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The Role of Learning in Investment Decisions

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Abstract

If information arrives over time, there is some value in waiting to make an investment involving a sunk cost. Recent work, which specifies the nature of the stochastic process for prices, has shown the standard investment rules are often quite wrong. I relate this emerging body of literature to previous work in other areas of economics which has examined problems where the prospect of learning is important.

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Decision - making

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1.0 Introduction

The study of making irreversible investments[1] under uncertainty is a rapidly expanding area of economic research. Most investments involve a sunk cost so are to some extent irreversible. Secondly, most investments can be postponed. If information affecting the value of the investment arrives over time there may be value in remaining flexible by postponing the investment while more information arrives. The net present value rule - invest whenever the expected net present value of an investment is positive, is inappropriate for irreversible investments in the presence of uncertainty. The *closed loop* optimal solution[2] to these problems, which incorporates the benefit of learning, can substantially alter standard investment rules.

The primary purpose of this paper is to relate this emerging body of literature to previous work in other areas of economics including Resource Economics, Macroeconomics, Financial Management, and the literature on economic search. This is done by characterizing the irreversible investment problems and other work as special cases of a broader class of economic problems which involve a prospect of learning, and therefore have a closed loop optimal solution. In addition to providing a framework to compare and contrast this work the paper should facilitate the development of the intuition required to understand these models. It is hoped that a general discussion of this class of problems will lead to treatment of the problem as a part of the basic training of an economist rather than specialized treatment in specific fields of economics. This survey may also act as a vehicle to assist the description of a particular investment problem in the context of the work which has already been done.

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[1]In this paper irreversible investment will refer to investments in which there is a transaction cost for either investment or divestment. Totally irreversible is used to describe investments which cannot be reversed at any cost.

[2]For a discussion of the between open and closed loop models see Rausser and Hochman.

Finally, by clarifying the properties of economic problems which belong to this class of economic problems, the paper may assist in the identification of areas for further research.

Due to the diversity of economic problems in which the prospect of learning is important, and the simultaneous treatment in many fields of economics, the literature is difficult to describe using a framework which is purely chronological without losing the cross sectional image of the literature. Alternatively, a purely cross sectional description of the literature is difficult without losing the thread of historical development. This paper will use an outline, albeit somewhat arbitrary, based on the properties of the investment opportunities and the nature and source of the future information affecting the investment decision. This outline, which is shown in Figure 1.0, is developed in Sections 2 and 3 of the paper, and is summarized in Section 4. Section 5 discusses areas for further research.

2.0 Decision making Under Uncertainty

The study of decision making in the presence of random variables has a long history in neoclassical economic theory. In the early part of this century Hicks, Knight, and others attempted to formalize decision making under risk and uncertainty. They believed that decision makers had to choose actions, which given their anticipations, would maximize the probability weighted sum of benefits. Knight wished to distinguish between two types of randomness. He defined *risk* as the situation in which the decision maker's anticipation of the random variable came from a distribution with known parameters. Knight defined *uncertainty* as the situation in which the decision maker's anticipation of the random variable came from a distribution with unknown parameters. Knight's definition has become the standard distinction between risk and uncertainty in economics[3].

[3]Another some what parallel distinction is based on the notion that risk involves an objective assessment of probabilities whereas uncertainty involves only a subjective assessment of probabilities. Leroy and Singell suggest that Knight wished to distinguish between insurable risk which were free of moral hazard from uninsurable risk which were subject to market failure.

At that time economists felt this distinction wasn't particularly important for many economic problems. They argued even under uncertainty a probability distribution with known parameters could be derived by integrating over all random parameters of the probability distribution. Thus many problems involving uncertainty could be reduced to a problem of decision making under risk.

Stigler (1939), Tintner, and Hart, were among the first to recognize the importance of the prospect of learning in economic decisions. They found the reductionist approach transforming problems involving uncertainty to risk problems to be unsatisfactory. They believed this approach masked the fundamental economic difference between risk and uncertainty. They pointed out that with risk the individual believes he knows the parameters of the distribution and therefore has no anticipation of learning about these parameters. With uncertainty, the decision maker anticipates learning about these parameters through additional information or the passage of time. Stigler indicated that with uncertainty there would be some value in remaining flexible during the time in which the learning was taking place. He showed that fixed inputs which generated a more flexible productive capacity, i.e. flatter short run marginal cost curves, were more valuable in the presence of price uncertainty than a fixed input which generated fixed plant capacity[4].

Hart also believed the anticipation of learning was very important in many economic problems. *However, Hart pointed out the anticipation of learning has no economic consequence if there is either complete flexibility after learning, or there is no flexibility after learning*. With complete flexibility the decisions are made with the existing knowledge at a point in time, knowing that as more information arrives the decisions can be costlessly adjusted to reflect the new information. With complete inflexibility, the decision maker is unable to make any adjustment to his decision in response to future information. The

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[4]This argument was later refined by Wright in 1985.

probability distribution which exists at the time the decision has to be made becomes the only relevant distribution. Thus, investment under uncertainty can be reduced to a problem of risk, by integrating over the unknown parameters of the probability distribution at the time of investment, if either complete flexibility or complete inflexibility exists. To summarize, closed loop solutions for economic problems are required when the decision maker anticipates receiving more information and the ability to react to this new information is affected by current decisions. Stated another way, the prospect of learning will affect optimal behavior if current actions affect the ability to exploit anticipated learning.

Despite this early work which emphasized the importance of the prospect of learning in decision making, the subsequent study of investment decisions became so dominated with two period comparative statics with either complete flexibility or inflexibility, that decision making under risk and uncertainty became known as an open loop problem (Arrow, Pratt, Sandmo). It is clear that Hart, Stigler, and Tintner believed in an important distinction between situations which included a prospect of learning from those which did not. Perhaps these early arguments received limited attention because the distinction they wished to draw between risk and uncertainty did not fit with the standard distinction in the literature. In particular they used risk versus uncertainty to distinguish between situations where learning was not anticipated versus those where it was. Subsequent work has shown that the prospect of learning can be important even in problems involving only risk. The classic example is a search problem where the decision maker faces several prospects each with a known probability distribution of rewards. The decision maker must anticipate what he will learn from searching an additional prospect. As long as the decision maker has the ability to react to this information and this ability to react is influenced by previous actions the solution to the problem is closed loop. *Thus closed loop solutions can be important for decisions involving either risk or uncertainty.*[5]

Rather than being a special circumstance, a wide range of economic problems have these characteristics and therefore require a closed loop solution. Current actions will influence future flexibility in many circumstances. This linkage between current actions and future flexibility can exist in at least three different forms. If a current decision involves an investment and the process of investment or divestment involves a transaction cost[6] this will affect the expected average cost of an investment. The investment may have to be reversed due to particular adverse realizations of a random variable. The fixed transaction cost will be spread over an expected period of time which decreases with the amount of uncertainty. Some examples of transaction costs would include installation costs, brokerage fees, and asymmetric information which create a wedge between acquisition cost and salvage value. Secondly, the search for more information may involve a sunk cost. In this case the ability to use potential information is a direct function of a decision to seek information. This would apply to simple search problems as well as more complex exploration or research projects. The third case where current actions can affect the ability to respond to future information is where the decision makers ability to respond to new information is constrained. If current actions affect the probability that the constraint will be binding these actions will have an indirect influence on future flexibility. Examples of this type of situation are storage models where there is a "stock out" constraint or a storage capacity constraint (Wright and Williams)

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[5]Given that that both open and closed loop decisions can involve either risk or uncertainty, attempting to distinguish between open and closed loop problems on this basis may have weakened the interpretation of Hart's paper. Uncertainty would become a necessary condition for the closed loop solution only if the definition of uncertainty was expanded to include situations of risk where realizations could be acted upon.

[6]An extreme transaction cost is a totally irreversible investment. Note however, even with a totally irreversible investment the decision maker has some flexibility because if the investment can be postponed.

or in finance models where the behavior of a firm is influenced by a bankruptcy constraint (Foster and Rausser, Gray).

The wide range of economic problems requiring closed loop solutions has lead to a separate and specialized treatment of the problems in many different fields of economics. This separate treatment has masked the similarity of these problems which when revealed, assists the development of the intuition required to understand these models. The remainder of this paper briefly summarizes of the closed loop models in each of the areas.

3.1 Economic Search Models - Learning Given Static Distributions

Some of the earliest applications of closed loop solutions dealt with models of economic search. In these models the distributions of the random variables are static and do not change over time[7]. In this situation the decision maker would acquire the optimal amount of information and then make the investment decision. These closed loop decision problems tend to be simpler than fully dynamic problems.

The large amount of effort in examining this type of problem is outlined in the literature on economic search. In this type of problem an economic agent must search across many prospects for an economic reward such as a higher salary or or a lower price for some good he is purchasing. The seminal piece in this area of work was done by Stigler (1961, 1962). Subsequent work examined more complex search problems. For example, Weitzman examined the case where the prospects varied in nature and Rothschild examined the case of uncertainty where the probability distribution of the prospects was unknown to the agent. The solution to these search problems are characterized by a decision rule in which the search is terminated when a reward exceeds some reservation price. At the reservation price the expected gain from searching an additional prospect is just equal to the cost of doing so.

[7]Some of the complex search problems do contain more dynamics compenents for example see Landsberger and Peled consider the affect of the duration of offer on the value of search and optimal behavior.

These decision problems are not fully dynamic. The degree of uncertainty would affect the amount of information collected and the optimal investment behavior but in general would not affect the timing of the investment project. A decision of how much to spend on a research project or on exploration may fall into this category of decision making. In a Bayesian context this would be the problem of determining the optimal stopping rules for sampling a static distribution when there is a cost to sampling.

3.2 Learning Over Time and the Value of Waiting to Invest

In many economic situations information arrives over time. *When information arrives over time, the prospect of additional information in the future creates a value of waiting to invest for irreversible investments.* Recent developments in this area have come from concepts and economic tools which were developed and refined in the fields of Resource Economics and Financial Management.

There have been several papers dealing with irreversible investments and the theory of the firm done very much in the spirit of Hart, Stigler and Tintner. Until recently this work has been sparse and spread over several decades. In 1959 Bellman outlined the principles of dynamics programming and in particular introduced the Bellman equation which characterizes the closed loop solution. Howard described the closed loop solution to the toy makers problem. Marglin applied the concept to other economic investments. Cukierman, and Bernanke both demonstrated that if information arrives over time there will be some incentive for the firm to postpone irreversible investments. Bernanke suggests that this incentive to wait could play a large role in business cycles. Baldwin examined the optimal sequencing of irreversible investments. Jones and Ostroy derived several propositions regarding the value of waiting to invest and showed the value is increasing with the amount of uncertainty.

In Resource Economics, in what has become a seminal paper, Arrow and Fisher considered the case of making a totally irreversible decision to develop a natural wilderness area for commercial use[8]. They showed, given the decision to develop was totally irreversible, and the decision to postpone was reversible at some future date, the net benefits from developing the area are reduced, and in general, less area should be developed. They used a two period closed loop dynamic programming model under the assumption of risk neutrality[9]. They found if the benefits in period one are correlated with the benefits in period two--thus some learning was possible, there was a 'quasi-option' value to retaining the natural habitat. This is consistent with Hart's assertion that risk aversion is an unnecessary assumption in order to study investment under uncertainty. Arrow and Fisher state:

> "An interpretation of this result might be, that if we are uncertain about the payoff to investment in development, we should err on the side of under investment, since development is irreversible. Given an ability to learn from experience, under investment can be remedied before the second period whereas mistaken over investment cannot.." pp.317.(Arrow and Fisher)

All of the work above demonstrated, using simple discrete examples, that the prospect of learning created a value in waiting to invest. Until recently, attempts to apply these models in a more general way to real world investment problems failed. This failure was in large part due to the difficulty in deriving closed loop solution to more complex problems. In particular it may have been the failure to derive analytic solutions general enough in nature to be applied to actual investment problems which could demonstrate the magnitude of the difference between the open and closed loop solutions to the same problem. Fortunately, recent work has been successful in deriving analytic solutions to more general problems. These break-throughs have occurred in the area of Financial Management and have subsequently been applied to a wide range of investment problems.

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[8]Henry independantly introduced a very similar augurment to Arrow and Fisher also in 1974.

[9]This result was particularly interesting given that the Arrow Lind had shown that public investments should be made in a risk neutral context because of the risk pooling effect (Fisher).

3.3 Learning Through Price Changes Over Time

In the field of Financial Management early work employed the risk framework, with Portfolio Analysis becoming a standard tool. This was basically a reduced form of the mean-variance approach to risk where the expected rate of return on an asset was determined by the riskless interest rate, the market price for risk and the co-variance of the asset with other assets in the portfolio. In this framework the probability distribution for price in some future period was a known distribution which was invariant to time. This implied no learning was possible about this price distribution.

Muth and later Samuelson pointed out that if prices of assets or commodities had this type of probability distribution for prices an investor could get infinitely wealthy by arbitraging the market. To do this the investor could buy at a low price and sell at a high price realizations. The only equilibrium condition which eliminates systematic profitable arbitrage for assets or storable commodities is that the current price of the asset plus interest and storage costs, always reflects the expected value of prices in the future[10]. This implies realized future prices of a dividendless stock P_{t+1} will be equal to:

(1) $$P_{t+1} = P_t(1+r) + \varepsilon_t \qquad or \;\; E[P_{t+1}] = P_t(1+r).$$

where ε_t is an error term independently distributed over time with a zero mean, r is the one period risk adjusted interest rate, and $E[P_{t+1}]$ is the expected price in the next period. This well known random walk model of stock prices has added a great deal of structure to individual expectations and price movements over time. In the continuous time version of Sameulson's model prices are assumed to follow a process of Brownian motion. In these models the variance in the price forecast is equal to the sum of the variance the error terms between the current period and the forecast period. As the forecast period is approached,

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[10]Some commodities exhibit expected future prices which are less then the current price plus interest and yet the commodity is held in stocks. One explanation is that the stock have a convenience yield to their holders or there is some probability of a stock out between the current period and a distant period.

these errors in forecast naturally diminish. In addition, changes in the current price change the expectation of prices for all future periods. Therefore, in an arbitragable market, the anticipation of learning and waiting are synonymous. The flow of information is characterized by the stochastic process of prices.

As financial instruments evolved to include contingent claims and financial options, the need to model investment under uncertainty with incomplete flexibility became important. Contingent claims are contracts which give the holder of the contract an option to undertake a transaction which has a value which varies with states of nature. An example of such a contingent claim is a commodity option. An American Call Option gives the holder of the contract the option to obtain a commodity futures Buying Contract at a specified price called the *strike price*. The holder of this contract has this option up until one month before the particular Buying Contract is due. This clearly gives the holder of the option some flexibility. If the futures price remains below the strike price she will not exercise her option. If the futures contract price exceeds the strike price he can buy the futures at the specified price and resell them for an immediate profit. Thus, the realized value of an option is a non-linear (convex) function of realized futures prices. Black and Scholes developed a continuous time model which assumed price followed Brownian motion, and thus were able to value a simple (European) option. To derive the equilibrium conditions they used a technique of constructing a riskless portfolio. They then used Ito's lemma, from stochastic calculus, to derive an expression for the change in the value of the contract over time. Given that they had constructed a riskless portfolio the value of the option must be expected to increase at the riskless rate of interest. From this relationships they were able to derive the value of contract at any point in time or price level.[11]. This was the first of several papers which used no arbitrage equilibrium conditions to determine the value of financial

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[11]For a complete description of the Black and Scholes formula, its derivation and uses see Milliaris and Brock.

instruments. Later work was able to derive analytic solutions for more complex options. In particular it became possible to calculate the optimal *exercise price* for some options. The exercise price defines the price at which it is optimal to exercise the option. At the exercise price the marginal cost of holding the option another instant is its value multiplied by the riskless rate of interest. This cost must be just equal to the marginal expected gain of holding the option another instant. Therefore, at the exercise price the value of the option must be increasing at the rate of interest. This is analogous to the condition which defines the the reservation price in the standard search models. Other important work in Finance included Merton who derived equilibrium returns for a portfolio of assets and Cox, Ingersoll and Ross who derived the value all contingent claims within a portfolio.

In addition to the analytic approaches, numerical procedures for evaluating contingent claims have been developed in Financial Management using dynamic programming based on the Bellman equation. For an exposition of some of these techniques see Ho and Lee. These techniques, although not as elegant, are accurate and have the important added advantage of being able to handle a much wider range of functional forms, time horizons and stochastic processes. These approaches can be utilized to evaluate the value of an option for a variety of stochastic processes as long as they can be written down as a Markov process[12].

The tools developed in Financial Management have allowed the theory of investment under uncertainty with incomplete flexibility to be applied to investment problems. By specifying the stochastic processes which prices follow, it is possible to derive a solution to the closed loop problem. The natural resource literature continued to explore the effects of irreversible investments and learning. The early models which extended the work of Arrow and Fisher were discrete dynamic programming models used for illustrative purposes. The study of irreversible investments in Resource Economics generally dealt with physical

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[12] For a introduction to dynamic programming and Markov process see Howard 1960.

irreversibilities, but as Hanemann points out in his survey article, the results derived would also apply to economic irreversibilities. Generally in most environmental problems dealing with irreversible investments the form of learning cannot be well defined and as such the value of waiting to invest can only be described qualitatively.

Many of the later models employed in Resource Economics have borrowed techniques developed in Financial Management to examine topics involving uncertainty including optimal extraction of resources in the presence of either supply or demand uncertainty, optimal exploration, etc. Brennan and Schwartz consider a decision of opening or closing a mine given the mine will rapidly deteriorate and will be subject to investment cost to resume production if at any point it is shutdown. They find using a numerical solution that the price at which a mine will be opened and the price at which it will be shutdown will be separated by a large band of investment inactivity in between. Using a slightly more restrictive set of assumptions the papers by McDonald and Seigel derive some general analytic results for problems of irreversible investments. They examine a real investment opportunity which is totally irreversible and cast the problem in continuous time assumming Geometric Brownian motion in prices. They then use Ito's lemma to derive a stochastic differential equation describing how the value of the states evolve over time assuming an infinite planning horizon. In order to find the specific price (the exercise price) at which it is optimal to move from one state to another they impose the *smooth pasting condition.* The smooth pasting condition utilizes the concept that at the exercize price if the firm is indifferent between two states at a point in time it must also be indifferent a moment later. This means the value of the two states must be increasing at the same rate at the exercize price. This is analogous to condition which defines the reservation price in the standard search models. They show the value of waiting is very large and, in many situations, the investment should be made only when the expected present value of the investment is more than double the cost of the investment. Another important point which they make is that if both the cost and the present value of the investment are stochastic it is the variance of this ratio which determines the value of waiting to invest. *Thus if the cost of the investment always reflects the present value of the investment (which it would in the case of a fixed resource) standard investment rules apply.*

These conclusions have been supported for a variety of irreversible investments. McDonald and Seigel, and Ekern examine project feasibility where the the project is a lumpy investment. Pindyck builds on the analysis developed by and McDonald and Seigel to examine the effect of the value of waiting on the marginal decisions of a firm to expand capacity. Dixit (1989a) explores the conditions of entry and exit when investments involves a small sunk cost. In the case of sunk costs Dixit finds a considerable gap between the price of entry and exit even when transaction costs are relatively small. This band of price inaction will create *hysteresis* which means the equilibrium level of capital in an industry will be function of past prices as well as current prices. In Dixit (1989b) he shows that temporary change in exchange rates can have lasting effects on trade even after exchange rates return to their historical levels. Perhaps the most general case for examining the value of waiting to invest is presented by Ingersoll and Ross where they explicitly look at the effect of stochastic interest rates on a variety on investment projects. They show using numerical methods that the effect of stochastic interest rate on the delay of projects is sizeable and an increase in the uncertainty of interest rates will reduce the optimal level of investment.

4.0 Summary and Conclusions

For some investment problems, where either complete flexibility or inflexibility exists the prospect of learning has no influence on the decision and standard investment rules apply. However, for many economic decision the prospect of learning may affect optimal behavior.

The solution to this closed loop problem can be very different than the standard investment rules. If information arrives over time there is some value in waiting to invest which increases with the amount of uncertainty.

By specifying the nature of the stochastic process which affects the investment decision and borrowing the necessary tools for the Financial Management literature, it has recently been possible to derive quantitative analytic solutions describing the optimal investment behavior for a variety of investment problems. A quote from the abstract of Ingersoll and Ross which summarizes the results from this growing body of literature:

> "The textbook analysis that accepts projects when their net present value is positive is generally quite wrong. The ability to delay committing to a project means that almost every project is in competition with itself postponed."

The recent work has shown that if the stochastic process is defined it is possible to derive the closed loop solution for investment problems with incomplete flexibility. Given that almost any investment involves some transaction cost there is a need to re-examine much of the existing literature based on standard investment rules and ignore the prospect of learning. Perhaps for most investments the prospect of learning will have only a small influence on the investment rule. However, the work that has been done to date suggests that the standard investment rules are substantually wrong for many investments.

5.0 Further Work

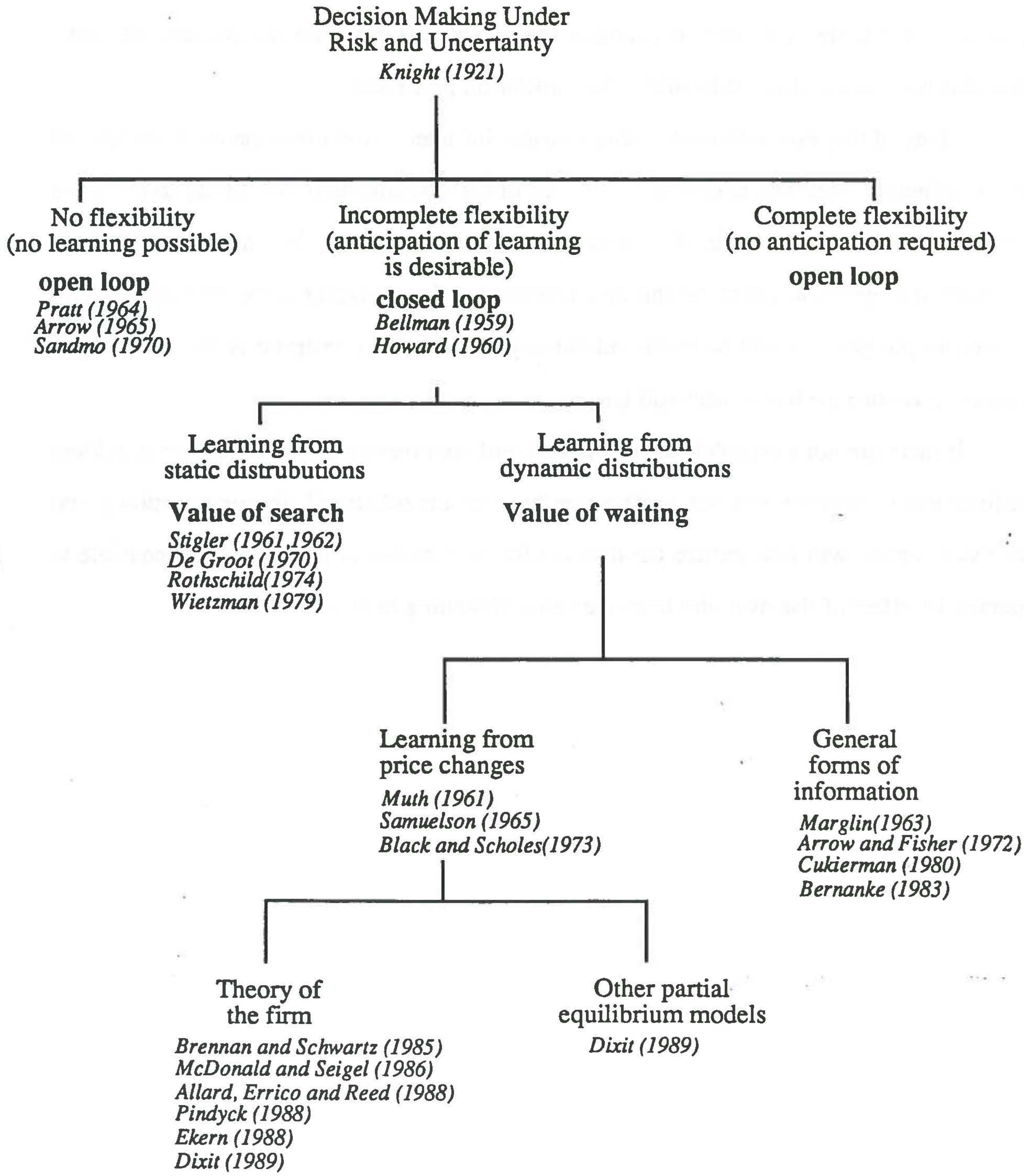
The assumptions of the existing work, which derive optimal investment behavior, require further examination before the results are broadly applied. Currently, there are many questions with unsatisfactory answers representing areas for further theoretical and applied research. For example, are the assumptions consistent with equilibrium in the goods market? In particular, existing investment models examine the behavior of an individual firm assuming the investment behavior has no influence on the prices of the investment or the output prices. Even in a perfectly competitive industry if all firms invest in response to the same stimulus this assumption may be unrealistic.

A related question is; do prices follow a pure diffusion process or do relative prices tend to revert to some underlying mean? There is considerable study and debate on this issue, however, given the possibilities of stock outs and underlying technical relationships there is some evidence that prices of commodities do not reflect pure Brownian motion. This being the case there is a need to examine the extent to which the results derived under Brownian motion are affected by alternative stochastic processes.

Many of the results derived to date consider infinitely lived investments. If investment are not infinitely lived this may affect the value of the investment an the ability to postpone new investments. For example, if someone is considering purchasing a new automobile he may have the option to postpone the new investment by spending more on repairs. This decision to postpone cannot be made indefinitely. Given this constraint is the value of the option to postpone the investment still large?

If there are not a complete set of markets and the investor is unable to form a riskless portfolio what influence will risk preferences have on the outcome? In this situation given that risk aversion will also reduce the demand for investment, is it empirically possible to separate the effect of risk aversion from the value of waiting to invest?

Figure 1.0: Decision Making Under Risk and Uncertainty



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