Risk

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DECISION TREES FOR HORTICULTURAL DECISION–MAKING UNDER NON–CERTAINTY

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DECISION TREES FOR HORTICULTURAL

DECISION-MAKING UNDER NON-CERTAINTY

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PREFACE

The complex problems involved in managing horticultural enterprises are receiving increasing attention from advisory officers and agricultural educationalists. Massey University introduced a new course in Horticultural Management in 1972 in the fourth year of the Bachelor of Horticultural Science degree. In 1973, a new course in Horticultural Economics will be introduced in the third year of that degree.

Dr. A.N. Rae, who is teaching these two courses, has developed this Discussion Paper from an address he gave at a Ministry of Agriculture and Fisheries seminar for Horticultural Advisers held in Christchurch in June 1972. The paper discusses an approach to decision making which may be particularly useful to horticultural producers and their advisers. Readers should be aware that the management methods outlined have not yet been comprehensively tested in the field in realistic farming situations. However, as Dr. Rae says:

> "Just the exercise of constructing a decision tree can be of immense value to both grower and adviser alike".

I hope that interested readers will be stimulated to consider the potential applications of the analysis presented. Dr. Rae would welcome comment and constructive criticism.

A.R. Frampton,
Professor of Agricultural Economics
and Farm Management.

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CONTENTS

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1. INTRODUCTION

The great majority of decisions facing horticulturists contain some element of risk, or non-certainty.[1] In deciding whether to invest in a new glasshouse, whether to plant a further 20 acres of apples, whether to store pumpkins, to which mart should produce be consigned, how much cash should be allocated to advertising house plants, and so on, the horticulturist may not be sure of the outcome of his decision.

Traditionally, many such risky decision problems have tended to be analysed as if risk did not exist, for example by using budgeting to compare the profitability of different production and/or marketing plans. The answers so obtained may be acceptable if the consequences of risk are not too great; say, if only small differences exist between the possible payoffs that might be obtained from a new crop planting. These answers may not be acceptable where such differences are large; for instance

1 The term 'risk' is used to refer to any non-certain situation, rather than in the sense of Knight's classification of risk versus uncertainty (Knight (1921)). Our approach is that, even in Knight's 'uncertain' situation, subjective probabilities can always be estimated so that for analytic purposes, risk and uncertainty become synonymous.

a combination of floods, heavy disease infestation and low prices may result in bankruptcy for the manager, and he may then prefer to choose an action that somehow 'guards against' such effects.

Non-certainty might be recognised in traditional budgeting analyses to the extent of using conservative estimates in the budgets, say higher-than-expected costs and lower-than-expected prices and yields. Alternatively, parametric budgeting might be employed to examine the sensitivity of profit to non-certain future events. If the grower feels the sensitivity is 'too high' changes to his production schedule might be made. Such approaches to incorporating risk elements into the analysis of a farm management problem can only be approximate, though, and really 'miss the heart' of the decision-making problem. It is suggested that the approach to be described in this paper has the potential of coming nearer to the formulation of decisions that are in accord with the manager's beliefs about the future and his attitudes towards the taking of risks.

Opportunities for diversification are often included in budgeted plans, or such plans are constructed with some degree of flexibility, to act as a hedge against non-certainty. Flexibility might enable the operator to take advantage of high prices in some future season, or to modify his management to avoid large losses in times of low prices. A desire for a diversified cropping programme might reflect a situation where crop incomes are less than perfectly correlated, and low incomes from some crops might be offset to some degree by high incomes from other crops. Provided that all such opportunities are specified, the analysis to be described is capable of providing the manager with a strategy that will indicate the optimal decision to take at any point in time, given the sequence of events that has occurred up to that time. This strategy will be optimal in terms of the manager's beliefs, preferences and objectives, and will thus exhibit the optimal degree of flexibility and diversification. The remainder of this paper illustrates how budgeting analyses might be modified to allow such strategies to be derived.

Before discussing how rational decisions can be formulated in the face of non-certainty, it is necessary that we understand the difference between 'good' decisions and 'right' decisions. The evidence upon which decisions are based relates to the past, whereas the consequences of our present decisions lie in the future, and we cannot be sure what the future will bring. A 'good' decision is one that is consistent with the assembled evidence and the beliefs and preferences of the decision maker at the time he must make the decision. Whether or not the decision is 'right' will not be known until its consequences have been observed some time in the future. If a poor outcome results from a 'good' decision, we say the outcome, <u>not</u> the decision, was bad.

2. ANATOMY OF A DECISION TREE

The principal components of a risky decision problem are possible courses of action, possible events that might occur in the future over which we have no control, and the payoff received given any combination of action and event. A decision tree (Raiffa (1968), Schlaifer (1969)) is a convenient way of providing a diagrammatic representation of the problem. The tree is made up of act forks, event forks, and branches, and Figure 1 presents a simple decision tree. Here, a grower may contract to grow beetroot at a price of $40 per ton, or he may grow the crop for the market, where price may be either $30 or

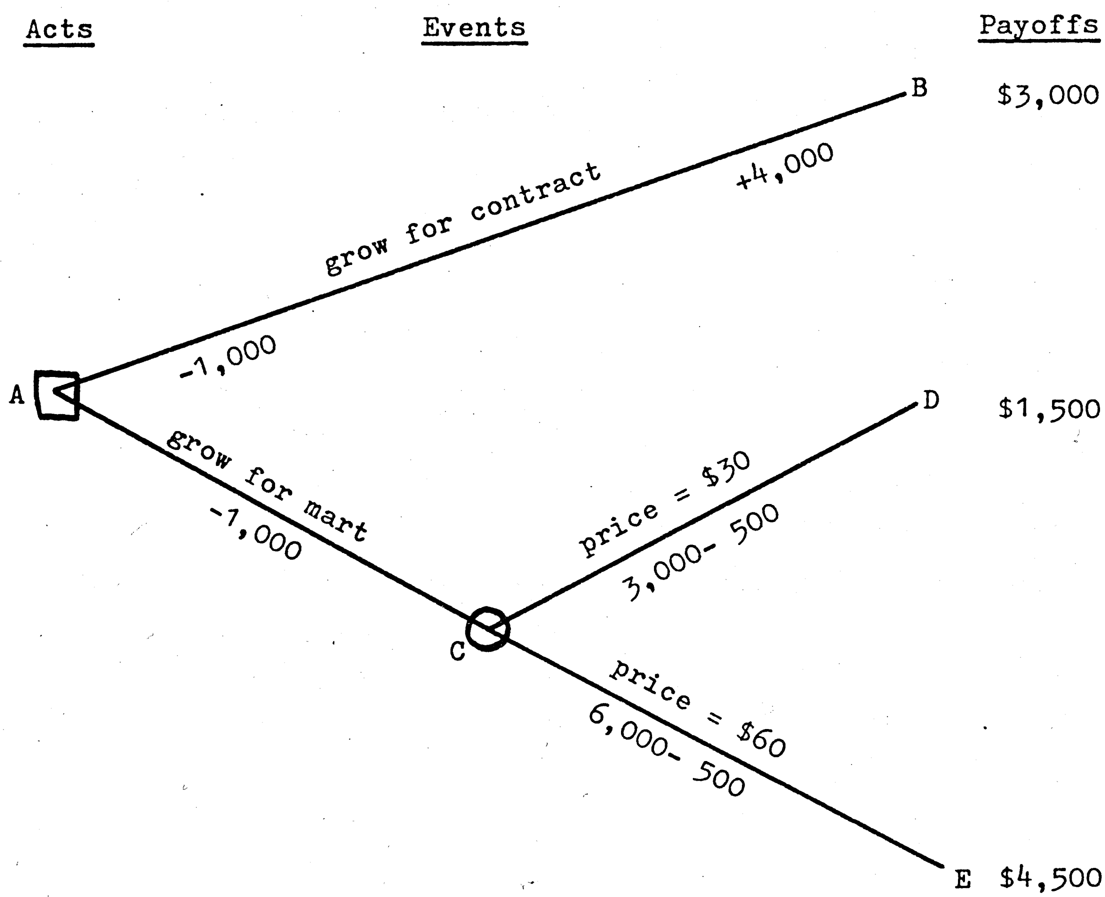

Figure 1. A simple decision tree

$60 per ton,[2] depending on demand/supply conditions. Yield is fairly certain to be 10 tons per acre, production costs are known to be $100 per acre, and a marketing cost of $5 per ton must be met if the crop is sold on the fresh market.

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2 Alternatively, imagine (i) that all possible price per ton outcomes have been partitioned into two sets, say $45.00 and over ('high') and $44.99 and under ('low') and (ii) that $60 was chosen to represent the 'high' price event and $30 to represent the 'low' price event.

Let us assume he has already decided to grow 10 acres of the crop, so that the only two alternative courses of action in this problem are to grow under contract or to grow for the market. These are represented by the two branches emanating from point A in the figure, namely AB and AC, and point A represents the node of an <u>act fork</u>. The branches labelled CD and CE represent the source of uncertainty in this problem, and point C is the node of an <u>event fork</u>. The terminal payoffs $3,000, $1,500 and $4,500 are the possible outcomes of the problem. A little budgeting will show that the crop will net $3,000 if grown under contract, $1,500 if grown for the market and a price of $30 per ton is received, and $4,500 if grown for the market and a price of $60 is received. The actual flow of cash is represented by the figures entered underneath the branches.

An important point to note is that act forks and event forks represent two quite distinct things, and the node of an act fork is often surrounded by a square, and that of an event fork by a circle, to avoid confusing the two. It is also important that act forks and event forks appear in the decision tree in the actual order in which they will occur. In the example, market prices will not be known until some time after the crop has been sown, and possibly not until the crop has been sold.

The decision tree, correctly drawn, gives a clear picture of the situation and may be worth constructing just for the purpose of clarifying the problem and indicating possible actions and outcomes. However, we will use the decision tree in order to find the best solution to the problem. To achieve this, we must first of all consider people's preferences among gambles and the notion of certainty equivalents.

3. PREFERENCES AMONG RISKY PROSPECTS

A course of action whose outcome is uncertain at the time of choosing that action can be called a risky prospect. Here, we want to show how people's preferences among risky prospects can be measured and that what is best for one person need not be best for another. To illustrate the principle involved, we will consider a person who has the opportunity to participate in one of two coin-toss gambles. The first will return him $10 if heads comes up, but he will lose his $10 if the coin turns up tails. In the second, he will win $1,000 if heads comes up, but will lose $1,000 if the outcome is tails. Now which gamble would he rather participate in? If he chooses the first gamble this means the satisfaction, or 'utility', to be expected from this gamble exceeds that of the second gamble. An interesting fact, though, is that the <u>expected</u> payoff[3] from each gamble is $0:

Expected payoff, gamble 1 = (.5 x $10) + (.5 x -$10)
= $0.

Expected payoff, gamble 2 = (.5 x $1,000) + (.5 x -$1,000)
= $0.

This means, then, that we cannot necessarily calculate the expected monetary values of alternative risky prospects and choose that whose expected value is highest.

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3 The expected payoff is simply the weighted average of all possible payoffs, where the weights are the relevant probabilities. In this example (provided the coin is 'fair') the probability of 'heads' is 0.5, as is the probability of 'tails'.

To get over this problem, we work with certainty equivalents[4] (Schlaifer (1969)), where the certainty equivalent of a risky prospect is that sum of money which, if received by the decision maker for certain, would make him just as happy as having to face the risky prospect. That is, if he were offered that sum with no risk he would be indifferent between accepting it, or rejecting it and participating in the gamble. In the coin toss problem, then, he would have to decide on a sum of money that would make him just as happy as participating in the +$10/-$10 gamble - say $3. Likewise for the +$1,000/-$1,000 gamble, we will say the certainty equivalent is -$100, which means that just before the coin is tossed, he is indifferent between calling "toss" and paying <u>out</u> $100 to be released from the bet (and getting his $1,000 stake back). Since the certainty equivalent of the first gamble exceeds that of the second, then the first gamble is to be preferred. The important points to note are that since different people have different attitudes to risk-taking so different people may choose differently among risky prospects, and that the act for which expected money payoff is greatest need not be the best act.

4. SOLUTION OF THE BEETROOT CONTRACT PROBLEM

Now that we have seen that a person's preference among risky prospects can be indicated through estimation of certainty equivalents, we are in a position to solve the beetroot contract problem of Figure 1. The method involves starting at the <u>final</u>

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4 Raiffa (1968) uses the term 'certainty money equivalent', rather than 'certainty equivalent'. Both concepts are identical, however.

stage of the decision tree (or the right-hand edge) and then moving to the left, or backwards through the tree, calculating certainty equivalents as we go. There is only one event fork in Figure 1, reflecting the two possible beetroot prices. If beetroot is grown for the mart, the terminal payoff will be either \$1,500 or \$4,500. Now we say to the grower "If the price turns out to be \$30 per ton you will receive a payoff of \$1,500, but if \$60 per ton, you will receive \$4,500. Would you rather take a chance on the market price being high, or take a certain payment of, say \$4,000?" The odds are that he will prefer the certain \$4,000, so we know our first attempt at specifying the certainty equivalent (\$4,000) was too high. We then repeat the question until he is indifferent between receiving the certain payment or participating in the market price gamble. Let us say that the certainty equivalent turned out to be \$2,500. The decision tree can now be redrawn in a simpler but <u>equivalent form</u>, as in Figure 2.

It can be seen that the certainty equivalent takes the place of the risky prospect over which it was estimated. This is because the certain \$2,500 gives the same satisfaction to the grower as does participating in the gamble. Thus, payoffs from other acts may be compared directly with the certainty equivalent. In this case, the payoff from growing for contract of \$3,000 exceeds the 'equivalent' payoff from growing for the fresh mart, of \$2,500, so growing for contract is the optimal or 'best-bet' decision. Of course, another grower may have a different attitude to risk, and may have come up with a certainty equivalent of, say, \$3,500. Then, <u>his</u> optimal decision would be to grow for the mart.

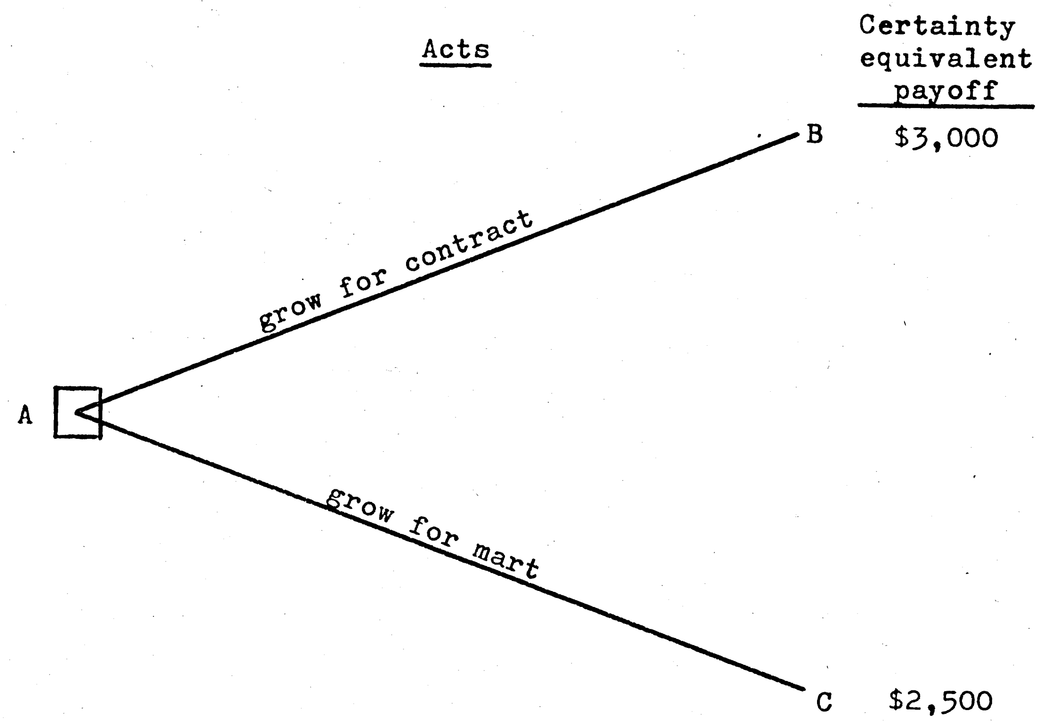

Figure 2. Simplified decision tree - beetroot contract problem

It should be realised that when certainty equivalents are estimated, the decision-maker does two things:

a) he 'weighs up the odds' involved in the gamble; and

b) he considers his preference for the terminal payoffs associated with the risky prospects.

In assessing the 'odds', the grower would make some estimate of the likelihood of each price arising. Put another way, he makes an estimate of the probabilities involved. These probabilities are simply weights representing degrees of rational belief that the decision-maker holds in the various events occurring, and should be made with recognition of whatever objective evidence happens to be available. Thus if a series of prices received over some past time period is available, probabilities may be estimated by applying a smoothing procedure to the relative frequencies as described, for example, by Schlaifer (1969). However, if the marketing system had recently undergone some change so that prices may not be expected to behave in the same way as in the past, then the historical data becomes less important, and the subjective beliefs of the decision-maker more important, in determining the probability distribution of future prices. In the latter case, then, the probabilities may be estimated subjectively, rather than objectively.[5]

The decision-maker's preference for terminal payoffs refers to the satisfaction or dissatisfaction he would experience if a given outcome occurred. We have already seen that this satisfaction need not be in direct proportion to the value of the money sum, and of course a given money sum will give more satisfaction to some people than to others.

The solution procedure requires that we work only with the terminal payoffs, and the cash flow is important only in deriving the terminal payoffs.[6] To summarise the method, we commence computations at the right-hand edge of the decision tree and move 'backwards' through the tree, replacing event forks with their certainty equivalents. When we come to an act fork, we select the act branch with the highest certainty equivalent payoff, until the complete strategy has been enumerated.

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5 Raiffa (1968, Ch. 10) reviews the historical development of the concept of probability.

6 Depending on the length of the planning period, it might be necessary to express terminal payoffs as discounted present values.

5. A SOLUTION PROCEDURE FOR MORE COMPLEX PROBLEMS

For many problems, we can expect the number of branches in some event forks to be much greater than two. In such cases, certainty equivalents would have to be estimated over gambles with a large number of possible payoffs, and we might expect difficulty simply in comprehending such gambles. Fortunately, these problems can still be solved provided that we can estimate the probabilities of the various events, and the decision-maker's utility function. The estimation procedures themselves are presented by many authors such as Halter and Dean (1971), Schlaifer (1969) and Makeham *et al*. (1968) and here we shall be concerned with solving the beetroot contract problem making use of probabilities and utilities.

First, suppose the grower believed that the likelihood of the market price being \$30 was 0.4 and that the likelihood of the price being \$60 was 0.6. Second, making use of the certainty equivalent already estimated, we know that the utility (satisfaction) provided by a \$2,500 payment equals the utility to be expected from the gamble involving a \$1,500 payoff with probability 0.4 and a \$4,500 payoff with probability 0.6. Since \$1,500 is the smallest, and \$4,500 the largest payoff in this problem, we can arbitrarily fix the scale of the utility function by assigning a utility value of zero to \$1,500 and a utility value of 100 to \$4,500. That is:

$$u(\$1{,}500) = 0,$$
$$u(\$4{,}500) = 100.$$

Making use of the certainty equivalent relation, then, we know

$$
\begin{aligned}
u(\$2{,}500) &= (0.4 \times u(\$1{,}500)) + (0.6 \times u(\$4{,}500)) \\
&= (0.4 \times 0) + (0.6 \times 100) \\
&= 60.
\end{aligned}
$$

Thus the payoff of $2,500 would be given a value of 60 on the zero-to-100 utility scale. These three utility-payoff points are plotted in Figure 3, and a smooth curve may be fitted. Having established the utility function, each terminal payoff in the beetroot contract problem can be converted to its utility value. Thus $1,500 has a utility of zero, $4,500 has a utility of 100, and the utility of $3,000 may be read off the graphed function as 75. The decision tree can now be redrawn as in Figure 4, entering probabilities under event branches, and utilities rather than terminal payoffs.

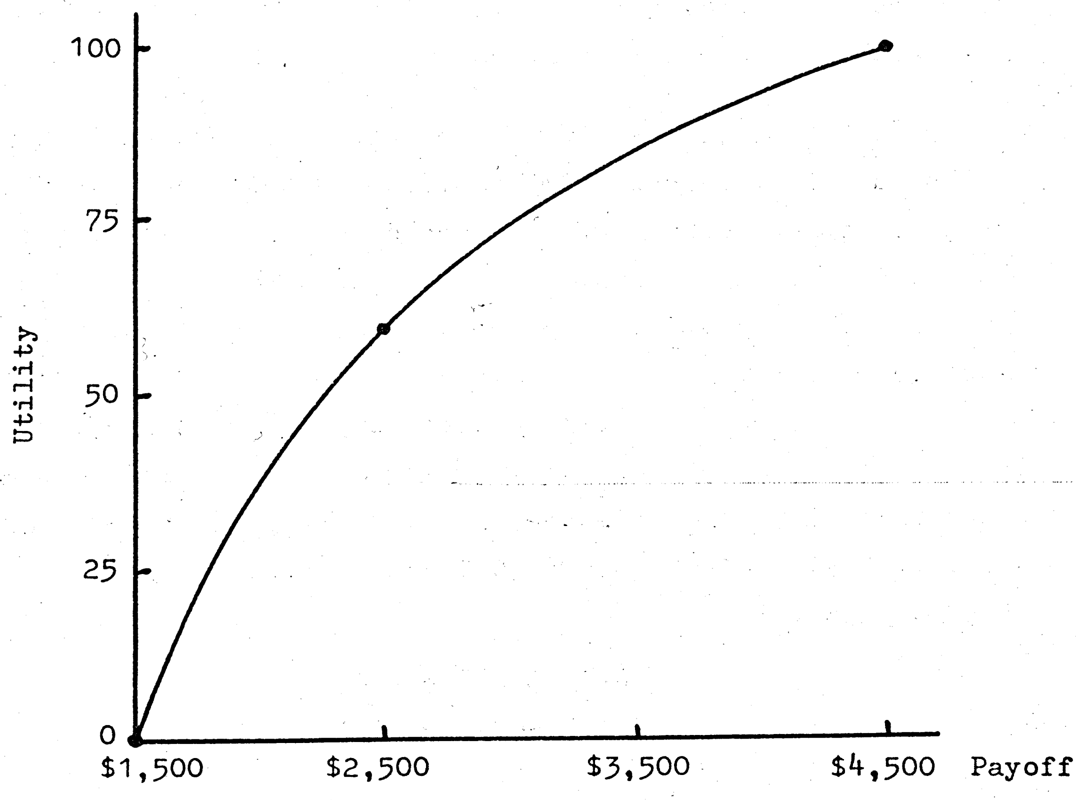

Figure 3. The utility function

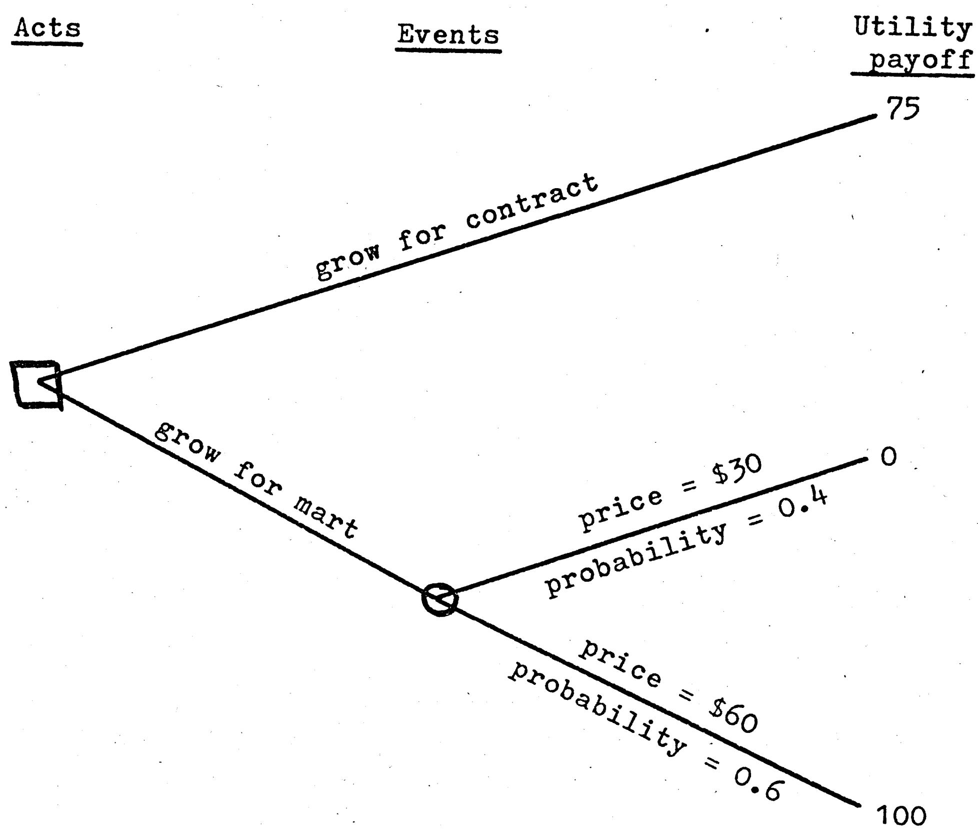

Figure 4. Beetroot contract problem with probabilities and utilities

The solution procedure (Hadley (1967)) is rather similar to that already described except, as we work from right to left through the tree, expected utility payoffs rather than certainty equivalents are estimated at each event fork, and at act forks, we choose the act branch with the highest expected utility. Thus in Figure 4, the event fork may be replaced by an expected utility value of 60:

$$(0.4 \times 0) + (0.6 \times 100) = 60.$$

Then, growing for contract is the optimal decision since its utility value of 75 exceeds the utility to be expected from growing for the mart, of 60.

6. A COMPUTERISED APPLICATION

In this example, a vegetable grower has the opportunity of leasing an acre of cropping land in addition to his own area of land. The lease will cost him $200 and runs for 12 months from April. His immediate decision, then, is whether or not to lease the land. The grower realises the return from the land is highly uncertain, so he sets down the crops that he would consider growing and the major sources of risk, and he budgets a number of possible terminal payoffs. Then he is in a position to sketch a decision tree similar to that of Figure 5.

If he decides not to lease the land, he can invest the $200 plus the $78 he has available to grow the initial crop at 4½%, giving him around $290 at the end of 12 months, or a net earning of $12.

He considers only two crops for winter production, either cauliflower or lettuce, to be planted in April. Should the winter be excessively wet and cold, the lettuce crop would be harvested a month later than usual, yield would be reduced, and a lower price would be realised because of the lower grade produce. If the winter weather is average or good, the lettuce crop would be harvested by September in time for a crop of cabbage to be grown during spring, to be followed by either pumpkin or summer

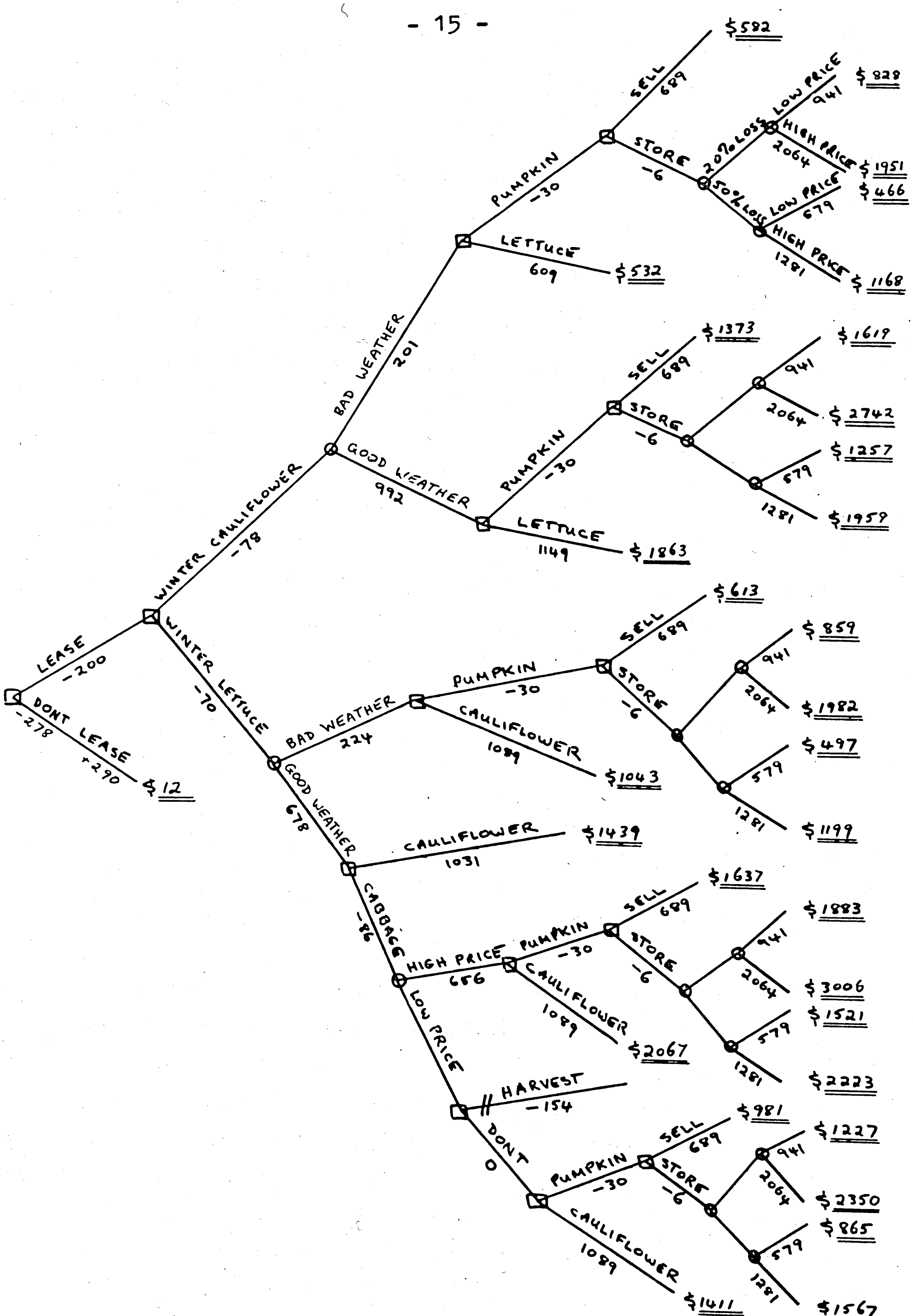

Figure 5. Decision tree for leasing problem

cauliflower. If pumpkin is grown, the crop may be sold once it is harvested, or it may be stored for later sale. In the latter case, however, storage losses, as well as the price received ex store, provide a major source of uncertainty. Cabbage prices are also considered to be "a bit of a gamble" and if too low it might be better to plough in the crop rather than harvest it. If winter lettuce is planted and a very wet winter follows, then there is no time in which to grow a spring cabbage crop before either the pumpkin or cauliflower crops.

The alternative winter crop is cauliflower. If the season is favourable to crop growth, the crop will be harvested during September and may be followed by an October planting of lettuce, or by pumpkin sown in November. Should the winter be very wet, cauliflower yield and the price received are likely to be lower than normal and the crop would not be harvested until October due to slower growth. Then, crops to follow may be either pumpkin or a November planting of lettuce. The disadvantage of the November lettuce crop, as compared with that sown in October, is that it is not ready to harvest until January, thus missing out on the high prices in the pre-Christmas market that the October-planted crop would obtain.

The decision tree in Figure 5 shows all act forks, event forks, and cash payments or receipts at each stage, with all terminal cash surpluses (one of which will be received at the end of the year) underlined. The probability estimates for all event branches are shown in Table 1. Since the cabbage price chosen to represent the 'low' price event did not allow returns to cover the harvesting cost, it was apparent that the 'harvest' option following this event would be the wrong decision. This is indicated by the 'double slash' through the appropriate branch of the tree.

Table 1. Probability Estimates - Leasing Problem

Event branch	Probability
'Bad' weather	0.3
'Good' weather	0.7
'High' cabbage price	0.8
'Low' cabbage price	0.2
20% storage loss	0.5
50% storage loss	0.5
'Low' price ex store	0.5
'High' price ex store	0.5

In analysing this problem, we shall assume that the grower's utility for money income derived from cropping the leased land (or alternatively, banking the available cash) can be measured as a linear function of that income. Thus the utility function would appear as a straight line when graphed against money income, the form of the function would be

$U = aX$, where X is money income and U is utility,

and by setting $a = 1$ we see that the dollar values can actually be used as the utility values.

The optimal strategy was obtained by use of the computer programme listed in Appendix I. To enable the input data to be assembled, the decision tree of Figure 5 must be redrawn to a uniform format as in Figure 6. Here, each decision stage must have one set of decision nodes and one set of event nodes, and every decision branch emanating from the initial decision node must continue through to the final stage. To accomplish

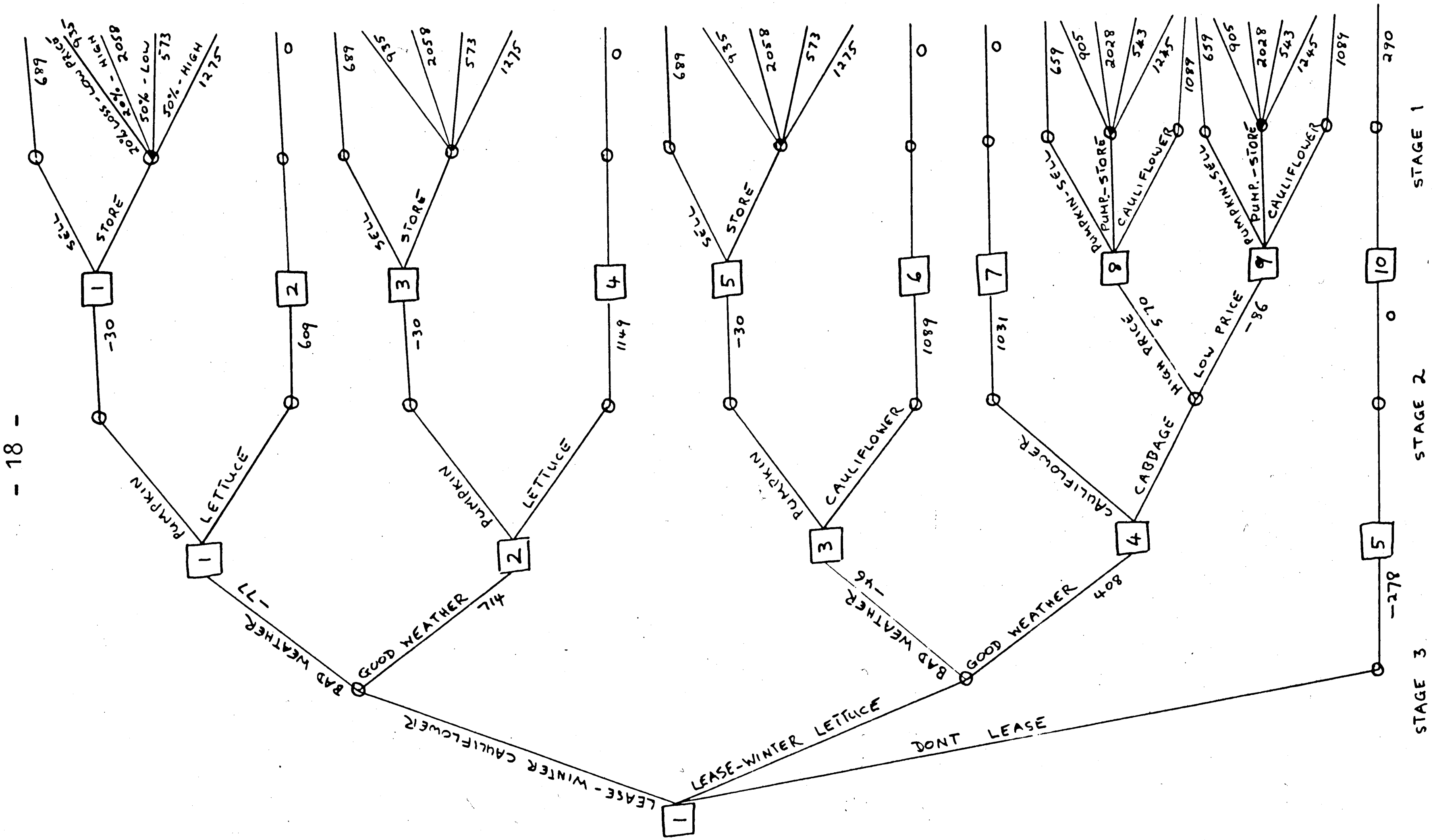

Figure 6. Decision tree for computer coding

this, 'dummy' decision and event nodes might be required with a zero monetary return, and with the 'dummy' event branches having a probability of one.

The problem consists of three stages, and decisions must be made at three points within the planning period. Since computation begins at the final stage, this stage is numbered as stage one for data preparation purposes, and thus the initial stage in the decision-making process appears as stage three. Also, the decision nodes within each stage are numbered, starting at the top of the decision tree with node one.

All costs and returns in the tree refer to a complete stage, and might be the summation of costs associated with a decision branch and an event branch within that stage. If the land is leased and planted with winter cauliflower, then a return of -$77 will result if the season is 'bad', this being equal to the cost of $278 plus the return of $201.

If lettuce is grown in the second stage of the period rather than pumpkin, the pumpkin marketing decision of the third stage is not applicable. This provides an example of the use of dummy nodes. To allow returns to be summed to the right-hand edge of the tree, two dummy event branches and one dummy decision branch are required. As explained before, these events would have a probability of unity.

The payoff matrix referred to in the programme listing contains, row by row, the individual components of the payoff streams, with one stream for each path through the decision tree. As the tree of Figure 6 has 32 end points and three stages, these are the dimensions of the payoff matrix, which may be written as:

$$\begin{bmatrix} -77 & -30 & 689 \\ -77 & -30 & 935 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 408 & -86 & 1089 \\ -278 & 0 & 290 \end{bmatrix}$$

Following cards in the input deck give the discount rate, the coefficients of the utility function, and information on the structure of the decision tree. A complete listing of the data deck is given in Appendix II.

Appendix III gives the solution to the leasing problem. The solution procedure is exactly the same as that given in Section 5. At each stage the expected payoff associated with each decision branch is calculated, and the optimal decision branch for any decision node is indicated. By working back through the solution output, then, the optimal strategy can be pieced together. This is presented below in Figure 7; as indicated in the print-out, the optimal strategy has an expected income of $1759.20.

The 'best-bet' decision, then, is to lease the land and plant it initially in lettuce. Should the winter be 'bad' a stored pumpkin crop will follow; given less severe winter conditions, a cabbage planting would follow the lettuce crop, to be followed in turn by a stored pumpkin crop.

In order to determine the 'best-bet' <u>initial</u> decision, we first of all had to determine the 'best-bet' acts for the future. This necessity of having to examine future acts and

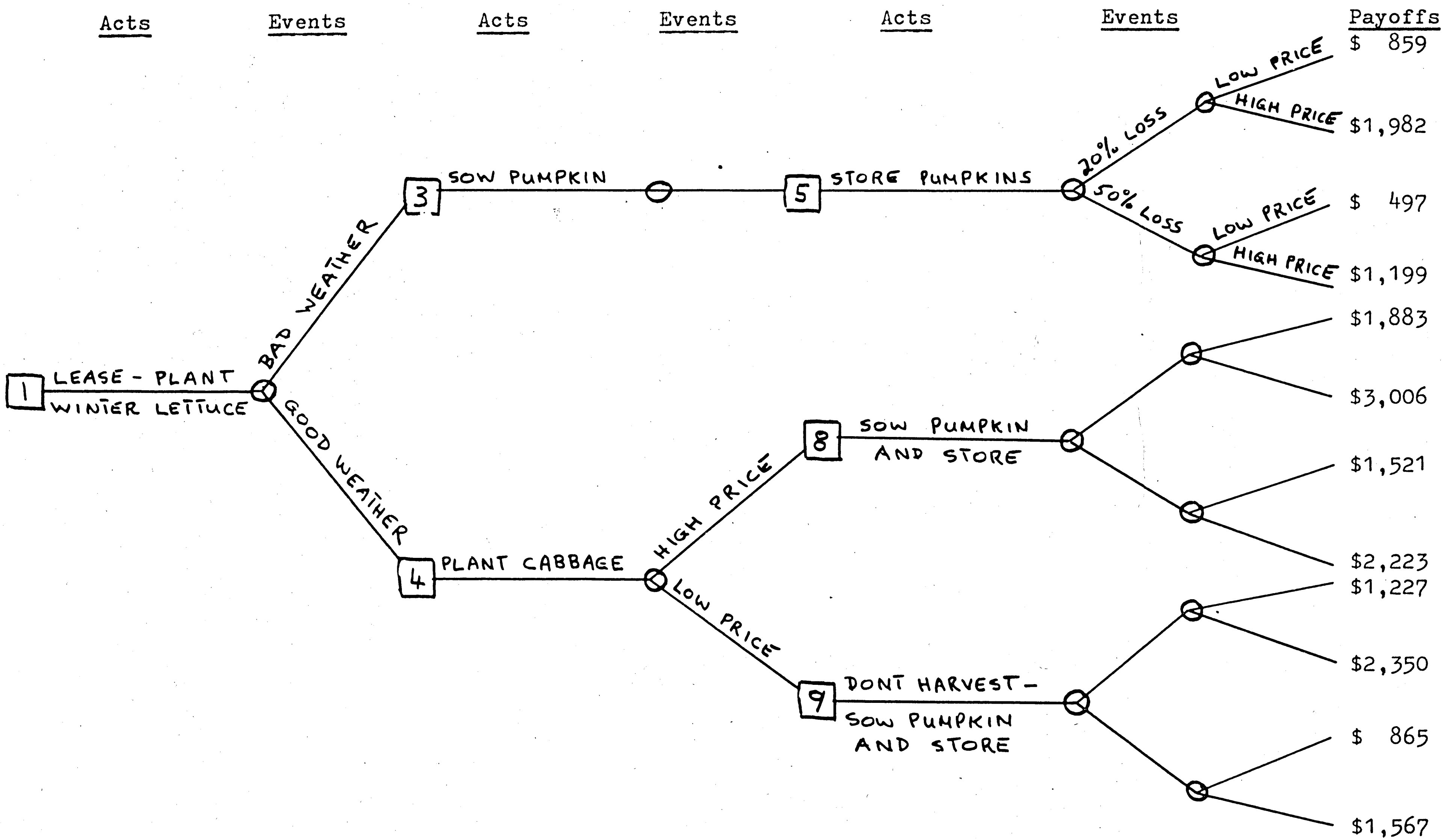

Figure 7. Optimal strategy - leasing problem

events in order to arrive at the best initial decision is a general feature of this type of problem and, of course, complicates the solution procedure. We should also realise that although the solution tells us what to do throughout the entire planning period, it is only the initial decision that needs to be implemented now. The future acts specified in the 'best-bet' strategy are in fact the best acts in terms of the information currently available. Once the winter lettuce crop has been harvested, we will probably have more (or better) information available to us than we included in the decision tree, and we may even think it worthwhile to re-analyse the problem, again, of course, making use of all available information.

7. CONCLUDING COMMENTS

When constructing a decision tree for a real problem that might confront us, we will soon realise that it is just not possible to put down and analyse all possible acts and all possible eventualities because of their complexity (even if we could find a piece of paper large enough!) As with any type of model building, then, we must learn to cultivate the art of efficient model building. That is, we want to be able to construct a model of the decision problem that is an adequate representation of the underlying problem in its real-world setting. Now, just what is 'adequate' is probably a matter of personal judgement, and here skill and experience is all-important in guiding us to the essential features of the management problem. Thus, acts that seem almost certain to be unprofitable or technically infeasible may be withdrawn from further consideration.

Likewise, some events may not seem to exert as great an impact on payoffs as some other events, or some future acts and events may be considered to have little influence on the choice of the initial action, so may also be deleted from the problem.

An important question is just how many events need to be specified at each event fork, for example, two, three, four or five possible price levels? The greater the number of events, the better will the problem approach reality, but the bigger also will be the decision tree. Also, it is often more efficient in terms of problem-solution to represent random variables by continuous, rather than discrete, distributions as explained, for example, by Hadley (1967). It is up to the analyst and horticulturist to determine the minimum number of acts and events that allow the decision tree to capture the essence of the decision problem. Although this will almost certainly involve some gross simplifications, the writer believes that sufficient of the original problem will still remain to allow a better decision to be made than if risk was completely ignored.

We have seen that the estimation of certainty equivalents requires recognition of beliefs and preferences. Now, where a risky prospect (or event fork) is composed of more than two or three possible outcomes, the specification of certainty equivalents becomes rather difficult. Preferences for gambles with a large number of possible payoffs may be hard to perceive. To get over this problem, we can examine historical data and question the grower in order to estimate the relevant probabilities, and obtain a numerical estimate of the utility function that allows us to measure the satisfaction he obtains from the various payoffs by employing a suitably-designed questioning procedure (Makeham, <u>et al</u>. (1968)). If this is done successfully we can then solve the problem 'away from the farm', so taking up less of the grower's time.

All relevant diversification and flexibility opportunities that exist should be identified in the problem-formulation stage, and included in the framework of the decision tree. The optimal strategy will then contain the optimal degree of diversification and flexibility, in accord with the decision-maker's preference for, or aversion against, the risks that exist.

What is the relationship between the decision tree technique and other planning techniques? Decision tree analysis can be considered as a way of thinking, rather than as a technique separate from others such as budgeting and linear programming. In fact, decision trees can be incorporated into these other techniques. First, it should be clear that the leasing problem was of the type that is often analysed by partial budgeting, and that is what we really did with the decision tree. However, there was not just one partial budget but several, since we recognised that one of several outcomes could occur. In this sense, the technique of decision tree analysis can be looked upon as 'budgeting for uncertainty' or, perhaps, 'probabilistic budgeting'. Second, a decision tree analysis can be put into a mathematical programming framework, in which case we could be talking about quadratic risk programming (Rae (1970)) or stochastic linear programming (Rae (1971)). This might be relevant if we wish to take specific account of production restraints, or if acts are continuous rather than discrete. In the first situation, inclusion of production (and marketing) restraints within the decision model will ensure that any solution provided by the model will be operational. When the decision tree model is used, however, some initial budgeting will be required during construction of the tree to check that all acts, as represented by the decision branches, are in fact feasible. In the second situation, acts can be described as 'continuous' when levels of the decision variables can take on <u>any</u> value within some range, rather than various discrete levels. Rather than specify an act as "accept beetroot contract", it might be possible to "accept beetroot contract for any tonnage up to 100 tons". A further advantage of the mathematical programming approach is that marginal value products are easily imputed to the resources, which is valuable information in studying, say, factor acquisition involving risk.

Finally, just the exercise of constructing a decision tree can be of immense value to both adviser and grower alike. What may have seemed an incomprehensible and highly complex problem becomes clearer after being stripped to its essentials. This should then lead to both a better understanding of the choices that are available and of the risks that must be faced.

8. REFERENCES

Hadley, G. (1967), Introduction to Probability and Statistical Decision Theory, Holden-Day, San Francisco.

Halter, A.N. and G.W. Dean (1971), Decisions under Uncertainty with Research Applications, South-Western, Cincinatti.

Knight, F.H. (1921), Risk, Uncertainty and Profit, Houghton-Mifflin, Boston.

Makeham, J.P., A.N. Halter and J.L. Dillon (1968), Best-Bet Farm Decisions, Farm Management Guidebook 6, University of New England, Australia.

Marien, E.J. and L.C. Jagetia (1972), A Solution Approach to the Calculation of Expected Discounted Decision Values for New-Product Sequential-Decision Processes, presented at the XIX International Meeting of T.I.M.S., Houston, Texas.

Rae, A.N. (1970), Quadratic Programming and Horticultural Management, Technical Discussion Paper No. 8, Dept. of Agricultural Economics and Farm Management, Massey University.

Rae, A.N. (1971), "An Empirical Application and Evaluation of Discrete Stochastic Programming in Farm Management", Amer. J. Agr. Econ. 53, pp. 625-638.

Raiffa, H. (1968), Decision Analysis, Addison-Wesley, Reading, Massachusetts.

Schlaifer, R. (1969), Analysis of Decisions Under Uncertainty, McGraw-Hill, New York.

Appendix I COMPUTER PROGRAMME DESCRIPTION

This Appendix contains a listing of a FORTRAN computer programme that can be used to solve decision tree problems.[7] The following notes, plus the data listing of Appendix II and the decision tree example of Figure 6, should explain the data input requirements of the programme.

The cards consist of six groups, as follows:

i) Payoff-stream matrix definition card;
ii) The payoff-stream matrix cards;
iii) Discount rate card;
iv) Utility function coefficients card;
v) Decision tree definition cards:
   (a) number of stages card;
   (b) number of decision node cards;
   (c) number of decision branches cards;
   (d) number of event branches cards;
   (e) event probability cards;
vi) Last card.

Payoff-stream matrix definition card

Columns	Format	Variable name	Purpose
1-3	I3	N	Number of rows in matrix.
4-6	I3	K	Number of columns in matrix.

7 Parts of the programme are modifications of the programme of Marien and Jagetia (1972), although their programme is designed to solve 'new product' decision problems through a dynamic programming approach.

Payoff-stream matrix cards

Columns	Format	Variable name	Purpose
1-80	10F8.0	ARRAY	Matrix coefficients, read in row by row.

Discount rate card

Columns	Format	Variable name	Purpose
1-5	F5.0	RATE	Discount rate as a decimal.

Utility function coefficients card

Columns	Format	Variable name	Purpose
1-10	F10.0	AUT	Coefficient a,
11-20	F10.0	BUT	Coefficient b,
21-30	F10.0	CUT	Coefficient c, of the utility function $U = aX + bX^2 + cX^3$.

Decision tree definition cards

Number of stages card

Columns	Format	Variable name	Purpose
1-2	I2	NMAX	Number of decision stages in the tree.

Number of decision nodes cards

Columns	Format	Variable name	Purpose
1-3	I3	NSTAG	Identification number of present stage.
4-6	I3	LMAX	Maximum number of decision nodes in present stage.

Number of decision branches cards

Columns	Format	Variable name	Purpose
1-3	I3	NSTAG	As above.
4-6	I3	NXN	Identification number of present decision node.
7-9	I3	JMAX	Maximum number of decision branches extending from present decision node.

Number of event branches cards

Columns	Format	Variable name	Purpose
1-3	I3	NSTAG	As above.
4-6	I3	NXN	As above.
7-9	I3	JXN	Identification number of present decision branch.
10-12	I3	KMAX	Maximum number of event branches extending from present decision branch.

Event probability cards

Columns	Format	Variable name	Purpose
1-3	I3	NSTAG	As above.
4-6	I3	NXN	As above.
7-9	I3	JXN	As above.
10-12	I3	KXN	Identification number of present event branch.
13-20	F8.0	PROBK	Probability of present event branch.

Last card

Columns	Format	Variable name	Purpose
1-3	I3	N	Directs programme to END, and may be any negative integer, or else blank.

The following points regarding the organisation of the decision tree definition cards should be noted:

i) The final stage in the decision tree is stage number 1.

ii) All cards relating to any one stage must be read in together, with stage 1 read in first.

iii) All cards relating to any one decision node within a given stage must be read in together.

iv) All cards relating to any one decision branch from a given decision node must be read in together.

A FORTRAN PROGRAMME FOR DECISION TREE ANALYSIS

```
C       DECISION TREE ANALYSIS
C       THIS PROGRAM SOLVES DECISION TREES FOR THE OPTIMAL STRATEGY BY THE
C       METHOD OF BACKWARDS INDUCTION - PAYOFFS MAY BE DISCOUNTED TO A
C       PRESENT VALUE AND UTILITY MAY BE EXPRESSED AS A LINEAR,QUADRATIC
C       OR CUBIC FUNCTION OF (DISCOUNTED) PAYOFFS
        DIMENSION ARRAY(200,20),TPO(200),UTPO(200),ENV(200),BEST(200)
        NI=2
        NO=3
C       READ IN NUMBER OF ROWS AND COLUMNS IN PAYOFF STREAM MATRIX
      1 READ (NI,10) N,K
     10 FORMAT (2I3)
        IF (N) 127,127,15
C       READ IN PAYOFF STREAM MATRIX,ROW BY ROW
     15 READ (NI,20) ((ARRAY(I,J),J=1,K),I=1,N)
     20 FORMAT (10F8.0)
C       READ IN DISCOUNT RATE AS A DECIMAL
        READ (NI,30) RATE
     30 FORMAT (F5.0)
        WRITE (NO,31) RATE
     31 FORMAT (1H1,17H DISCOUNT RATE = ,F5.2,14H X 100 PERCENT,/)
C       PAYOFF STREAMS ARE SUMMED TO A (DISCOUNTED) TERMINAL VECTOR, TPO(I)
        DO 40 I=1,N
        TPO(I)=0
        DO 40 J=1,K
        TPO(I)=TPO(I)+ARRAY(I,J)/(1.0+RATE)**J
     40 CONTINUE
C       READ IN LINEAR, QUADRATIC, AND CUBIC COEFFS. OF UTILITY FUNCTION
        READ (NI,50) AUT,BUT,CUT
     50 FORMAT (3F10.0)
        WRITE (NO,51)
     51 FORMAT (' UTILITY FUNCTION COEFFS ARE')
        WRITE (NO,52) AUT
     52 FORMAT (17X,13HLINEAR      = ,F10.5)
        WRITE (NO,53) BUT
     53 FORMAT (17X,13HQUADRATIC   = ,F10.5)
        WRITE (NO,54) CUT
     54 FORMAT (17X,13HCUBIC       = ,F10.5)
C       TERMINAL PAYOFFS ARE TRANSFORMED TO TERMINAL UTILITIES
        WRITE (NO,60)
     60 FORMAT (  '0JOINT EVENT  TERMINAL PAYOFF  UTILITY PAYOFF')
        DO 70 I=1,N
        UTPO(I)=0
        UTPO(I)=AUT*TPO(I)+BUT*TPO(I)**2+CUT*TPO(I)**3
     70 WRITE (NO,80) I,TPO(I),UTPO(I)
     80 FORMAT (4X,I3,9X,F9.2,7X,F9.2)
C       STAGE CYCLE
C       READ IN NUMBER OF STAGES CARD
        READ (NI,90) NMAX
     90 FORMAT (I2)
```

```
      WRITE (NO,91) NMAX
   91 FORMAT (/,12H PROBLEM HAS,I4,7H STAGES)
      DO 125 N=1,NMAX
      I=1
C     READ IN STAGE DEFINITION CARD AND NO OF DECISION NODES
      READ (NI,100) NSTAG ,LMAX
  100 FORMAT (2I3)
      IF (NSTAG -N) 101,102,101
  101 WRITE (NO,103)
  103 FORMAT (' DATA CARD OUT OF ORDER')
      GO TO 127
C     STATE CYCLE
  102 DO 121 L=1,LMAX
      WRITE (NO,1102)
 1102 FORMAT (/,' STAGE NO  DECN NODE NO  DECN BRANCH NO  EXPECTED VALUE
     1')
      BEST(L)=-9999999.
C     BEST IS THE CURRENT OPTIMUM VALUE FOR THIS DECISION NODE
      BRNCH=0
C     READ IN STAGE NUMBER-LEVEL NUMBER-NO OF DECISION BRANCHES
      READ (NI,104) NSTAG ,NXN,JMAX
  104 FORMAT (3I3)
      IF (NSTAG -N) 101,105,101
  105 IF (NXN-L) 101,106,101
  106 DO 120 M=1,JMAX
      ENV(M)=0
C     ENV(M) IS THE COMPUTED EXPECTED VALUE FOR AN EVENT NODE
C     READ IN STAGE NO-LEVEL NO-DECISION BRANCH NO-NO OF EVENT BRANCHES
      READ (NI,107) NSTAG ,NXN,JXN,KMAX
  107 FORMAT (4I3)
      IF (NSTAG -N) 101,108,101
  108 IF (NXN-L) 101,109,101
  109 IF (JXN-M) 101,110,101
  110 DO 116 K=1,KMAX
C     READ IN STAGE NO-LEVEL NO-DECISION BRANCH NO-EVENT BRANCH NO AND
C     EVENT PROBABILITY
      READ (NI,111) NSTAG ,NXN,JXN,KXN,PROBK
  111 FORMAT (4I3,F8.0)
      IF (NSTAG -N) 101,112,101
  112 IF (NXN-L) 101,113,101
  113 IF (JXN-M) 101,114,101
  114 IF (KXN-K) 101,115,101
  115 ENVK=PROBK*UTPO(I)
      ENV(M)=ENV(M)+ENVK
  116 I=I+1
      WRITE (NO,118) NSTAG ,NXN,JXN,ENV(M)
  118 FORMAT (2X,I3,9X,I3,12X,I3,10X,F9.2)
      IF (ENV(M)-BEST(L)) 120,120,119
  119 BEST(L)=ENV(M)
      BRNCH=M
  120 CONTINUE
```

```
      WRITE (NO,122) NSTAG ,NXN,BEST(L)
  122 FORMAT (26H MAX EXPECTED VALUE STAGE ,I3,11H DECN NODE ,I3,4H IS ,
     1F9.2)
  121 WRITE (NO,123) BRNCH
  123 FORMAT (12H AND BRANCH ,F3.0,53H IS THE BEST DECISION BRANCH EXTEN
     1DING FROM THIS NODE)
C     END OF DECISION NODE LEVEL CYCLE
C     BEST(L) IS THE ARRAY FOR KEEPING OPTIMUM VALUES FOR EACH DECISION
C     NODE IN A CERTAIN STAGE
      DO 124 I=1,LMAX
  124 UTPO(I)=BEST(I)
  125 CONTINUE
C     END OF STAGE CYCLE
      WRITE (NO,126)
  126 FORMAT (  '0THIS IS AN OPTIMAL SOLUTION')
C     BACK TO READ ANOTHER TREE
      GO TO 1
  127 STOP
      END
```

1–10 11–20 21–30 31–40 41–50 51–60 61–70 71–80

32	3								
-77.	-30.	689.	-77.	-30.	935.	-77.	-30.	2058.	-77.
-30.	573.	-77.	-30.	1275.	-77.	609.	0.	714.	-30.
689.	714.	-30.	935.	714.	-30.	2058.	714.	-30.	573.
714.	-30.	1275.	714.	1149.	0.	-46.	-30.	689.	-46.
-30.	935.	-46.	-30.	2058.	-46.	-30.	573.	-46.	-30.
1275.	-46.	1089.	0.	408.	1031.	0.	408.	570.	659.
408.	570.	905.	408.	570.	2028.	408.	570.	543.	408.
570.	1245.	408.	570.	1089.	408.	-86.	659.	408.	-86.
905.	408.	-86.	2028.	408.	-86.	543.	408.	-86.	1245.
408.	-86.	1089.	-278.	0.	290.				
0.									
1.	0.	0.							
3									
1	10								
1	1	2							
1	1	1	1						
1	1	1	1	1.					
1	1	2	4						
1	1	2	1	.25					

Appendix II INPUT DATA LISTING FOR EXAMPLE PROBLEM

1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80
1 1 2	2 .25						
1 1 2	3 .25						
1 1 2	4 .25						
1 2 1							
1 2 1	1						
1 2 1	1 1.						
1 3 2							
1 3 1	1						
1 3 1	1 1.						
1 3 2	4						
1 3 2	1 .25						
1 3 2	2 .25						
1 3 2	3 .25						
1 3 2	4 .25						
1 4 1							
1 4 1	1						
1 4 1	1 1.						
1 5 2							
1 5 1	1						
1 5 1	1 1.						

1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80
1 5 2	4						
1 5 2	1 .25						
1 5 2	2 .25						
1 5 2	3 .25						
1 5 2	4 .25						
1 6 1							
1 6 1	1						
1 6 1	1 1.						
1 7 1							
1 7 1	1						
1 7 1	1 1.						
1 8 3							
1 8 1	1						
1 8 1	1 1.						
1 8 2	4						
1 8 2	1 .25						
1 8 2	2 .25						
1 8 2	3 .25						
1 8 2	4 .25						
1 8 3	1						

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21–80
		1			8			3			1							1	.	
		1			9			3												
		1			9			1			1									
		1			9			1			1							1	.	
		1			9			2			4									
		1			9			2			1						.	2	5	
		1			9			2			2						.	2	5	
		1			9			2			3						.	2	5	
		1			9			2			4						.	2	5	
		1			9			3			1									
		1			9			3			1							1	.	
		1		1	0			1												
		1		1	0			1			1									
		1		1	0			1			1							1	.	
		2			5															
		2			1			2												
		2			1			1			1									
		2			1			1			1							1	.	
		2			1			2			1									
		2			1			2			1							1	.	

1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80
2 2 2							
2 2 1	1						
2 2 1	1 1.						
2 2 2	1						
2 2 2	1 1.						
2 3 2							
2 3 1	1						
2 3 1	1 1.						
2 3 2	1						
2 3 2	1 1.						
2 4 2							
2 4 1	1						
2 4 1	1 1.						
2 4 2	2						
2 4 2	1 .8						
2 4 2	2 .2						
2 5 1							
2 5 1	1						
2 5 1	1 1.						
3 1							

```
  3  1  3
  3  1  1  2
  3  1  1  1      .3
  3  1  1  2      .7
  3  1  2  2
  3  1  2  1      .3
  3  1  2  2      .7
  3  1  3  1
  3  1  3  1      1.
-99
```

Appendix III **COMPUTER OUTPUT SOLUTION**

DISCOUNT RATE = 0.00 X 100 PERCENT

UTILITY FUNCTION COEFFS ARE

LINEAR	=	1.00000
QUADRATIC	=	0.00000
CUBIC	=	0.00000

JOINT EVENT	TERMINAL PAYOFF	UTILITY PAYOFF
1	582.00	582.00
2	828.00	828.00
3	1951.00	1951.00
4	466.00	466.00
5	1168.00	1168.00
6	532.00	532.00
7	1373.00	1373.00
8	1619.00	1619.00
9	2742.00	2742.00
10	1257.00	1257.00
11	1959.00	1959.00
12	1863.00	1863.00
13	613.00	613.00
14	859.00	859.00
15	1982.00	1982.00
16	497.00	497.00
17	1199.00	1199.00
18	1043.00	1043.00
19	1439.00	1439.00
20	1637.00	1637.00
21	1883.00	1883.00
22	3006.00	3006.00
23	1521.00	1521.00
24	2223.00	2223.00
25	2067.00	2067.00
26	981.00	981.00
27	1227.00	1227.00
28	2350.00	2350.00
29	865.00	865.00
30	1567.00	1567.00
31	1411.00	1411.00
32	12.00	12.00

PROBLEM HAS 3 STAGES

```
STAGE NO   DECN NODE NO   DECN BRANCH NO   EXPECTED VALUE
   1           1               1                582.00
   1           1               2               1103.25
MAX EXPECTED VALUE STAGE    1 DECN NODE    1 IS    1103.25
AND BRANCH  2. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO   DECN NODE NO   DECN BRANCH NO   EXPECTED VALUE
   1           2               1                532.00
MAX EXPECTED VALUE STAGE    1 DECN NODE    2 IS     532.00
AND BRANCH  1. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO   DECN NODE NO   DECN BRANCH NO   EXPECTED VALUE
   1           3               1               1373.00
   1           3               2               1894.25
MAX EXPECTED VALUE STAGE    1 DECN NODE    3 IS    1894.25
AND BRANCH  2. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO   DECN NODE NO   DECN BRANCH NO   EXPECTED VALUE
   1           4               1               1863.00
MAX EXPECTED VALUE STAGE    1 DECN NODE    4 IS    1863.00
AND BRANCH  1. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO   DECN NODE NO   DECN BRANCH NO   EXPECTED VALUE
   1           5               1                613.00
   1           5               2               1134.25
MAX EXPECTED VALUE STAGE    1 DECN NODE    5 IS    1134.25
AND BRANCH  2. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO   DECN NODE NO   DECN BRANCH NO   EXPECTED VALUE
   1           6               1               1043.00
MAX EXPECTED VALUE STAGE    1 DECN NODE    6 IS    1043.00
AND BRANCH  1. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO   DECN NODE NO   DECN BRANCH NO   EXPECTED VALUE
   1           7               1               1439.00
MAX EXPECTED VALUE STAGE    1 DECN NODE    7 IS    1439.00
AND BRANCH  1. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO   DECN NODE NO   DECN BRANCH NO   EXPECTED VALUE
   1           8               1               1637.00
   1           8               2               2158.25
   1           8               3               2067.00
MAX EXPECTED VALUE STAGE    1 DECN NODE    8 IS    2158.25
AND BRANCH  2. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO   DECN NODE NO   DECN BRANCH NO   EXPECTED VALUE
   1           9               1                981.00
   1           9               2               1502.25
   1           9               3               1411.00
MAX EXPECTED VALUE STAGE    1 DECN NODE    9 IS    1502.25
AND BRANCH  2. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE
```

```
STAGE NO  DECN NODE NO  DECN BRANCH NO  EXPECTED VALUE
   1          10              1               12.00
MAX EXPECTED VALUE STAGE   1 DECN NODE  10 IS     12.00
AND BRANCH  1. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO  DECN NODE NO  DECN BRANCH NO  EXPECTED VALUE
   2           1              1             1103.25
   2           1              2              532.00
MAX EXPECTED VALUE STAGE   2 DECN NODE   1 IS   1103.25
AND BRANCH  1. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO  DECN NODE NO  DECN BRANCH NO  EXPECTED VALUE
   2           2              1             1894.25
   2           2              2             1863.00
MAX EXPECTED VALUE STAGE   2 DECN NODE   2 IS   1894.25
AND BRANCH  1. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO  DECN NODE NO  DECN BRANCH NO  EXPECTED VALUE
   2           3              1             1134.25
   2           3              2             1043.00
MAX EXPECTED VALUE STAGE   2 DECN NODE   3 IS   1134.25
AND BRANCH  1. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO  DECN NODE NO  DECN BRANCH NO  EXPECTED VALUE
   2           4              1             1439.00
   2           4              2             2027.05
MAX EXPECTED VALUE STAGE   2 DECN NODE   4 IS   2027.05
AND BRANCH  2. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO  DECN NODE NO  DECN BRANCH NO  EXPECTED VALUE
   2           5              1               12.00
MAX EXPECTED VALUE STAGE   2 DECN NODE   5 IS     12.00
AND BRANCH  1. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

STAGE NO  DECN NODE NO  DECN BRANCH NO  EXPECTED VALUE
   3           1              1             1656.94
   3           1              2             1759.20
   3           1              3               12.00
MAX EXPECTED VALUE STAGE   3 DECN NODE   1 IS   1759.20
AND BRANCH  2. IS THE BEST DECISION BRANCH EXTENDING FROM THIS NODE

THIS IS AN OPTIMAL SOLUTION
```

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