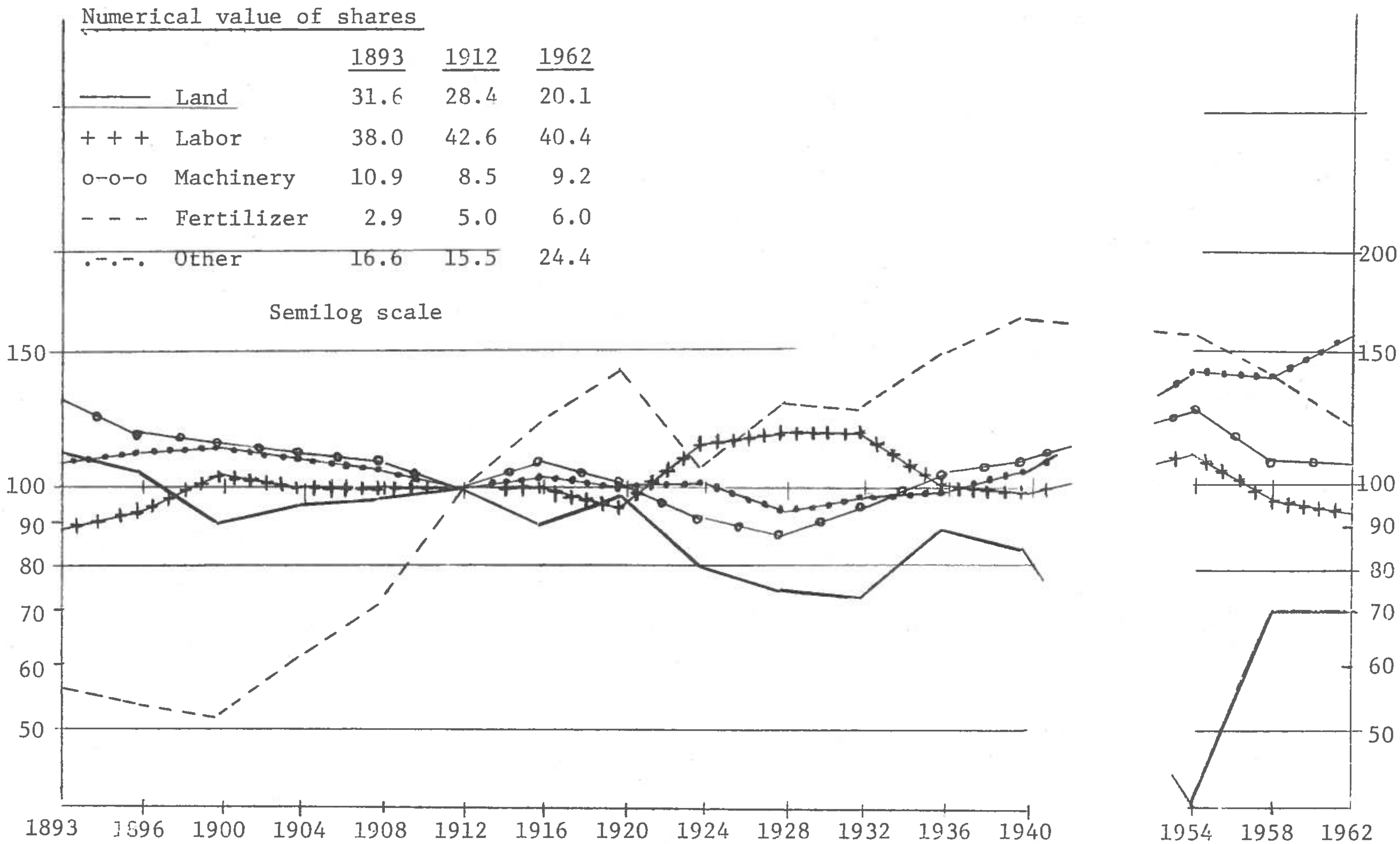

Figure 7. Japanese actual development of the factor shares, in percent of their 1912 value

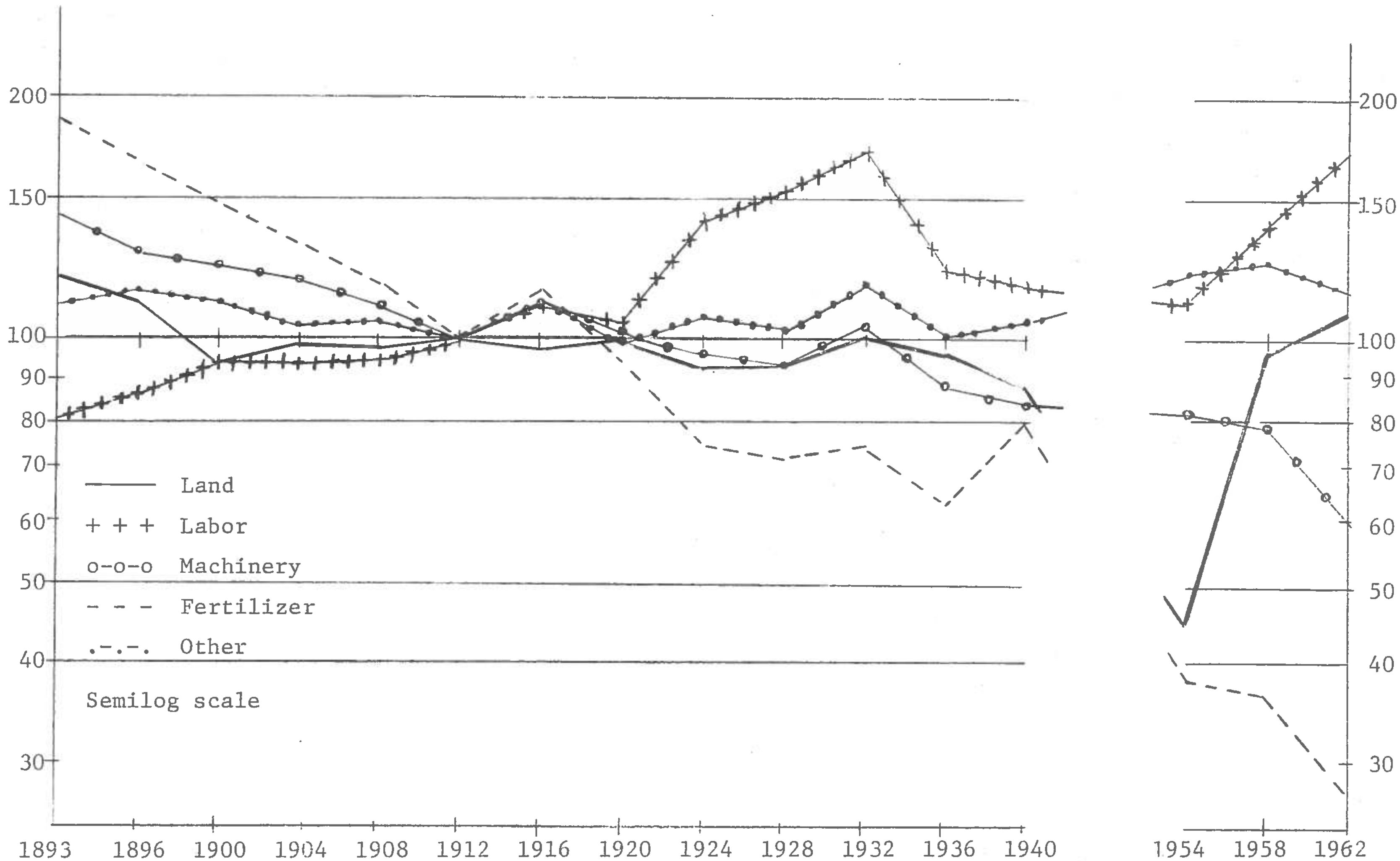

Figure 8. Japanese indices of input prices relative to aggregate output prices, 1912 = 100

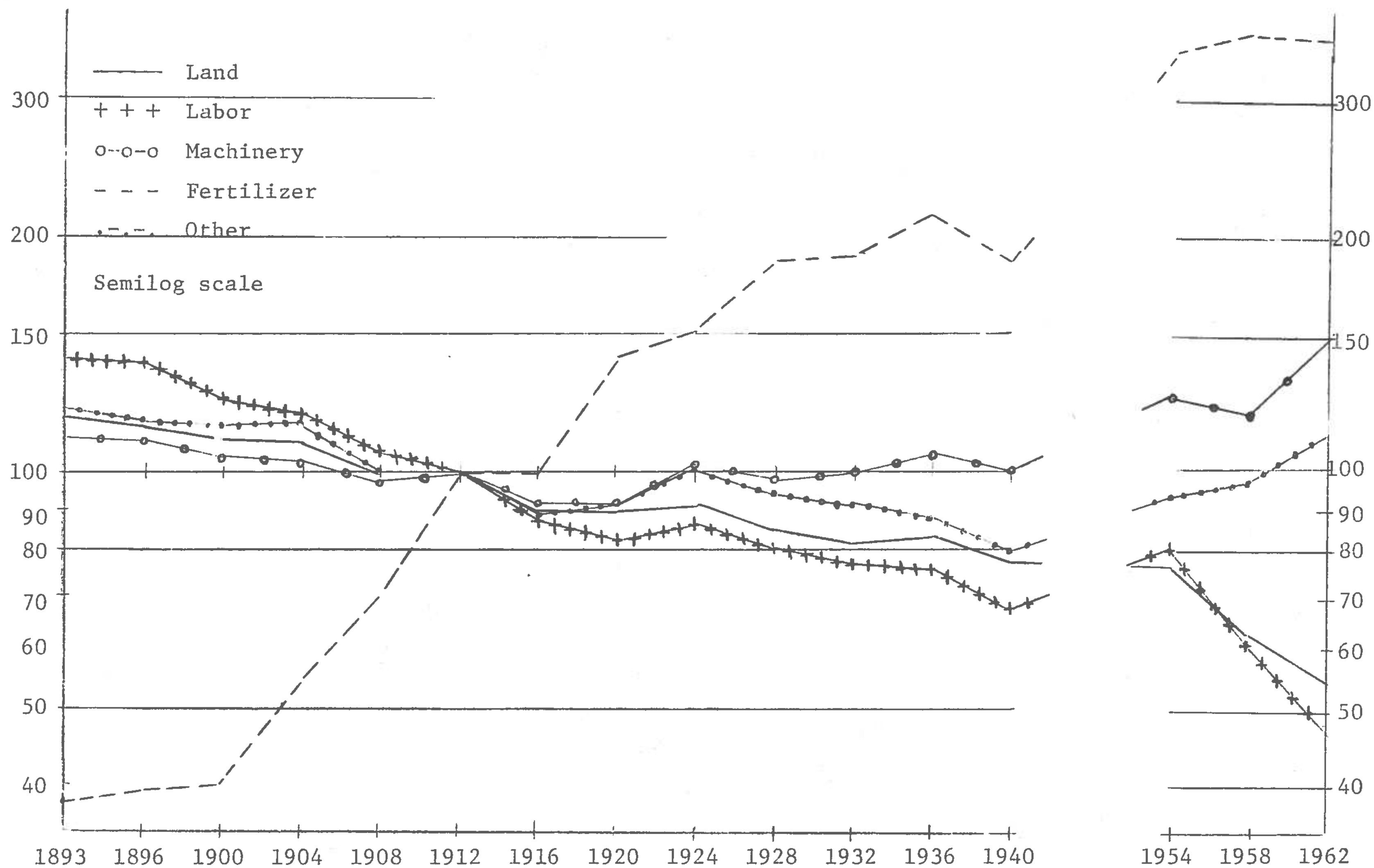

Figure 9. Japanese quantity indices of inputs per unit of output, 1912 = 100

positive U. S. bias in machinery. Labor was used until 1928 and then saved while land had a slight overall negative bias. Other inputs were saved until 1928 and then used.

Another conclusion which can be drawn is that biases are very important forces in the determination of factor shares. Of the 60 percent drop in the labor share in the United States between 1944 and 1968, the labor-saving bias accounts for about 35 percent whereas the direct price influence accounts for the remaining 25 percent (neglecting any influence which the prices might have had in determining the biases themselves through induced innovation).

How much confidence can we have in the quality of the α^* series? This is a critical question before any interpretative work can be done. The series of the biases, shares, and factor prices are internally consistent with the elasticities of derived demand and the elasticities of substitution measured in the last chapter. But this internal consistency cannot be used to evaluate the series because the elasticities are estimated from the same $\hat{\gamma}_{ij}$ which relate the series of biases, shares and factor prices to each other.

If the estimates of the γ_{ij} were really far off, chances would be that over the long periods involved, which include two world wars and the depression, some strange result would be immediately apparent in the $\hat{\alpha}^*$ series. Such a result might be if one of the $\hat{\alpha}^*$ series became negative. Smaller errors in the $\hat{\alpha}^*_{ij}$ are, of course, not ruled out by such consider ations. The errors could even be large enough to make inferences from small direction changes of the series impossible. That some such errors are present in the $\hat{\gamma}_{ij}$ became apparent when the matrix was inverted and estimates of the series of augmentation coefficients were derived according to (25). These estimates showed the strange result that the augmentation coefficient of fertilizer became negative in both the Japanese and the U. S. case.

Does this result also invalidate the estimated $\hat{\alpha}^*$ series? Not necessarily, for the following reasons. First, the properties of an estimator of the true elements of the γ^{-1} matrix, which inverts unbiased estimates of $\hat{\gamma}_{ij}$ are unknown (Theil, 1971, p. 322). Further, an error in just one of the $\hat{\gamma}_{ij}$ can lead to erroneous estimates of all the elements of the inverse. Table 11 shows γ and γ^{-1} matrices. The largest elements of the inverse correspond to the smallest values of γ_{ij}, which also are the values with the smallest t-ratio (see Table 6), so that we have no assurance that they have the correct signs.[12] The negative augmentation coefficient of fertilizer was therefore not judged important enough to also invalidate the $\hat{\alpha}^*$ series.

Table 11. The γ matrix and its inverse

$$\gamma = \begin{bmatrix} .0775 & -.0361 & .0048 & .0107 \\ & -.0637 & -.0066 & -.0281 \\ \text{(symmetric)} & & -.0349 & -.0088 \\ & & & .0007 \end{bmatrix}$$

$$\gamma^{-1} = \begin{bmatrix} 18.114 & 2.942 & 10.079 & -32.383 \\ & -2.812 & 9.839 & -35.352 \\ \text{(symmetric)} & & -21.565 & -30.237 \\ & & & 129.076 \end{bmatrix}$$

[12] To obtain a series of draft animal capital in Japan, an 8×8 matrix of animal numbers had to be inverted and the inverse postmultiplied with vector to obtain the prices of the animal classes. There was a small

The time coefficients $\hat{\omega}_i$ estimated in the last chapter allow the estimation of biases using Model B of Chapter 3. The price-corrected share changes, $\Delta\alpha^{**}$, are computed for the period 1948-1964 for the United States as follows: $\Delta\alpha_i^{**} = \hat{\omega}_i \, d\ln t$ under the assumption that the rate of the bias remained constant during that particular period or alternatively that $\hat{\omega}_i$ measures an average rate of bias. Apart from the fact that the $\hat{\gamma}_{ij}$ and the $\hat{\omega}_i$ were estimated in the same equations, the Model A estimates, $\Delta\alpha_i^{*}$, have nothing to do with Model B estimates, $\Delta\alpha_i^{**}$, and therefore there is no reason, apart from chance, that they would come out to be the same if either set of estimates were wrong. Table 12 shows the comparison of the Model B estimates with the Model A estimates reported in the graphs.

Table 12. Comparison of Model A and Model B estimates of biases for the period 1948-1964 for the United States

Factor	1948 level of shares	Estimates share change due to technical change alone	
		Model A 1948-1964 $\Delta\alpha^{*}$	Model B 1948-1964 $\Delta\alpha^{**}$
		(percent)	(percent)
Land	9.4	+ 2.3	+ .7
Labor	37.7	-15.1	-11.4
Machinery	12.2	+ 6.9	+ 8.5
Fertilizer	2.8	+ .5	+ 1.6

printing error in the data source with the animal numbers which was detected because the solution showed negative prices. This is just an example of the sensitivity of inversion to small errors in the original matrix.

Both series estimates biases are of the same sign and of about the same magnitude. They cannot be expected to agree perfectly because of the differences in the underlying assumption. This comparison provides strong support of the α^* series measured by Model A.

The conclusion of this chapter, therefore, is that the quality of the estimated series of biases is high enough for their use in a test of the induced innovation hypothesis. The next chapter will first set forth the framework for such a test by discussing the literature on the induced innovation hypothesis and then draw conclusions about the conformity of the measured biases with the hypothesis.

CHAPTER 6
INDUCED INNOVATION

The previous chapters were devoted to measuring the biases in technical change as they have actually occurred in the past. This leaves completely unanswered the question of what determined the biases.

One way of thinking of the biases and, more generally of the rate of technological change, is to treat them as given from outside of the economic system. This view in a way likens the discovery of new methods or production to geographic discoveries. The physical, chemical, and biological world has certain properties which are given and can be discovered. Once they are discovered, they will uniquely determine both the rate and the biases of technological change. Similarly, in geographic exploration you can only find what is there: Columbus set out for India; what he found was America.

While it is certainly true that one can only discover the existing properties of the real world, technological possibilities of these properties might be much more flexible than the view of exogenous determination of rate and biases of technological change might hold. Given a certain amount of research expenditures, one can develop a large variety of processes, each with a different impact on the cost of production and on factor intensities. If this view is true, then the rate and the biases would be determined within the economic system and to find out more about it, one would need an investment theory of technological change. Schmookler (1966) and Nelson (1959a and 1959b) have loosely discussed invention in such a framework, but not much progress has been made in this area toward developing a rigorous model.

To facilitate later discussion, therefore, the elements which such a model should include are sketched out, first in terms of the rate of technological change or innovation in a particular industry and then in terms of the biases.

Leaving product innovation aside, the rate of efficiency growth would be governed by the following elements to which one can assign pseudo-mathematical symbols for further reference.

1. Physical, chemical and biological possibilities, i.e., the state of the basic sciences which one might assume to be exogenous. Let this complex be denoted by S.
2. The cost of developing actual production processes from S, i.e., the research and development costs, C.
3. The expected rate of return obtainable from the innovation which will be governed by
    a. the size of the process to which an innovation is applied, M. The bigger the process, the larger the market potential of the innovation;
    b. the prices of other factors of production, P;
    c. the interest rate, r; and
    d. other factors such as the state of competition in the industry, patentability or other protection of the innovators' rights, etc. Let this be O.

One can then write the rate of efficiency growth, T, due to technological advance as the following general relationship:

$$T = f(S, C, M, P, r, O) . \tag{49}$$

This is the framework which is very similar to the human capital approach of labor quality improvements or to investments in soil improvements, apart from its similarity to any investment problems.

Given such an investment theory, the question of endogeneity or exogeneity of the rate is an empirical question of the relative importance of the different variables in f. If the S complex dominates all other elements, then the rate will be mainly exogenous while it will be endogenous if the economic variables are more relevant than the S complex. In an outstanding empirical investigation of U. S. patent statistics and of hundreds of important inventions in four industries, Schmookler (1966) has come to the conclusion that the rate of return to inventions is of far greater importance than the state of knowledge. He shows that it is market forces and not the availability of all the necessary elements of S which trigger inventions. While the availability of all necessary basic knowledge may be a condition for an invention, he has found no instance where this alone has brought about an important invention. In most cases considered, the necessary basic knowledge was available decades before the innovation was actually made. He goes as far as to say that if the market potential of a given invention is large, there are often several ways to achieve an innovation which will satisfy it, each of which might draw on different basic knowledge so that the importance of scientific knowledge as a precondition is even lower than what it seems on the basis of his evidence. Therefore, the rate of innovation in an industry is endogenously determined. Schmookler does not discuss the importance of the costs of achieving an invention nor does he disaggregate the rate of return into several components.

It is a small step from function (49) to the formulation of an analogous investment model of the biases. Clearly, if labor prices are rising and it is possible and equally as expensive to achieve a labor-saving innovation as a capital-saving one (reducing total factor requirements by the same amount), the rate of return from the labor-saving innovation will be larger than from the capital-saving one. Therefore, one can write

$$\text{Biases} = f(S, C, M, P, r, O) \; . \tag{50}$$

Of course, in the actual world, the biases and the rate will be determined simultaneously, but it is analytically convenient to separate the two.

The questions to be asked in this investment model are the same as before. Does S constrain the possibilities of biases such that all other arguments become empirically irrelevant? If that is the case, biases (or neutrality) are given exogenously, even if entrepreneurs tried to allocate their research expenditures according to an investment model. _De facto_ exogeneity would also exist if the cost of achieving labor-saving biases was small while the cost of capital-saving biases was exorbitant. This is similar to the problem of planting bananas in Quebec. While it is not impossible to build vast heated greenhouses there, no one will do it commercially because of the exorbitant costs associated with it.

The theoretical discussions of induced biases in the mid-sixties and the empirical research based on it have centered on the following aspects:

1. They consider only biases due to technological change and not biases which might result from investments in human capital

or soil improvements. The measured biases in the last chapter, however, result from all sources of efficiency gains.

2. The discussions have centered on whether the S elements determine the biases exclusively or not.
3. They have completely neglected the cost aspects.
4. On the return side, they have looked almost exclusively at relative prices of factors.
5. They have not treated the benefits from an innovation as a flow over several production periods but largely as one-production period models.

Empirical work has concentrated on the question of whether there have been biases in the U. S. economy and, in tests of the induced innovation hypothesis, considered only relative factor prices as determinants.

The first reference to factor prices as a source of biases is made by Hicks (1964) in his Theory of Wages (originally published in 1932). He argues that changes in factor prices will induce biases which will save the progressively more expensive factor. (Of course, the biases themselves will influence the factor prices.) Hicks did not specify the mechanism by which this would occur.

Ahmad (1966) has a very careful exposition of this idea. He uses the concept of a historic innovation possibility curve which he defines as follows. At a given time there exists a set of potential production processes to be developed. This set might be thought of as the state of the basic sciences. Each process in the set is characterized by an isoquant with a relatively small elasticity of substitution, and each of the processes in the set requires a given amount of resources to be developed to the point where it actually can be used. The IPC is the develop the process I_{t+1} for the next period. If the IPC shifts inward neutrally, technical change will be neutral. (But Ahmad recognizes it is possible the IPC shifts inward non-neutrally, which would result in biases even at constant factor prices.) If, however, factor prices change to P_{t+1} P_{t+1}, it is no longer optimal to develop I_{t+1}; the process corresponding to I'_{t+1} becomes optimal. In the graph, P_{t+1} P_{t+1} corresponds to a rise in the relative price of labor. If the IPC has shifted neutrally, I_{t+1} will be relatively labor saving in comparison to I_t.

Because of the way in which IPC is defined, and given full knowledge of entrepreneurs about factor prices and the possible alternative processes, it is irrefutable that induced innovation will occur. But the assumptions have to be examined.

First, the theory assumes the further shift of IPC is independent of the process developed in period t. This may or may not be true.

Second, the theory does not consider the possibility of spending resources to influence the shift of the IPC. It is conceivable that resources could be spent either to increase the elasticity of substitution of the IPC or to have it shift non-neutrally.

Further, the theory might become irrelevant if the elasticity of the IPC were not much larger than the isoquants corresponding to the individual processes. If, moreover, the IPC was biased, a fundamental bias would be obtained. The same conclusion would apply if it were nevertheless possible to increase the elasticity of substitution of the IPC, but required an exorbitant amount of resources.[13]

---

[13]Before Ahmad developed his theory, Salter (1960) criticized the price-induced innovation hypothesis as follows. He distinguishes between

Jumping a little bit ahead--to test the relevance of the induced innovation hypothesis requires that one test whether the IPC has a substantially larger elasticity of substitution than the individual processes. Even if the IPC were fundamentally biased, a large elasticity of substitution would still make induced innovation empirically relevant because it would substantially increase or offset the fundamental biases. A direct measure of the elasticity of substitution of the IPC is not attempted. Indirect evidence can be obtained by considering the biases in Japanese and U. S. agriculture. Since they had differing trends in factor prices and other economic variables, they must have had differing biases in the same time periods if the elasticity of substitution of the IPC is large and induced innovation is true. The differences in the

---

fundamental knowledge (S factors) and applied knowledge and argues that no firm could be induced to develop new fundamental knowledge. If innovation is defined as the development of such new knowledge, it has to be rejected because (Salter, 1960, p. 43)

> If ... the theory implies that dearer labor stimulates the search for new knowledge aimed specifically at saving lavor, then it is open to serious objections. The entrepreneur is interested in reducing costs in total, not particular costs such as labor costs or capital costs. When labor's costs rise, any advance that reduces total costs is welcome, and whether this is achieved by saving labor or capital is irrelevant. There is no reason to assume that attention should be concentrated on labor-saving techniques, unless, because of some inherent characteristic of technology, labor-saving knowledge is easier to acquire than capital-saving knowledge.

But then he states that engineers, given the fundamental knowledge, design machines so that they use optimal amounts of factors, given the existing factor prices. But this, he says, is not induced innovation. In a way this amounts to defining away induced innovation. The mechamisms by which engineers will respond to existing factor prices in the design state have still to be defined and would probably be similar to induced innovation.

biases must, moreover, be large if the theory is also to be relevant. Note also that because Ahmad's theory neglects all other determinants of the rate of return to biases, the biases must not necessarily conform to the simple price inducement mechanisms for the theory to be true. Other economic factors might have influenced the biases and overshadowed the factor price influences.

Other shortcomings of Ahmad's theory are precisely that no other economic factors governming the rate of return to biases are considered and that the time dimension of the benefits to biases is neglected. In particular, if biases were only obtainable at a cost, the relative importance of a factor to which a given saving applies would make a difference in the rate of return.

Kennedy (1964) and later Samuelson (1965b) developed a version of the induced innovation theory which takes account of the relative importance of factors and, in some sense, of the cost of obtaining biases.

The basic idea of this theory can best be explained with an example. Suppose it were equally expensive to develop a new technology which reduces labor requirements by 10 percent as one which reduces capital requirements by 10 percent. If the capital share is equal to the labor share, the entrepreneurs will be indifferent between the two and half will choose the one and the other half the other. The outcome will be neutral technical change. If, however, the labor share were 60 percent, then all would choose the labor-reducing version. If the elasticity of substitution were less than 1, this would go on until the labor and the capital shares became equal again, provided the induced biased technical changes does not alter the tradeoff relationship between labor requirement and capital requirement reducing (augmenting) technical change.

Therefore, shares can be stable even if the capital-labor ratio changes historically. This implication of shares stability is what interested the authors.

The following section discusses the theory in mathematical detail and the objections which might be raised against it.

Write total unit costs as follows:

$$U = KR + LW \qquad \text{s.t.} \quad Y(A_K K,\ A_K L) = 1 \ , \qquad (51)$$

where W is the wage rate and R the capital rental rate and the As are augmentation coefficients. The instantaneous proportional rate of reduction in unit costs (see Appendix E for the derivation) can be written

$$\dot{u} = \frac{\dot{U}}{U} = - \dot{a}_K \alpha_K - \dot{a}_L \alpha_L + \text{terms involving price changes} \ , \qquad (52)$$

where α_K and α_L are the factor shares

$$\dot{a}_K = \frac{1}{A_K} \frac{\partial A_K}{\partial t}$$

and

$$\dot{a}_L = \frac{1}{A_L} \frac{\partial A_L}{\partial t} \ .$$

Now assume:

1. given factor prices,
2. an exogenously given budget for research and development of new techniques, and
3. a fundamental tradeoff between the rate of proportional reduction in labor requirements, $\dot{a}_L$, and the rate of proportional reduction in capital requirements, $\dot{a}_K$.

This relationship, which is simply an assumption about the underlying possibilities of technical change, can be written as:

$$\dot{a}_L = f(\dot{a}_K)$$

or

$$\phi(\dot{a}_K, \dot{a}_L) = 0. \quad (53)$$

Assume that this "transformation" function or as Kennedy (1964) called it, this "innovation possibility frontier" (IPF) has the usual characteristic of economic transformation functions, i.e.,

$$\frac{d\dot{a}_K}{d\dot{a}_L} < 0, \quad \frac{d^2\dot{a}_K}{d\dot{a}^2_L} < 0 .$$

Graphically, this transformation function will look as in Figure 11. The IPF is assumed to be invariant over time. Neither Kennedy nor Samuelson discusses in detail what determines the position of the IPF, which, in a way, governs the growth rate. The farther out the IPF lies, the faster will be the reduction in input requirements per unit of output at a given ratio of $\dot{a}_K$ to $\dot{a}_L$.

Ahmad (1966) has shown that (53) is a very restrictive assumption in that it assumes a stable relationship of the tradeoffs between capital and labor augmentation (i.e., the cost of the one in terms of the other), which is completely independent of the initial factor use. It might be argued that, if the capital labor ratio is high, the tradeoff relationship is different from the one which would be obtained at a low capital-labor ratio. Ahmad (1966) shows graphically that this assumption of

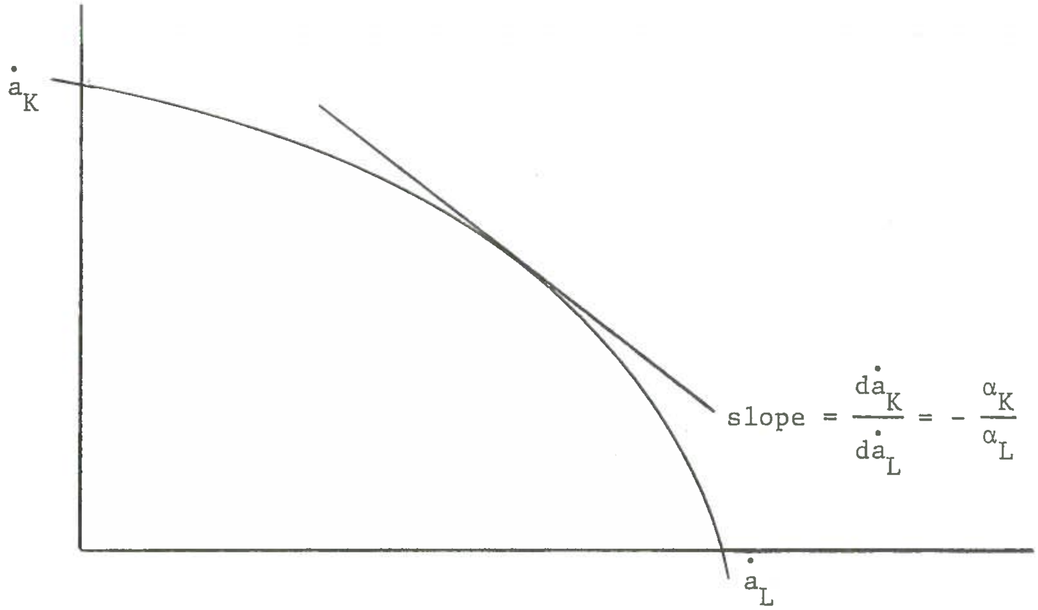

Figure 11. Kennedy's innovation possibility frontier

independence from the initial capital-labor ratio is equivalent to an assumption that the innovation possibility curve (Figure 10) is Cobb-Douglas. This proposition is proved mathematically in Appendix E. Therefore, the implication of shares stability, which emerges from the Kennedy-Samuelson approach, is entirely based on the fundamental neutrality implied in a Cobb-Douglas IPC. Nevertheless, the derivation of the Kennedy-Samuelson approach is sketched out here because it shows how one would have to proceed to derive implications of a more general framework, once better tradeoff relationships have been empirically tested.

Given Equation (52) and Equation (53), one can set up a maximization problem. Maximize the rate of instantaneous unit cost reduction subject to the tradeoff relation of factor augmentation.

$$\text{Minimize } \dot{u} = -\dot{a}_K\alpha_K - \dot{a}_L\alpha_L \tag{54}$$

subject to $\phi(\dot{a}_K, \dot{a}_L) = 0$.

The solution is completely analogous to the solution of the similar system of minimizing cost subject to a given output, where α_K and α_L now have the same role as factor prices. Hence, the rate of cost reduction is maximized at a point where

$$\frac{d\dot{a}_K}{d\dot{a}_L} = -\frac{\alpha_L}{\alpha_K} . \tag{55}$$

The slope of the IPF has to be equal to the inverse ratio of the shares (see Figure 11). Hence, the higher the labor share, the higher will be $\dot{a}_L$ relative to $\dot{a}_K$ or technical change will be relatively labor-augmenting. (According to (5) on page 11, it will be labor-saving if, in addition, $\sigma < 1$.

The mechanism implied in Equation (55) can explain the constancy of relative factor shares even if the capital-labor ratio increases, provided that σ of the individual production process is less than 1. In the absence of technical change, an increase in the capital-labor ratio would increase the labor share if $\sigma < 1$. But as the labor share increases, resources are shifted by the above mechanism to the development of labor-augmenting technology, which will offset the tendency of the labor share to increase. Dynamic properties of this system under various assumptions can be found in Samuelson (1965b, 1966) and Drandakis and Phelps (1966).

As discussed before, the weakness of the theory lies in its assumption about the tradeoff relationship between $\dot{a}_K$ and $\dot{a}_L$. Also, nothing is said about what research efforts will do to it. It is clear, however, that if the true, and certainly more complex, tradeoff relationship between the various augmentation coefficients were known, a similar analytical procedure could be used to derive the properties of the system. But only a joint empirical and theoretical investigation could find which arguments to include in such a tradeoff relationship and what form it should take. As the previous discussion shows, the theoretical part of this still remains to be done.

Little empirical evidence on induced biases is available. Solow (1957), Sato (1970) and Fellner (1971) consider the question of whether there has been an aggregate labor-saving bias in technological change in the U. S. economy. All three attempts impute biases, if any, to the effect of technical change alone and neglect the human capital aspect as a possible source of bias. But for their argument it is immaterial whether the human capital investment is a source of bias. Solow's test is based on the mathematicl fact that, if biases occur, the rate of technological change (his residual) cannot be independent of the capital-labor ratio. Since he fails to find such a relationship, he concludes that technical change must have been neutral.

This conclusion is contradicted by Sato (1970), who measured changes in factor-augmentation coefficients using Equation (7), and the assumption that the elasticity of substitution is less than 1, which is empirically supported by most attempts to measure this parameter. He finds that technical change has been almost exclusively labor augmenting. If σ is less than 1, this implies that technical change has been labor saving (Equation (5)).

This conclusion is supported by Fellner (1971) who shows that during the period 1948-1957 the labor share rose from approximately 60 percent to 65 percent, while it remained constant during the rest of the period 1920-1966. Between 1948 and 1957 the capital-labor ratio rose at a much faster rate (3.7 percent per annum) than during any part of the period 1920-1966. This is interpreted as follows: Given an elasticity of substitution of less than 1, the rise in the capital-labor ratio during the whole of the period 1920-1966 should have had a tendency to increase the labor share during the entire period. That it did not do so must have been due to an exactly offsetting labor-saving bias, except between 1948 and 1957. In this subperiod, the rise in the capital-labor ratio was so large that the bias was no longer sufficient to hold shares constant.

The exactly offsetting bias except between 1948 and 1957 would be consistent with a share-induced innovation process according to Kennedy and Samuelson. It is, however, also consistent with the idea of a fundamental bias during the entire period.

Also, that the labor share actually increased between 1948 and 1957 and stayed constant afterwards would indicate to me that the inducement mechanisms to hold shares stable either did not work at all during that period or were so weak as to have only a small impact. If the share-inducement mechanism had been very responsive, the labor share would not have risen between 1948 and 1957 despite the strong rise in the capital-labor ratio. But constancy of the labor share throughout the period might then again have been consistent with the opposing hypothesis that technical change was neutral throughout the period. This is just an example of the impossibility of inferring something about the source of biases on the basis of actual share behavior in only one country without measuring the biases first.

Solow's finding of neutrality is inconsistent with Sato's and Fellner's finding of non-neutrality. The inconsistency remains unresolved. If Sato and Fellner are right, then we still do not know whether the bias was fundamental.

Hayami and Ruttan (1970) followed an entirely different approach. From a comparison of agricultural time series data on labor, land, and capital (machinery) productivity in Japan and the United States and from supplementary evidence of fertilizer use, they concluded that both countries had experienced biased efficiency growth. The differences in the development of these series between the two countries are indeed so striking that one has the impression that the differences must be due to biases in different directions rather than to ordinary factor substitution along the production function of the neutrally changing individual production process.

Hayami and Ruttan then assume that at each moment of time the elasticities of substitution among factors in agricultural production are very small so that almost fixed proportions prevail. As support they cite evidence from experimental studies on fertilizer response which indicates that the optimal fertilizer use in each crop does not change very much with changes in prices. Examples of mechanical processes such as harvesting of grain are also presented. However, while it may be true that for individual crops or tasks the elasticities of substitution are quite small, this may no longer hold for the farm level where much more flexibility is likely to exist, as linear programming studies in general show.

Given the assumption of almost fixed proportions of individual production processes, the induced innovation hypothesis can be proved as follows. Estimate the elasticities of substitution using time series data. If they are large, the ex post observed substitution must have been due to biased technical change rather than to substitution along a given production function which was assumed to be very difficult. The advantage of this method is that it would prove both the endogeneity of the biases and the predominant role of factor prices in explaining them.

The estimation equation which Hayami and Ruttan use to estimate the elasticities of substitution has certain problems which are reviewed in Appendix E. Their largest measured elasticity of substitution is 1.3 between machinery and labor in the United States. All other elasticities of substitution are estimated to be less than 1. Therefore, if one rejects their hypothesis of almost fixed proportions at each moment of time, one cannot consider their estimates as conclusive evidence for the induced innovation hypotheses.

In the light of this, it seemed to be necessary actually to measure the biases for U. S. and Japanese agriculture. This was the starting point for the work described in the previous chapter to measure biases in the n-factor case. Once the biases are measured, the question of endogeneity or exogeneity can be answered. If the biases behave substantially alike in the two economies, they must have been exogenous, given the different behavior of economic variables in the two countries. If, however, they differ strongly, some sort of endogeneity must have prevailed.

Note that from mere inspection of the series it will not be possible to distinguish which economic elements in f (Equation (50)) have had the strongest impact on the biases. More detailed research and model building is necessary for that. Also note again that the measured biases probably are not due to technical change alone but to a complicated interaction of technical change, investment into human capital and other components of factor quality. This sets the stage to look back at the measured series of biases in Chapter 5.

Figures will be presented which compare the U. S. and Japanese series of biases for the factors, land, labor, machinery, and fertilizer. The graphs will also contain standardized series of the actual shares (except when the indices of the bias and of the actual share moved closely together). The comparison for other inputs is not shown because it contains no independent information due to the homogeneity constraints.

Figure 12 may be some help in interpreting the graphs. Suppose the line OA depicted the Japanese series of fertilizer bias, while the line OBC represented the fertilizer bias in the United States. Both series originate at t at a level of 100, which corresponds to the level of the actual factor share in each country at that time. The actual shares at t will in general be different between the countries. This initial difference is not explained. It may result from differences in factor prices at that time and from differences in biases which occurred prior to the investigation. The graph would tell us that Japan had experienced a fertilizer-using bias at constant rate during the entire period which would have tended to double the actual factor share if price changes had not deviated the actual share from that path. The United States, on the other hand, would first have experienced a fertilizer-saving bias with a corresponding tendency of the actual share to decline by 30 percent. After time t + i, however, the bias would have been positive with a rate (slope) equal to Japanese bias. The total impact of the

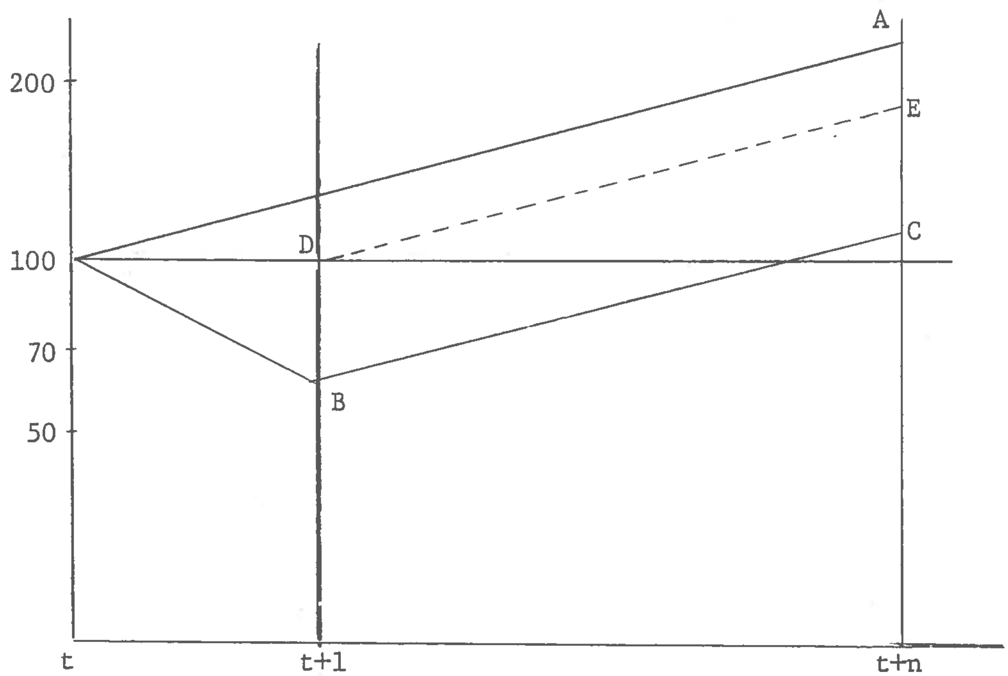

Figure 12. Example of graph, semilog scale

U. S. bias during the period would have been a tendency of the share to rise by 10 percent.

Suppose we only had the data from t + i to t + n. Then both series would originate at the level of 100 at t + i and be presented by an identical line DE with equal slope as the other ones. From that evidence alone we would conclude that both countries had experienced identical fertilizer biases, which would lead us to believe that the bias was exogenous. Given, however, the strong divergence of the biases between t and t + i, we would reach the opposite conclusion that biases have been endogenous, at least between t and t + i.

This example is given to show that equal development of biases for one factor share during a long period does not necessarily disprove the endogeneity hypothesis. The economic forces on that particular factor during that time might have been similar and caused similar biases. A strong case for exogeneity could, however, have been made if all biases showed similar slopes during most of the time.

By a similar reasoning, it is clear that neutrality of a bias for one factor alone does not mean that this neutrality is exogenous. If an induced innovation process has occurred long before t, the rate of return to further saving or using biases in this factor might have been no larger than the rate of return to neutral efficiency growth of this factor, i.e., the possibilities of further biases might have come close to exhaustion precisely because induced biases have previously been strong.

Turning now to the evidence (Figures 13, 14, 15 and 16), note that the Japanese series start in 1893 while the U. S. series start in 1912. Both series are standardized for the 1912 value of the actual share. Note also that there is a data break for the Japanese series from 1940 to 1954. While we still know what the total impact of the bias has been in this interval, possible departures from a straight line during this time are unknown.

From the evidence for land and labor alone, the conclusion would probably have been that, while biases did occur, they were of essentially the same nature in both countries during the period of overlap. This would have led to a conclusion that some exogenous force was at work. The only evidence for endogeneity would have been the following observations. In both countries the labor biases were labor-saving after World War II, which coincides with a strong wage rate rise in both countries. Also, the labor-saving bias in the United States was

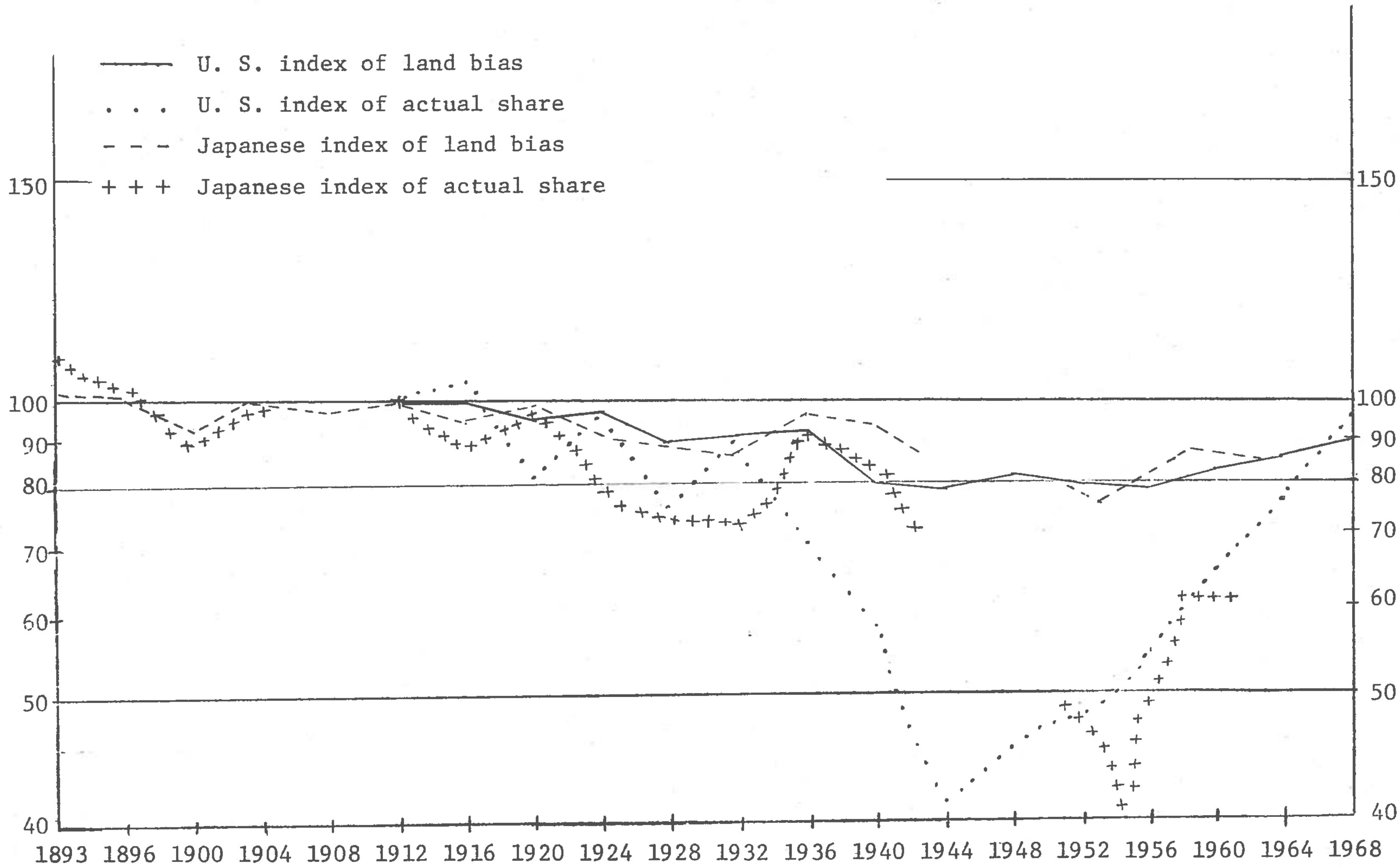

Figure 13. Comparison of U. S. and Japanese land biases, semilog scale, 1910-1912 = 100

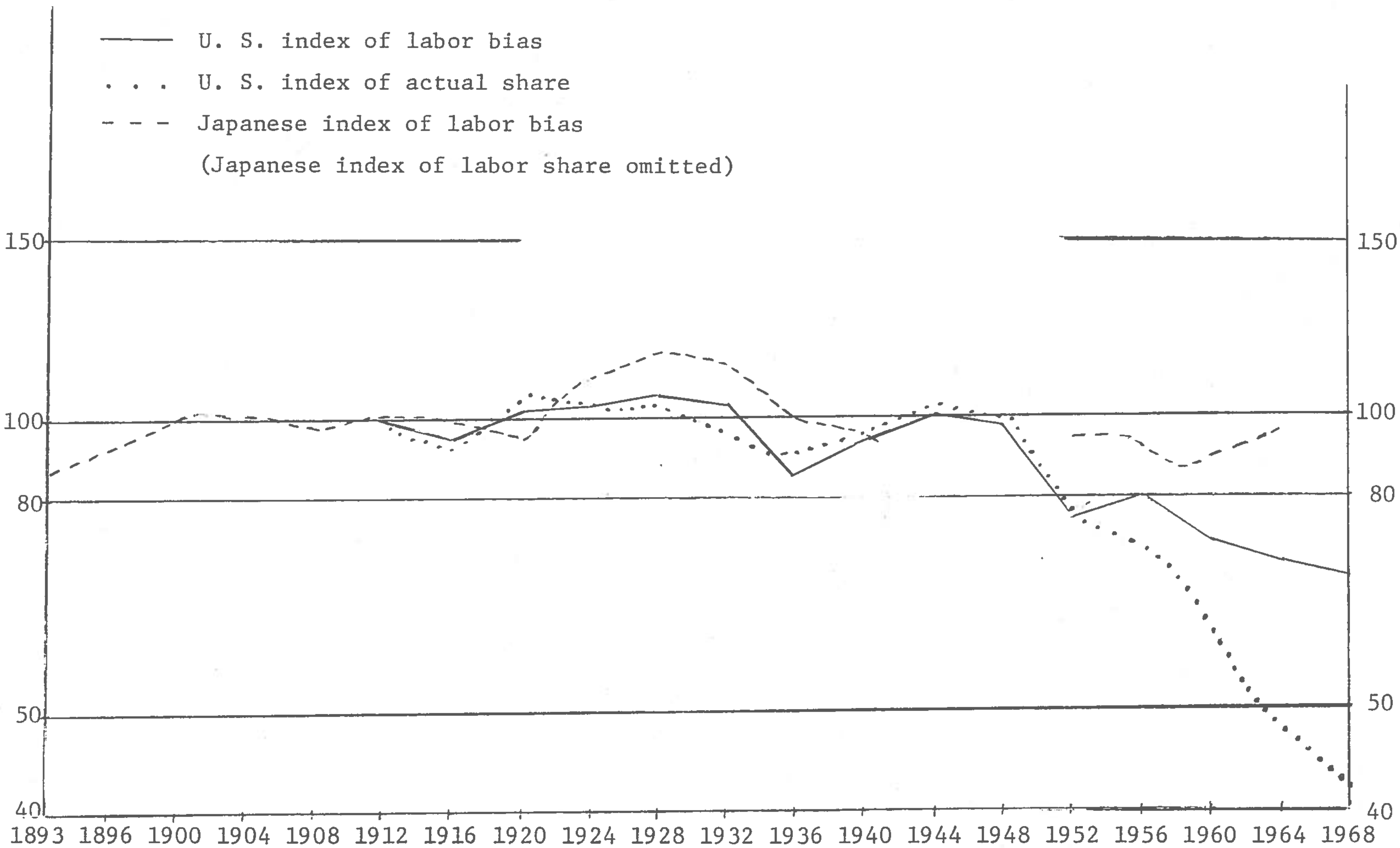

Figure 14. Comparison of U. S. and Japanese labor biases, semilog scale, 1910-1912 = 100

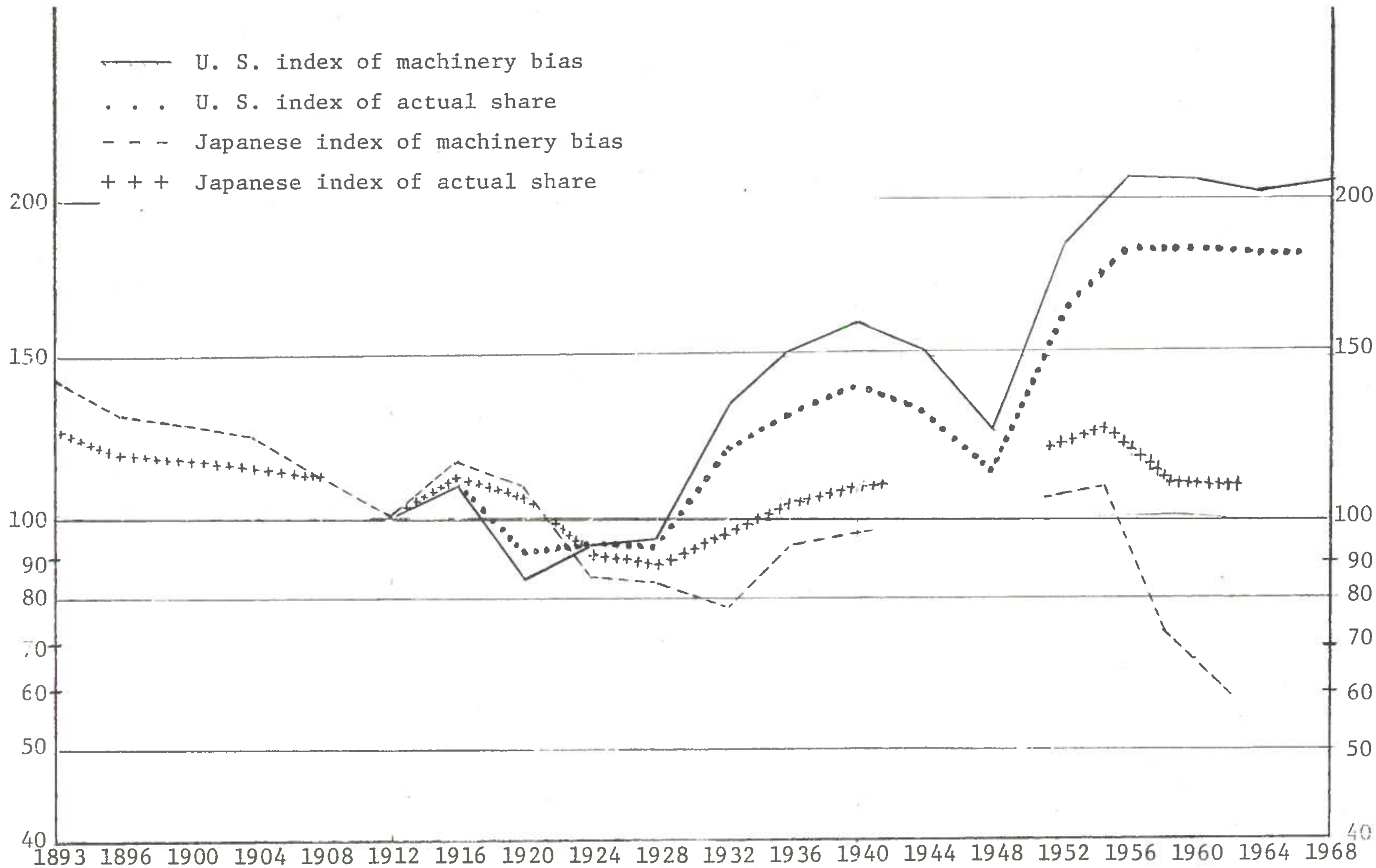

Figure 15. Comparison of U. S. and Japanese machinery biases, semilog scale, 1910-1912 = 100

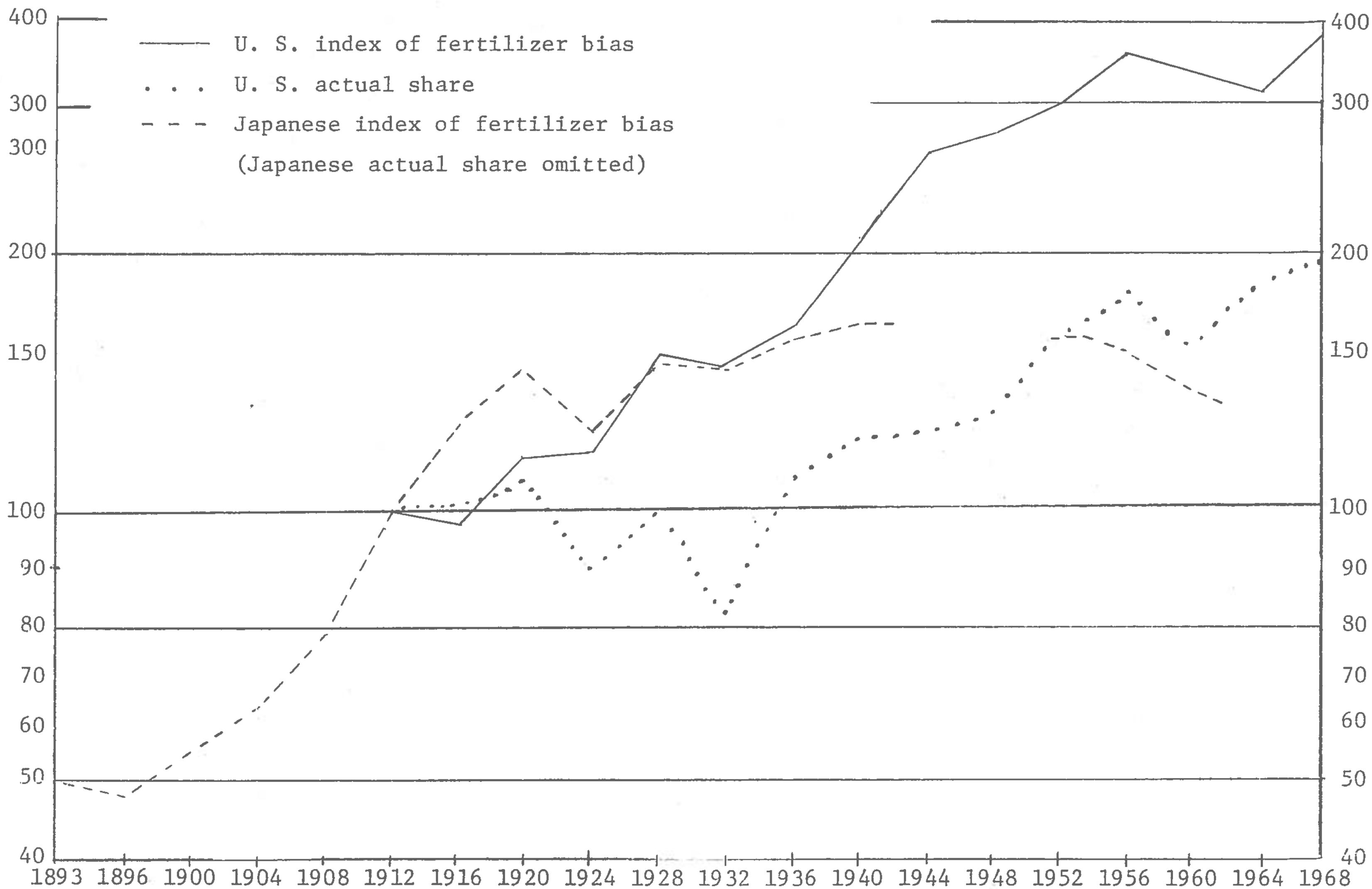

Figure 16. Comparison of U. S. and Japanese fertilizer biases, semilog scale, 1910-1912 = 100

much stronger than in Japan, which tends to confirm the endogeneity because labor price rises were stronger in the United States than in Japan during the 1950s. But this would be only weak evidence for the endogeneity hypothesis. Also the biases, contrary to what one might expect initially, were rather weak. A priori we might ahve expected a strong land-saving bias in Japan. The only ex post explanation that this did not occur might be that Japan started in 1893 already at a point where land-saving biases had occurred previously and driven the rate of return from further biases down or the cost of the biases up.[14]

Another point to be noticed in the series is that actual shares would have been poor indicators of biases. (The actual labor share for Japan is not shown because it practically coincides with the series for the Japanese bias.)

Turning now to the evidence from machinery and fertilizer, it becomes clear that the biases were endogenously determined to a very large extent. The United States experienced a strong machinery-using bias while Japan experienced a machinery-saving one. This is what would be expected from the induced innovation hypothesis.

The fertilizer biases strengthen this conclusion. From 1932 to 1962 the United States experienced a strong fertilizer-using bias while Japan had neutral fertilizer efficiency growth. This can only be

---

[14]The peculiar behavior of the U. S. land share should be noted. It declined by more than 50 percent between 1932 and 1944 only to increase again to its 1912 value after World War II. This dramatic change is almost entirely a price phenomenon and is not caused by corresponding biases. It is a nice demonstration of the strong influence of agricultural prices and policies on land rents.

explained if biases are endogenous. It is also interesting to note that the Japanese period of neutrality followed after a period of strong fertilizer-using bias. This lends support to the hypothesis that after a prolonged period of bias in one direction, further gains from biases become exhausted despite a further drop in the input price. This is turn suggests that not too much should be made of the almost neutrality of labor and land series.

How would one, a posteriori, explain the fact that biases were much weaker for labor and land (except for labor in the period after World War II)? Both fertilizer and machinery went through strong structural changes in their form and production methods. Before the period under consideration, the source of plant nutrients was primarily organic fertilizer produced by farmers themselves. Chemical fertilizer changed the form of plant nutrients and their production takes place outside the farm sector. This releases farm labor for other purposes. Essentially the same is true for machinery. This factor consisted originally of draft animals and tools and implements which could be produced on the farm or in small-scale rural industry. Toward the end of the period, mechanical traction replaced the animals, and the tools and machinery became more complex and were produced largely outside the farm economy. It seems clear that the strongest biases would be expected with respect to factors undergoing such strong changes. But this is an a posteriori explanation which was not considered at the outset.

Another conclusion can be made from the series. Where strong biases occurred, the absolute difference between the Japanese and the U. S. series is equal to or larger than the larger of the absolute biases. This means that the total extent of the large biases must be explained by endogenous forces rather than a fundamental bias in any direction. This not only strengthens the endogeneity hypothesis but means that endogenous biases are empirically important in explaining shares and wage rates of factors.

Conclusions with respect to the precise inducement mechanisms are all negative. Which element is most important in terms of function (49), the factor prices, interest rates, size of markets, or cost of obtaining the innovation? Data are only available on factor prices and factor shares and both fail if considered to be the sole empirically relevant source of bias. From the graphs, no clear relationship emerges between actual shares and biases. Turning back to Figures 4 and 8 on prices, we see that the price of fertilizer relative to the output price declines dramatically in both countries. This is consistent with the observed biases. The strong increase of labor prices after World War II in both countries is consistent with labor-saving biases during that period as well. But the puzzle lies in the behavior of machinery prices and the machinery biases--machinery prices rose as fast in the United States as labor prices while they declined in Japan. But it was the United States which experienced machinery-using bias while Japan experienced a machinery-saving bias. If factor prices had been the single most important factor-determining biases, this could not have happened. Explanations might be found in the behavior of interest rates or the absolute size of the markets for machinery in dollar terms, for example.

CHAPTER 7
CONCLUSIONS

This chapter briefly summarizes the empirical and theoretical conclusions of this thesis for the induced innovation hypothesis and tries to show some policy implications.

The comparison of the biases in the agricultural sectors of Japan and the United States shows that the biases are endogenous to a very large extent. This does not mean that advances in basic sciences are unimportant. Without such advances the fertilizer-using biases in both countries would not have been possible. But the basic sciences are only a necessary condition for technical change. They leave the options open as to the timing of technical change and the direction of the biases which are determined by economic forces.

Can this conclusion be generalized to the economy as a whole? Sato's work on measuring biases of the U. S. private nonfarm sector seems to support this. The labor-saving biases which he measured are clearly consistent with induced innovation because the rise in wage rates has been one of the most important features of recent U. S. economic history. So the burden of the proof is now on those who argue that biases are exogenously determined.

On the other hand, it has not been possible to find out how the different economic variables interact in determining the biases. Simple hypotheses that just one set of variables is all important seem to be doomed to failure. Therefore, it will be necessary to build a better formal model of induced innovation capable of generating refutable empirical hypotheses. Such a model will have to be an investment model.

Ahmad's graphical technique is unable to take into account the time dimension of the costs and benefits of efficiency gains. I also believe that attempts to generalize Kennedy's innovation possibility frontier will not lead anywhere for the following reasons. Factor-augmenting technical change has a tremendous appeal because of its mathematicl simplicity. Changes in the factor-augmenting coefficients have the same effect on output as an equiproportional increase in the corresponding factor of production. All one has to know is the changes in the factor-augmenting coefficients and the parameters of the production function to determine what will happen to output. But the problem with this approach is that while we may be able to measure the changes in factor-augmenting coefficients *a posteriori*, we have no way to know how they have been generated. Have they been due to investments in human capital, quality improvements in capital equipment or intermediate inputs, new production techniques or organizational improvements? There is no simple relationship between any one of these changes and particular augmentation coefficients. Human capital affects not only the augmentation coefficient of labor, but all cooperating factors; however, we do not know to what extent. The same holds for new production techniques, *etc*.[15] In Kennedy's framework the benefit of efficiency gains is in the augmentation of the factors. But the cost is in some real investment activity. Any businessman or economist could not answer *a priori* the question of which economic activity or investment leads to labor-augmenting technical change. And unless we

---

[15] A good example is a new seed variety. The efficiency gain is embodied in the new seeds. Unless you have the new seeds, there is no access to the efficiency gain. But not only the augmentation coefficients of the seeds will be altered, but also the augmentation coefficients of all cooperating factors, and probably in varying degrees.

know that, there is no way in which useful policy guidelines can come out of a factor-augmenting induced innovation hypothesis.

The task of building an investment model will be further complicated by the presence of externalities in most activities which lead to efficiency gains.

With respect to development policy, the conclusion of this paper strengthens the conclusions of Schultz (1964) because optimal technology is clearly shown to be location specific. Unless the less devleoped countries are able to set up institutions which are capable of responding to local factor scarcities in developing and distributing modern production inputs, they will not be able to achieve growth. Also, factor prices should be such that producers are given the incentives to adopt these locally developed production methods. Some imitation of production methods of more advanced countries is, of course, possible and desirable. However, the countries will be more successful if they copy methods from countries which have had factor endowments similar to their own rather than from the western countries which are rich in physical and human capital.

LIST OF REFERENCES

Ahmad, S. 1966. On the theory of induced invention. Economic Journal 76(302):344-357.

Allen, R. G. D. 1938. Mathematical Analysis for Economists. St. Martin's Press, Inc., New York, New York.

Bank of Japan. 1966. Hundred Year Statistics of the Japanese Economy. Tokyo, Japan.

Denison, E. F. 1969. Some major issues in productivity analysis: An examination of estimates by Jorgenson and Griliches, pp. 1-27. In Survey of Current Business, 49(5, Part II), U. S. Government Printing Office, Washington, D. C.

Diewert, W. E. 1971. An application of the Shephard duality theorem: A generalized Leontief production function. Journal of Political Economy 70(3):481-505.

Drandakis, E. M. and E. S. Phelps. 1966. A model of induced invention, growth and distribution. Economic Journal 76(304):823-840.

Ethier, W. 1971. General equilibrium and the concept of effective protection, pp. 17-44. In H. G. Grubel and H. G. Johnson (eds.), Effective Tariff Protection, GATT, Geneva, Switzerland.

Farrell, M. J. 1957. The measurement of productive efficiency. Journal of the Royal Statistical Society, Series A (General), 120(3):253-290.

Fellner, W. 1971. Empirical support for the theory of induced innovation. Quarterly Journal of Economics 85(4):580-604.

Fishelson, G. 1968. Returns to human and research capital, United States agriculture, 1949-1964. Unpublished Ph.D. thesis, Department of Economics, North Carolina State University, Raleigh. University Microfilms, Ann Arbor, Michigan.

Griliches, Z. 1957. Specification bias in estimates of production functions. Journal of Farm Economics 39(1):8-20.

Griliches, Z. 1964. Research expenditures, education and the aggregate agricultural production function. American Economic Review 54:961-974.

Hanoch, G. 1970. Generation of new production functions through duality. Discussion Paper 118, Harvard Institute of Economic Research, Harvard University, Cambridge, Massachusetts.

Hayami, Y. and V. W. Ruttan. 1970. Factor prices and technical changes in agricultural development: The United States and Japan, 1880-1960. Journal of Political Economy 78(5):1115-1141.

LIST OF REFERENCES (continued)

Hayami, Y. and V. W. Ruttan. 1971. Agricultural Development: An International Perspective. Johns Hopkins Press, Baltimore, Maryland.

Hicks, J. R. 1964. The theory of wages. Macmillan & Company, Ltd., London, England.

Hoover, D. 1961. Land prices in United States agriculture, 1910-1950. Unpublished Ph.D. thesis, Department of Economics, University of Chicago, Chicago, Illinois. University Microfilms, Ann Arbor, Michigan.

Jorgenson, D. W. and Z. Griliches. 1967. The explanation of productivity changes, Review of Economic Studies 34(3):249-283.

Kennedy, C. 1964. Induced bias in innovation and the theory of distribution. Economic Journal 74:541-547.

Mundlak, Y. 1968. Elasticities of substitution and the theory of derived demand. Review of Economic Studies 35(2):225-236.

Nadiri, M. I. 1970. Some approaches to the theory and measurement of total factor productivity. Journal of Economic Literature 8(4): 1137-1177.

Nakamura, T. I. 1966. Agricultural Production and the Economic Development of Japan, 1873-1922. Princeton University Press, Princeton, New Jersey.

Nelson, R. R. 1959a. The economics of invention. The Journal of Business 32(2):101-127.

Nelson, R. R. 1959b. The simple economics of basic scientific research. The Journal of Political Economy 67(3):297-307.

Okawa, K., M. Shinohara and M. Umemura, (eds.). 1966. Estimates of long term economic statistics of Japan since 1868. Toyo Keizai Shinposha, Tokyo, Japan.

Salter, W. E. G. 1960. Productivity and Technical Change. Cambridge University Press, Cambridge, England.

Samuelson, P. A. 1965a. Foundations of Economic Analysis. Atheneum, New York, New York.

Samuelson, P. A. 1965b. A theory of induced innovation along Kennedy-Weisäcker lines. Review of Economics and Statistics 47:343-356.

Samuelson, P. A. 1966. A theory of induced innovation along Kennedy-Weisäcker lines: Rejoinder. Review of Economics and Statistics, 48:444.

Sato, R. 1970. The estimation of biased technical progress. International Economic Review 11(2):179-207.

Schmookler, J. 1966. Invention and Economic Growth. Harvard University Press, Cambridge, Massachusetts.

Schultz, T. W. 1964. Transforming Traditional Agriculture. Yale University Press, New Haven, Connecticut.

Solow, R. M. 1957. Technical change and the aggregate production function. Review of Economics and Statistics 39:312-320.

Solow, R. M. 1967. Some recent developments in the theory of production, pp. 55-76. In M. Brown (ed.), The Theory and Empirical Analysis of Production. National Bureau of Economic Research, Studies in Income and Wealth, Vol. 31.

Theil, H. 1971. Principles of Econometrics. John Wiley & Sons, Inc., New York, New York.

Timmer, C. P. 1970. On measuring technical efficiency. Food Research Institute Studies in Agricultural Economics, Trade and Development, Vol. 9, No. 2.

Triangle Universities Computing Center. 1972. Program Manual for Two and Three Stage Least Squares. Research Triangle Park, North Carolina.

U. S. Bureau of the Census. 1950-1964. U. S. census of agriculture, 1950, 1954, 1959, 1964. U. S. Government Printing Office, Washington, D. C.

U. S. Department of Agriculture. 1936-1972. Agricultural statistics. U. S. Government Printing Office, Washington, D. C.

U. S. Department of Agriculture. 1945-1972. Farm labor. U. S. Government Printing Office, Washington, D. C.

U. S. Department of Agriculture. 1953-1961. Farm real estate market. U. S. Government Printing Office, Washington, D. C.

U. S. Department of Agriculture. 1954-1972. Farm income situation. July issue and annual supplement to July issue. U. S. Government Printing Office, Washington, D. C.

U. S. Department of Agriculture. 1958-1971. Changes in farm production and efficiency. Statistical Bulletin 233. U. S. Government Printing Office, Washington, D. C.

U. S. Department of Agriculture. 1961-1972. Farm real estate market developments. U. S. Government Printing Office, Washington, D. C.

Ozawa, H. 1962. Production functions with constant elasticities of substitution. Review of Economic Studies 29:291-299.

Wallace, T. D. and V. G. Ashar. 1972. Sequential methods in model construction. Review of Economics and Statistics 54(2):172-178.

Zellner, A. 1962. An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias. Journal of the American Statistical Association 57:348-368.

Zellner, A. 1963. Estimators for seemingly unrelated regression equations: Some exact finite sample results. Journal of the American Statistical Association 58:977-992.

APPENDICES

Appendix A. Estimation of Factor-Augmenting Coefficients with n-Factors: General Case

Let capital letters denote variables and their quantities, while lower case letters denote the logarithms of the variables. The production function is specified in factor-augmenting form as follows:

$$Y = \phi(Z_1, Z_2, \ldots, Z_n) = \phi[(A_1X_1), (A_2X_2), \ldots, (A_nX_n)], \quad (A.1)$$

where $Z_i = A_iX_i$ is factor i in augmented units, A_i is the augmentation coefficient of factor i, and X_i = quantity of factor i.

Total per unit costs are

$$U = \sum_i X_iW_i = \sum_i (A_iX_i)(\frac{W_i}{A_i}) = \sum_i Z_iR_i \ , \quad (A.2)$$

where $R_i = \frac{W_i}{A_i}$ is the price concept corresponding to the augmented units.

Set up a Lagrangean expression for cost minimization per unit of output.

$$G = \sum_i Z_iR_i - \Lambda[\phi(Z_i, \ldots, Z_n) - 1] \ . \quad (A.3)$$

This is an "as if" approach since producers do not know the Z_i and R_i, but only the X_i and W_i. Why they do not need to know these parameters is discussed on pages 9 and 10.

The first order conditions of (A.3) are as follows.

$$\phi(Z_1, \ldots, Z_n) = 1 \quad (A.4)$$

and

$$\Lambda\phi_i = R_i \qquad i = 1,\ldots,n \quad (A.5)$$

where

$$\phi_i = \frac{\partial \phi}{\partial Z_i} \quad .$$

To solve for the displacement, differentiate all n + 1 equations totally.

$$\frac{\partial \phi}{\partial Z_1} dZ_1 + \frac{\partial \phi}{\partial Z_2} dZ_2 + \ldots + \frac{\partial \phi}{\partial Z_n} dZ_n = 0$$

or

$$\sum_j \phi_j \, dZ_j = 0 \; .$$

Dividing the whole equation by Y and multiplying each summand by $\frac{Z_j}{Z_j}$,

$$\sum_j \phi_j \frac{Z_j}{Y} \frac{dZ_j}{Z_j} = 0$$

or

$$\sum_j \psi_j \, dz_j = 0 \; , \qquad \text{(A.6)}$$

where $\psi_j = \frac{\partial \phi}{\partial Z_j} \frac{Z_j}{Y}$ is the output elasticity of factor j in augmented units, and $dz_j = d \log Z_j$. The total differential of (A.5) is:

$$d\Lambda \, \phi_i + \sum_j \Lambda \frac{\partial \phi_i}{\partial Z_j} dZ_j = dR_i \qquad i = 1, \ldots, n \; . \qquad \text{(A.7)}$$

Dividing the equation by Λ and ϕ_i and each element in the sum by $\frac{Z_j}{Z_j}$ and the right-hand side by $\frac{R_i}{R_i}$,

$$\frac{d\Lambda}{\Lambda} + \sum_j \frac{\partial \phi_i}{\partial Z_j} \frac{Z_j}{Z_j} \frac{1}{\phi_i} dZ_j = \frac{dR_i}{R_i} \frac{R_i}{\Lambda \phi_i} .$$

By (A.5) $\frac{R_i}{\Lambda \phi_i}$ is equal to 1, so we can rewrite:

$$d\lambda + \sum_j \psi_{ij} \, dz_j = dr_i \qquad i = 1, \ldots, n \quad , \tag{A.8}$$

where everything now is in proportional changes and elasticity form and

$$d\lambda = d \ln \Lambda \qquad \text{and} \qquad dr_i = d \ln R_i$$

and

$$\psi_{ij} = \frac{\partial \left(\frac{\partial \phi}{\partial Z_i}\right)}{\partial Z_j} \cdot \frac{Z_j}{\frac{\partial \phi}{\partial Z_i}} = \frac{\partial \phi_i}{\partial Z_j} \frac{Z_j}{\phi_i} .$$

Rewriting (A.6) and (A.8) in matrix form, we have the basic system out of which all estimating equations will come.

$$\begin{bmatrix} 0 & \psi_1 & \psi_2 & \cdots\cdots\cdots & \psi_n \\ 1 & \psi_{11} & \psi_{12} & \cdots\cdots\cdots & \psi_{1n} \\ 1 & \psi_{21} & \psi_{22} & \cdots\cdots\cdots & \psi_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \psi_{n1} & \psi_{n2} & \cdots\cdots\cdots & \psi_{nn} \end{bmatrix} \begin{bmatrix} d\lambda \\ dz_1 \\ dz_2 \\ \vdots \\ dz_n \end{bmatrix} = \begin{bmatrix} 0 \\ dr_1 \\ dr_2 \\ \vdots \\ dr_n \end{bmatrix} , \tag{A.9}$$

or in shorter form

$$[\psi] \begin{bmatrix} d\lambda \\ \hline dz \end{bmatrix} = \begin{bmatrix} 0 \\ \hline dr \end{bmatrix} , \tag{A.10}$$

where ψ is $(n+1) \times (n+1)$ and is a transformation of the bordered Hessian matrix to elasticity form. The terms dz and dr are (nx1) vectors. The term dz_i is the proportional change of the product A_iX_i and therefore $dz_i = dx_i + da_i$. The term dr_i is the proportional change in the quotient $\frac{W_i}{A_i}$ and therefore $dr_i = dw_i - da_i$.

We now can write (A.10) as follows:

$$\psi \begin{bmatrix} d\lambda + 0 \\ dx + da \end{bmatrix} = \begin{bmatrix} 0 - 0 \\ dw - da \end{bmatrix} . \tag{A.11}$$

Matrix multiplication is associative, hence

$$\psi \begin{bmatrix} d\lambda \\ dx \end{bmatrix} + (\psi + I) \begin{bmatrix} 0 \\ da \end{bmatrix} = \begin{bmatrix} 0 \\ dw \end{bmatrix} ,$$

and

$$\begin{bmatrix} 0 \\ da \end{bmatrix} = (\psi + I)^{-1} \begin{bmatrix} 0 \\ dw \end{bmatrix} - (\psi + I)^{-1} \psi \begin{bmatrix} d\lambda \\ dx \end{bmatrix} . \tag{A.12}$$

Now

$$(\psi + I)^{-1}(\psi + I) = I ,$$

$$(\psi + I)^{-1}\psi + (\psi + I)^{-1} = I ,$$

$$(\psi + I)^{-1}\psi = I - (\psi + I)^{-1} . \tag{A.13}$$

Substituting (A.13) into (A.12)

$$\begin{bmatrix} 0 \\ da \end{bmatrix} = (\psi + I)^{-1} \begin{bmatrix} 0 \\ dw \end{bmatrix} + (\psi + I)^{-1} \begin{bmatrix} d\lambda \\ dx \end{bmatrix} - \begin{bmatrix} d\lambda \\ dx \end{bmatrix} ,$$

and collecting terms

$$\begin{bmatrix} 0 \\ da \end{bmatrix} = (\psi + I)^{-1} \begin{bmatrix} 0 + d\lambda \\ dw + dx \end{bmatrix} - \begin{bmatrix} d\lambda \\ dx \end{bmatrix}. \tag{A.14}$$

Out of time series we will have data for dw and dx and $d\lambda$ (the proportional change in marginal cost). If the values of the elements of $(\psi + I)^{-1}$ are known, the values of da are found simply by substituting into (A.14).

The nature of technical change is judged by solving for dx^*, the proportional change of inputs per unit of output, given that factor prices had remained constant. Using (A.12) and (A.13)

$$\begin{bmatrix} d\lambda^* \\ \hline dx^* \end{bmatrix} = -\begin{bmatrix} I - (\psi + I)^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ da \end{bmatrix}. \tag{A.15}$$

The biases between each factor pair are found solving for $B_{ij} = \dfrac{dx_i^*}{dx_j^*}$ where B_{ij} is a measure of the bias.

Evidently it is impossible to estimate the values of the $(\psi + I)^{-1}$ matrix at the same time as the values of da. There are only n + 1 equations but n values of da and $(n + 1) \times (n + 1)$ values of the $(\psi + I)$ matrix to be estimated. In the literature this fact is known as the impossibility theorem of factor-augmenting technical change. Hence, the values of the $(\psi + I)^{-1}$ matrix have to be found from another relationship.

It is obvious that if we knew the values of the ψ^{-1} matrix, we could obtain from it the values of the $(\psi + I)^{-1}$ matrix from the computer.

Consider the system (A.11). In a cross section we may assume that at a particular moment of time, all da are equal to zero. Therefore, the system collapses to the following one:

$$\psi \begin{bmatrix} d\lambda \\ dx \end{bmatrix} = \begin{bmatrix} 0 \\ dw \end{bmatrix} . \tag{A.16}$$

But this is also the system which one obtains by setting up the Lagrangean expression in natural units when technical change does not occur and deriving the displacements from equilibrium (Samuelson, 1965a). This shows the validity of the "as if" approach in the Lagrangean expression (A.3).

Solving (A.15), we have

$$\begin{bmatrix} d\lambda \\ dx \end{bmatrix} = \psi^{-1} \begin{bmatrix} 0 \\ dw \end{bmatrix} , \tag{A.17}$$

and the elements of the ψ^{-1} matrix can be estimated in this system. But for this, the economic meaning of the elements of the ψ^{-1} matrix must be known. The marginal products of the factors X_i in the production function (A.1) are related to ψ_i as follows. Let

$$f_i = \frac{\partial Y}{\partial X_i} .$$

Then

$$\frac{\partial Y}{\partial X_i} \frac{X_i}{Y} = \frac{\partial Y}{\partial Z_i} \frac{\partial Z_i}{\partial X_i} \frac{X_i}{Y} = \phi_i \frac{A_i X_i}{Y} = \psi_i . \tag{A.18}$$

The second derivatives with respect to the factors X_i are related to ψ_{ij} as follows. Let

$$f_{ij} = \frac{\partial^2 Y}{\partial X_i \partial X_j} = \frac{\partial f_i}{\partial X_j}$$

and

$$\phi_{ij} = \frac{\partial^2 Y}{\partial Z_i \partial Z_j} = \frac{\partial \phi_i}{\partial Z_j} .$$

Then

$$\frac{\partial f_i}{\partial X_j} \frac{X_j}{f_i} = \frac{\partial(\phi_i)A_i}{\partial Z_j} \frac{\partial Z_j}{\partial X_j} \frac{X_j}{\phi_i A_i} = \phi_{ij} \frac{A_j X_j}{\phi_i} = \psi_{ij} . \qquad \text{(A.19)}$$

This proves the elementwise identity of ψ in the systems (A.10) and (A.16). Therefore, the meaning of the elements of ψ^{-1} can be established entirely in the system (A.16).

Let f be the bordered Hessian matrix of the production function with respect to the factors in natural units,

$$f = \begin{bmatrix} 0 & f_1 & f_2 & \cdots\cdots & f_n \\ f_1 & f_{11} & f_{12} & \cdots\cdots & f_{1n} \\ \vdots & & & & \\ f_n & f_{n1} & f_{n2} & \cdots\cdots & f_{nn} \end{bmatrix}$$

and $F = \det[f]$.

Let $\left[F_{ij}\right]$ be the corresponding matrix of cofactors of f.

$$[F_{ij}] = \begin{bmatrix} F_{00} & F_{01} & \cdots\cdots & F_{0n} \\ F_{10} & F_{11} & \cdots\cdots & F_{1n} \\ \vdots & & & \vdots \\ F_{n0} & F_{n1} & \cdots\cdots & F_{nn} \end{bmatrix} .$$

Recall from (A.9)

$$[\psi] = \begin{bmatrix} 0 & \psi_1 & \psi_2 & \cdots\cdots\cdots & \psi_n \\ 1 & \psi_{11} & \psi_{12} & \cdots\cdots\cdots & \psi_{1n} \\ \vdots & & & & \vdots \\ 1 & \psi_{n1} & \psi_{n2} & \cdots\cdots\cdots & \psi_{nn} \end{bmatrix},$$

or expanded,

$$[\psi] = \begin{bmatrix} 0\ , & f_1 \frac{X_1}{Y}\ , & f_2 \frac{X_2}{Y}\ , & \cdots\cdots\cdots, & f_n \frac{X_n}{Y} \\ \frac{f_1}{f_1}\ , & f_{11} \frac{X_1}{f_1}\ , & f_{12} \frac{X_2}{f_1}\ , & \cdots\cdots\cdots, & f_{1n} \frac{X_n}{f_1} \\ \frac{f_2}{f_2}\ , & f_{21} \frac{X_1}{f_2}\ , & f_{22} \frac{X_2}{f_2}\ , & \cdots\cdots\cdots, & f_{2n} \frac{X_n}{f_2} \\ \vdots & & & & \\ \frac{f_n}{f_n}\ , & f_{n1} \frac{X_1}{f_n}\ , & f_{n2} \frac{X_2}{f_n}\ , & \cdots\cdots\cdots, & f_{nn} \frac{X_n}{f_n} \end{bmatrix}.$$

Row zero is divided by Y. Rows 1 to n are divided by f_i, respectively. Columns 1 to n are multiplied by X_j, respectively. ψ is the bordered Hessian matrix in elasticity form and is the same whether with respect to X or to Z. It can be proved that, if f is of full rank, so is ψ.

Let Ψ be det $[\psi]$ and $[\Psi_{ij}]$ the matrix of the cofactors corresponding to ψ.

$$[\Psi_{ij}] = \begin{bmatrix} \Psi_{00} & \Psi_{01} & \cdots\cdots\cdots & \Psi_{0n} \\ \Psi_{10} & \Psi_{11} & \cdots\cdots\cdots & \Psi_{1n} \\ \vdots & \vdots & & \vdots \\ \Psi_{n0} & \Psi_{n1} & \cdots\cdots\cdots & \Psi_{nn} \end{bmatrix} .$$

Hence,

$$\Psi = \det \psi = \frac{1}{Y} \frac{X_1 \quad \cdots\cdots\cdots \quad X_n}{f_1 \quad \cdots\cdots\cdots \quad f_n} F , \tag{A.20}$$

$$\Psi_{ij} = \frac{1}{Y} \frac{X_1 \ \cdots \ X_{j-1} X_{j+1} \quad \cdots\cdots\cdots \quad X_n}{f_1 \ \cdots \ f_{i-1} f_{i+1} \quad \cdots\cdots\cdots \quad f_n} F_{ij} , \tag{A.21}$$

$$\psi^{-1} = \frac{[\Psi_{ij}]^T}{\Psi} . \tag{A.22}$$

Allen (1938, p. 504) defines partial elasticities of substitution as follows:

$$\sigma_{ij} = \frac{X_1 f_1 + X_2 f_2 + \ldots + X_n f_n}{X_i X_j} \quad \frac{F_{ij}}{F} \tag{A.23}$$

Therefore,

$$\sigma_{ij} = \frac{\sum X_i f_i}{X_i X_j} \frac{\frac{1}{Y} \frac{X_1 \cdots\cdots X_n}{f_1 \cdots\cdots f_n}}{\frac{1}{Y} \frac{X_1 \cdots X_{j-1} X_{j+1} \cdots X_n}{f_1 \cdots f_{i-1} f_{i+1} \cdots f_n}} \frac{\Psi_{ij}}{\Psi}$$

$$= \frac{\sum X_i f_i}{X_i f_i} \frac{\Psi_{ij}}{\Psi} = \frac{1}{\alpha_i} \frac{\Psi_{ij}}{\Psi} , \qquad \text{(A.24)}$$

where α_i is the share of factor i.

But $\frac{[\Psi_{ij}]^T}{\Psi}$ is the ji element of ψ^{-1}, i, j $\neq$ 0, hence,

$$[\psi^{-1}]_{ij} = \alpha_j \sigma_{ij} \qquad , \text{ i, j} \neq 0 . \qquad \text{(A.25)}$$

Solving the system (A.14) by Cramer's rule for dx_k reveals the economic meaning:

$$dx_k = \frac{\Psi_{0k} dy + \sum_{i=1}^{n} \Psi_{ik} dw_i}{\Psi} . \qquad \text{(A.26)}$$

Solving this for $\frac{\partial x_k}{\partial w_i}$, the elasticity of derived demand for input k with respect to a change in the price of input i, and by (A.25),

$$\eta_{ki} = \frac{\partial x_k}{\partial w_i} = \frac{\Psi_{ik}}{\Psi} = \alpha_i \sigma_{ik} . \qquad \text{(A.27)}$$

From (A.26) we also have the reciprocal of the output elasticity with respect to each input

$$\frac{1}{\varepsilon_{ky}} = \frac{\partial x_k}{\partial y} = \frac{\Psi_{0k}}{\Psi} = [\Psi^{-1}]_{k0} \quad . \tag{A.28}$$

Solving (A.14) by Cramer's rule for $d\lambda$,

$$d\lambda = \frac{\Psi_{00}dy + \sum \Psi_{i0}dw_i}{\Psi} , \tag{A.29}$$

$$\frac{d\lambda}{dy} = \frac{\Psi_{00}}{\Psi} = [\psi^{-1}]_{00} \quad .$$

But it can be proved that Λ is marginal cost. Under competition, $P = MC$. Therefore, $d\lambda = dp = \frac{dP}{P}$. Now $\frac{\partial\lambda}{\partial y}$ is the proportional change in the scale of output with constant returns to scale where the homogeneity parameter $\gamma = 1$, $\frac{\partial\lambda}{\partial y} = 0$. Therefore,

$$\frac{\partial\lambda}{\partial y} = [\psi^{-1}]_{00} = 1 - \gamma \quad . \tag{A.30}$$

Also from (A.29),

$$\frac{\partial\lambda}{\partial w_i} = \frac{\Psi_{i0}}{\Psi} = [\psi^{-1}]_{0j} \quad .$$

But under homogeneity of degree 1, $\Lambda = P$. Therefore,

$$\frac{\partial\lambda}{\partial w_i} = \frac{\partial P}{\partial W_i} \cdot \frac{W_i}{P} \quad .$$

It can be proved that $\frac{\partial\Lambda}{\partial W_i} = X_i$ (Samuelson, 1965a). Hence

$$\frac{\partial\lambda}{\partial w_i} = [\psi^{-1}]_{0j} = \frac{X_i W_i}{P} = \alpha_i \quad . \tag{A.31}$$

Summarizing from (A.27), (A.28), (A.30) and (A.31),

$$\psi^{-1} = \begin{bmatrix} 1-\gamma & \alpha_1 & \alpha_2 & \cdots\cdots & \alpha_n \\ \frac{1}{\varepsilon_{1y}} & \alpha_1\sigma_{11} & \alpha_2\sigma_{12} & \cdots\cdots & \alpha_n\sigma_{1n} \\ \frac{1}{\varepsilon_{2y}} & \alpha_1\sigma_{21} & \alpha_2\sigma_{22} & \cdots\cdots & \alpha_n\sigma_{2n} \\ \vdots & & & & \vdots \\ \frac{1}{\varepsilon_{ny}} & \alpha_1\sigma_{n1} & \alpha_2\sigma_{n2} & & \alpha_n\sigma_{nn} \end{bmatrix} = \begin{bmatrix} 1-\gamma & \alpha_1 & \alpha_2 & \cdots\cdots & \alpha_n \\ \frac{1}{\varepsilon_{1y}} & \eta_{11} & \eta_{12} & \cdots\cdots & \eta_{1n} \\ \frac{1}{\varepsilon_{2y}} & \eta_{21} & \eta_{22} & \cdots\cdots & \eta_{2n} \\ \vdots & & & & \\ \frac{1}{\varepsilon_{ny}} & \eta_{n1} & \eta_{n2} & & \eta_{nn} \end{bmatrix} .$$

This means that (A.17) can be estimated in a system of linear factor demand equations where the coefficients are estimates of factor demand elasticities. This assumes, however, that these elasticities are constant which is not the case except in the Cobb-Douglas case. If more than local approximation is desired, a specific functional form has to be chosen, which will generally alter the form of the estimation equation such that the parameters of that particular function can be stable.

Appendix B. Restricted Generalized Least Squares (RGLS)

This appendix lists the estimators and test statistics used, following Theil (1971).

$$\text{Let } Y_i = \begin{bmatrix} Y_{i1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Y_{iT} \end{bmatrix}_{T \times 1}, \qquad \varepsilon_i = \begin{bmatrix} \varepsilon_{i1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{iT} \end{bmatrix}_{T \times 1},$$

the vectors of T observations on the share i in one particular cross section, and the corresponding error terms.

$$\beta_i = \begin{bmatrix} \beta_{i1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_{im} \end{bmatrix}_{m \times 1},$$

the vector of coefficients of the m exogenous variables (including the intercept).

Let X_i be the T x m observation matrix on the exogenous variables. In each cross section, all X_i are equal for all shares, say $\bar{X}$, but if several cross sections are involved, the X differ.

$$b = \begin{bmatrix} b_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix}_{nm \times 1} = \begin{bmatrix} (X_1'X_1)^{-1} \quad X_1'Y_1 \\ \\ \\ (X_n'X_n)^{-1} \quad X_n'Y \end{bmatrix} . \qquad (B.1)$$

This is the OLS estimator of the coefficients of all equations in a particular system.[16]

$$\text{Let } Y = \begin{bmatrix} Y_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Y_n \end{bmatrix}_{nT \times 1}, \quad \beta = \begin{bmatrix} \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_n \end{bmatrix}_{nm \times 1}, \quad \text{and } \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_n \end{bmatrix}_{nT \times 1}.$$

The nT-element disturbance vector has the variance-covariance matrix

$$V = E(\varepsilon'\varepsilon) = \Omega \otimes I , \qquad \text{(B.2)}$$

where Ω is the n x n variance-covariance matrix of the error term of the n shares in the system.

$$\text{Let } X = \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & X_n \end{bmatrix} \quad \text{or } X = \begin{bmatrix} \bar{X} & 0 & \dots & 0 \\ 0 & \bar{X} & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & \bar{X} \end{bmatrix},$$

depending on whether the system contains data for several cross sections. The unconstrained GLS estimator then is the Aitkens GLS estimator applied to the whole system as if it were just one equation.

$$\hat{\beta} = (X'V^{-1}X)^{-1} X'V^{-1}Y = [X'(\Omega^{-1} \otimes I)X]^{-1}X'(\Omega^{-1} \otimes I)Y . \qquad \text{(B.3)}$$

Let R be the matrix corresponding to the constraint

$$R\beta = 0 . \qquad \text{(B.4)}$$

---

[16] If the system contains the factor shares of a particular cross section, then there is one less than the number of shares. Otherwise, the variance-covariance matrix of the error terms would be singular (Theil, 1971, p. 335).

The restricted GLS estimator then is (Theil, 1971, pp. 284 and 316),

$$\beta^* = \hat{\beta} - CR'(RCR')^{-1}R\hat{\beta} \ , \tag{B.5}$$

where

$$C = [X'(\Omega^{-1} \otimes I)X]^{-1} \ ,$$

which for equal exogenous variables simplifies to

$$C = \Omega \otimes (\bar{X}'\bar{X})^{-1} \ ,$$

where Ω is unknown and in actual estimation is replaced by an $\hat{\Omega}$ in the above formulas. Deviating slightly from Theil, the variance-covariance matrix Ω of the errors of the factor shares equations is estimated as

$$\hat{\omega}_{ij} = \frac{\hat{e}_i{}'\hat{e}_j}{T-m} \ , \tag{B.6}$$

where $\hat{e}_i$ and $\hat{e}_j$ are the OLS-residuals of the equations of share i and share j, respectively. m is the number of explanatory variables including intercept in each individual share equation.

Theil (1971, p. 341) in his "Consumer Allocation Problem" uses instead the residuals of a GLS regression which uses prior information on Ω. Also, he divides by T rather than T-m. In the case of identical explanatory variables, dividing by T-m gives unbiased estimates according to Triangle Universities Computing Center (1972).

The following test statistic has an F distribution with g and Tn-nm degrees of freedom, where g is the number of restrictions in R (Theil, 1971, Equation 3.6):

$$\frac{(R\beta^*)' \{R[X' (\Omega^{-1} \otimes I)X]^{-1}R'\}R\beta^*}{(Y-X\beta^*)'(\Omega^{-1} \otimes I)(Y-X\beta^*)} \cdot \frac{Tn - nm}{g} . \qquad (B.7)$$

If Ω^{-1} is replaced by $\hat{\Omega}^{-1}$, the above is an asymptotic test. The program used was the Two and Three Stage Least Squares (TTLS) program of the Triangle Universities Computing Center, Research Triangle Park, North Carolina.

When the data for one individual factor share from more than one cross section are combined into several equations, the same estimators and tests can be used.

However, the TTLS program estimates too many variance-covariance terms when the number of cross sections exceeds two. If we have four cross sections, Ω has the following structure in the case of simple autocorrelation:

$$\Omega = \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}, \qquad (B.8)$$

i.e., the different variance-covariance terms are related to each other by the constraint that the ρ value be equal for all of them.

TTLS cannot impose this constraint on Ω and estimates an individual variance and covariance term for each position. The resulting estimator might have undesirable and unknown properties.

For the same reason that no constraints on Ω can be imposed, no simultaneous estimation of all shares equations in all time periods

$$\frac{(R\beta^*)'\{R[X'(\Omega^{-1} \otimes I)X]^{-1}R'\}R\beta^*}{(Y-X\beta^*)'(\Omega^{-1} \otimes I)(Y-X\beta^*)} \cdot \frac{Tn - nm}{g} . \qquad (B.7)$$

If Ω^{-1} is replaced by $\hat{\Omega}^{-1}$, the above is an asymptotic test. The program used was the Two and Three Stage Least Squares (TTLS) program of the Triangle Universities Computing Center, Research Triangle Park, North Carolina.

When the data for one individual factor share from more than one cross section are combined into several equations, the same estimators and tests can be used.

However, the TTLS program estimates too many variance-covariance terms when the number of cross sections exceeds two. If we have four cross sections, Ω has the following structure in the case of simple autocorrelation:

$$\Omega = \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}, \qquad (B.8)$$

i.e., the different variance-covariance terms are related to each other by the constraint that the ρ value be equal for all of them.

TTLS cannot impose this constraint on Ω and estimates an individual variance and covariance term for each position. The resulting estimator might have undesirable and unknown properties.

For the same reason that no constraints on Ω can be imposed, no simultaneous estimation of all shares equations in all time periods was done by defining 16 individual equations. This system would also have exceeded the limits of TTLS.

Appendix C. Tables on Measured Biases in Efficiency Gains, Actual Shares, Factor Prices and Quantities for the United States and Japan

Appendix C Table 1. U. S. factor shares adjusted for factor price influence: indices of biases in technical change

Year	Land	Labor	Machinery	Fertilizer	Other
	Numerical values, as percent of total expenditures				
1912	21.0	38.3	10.9	1.9	28.0
1916	21.2	36.7	11.6	1.8	28.7
1920	19.6	39.3	9.3	2.1	29.7
1924	20.0	39.7	10.3	2.2	27.8
1928	18.1	41.4	10.4	2.7	27.4
1932	18.8	40.3	14.3	2.7	24.0
1936	18.9	32.5	16.3	3.0	29.3
1940	16.8	34.3	17.6	3.9	27.5
1944	16.5	38.4	16.1	4.8	24.2
1948	17.1	37.2	13.9	5.1	26.7
1952	16.5	29.8	19.7	5.7	28.3
1956	16.3	30.6	23.1	6.5	23.4
1960	17.1	27.2	23.4	6.1	26.1
1964	17.8	25.8	22.4	6.7	27.3
1968	19.1	25.3	23.1	7.2	25.3
	Standardized, as percent of their 1910-1912 value				
1912	100.0	100.0	100.0	100.0	100.0
1916	101.1	95.8	106.8	96.5	102.6
1920	93.5	102.6	85.6	113.4	106.1
1924	95.4	103.7	94.8	113.9	99.4
1928	86.3	108.1	95.8	144.0	97.9
1932	89.7	105.2	131.7	142.4	85.4
1936	90.1	84.9	150.1	159.8	104.7
1940	80.1	89.6	162.1	204.1	98.3
1944	78.7	100.3	148.3	253.2	86.5
1948	81.5	97.2	128.0	267.9	95.4
1952	78.7	77.9	181.4	298.0	101.1
1956	77.7	79.9	212.7	341.8	83.6
1960	81.5	71.0	215.5	323.3	93.3
1964	84.9	67.4	206.3	354.4	97.6
1968	91.1	66.1	212.7	379.9	90.4

Appendix C Table 2. Japanese factor shares adjusted for factor price influence: indices of biases in technical change

Year	Land	Labor	Machinery	Fertilizer	Other
	Numerical values, as percent of total expenditures				
1893	31.6	38.0	10.9	2.9	16.6
1896	31.2	39.4	9.4	2.8	17.3
1900	28.6	43.5	9.2	3.2	15.7
1904	29.6	43.0	9.1	3.8	14.5
1908	30.1	42.3	8.2	4.7	14.7
1912	30.8	42.9	7.4	5.8	13.0
1916	29.3	42.4	8.3	7.0	13.0
1920	30.6	41.1	7.8	8.2	12.3
1924	27.7	48.9	6.2	7.0	10.2
1928	26.4	51.4	6.1	8.4	7.7
1932	26.4	50.4	6.4	8.3	8.4
1936	29.2	43.1	7.1	8.8	11.8
1940	28.7	41.5	7.2	9.3	13.3
1954	23.1	40.3	7.8	9.3	19.5
1958	26.5	37.4	5.6	8.1	22.4
1962	26.2	38.5	4.9	7.5	23.0
	Standardized, as percent of their 1910-1912 value				
1893	102.6	88.6	147.3	50.0	127.7
1896	101.3	91.8	127.0	48.3	133.1
1900	92.9	101.4	124.3	55.2	121.0
1904	99.1	100.2	123.0	65.5	111.5
1908	97.7	98.6	110.8	81.0	113.1
1912	100.0	100.0	100.0	100.0	100.0
1916	95.1	98.8	112.1	120.7	100.0
1920	99.4	95.8	105.4	141.4	94.6
1924	89.9	114.0	83.8	120.7	78.4
1928	85.7	120.0	82.4	144.8	59.2
1932	85.7	117.5	86.5	143.1	64.6
1936	94.8	100.5	95.1	151.7	90.8
1940	93.2	96.7	97.2	160.3	102.3
1954	75.0	93.9	105.4	160.3	150.0
1958	86.0	87.1	75.7	139.7	172.3
1962	84.4	89.7	66.2	129.3	176.9

Appendix C Table 3. Calculated actual shares, United States[a]

Year	Land	Labor	Machinery	Fertilizer	Other
	Numerical values, as percent of total expenditures				
1912	21.0	38.3	10.9	1.9	28.0
1916	21.6	36.5	11.6	1.9	28.4
1920	17.3	40.5	10.1	2.0	30.1
1924	19.7	38.5	10.3	1.7	29.7
1928	15.9	40.9	10.2	1.9	31.1
1932	18.6	37.6	12.6	1.6	29.7
1936	14.9	34.7	14.5	2.2	33.7
1940	12.0	35.3	15.1	2.3	35.2
1944	8.5	39.5	14.0	2.3	35.6
1948	9.4	37.7	12.2	2.4	38.3
1952	9.8	29.7	17.5	3.0	40.0
1956	11.5	27.4	20.1	3.3	37.8
1960	15.6	21.3	19.8	2.9	40.4
1964	17.5	18.3	18.5	3.3	42.3
1968	20.4	15.8	19.1	3.6	41.1
	Standardized, as percent of their 1910-1912 value				
1912	100.0	100.0	100.0	100.0	100.0
1916	103.0	95.3	106.8	101.3	101.5
1920	82.5	105.8	93.0	105.0	107.6
1924	93.9	105.5	94.8	89.1	106.1
1928	75.8	106.8	93.9	99.7	111.2
1932	88.7	98.2	116.0	83.3	106.1
1936	71.1	90.6	133.5	113.9	120.4
1940	57.2	92.2	139.0	121.3	125.8
1944	40.5	103.2	128.9	122.4	127.2
1948	44.8	98.5	112.3	127.1	136.9
1952	46.7	77.6	161.1	156.6	143.0
1956	54.8	71.6	185.1	173.5	135.1
1960	74.4	55.6	182.3	153.5	144.4
1964	83.5	47.8	170.3	175.6	151.2
1968	97.3	41.3	175.9	189.3	146.9

[a]For construction of the series and the data sources, see Appendix D.

Appendix C Table 4. U. S. input-price/output-price ratio indices[a]

Year	Land	Labor	Machinery	Fertilizer	Other
			(1910-12 = 100)		
1912	100.0	100.0	100.0	100.0	100.0
1916	113.3	106.8	110.0	105.7	103.8
1920	79.0	104.3	81.3	85.7	105.0
1924	119.0	134.5	111.7	93.1	106.6
1928	104.8	154.1	128.5	90.0	118.9
1932	160.8	194.7	231.5	128.6	101.5
1936	69.4	113.4	189.2	99.6	110.9
1940	87.3	179.0	288.8	103.4	160.1
1944	65.2	217.2	244.2	63.0	211.7
1948	73.0	247.8	226.6	50.4	222.8
1952	91.3	274.3	301.1	53.6	214.6
1956	145.8	407.9	423.7	65.9	229.6
1960	254.1	502.7	550.3	63.0	241.5
1964	338.1	610.0	651.2	63.2	270.9
1968	481.0	766.9	735.8	58.2	280.4

[a]For construction of the series and the data sources, see Appendix D.

Appendix C Table 5. U. S. input/unit of output quantity indices[a]

Year	Land	Labor	Machinery	Fertilizer	Other
			(1910-12 = 100)		
1912	100.0	100.0	100.0	100.0	100.0
1916	96.9	95.1	103.3	95.6	103.8
1920	102.6	99.5	111.7	114.8	100.4
1924	96.3	91.4	103.6	106.4	121.6
1928	88.3	84.5	89.0	112.3	113.7
1932	88.1	79.7	81.4	85.2	164.1
1936	104.8	81.4	72.0	108.0	112.5
1940	84.7	66.4	62.1	112.4	101.7
1944	75.2	57.4	63.8	152.7	72.2
1948	74.7	48.0	60.2	194.4	74.2
1952	71.2	39.3	74.1	245.6	92.3
1956	64.5	30.2	75.1	246.5	100.9
1960	58.9	22.2	66.5	244.3	119.9
1964	54.5	17.3	57.8	283.3	123.1
1968	50.6	13.5	59.6	347.1	130.2

[a]For construction of the series and the data sources, see Appendix D.

Appendix C Table 6. Calculated actual shares, Japan[a]

Year	Land	Labor	Machinery	Fertilizer	Other
		Numerical values, as percent of total expenditures			
1893	31.6	38.0	10.9	2.9	16.6
1896	30.1	40.0	9.9	2.7	17.2
1900	25.8	44.2	9.7	2.6	17.7
1904	27.5	43.0	9.5	3.1	16.9
1908	27.6	42.7	9.1	4.1	16.6
1912	28.4	42.6	8.5	5.0	15.5
1916	25.9	42.7	9.2	6.1	16.1
1920	28.0	40.6	8.7	7.1	15.5
1924	22.7	48.5	7.8	5.3	15.7
1928	21.4	50.1	7.5	6.4	14.6
1932	29.8	49.8	8.0	6.3	15.2
1936	25.2	43.2	8.8	7.4	15.3
1940	24.0	42.2	9.1	8.2	16.5
1954	11.6	44.7	10.8	7.9	22.0
1958	20.1	41.5	9.3	7.0	22.1
1962	20.1	40.4	9.2	6.0	24.4
		Standardized, as percent of their 1910-1912 value			
1893	111.2	89.2	128.4	57.8	107.1
1896	105.9	93.9	116.6	54.0	111.0
1900	90.7	103.8	114.3	52.4	114.3
1904	96.7	100.9	111.9	62.0	109.1
1908	97.1	100.2	107.2	82.0	107.2
1912	100.0	100.0	100.0	100.0	100.0
1916	91.1	100.2	108.4	122.0	103.9
1920	98.5	95.3	102.5	142.0	100.1
1924	79.8	113.8	91.9	106.0	101.4
1928	75.3	117.6	88.3	128.0	94.3
1932	73.2	116.9	94.2	126.0	98.1
1936	88.6	101.4	103.6	148.0	98.8
1940	84.4	99.1	107.2	165.0	106.5
1954	40.8	112.0	127.2	158.0	142.0
1958	70.7	97.4	109.5	140.0	142.7
1962	70.7	94.8	108.4	120.0	157.5

[a]For construction of the series and the data sources, see Appendix D.

Appendix C Table 7. Japanese input-price/output-price ratio indices[a]

Year	Land	Labor	Machinery	Fertilizer	Other
			(1910-12 = 100)		
1893	120.6	80.5	143.6	190.8	113.0
1896	112.9	85.4	129.4	165.2	115.9
1900	94.4	94.9	124.0	148.7	112.9
1904	99.0	93.7	120.5	127.9	104.8
1908	97.6	94.6	110.2	118.2	106.1
1912	100.0	100.0	100.0	100.0	100.0
1916	97.7	111.0	113.7	116.1	113.4
1920	99.2	105.0	102.8	93.8	99.2
1924	92.7	141.1	96.3	74.6	107.7
1928	93.8	153.7	93.6	71.9	104.2
1932	100.3	172.5	105.1	73.7	118.7
1936	96.4	122.4	88.8	62.6	101.1
1940	87.4	117.1	84.4	79.1	105.6
1954	44.0	113.9	82.5	38.2	122.7
1958	97.2	138.1	78.6	32.9	126.0
1962	109.1	170.5	60.9	27.6	116.3

[a]For construction of the series and the data sources, see Appendix D.

Appendix C Table 8. Japanese input/unit of output quantity indices[a]

Year	Land	Labor	Machinery	Fertilizer	Other
			(1910-12 = 100)		
1893	116.1	139.2	111.8	37.8	118.8
1896	113.7	133.3	109.1	38.9	116.1
1900	109.6	124.0	104.9	39.9	114.9
1904	108.1	119.1	102.8	53.7	115.2
1908	99.5	105.7	96.7	68.5	100.5
1912	100.0	100.0	100.0	100.0	100.0
1916	88.9	86.0	90.7	98.7	87.0
1920	89.0	82.5	90.4	137.8	91.5
1924	92.8	86.9	101.9	151.1	101.2
1928	83.8	80.0	98.1	185.7	94.1
1932	81.0	76.1	99.9	188.7	91.7
1936	82.9	74.6	105.3	210.3	87.9
1940	76.3	66.9	100.4	185.2	79.1
1954	76.3	80.2	125.1	347.1	94.4
1958	62.8	60.8	119.5	365.2	97.2
1962	54.1	46.6	148.9	360.6	113.0

[a]For construction of the series and the data sources, see Appendix D.

Appendix D. Variable Construction and Data Sources

Three data sets were computed mainly from published sources: U. S. states cross-section data, U. S. time series and Japanese time series data. All three sets basically use definitions of factors given by Griliches (1964). Complete consistency of definitions among all three data sets could not be achieved due to differences in data collection or lack of data.

Cross-Section Data

Aggregate input quantity data and expenditure data were derived for the 39 states and groups of states (see Appendix D Table 1).

Quantity Data

Except for "other" inputs, the quantity data were taken from Fishelson (1968, pp. 75-81), who used Griliches (1964) data with some changes. Fishelson's discussion of the construction of the variables is reproduced here.

Output

Output (Y_{it}) was defined to be the sum of cash receipts from marketing (C_{it}), value of home consumption (HC_{it}), government payments (G_{it}) and the value of the net change in inventories (N_{it}), all measured in 1949 prices.

$$Y_{it} = C_{it1}/I_{pt1} + \ldots + \ldots + C_{itj}/I_{ptj} + \ldots$$

$$+ C_{it12} + (HC_{it} + G_{it} + N_{it})/I_{pit} ,$$

where C_{itj} is cash receipts from marketing the commodities of group j in state i in year t. (The 12 commodity groups were: (1) meat animals, (2) dairy products, (3) poultry and eggs, (4) miscellaneous livestock and livestock meats, (5) food grains, (6) feed crops, (7) cotton, (8) tobacco, (9) oil crops,

Appendix D Table 1. Listing of states by group

Number	Listing of states	Group[a]
1	Maine, New Hampshire, Vermont, Massachusetts, Rhode Island and Connecticut	MN
2	New York	MN
3	New Jersey	MN
4	Pennsylvania	MN
5	Ohio	MN
6	Indiana	MN
7	Illinois	GR
8	Michigan	MN
9	Wisconsin	MN
10	Minnesota	GR
11	Iowa	GR
12	Missouri	GR
13	North Dakota	GR
14	South Dakota	GR
15	Nevada	GR
16	Kansas	GR
17	Delaware, Maryland	SE
18	Virginia	SE
19	West Virginia	MN
20	North Carolina	SE
21	South Carolina	SE
22	Georgia	SE
23	Florida	SE
24	Kentucky	MN
25	Tennessee	MN
26	Alabama	SE
27	Mississippi	GS
28	Arkansas	GS
29	Louisiana	GS
30	Oklahoma	GS
31	Texas	GS
32	Montana	GR
33	Idaho	MW
34	Wyoming, Utah, Nevada	MW
35	Colorado	GR
36	New Mexico, Arizona	MW
37	Washington	MW
38	Oregon	MW
39	California	MW

[a]MN = mixed agriculture, North; GR = grain farming; SE = Southeast; GS = Gulf States; MW = mixed agriculture, West.

(10) vegetables, (11) fruits and nuts and (12) all other crops...) I_{ptj} is the same for all i at a given year t. I_{pit} is the price index (1949=100) of total agricultural output of state i in year t ...

Material Inputs

... Land. In the U. S. Census of Agriculture (U. S. Bureau of the Census, 1952, 1956, 1962 and 1966), the average value of land and buildings per farm in each state was reported. However, the land value represented not only the value of land to agricultural production but also included the site value of land. The value of buildings included both farm structures and dwellings. Hence, census data on value of land and buildings were inadequate for the purposes of this study. To measure land by the number of acres per farm (giving each acre a value of one) is also inadequate because of the diversity of soil quality, fertility and uses.

In this study the weighting procedure for measuring land value was based on a study by Hoover (1961). The value of each acre in each state at each cross section was measured by its 1940 price relative to that of an acre of pasture in the corresponding state. The value of an acre of pasture in each state in 1940 was calculated by dividing the total value of land in 1940 by the number of pasture equivalent units of the land in 1940. This value of an acre of pasture was kept constant over time. Since all prices were deflated to the 1949 price level in this study, the value of an acre of pasture in 1940 was also adjusted to the 1949 price level. The deflator used was total value of land in the United States agriculture sector in 1949, i.e., the value of agricultural land in 1949 measured in 1940 relative land prices ratio. The ratio was 2.2. The use of this method provided a measure of the stock of land in constant prices. According to this method, changes in the stock of land occurred only because of changes in the number of acres or their use. The stock of land was unaffected by changes in prices of agricultural products, site effects, or government programs.

Labor. The labor input was measured in physical flow units defined as the number of days worked per farm per year. The labor input was obtained from three sources, operator's labor, labor of other family members and unpaid workers, and hired labor. Physical labor was adjusted for age (.6 for operators above 65) and for labor supplied by other family members (.65). No adjustments were made for changes in labor's quality.

The computational equation for labor is given in Griliches (1964, p. 974).

Machinery. The machinery variable was a measure, in constant prices, of the cost of the flow of services obtained through the use of farm machinery and equipment. The variable

> was the sum of deflated expenditures on repairs and operation (1949=100) and 15 percent of the stock value (after adjusting to 1949 prices) of machinery and equipment on farms. The latter item was an attempt to approximate machinery services by the costs of interest and depreciation assuming a constant proportion, over states and time, between the stock value and the flow of services.

For the purposes of this thesis, a definition excluding depreciation and operating expenditures would have been preferable. Since the depreciation is taken proportional to the stock and since operating expenditures in this time period are probably in a fairly close proportionality relationship to the stock as well, this introduces only proportionality errors in the quantity and price variables which do not affect the estimates of the γ_{ij}. It was felt that the quality of this quantity variable was far superior to anything which could have been constructed in a reasonable time span.

> Fertilizer. The fertilizer input was defined to be the weighted sum of the quantity of plant nutrients. The nutrients are nitrogen (N), phosphoric acid (P_2O_5) and potash (K_2O). The weights were their 1955 relative prices or 1.62, .93 and .45, respectively (Griliches, 1964, p. 967). Thus, the fertilizer input was measured in equivalent tons per year, *i.e.*, a flow measure. This measure provided a more accurate estimate of the real input than a cost measure because of the declining price per unit of nutrient over time and the changing nutrient content per ton of fertilizer over states.

The only change which was made in these quantity data was that whenever quantities per farm were used, the farm number was taken from the Census of Agriculture (U. S. Bureau of the Census, 1950-1964), rather than from Farm Labor (U. S. Department of Agriculture, 1945-1972).

Other Inputs

Since expenditure data corresponding to Fishelson's quantity data could not be constructed, new quantity data were defined as follows.

They are the sum of the explicit and implicit annual expenditures on all other material inputs used in production. The explicit expenditures were the cash expenditures on purchase of livestock, poultry, feed, seeds, plants and bulbs, operation and repairs of farm structures and other miscellaneous costs. The implicit expenditures were 8 percent interest (5 percent) on the value of farm structures, and the share of real estate taxes falling on buildings. Each of the expenditures was separately deflated to its 1949 price level to arrive at a quantity measurement (for taxes the agricultural output price index was used).

Expenditures and Factor Shares

The expenditure variables were defined, as far as possible, to correspond to the quantity variables. Income shares were then derived by dividing the expenditures by farm income defined as receipts from sales, home consumption, rental value of dwellings and change in inventories. Rental value of dwellings was included because expenditures for buildings could not be separated into expenditures for service structures and expenditures for dwellings. Expenditure shares were obtained by dividing through the sum of expenditures.

Land. Expenditures on land are simply 6 percent of the value of land plus the share of real estate taxes falling on land.

Labor. Expenditures for labor are the number of man-days of labor from Fishelson (1968) multiplied by a daily wage rate without room and board (U. S. Department of Agriculture, 1945-1972). This assumes that the opportunity cost of farm operators is the wage rate which they could earn as workers on other farms.

Machinery. Expenditures are assumed to be 15 percent of the value of farm machinery and equipment for interest and depreciation plus the current expenditures for operation and repair of machinery and equipment.

Fertilizer. Fertilizer expenditures are directly reported by USDA.

Other Expenditures. These expenditures were computed exactly as the quantity, except that the individual items were not deflated. Aggregate expenditures estimated in this way had a tendency to exceed aggregate income by up to 10 percent.

Prices

Prices were taken to be the expenditures divided by the quantities. They were then deflated to the 1949 price level using the U. S. agricultural output price index. Note that this procedure implies that the price of other inputs is equal to 1 for all states in the year 49. Appendix D Table 2 lists all the data sources.

U. S. Time Series Data

Basically the same approach was applied as for the cross-section data. The differences are the following.

Land

The total quantity of land for the United States for the census years 1910-1954 was taken from Hoover (1961). His approach was followed to compute the data for 1959 and 1964. For the missing years, interpolation was used. The approach is described in the previous section, except that the units were changed to 1910-14 prices.

Appendix D Table 2. Sources for the cross-section data

Variables	Source
Farm income, change in inventories, rental value of dwellings, all explicit current operating expenditures	Farm Income Situation, July supplement, USDA (1954-1972)
Annual average daily wage rate without board or room	Various issues of Farm Labor, USDA (1945-1972)
Farm numbers	Various issues of Census of Agriculture, U. S. Bureau of the Census (1950-1964)
Input and output price indices	Various issues of Agricultural Statistics, USDA (1936-1972)
Repairs and operation of farm dwellings and service structures, depreciation of dwellings, service buildings, motor vehicles, other machinery and equipment, value of farm machinery and equipment, value of crop inventories	USDA, unpublished

The expenditures on land are the interest charge on the value of land plus the part of real estate taxes falling on land. The interest rate used is the average rate on new loans by the Federal Land Bank. For the years 1910-16, this was approximated by subtracting 0.8 percent from the Federal Land Bank interest rate on mortgage loans.

Labor

The quantity of labor used is the USDA series of man-hours of labor used for farmwork multiplied by 1.06. The adjustment was made because the series does not account for standby time but is computed simply as hours needed to do the job (personal communication, USDA). The labor price used is the 1970 composite rate per hour from farm labor divided by the composite index of farm wage rates for each year. This treatment is better than the one in the cross section. But no hour of work statistics exist for states.

Machinery

The stock of machinery was estimated by deflating the value of motor vehicles and the value of other farm machinery and equipment by their respective price indices. To this was added the constant value of horses and mules derived by multiplying their total number by their average 1910-1914 price. This latter was found by dividing the value of horses and mules by their number. Neglecting animal power in a time series starting in 1910 would have been inappropriate since in the early years, it was an important power source. The drawback is that horses were dropped from agricultural statistics in 1962 which introduces a small break in the series. Including horses in the time series but not in the cross section assumes that their power substitutes for other factors in the same way as mechanical power.

Machinery expenditures are the amount of operating expenditures on machinery and equipment, depreciation thereof as computed by the USDA, 12 percent of the current value of horses and mules for depreciation and operating costs plus the current value of horses and mules times the interest rate on new loans increased by 2 percent.

Fertilizer

Expenditure data are published, as well as a price index for fertilizer. Therefore, a quantity index of fertilizer is derived as expenditures divided by the price index.

Other Inputs

Expenditures are computed as the sum of current expenditures (feed, livestock purchases, seed, lime, miscellaneous, repairs and operation of buildings) and implicit expenditures (building depreciation, accidental damage to buildings, the share of real estate taxes falling on buildings, the value of buildings times the interest rate and the value of livestock and crop inventory times the interest rate increased by 2 percent). From this were subtracted the estimated operating expenditures and depreciation of horses and mules.

The quantity was derived as the sum of the individually deflated current expenditures plus the implicit expenditures deflated by the output price index. The final data used were three-year moving averages of the above series.

For reporting purposes only, the price and quantity variables were transformed into indices. The quantity indices are indices of inputs per unit of output; the price indices are derived by deflating all prices by the output price index. The absolute magnitudes of these index numbers may be questionable. But they are unimportant. What is important is their trends relative to each other. The data sources for the U. S. time series data are summarized in Appendix D Table 3.

Appendix D Table 3. Data sources for U. S. time series data

Variables	Source
Farm income, rental value of dwellings, all current expenditures, depreciation of all capital items, accidental damage, taxes	Farm Income Situation, July 1971, (USDA, 1954-1972). Some data for early years were obtained from the Economic Research Service.
All price indices, value of land and buildings, of livestock and crop inventories, interest rates, number and value of horses and mules	Various issues of Agricultural Statistics (USDA, 1936-1972)
Hours of work	1964 and 1971 issues of Statistical Bulletin 233, Changes in Farm Production and Efficiency (USDA, 1958-1971)
Value of buildings	May 1959 issue of Farm Real Estate Market (USDA, 1953-1961). Later years: various issues of Farm Real Estate Market Developments, (USDA, 1961-1972).
Quantity of land, land class data	Up to 1954: Hoover (1961); 1959, 1964: 1964 Census of Agriculture, (U. S. Bureau of the Census, 1950-1964)

Japanese Time Series Data

This data set was taken almost entirely from Volume 9 (Agriculture) and Volume 3 (Capital Stock) of "Estimates of Long-Term Economic Statistics of Japan since 1868 (Okawa et al., 1966).[17] These will be referred

[17] Some controversies surround the LTES output statistics: Nakamura (1966) claims that for early years the data severely underestimate the output because the tax structure on output gave incentives not to report it. This is supported by the incidental finding that the total expenditures

to as LTES in what follows. The only other source used was "Hundred Year Statistics of the Japanese Economy" (Bank of Japan, 1966), from which interest rates were taken and which provided some checks of LTES.

LTES provides a tremendous wealth of data in great detail with a discussion in English of sources and methods used. It was quite extraordinary to find almost all data necessary for the extensive data requirements of this thesis collected in usable form in just one reference. Its data are annual and for some series in some years, based on inter- and extrapolation. In the case of agriculture, the data rely primarily on government statistics. The definitions of the variables follow as closely as possible the definitions used for the U. S. time series. Differences occur when no comparable data were available.

Land

LTES reports statistics on the area of paddy fields and the area of upland fields for Japan, as well as their rents. No statistics on pasture land were available. But Hayami and Ruttan (1971) report that in 1962 the area of permanent pasture was 10 percent of total arable land and that this could be assumed to hold for other years as well. The quantity of land therefore is the weighted sum of paddy fields, upland fields and permanent pasture. The latter two were converted in the paddy field units using as the weight for the upland fields their relative price to paddy fields in 1934-1936, the base years for most series of LTES. The weight of permanent pasture was found by assuming that the

---

computed here overexhaust aggregate output by a far greater margin in the early years of the analysis than in the later years. The output statistics of LTES were, however, not used in this thesis.

productivity of permanent pasture was one-third of the productivity of upland fields. This approach uses the key idea of Hoover (1961), but for lack of data cannot take account of regional efficiency differences.

Expenditures were computed by multiplying the land areas by the average rents per unit of land. Rents for post-World War years are not reported in LTES but were computed from the land prices using average ratios of prices for rents of pre-World War years. These ratios were 6.3 percent for paddy fields and 5.1 percent for upland fields.

Labor

LTES reports man-years of male and female workers. Unfortunately, no man-hour series is available so that no account could be taken of a possible decline or rise in hours worked per year. (A rise could be due to double cropping, etc.) Male and female workers were aggregated by using a factor of .775 for female workers, which is the long-run average wage ratio of female to male workers. Expenditures of labor were the man-years of male and female workers multiplied by the respective wage rate of annual workers.

Machinery

The power machinery and livestock capital series of LTES are computed by multiplying the number of machines in each machinery class (or livestock in each livestock class) by their 1934-36 unit prices which were derived from national wealth surveys. Since the numbers in each livestock class are reported, it was possible to solve for these unit prices used in constructing the capital series. Therefore, the livestock capital could be split into a series for draft animals and a series for other livestock.

All other capital series in LTES are also in 1934-36 prices. The quantity of machinery and draft animals is therefore the sum of capital in power machinery, other machinery and equipment, horses and draft animals plus beef cattle. Beef cattle could not be treated separately because it is not reported separately. In the early years, however, practically no beef was produced for meat consumption which parallels the extremely small number of milk cows at that time. From the series it is apparent, however, that this changed after 1951 when horses declined by 60 percent from 1951 to 1963, while milk cows increased four-fold. During the same time, the number of draft and beef cattle stayed about constant. The number of draft and beef cattle was therefore adjusted after 1951, assuming that the number of draft cattle declined at the same rate as the number of horses.

Expenditures on machinery and draft cattle were computed as follows, The capital items were inflated to their current values using their respective price indices. The same was done with the depreciation series which are also reported in constant 1934-36 prices. The current capital value is multiplied by the interest rate, which is the average Japanese rate for bank loans on deeds for the years 1891-1940. After the war, the rate on deeds of Tokyo banks is used since the rate for Japan was not available. When a comparison is possible, the Tokyo rate is slightly lower than the average Japanese rate.

The rate on deeds and the rates on real estate loans by private banks show little difference for the period in which a comparison can be made. All interest rates are substantially higher than their American counterparts.

The costs for feed of agricultural origin are prorated to draft animals according to their share in livestock capital. No operating costs for machinery are reported in LTES. To approximate them, the item "other expenditures for foods of nonagricultural origin" is prorated to livestock and machinery according to their combined share in livestock, machinery and building capital. It is assumed that this item contains mainly veterinary costs, fuel and repair costs, and building materials.

Fertilizer

The quantity is an index of primary nutrients with nitrogen as the unit contained in all fertilizers used. The weights for P_2O_5 and K_2O are their 1934-1936 unit values relative to N. These unit values, as well as the nutrient contents of fertilizer, are derived with great care in LTES. Expenditures on fertilizer are reported directly.

Other Inputs

The quantity of other inputs is the sum of the implicit expenditures on the remaining livestock capital, trees and shrubs, and buildings. To this are added current expenditures not accounted for previously, each of which is deflated separately to the 1934-36 price level. Expenditures are the same concept with the capital items inflated individually to the current price levels.

Prices

All prices are the expenditures divided by the quantities. The series were transformed into three-year moving averages. For reporting purposes, the prices and quantities were converted into index numbers with 1891-1893 taken as 100. The quantity indices are indices of inputs per unit of output. The same reservations apply to the absolute values of these series as to the ones of the U. S. time series.

The depreciation concept used in LTES differs from the one used in the U. S. agricultural statistics. Apart from differences in rates, the former use the straight-line method while the latter use the declining balance method.

Appendix E. Mathematical Appendix to Induced Innovation Chapter

Derivation of Equation (52)

The derivation of Equation (52) follows Samuelson (1965b).

$$\dot{u} = \frac{\dot{U}}{U} = - \dot{a}_K \alpha_K - \dot{a}_L \alpha_L + \text{terms involving price changes}$$

$$Y = F[(KA_K), (LA_L)] \; . \tag{E.1}$$

Corresponding to (1) is a dual minimum cost function (factor price frontier)

$$U = U[(\frac{R}{A_K}), (\frac{W}{A_L})] \; , \tag{E.2}$$

where R is the rental rate on capital and W the wage rate. Shephard's lemma holds.

$$\frac{\partial U}{\partial (\frac{R}{A_K})} = KA_K, \quad \frac{\partial U}{\partial (\frac{W}{A_L})} = LA_L \; ,$$

$$dU = \frac{\partial U}{\partial (\frac{R}{A_K})} \frac{\partial (\frac{R}{A_K})}{\partial A} dA + \frac{\partial U}{\partial (\frac{W}{A_L})} \frac{\partial (\frac{W}{A_L})}{\partial A_L} dA_L$$

$$+ \text{ terms involving dW and dR,}$$

$$\frac{dU}{U} = \dot{u} = - \frac{KR}{U} \frac{dA_K}{A_K} - \frac{LW}{U} \frac{dA_L}{A_L} + \ldots = -\alpha_K \dot{a}_K - \alpha_L \dot{a}_L \; \ldots . \quad .$$

Correspondence of Equation (52) with Innovation Possibility Curve

This section proves mathematically that, if the innovation possibility frontier (52), $\phi(\dot{a}_K, \dot{a}_L) = 0$, is independent of the initial factor levels, it corresponds to an innovation possibility curve (Figure 10), which is Cobb-Douglas.

It is possible to define a production function envelope corresponding to the innovation possibility curve as follows.

$$Y = f(X_1, X_2, \ldots, X_n, T) \ , \qquad \text{(E.3)}$$

where T stands for a technological change index which may shift neutrally or non-neutrally in terms of the Xs. (E.3) can be expanded in logs as a Taylor series to get an analogue of (32).

$$\ln Y = \ln \nu_0 + \sum_i \nu_i \ln X_i + \frac{1}{2} \sum_{ij}\sum \gamma_{ij} \ln X_i \ln X_j$$

$$+ \nu_t \ln t + \sum_i \omega_i \ln X_i \ln t + \omega_t (\ln t)^2$$

$$+ \text{higher order terms (neglected).} \qquad \text{(E.4)}$$

The biases are determined by the ω_i terms. If all γ_{ij} and ω_i are zero, then the function is Cobb-Douglas.

Setting output equal to 1 and taking the total differential in the two-factor case,

$$Q = \nu_1 \, d\ln X_1 + \nu_2 \, d\ln X_2 + \gamma_{11} \ln X_1 \, d\ln X_1 + \gamma_{22} \ln X_2 \, d\ln X_2$$

$$+ \frac{1}{2} \gamma_{12} \ln X_1 \, d\ln X_2 + \frac{1}{2} \gamma_{12} \ln X_2 \, d\ln X_1$$

$$+ \nu_t \, d\ln t + 2\omega_t \ln t \, d\ln t + \omega_1 \ln X_1 \, d\ln t$$

$$+ \omega_2 \ln X_2 \, d\ln t + \omega_1 \ln t \, d\ln X_1 + \omega_2 \ln t \, d\ln X_2 \; .$$

Collecting terms in $d\ln X_1$, $d\ln X_2$, and $d\ln t$, we have

$$d\ln X_1(\nu_1 + \gamma_{11} \ln X_1 + \frac{1}{2} \gamma_{12} \ln X_2 + \omega_1 \ln t)$$

$$= - d\ln X_2(\nu_2 + \gamma_{22} \ln X_2 + \frac{1}{2} \gamma_{12} \ln X_1 + \omega_2 \ln t)$$

$$- \frac{1}{t} dt(\nu_t + 2\omega_t \ln t + \omega_1 \ln X_1 + \omega_2 \ln X_2) \; .$$

And solving for $\frac{d\ln X_1}{dt}$,

$$\frac{d\ln X_1}{dt} = - \frac{d\ln X_2}{dt} \left(\frac{\nu_2 + \gamma_{22} \ln X_2 + \frac{1}{2} \gamma_{12} \ln X_1 + \omega_2 \ln t}{\nu_2 + \gamma_{11} \ln X_1 + \frac{1}{2} \gamma_{12} \ln X_2 + \omega_1 \ln t}\right)$$

$$- \frac{1}{t} \left(\frac{\nu_t + 2\omega_t \ln t + \omega_1 \ln X_1 + \omega_2 \ln X_2}{\nu_1 + \gamma_{11} \ln X_1 + \frac{1}{2} \gamma_{12} \ln X_2 + \omega_1 \ln t}\right) \; .$$

The only way in which $\frac{d\ln X_1}{dt}$ can be a function independent of the level of X_1 and X_2 is that all the following terms are zero: γ_{11}, γ_{12}, γ_{22}, ω_1 and ω_2. But this is the Cobb-Douglas case. Higher order terms would not change this finding because they would have to be zero as well. This proves that if the IPC corresponds to a Cobb-Douglas production function envelope, the corresponding IPF can be independent of factor levels. Ahmad (1966) has a graphic proof of the only if part.

Hayami-Ruttan Estimation Equations for the Elasticity of Substitution

Hayami and Ruttan (1970) chose the following two-factor price definition for the elasticity of substitution:

$$\sigma^*_{ij} = -\frac{\partial \log \frac{X_i}{X_j}}{\partial \log \frac{W_i}{W_j}} \ . \qquad \text{(E.6)}$$

This is just one way to generalize the elasticity of substitution for the two-factor case to the many-factor case and is not the concept which is used in the previous chapters. Instead, the Allen partial elasticity of substitution concept was used; although difficult to define (see Appendix Equation A.23), it is much easier to handle. The Allen concept, a measure which involves just one factor price is the symmetric concept corresponding to the elasticities of input demand and is related to it as follows:

$$\sigma_{ij} = \frac{\eta_{ij}}{\alpha_j} \ , \qquad \text{(E.7)}$$

where

$$\eta_{ij} = \frac{\partial \log X_i}{\partial \log W_j}$$

and

$$\alpha_j = \frac{X_j W_j}{\sum_i X_i W_i} \ .$$

σ and σ^* coincide in the two-factor case. In the many-factor case they are related as follows (Mundlak, 1967):

$$\sigma^*_{ij} = \frac{\alpha_i(\sigma_{ii} - \sigma_{ij}) \text{ d log } W_i + \alpha_j(\sigma_{ij} - \sigma_{jj}) \text{ d log } W_j}{\text{d log } W_i - \text{d log } W_j} . \tag{E.8}$$

They are variables which depend on the relative magnitude of the price changes involved. Neglecting this price change dependence, Hayami and Ruttan estimate the σ^*_{ij} in the following type of equations without choosing a functional form for the production function.

$$\log \frac{X_i}{X_j} = \beta_0 + \beta_1 \log \frac{W_i}{W_j} + \beta_2 \log \frac{W_k}{W_j} + \varepsilon , \tag{E.9}$$

where β_1 is a measure of σ^*_{ij}. Measuring σ^*_{ij} in this way implies that they are constants, which, in turn, implies that all elasticities of substitution are equal for all factor pairs.[18] This is a very restrictive assumption.

The assumption would not be too restrictive if only local correspondence is desired. Equation (E.9) might still provide the desired estimates. But then, the dependence of the σ^*_{ij} on the relative magnitudes of the factor price changes can probably not be neglected.

---

[18] Uzawa (1962) proves equality of all Allen partial elasticities of substitution if they are constant. Using this fact, the equality of the σ^*_{ij}, if they are constant, can be proved as follows. Allen (1938) proves $\sum_j \alpha_j \sigma_{ij} = 0$. Let $\sigma_{ij} = c$ for all $i \neq j$, then $\sigma_{ii} = c \frac{\alpha_i - 1}{\alpha_i}$ for all i. Setting this into (A), we find $\sigma^*_{ij} = -c$ for all $i \neq j$.