Vegetable

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QUADRATIC PROGRAMMING AND HORTICULTURAL MANAGEMENT.

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QUADRATIC PROGRAMMING AND

HORTICULTURAL MANAGEMENT

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INTRODUCTION

This paper represents a further development of research initially carried out in the Agricultural Economics and Farm Management Department at Massey University. Professor W.V. Candler, Head of the Department until September 1967, suggested the project and gave considerable assistance to Mr. Rae during 1967 and 1968.

The application of Mathematical Programming to the management of horticultural enterprises is well illustrated in this paper. I hope that horticultural producers will be encouraged to support further work in this field as the horticultural industry in New Zealand has considerable potential for growth in the future. Managers must be aware of modern methods of analysis and have some appreciation of their potential application in practice. Horticultural Advisory Officers should also find this paper of value when they try to assess the place of programming as a research method.

I wish to thank all those who gave of their time and knowledge in answering Mr. Rae's many questions. The assistance of members of the Vegetable Growers' Federation, Officers of the Department of Agriculture and the farmers who provided detailed information is especially appreciated.

January 1970

A.R. Frampton,
Professor of Agricultural Economics
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Preface

This paper discusses the role of quadratic programming in solving two distinct horticultural management problems. Parts A and B each deal, firstly, with the theoretical aspects of the problems and formulate the theoretical models, and secondly, each presents an empirical example from New Zealand horticulture. Attention will be concentrated on theory and the likely occurrence of the problems, with the practical examples discussed somewhat briefly. Readers wishing for greater detail, however, should refer to Chapters 5 and 6 of my M.Hort.Sc. thesis, "Applications of Mathematical Programming on Four New Zealand Horticultural Holdings", Massey University, 1968.

The first management problem discussed involves the determination of profit-maximising behaviour under conditions of imperfect competition in product and/or factor markets. Contrary to one assumption of perfect competition, necessary for linear programming to be appropriate, product prices may have to fall if producers wish to sell an increased level of output. In such a case the profit function to be maximised will no longer be linear and non-linear programming techniques are required.

The second situation discusses the formulation of cropping programmes under conditions of risk. After stating the general model, attention is then turned to one aspect of risk in production, namely the variability of prices and yields, (i.e., income). A model is then described which will allow a series of plans to be found, each of which minimises income variability for some level of average (or expected) income. The risk model is seen to be most appropriate to fresh vegetable production, since auction prices (and also yields) may be notoriously unstable, and managers may prefer a lower but more <u>stable</u> income rather than the highest possible income.

The empirical application of the risk model was intended as one of the programming studies of my thesis, but for reasons mentioned in the thesis, the solution was not available in time for inclusion in that work. The solution to the model is, however, presented in this paper, and I would like to express my thanks to the Department of Farm Management, University of New England, through whom the solution was obtained at the University of Melbourne.

Part C of the Discussion Paper summarises the situations discussed and the likely importance of quadratic programming in horticultural management.

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CONTENTS

Part A

PROFIT MAXIMISATION UNDER CONDITIONS OF IMPERFECT COMPETITION

A.1 INTRODUCTION

The behaviour of many farm firms may be realistically approximated as perfectly competitive in both factor and product markets. The New Zealand fresh vegetable industry, for example, comprises a large number of relatively small holdings and any single producer is but one of many sellers[1]. Under such conditions it may be assumed that individual firms cannot affect the price they receive by changing output levels, and are thus price-takers, the demand conditions facing such firms being perfectly elastic. These firms may also be near-perfect competitors in factor markets, since the larger the number of buyers, the less can any one firm influence the price it pays for a factor by changing the quantity it uses.

Two further market characteristics lend support to the supposition of a 'nearly' perfectly competitive fresh vegetable industry in New Zealand. Firstly, the wide variety of vegetable products generally available topurchasers suggests that individual firm's product demand curves will be elastic rather than inelastic, and the demand elasticity will be greater the larger the number of substitute products, and the greater the substitutability of one product for another.

Secondly, relatively little capital is required to enter the fresh vegetable industry, particularly since the average size of holdings tends to be small. Such ease of entry and exist effectively reduces the possibility of the industry earning anything above normal profits.

Therefore, as the above would indicate, it may be realistic to treat such firms as 'perfect competitors' when aspects of firm behaviour, such as changing output to maximise profits are under study. Thus linear programming may be employed to determine profit-maximising behaviour since it has been assumed that the market price of any product is independent of the amount produced, and that factor costs are unaffected by the quantity of resources purchased[2,3]. Along with the assumption that constant returns to scale exist (and therefore marginal and average costs remain constant as the firm's output of a product increases), the above implies the existence of a <u>linear</u> profit function.

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1. There are approximately 2600 growers in the industry, with about 40 percent of growers owning properties not larger than 10 acres. (See <u>Report on the Economic Position of the Fresh Vegetable Industry in New Zealand</u>, Enting, L.M., Philpott, B.P., and Ridler, D., New Zealand Vegetable & Produce Growers' Federation (Inc.), Wellington, 1965, p.1.
2. For an application of linear programming involving fresh vegetable production, see Rae, A.N., "Applications of Mathematical Programming on Four New Zealand Horticultural Holdings", unpublished M.Hort.Sc. thesis, Massey University, 1968, Ch.3.
3. Situations of falling average revenue and increasing average costs may be approximated by linear programming - see, for example, Candler, Wilfred and Musgrave, Warren F., "A Practical Approach to the Profit Maximisation Problems in Farm Management", <u>Journal of Agricultural Economics</u>, Vol.14, pp.208-223, 1960. Should the supply curve of a factor be downward-sloping, however, the convexity assumption of linear programming would be violated.

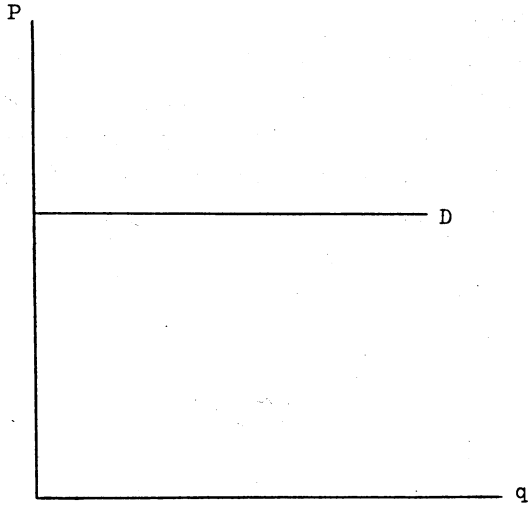

Fig.A.1(a) Individual Firm's Demand Curve - Perfect Competition

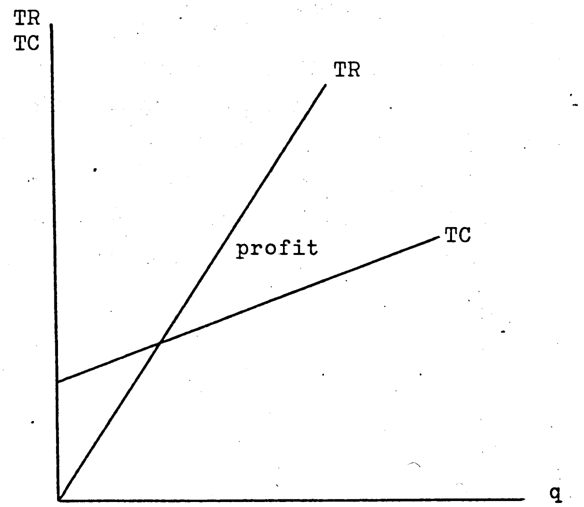

Fig.A.1(b) Total Revenue and Total Cost Curves - Perfect Competition

Fig.A.1 illustrates a single-product case, where profits will be maximised by increasing output until further expansion is prevented by some resource(s) being restrictive. This situation may be generalised to the typical multi-product case, with profit a linear function of output levels of all products.

So much for the assumptions of perfect competition and linear programming. Do situations exist in New Zealand horticulture where competition is less than perfect, and the behaviour of some firms can influence prices and thus the behaviour of other firms? The author believes such situations do exist, and will become more prevalent in future years. The emergence of relatively large vegetable-producing enterprises, for example, indicates that some assumptions of perfect competition may be violated, since changes in output levels by these firms may influence market prices. Such large firms may also have some monopsonistic influence in factor markets. Even relatively small vegetable producing firms may be able to influence market prices by a considerable expansion in output of some product and flooding the market, a situation which does occur from time to time.

The assumptions of perfect competition may not be appropriate for some nursery firms, since the price which the firm can charge if it hopes to sell its entire output may decrease as the quantity produced increases. Also, some of the larger nursery firms may be able to act as price leaders, so that prices received by smaller firms in the industry will depend upon the behaviour of the price leader.

Where such elements of imperfect competition exist in product and/or factor markets, non-linear programming techniques may provide more realistic solutions to profit-maximisation problems than would linear programming.

A.2 A QUADRATIC PROGRAMMING MODEL OF PROFIT MAXIMISATION

In linear programming, it is assumed that product prices remain constant regardless of the volume of output. Such a programming model may be converted into a somewhat more general model by allowing the prices of at least some products to decrease as their output is increased. In particular, if we assume that the firm's demand curves are linear and downward-sloping, the profit-maximisation problem has been converted into one of quadratic programming since the profit function is now of quadratic form (see Fig.A.2b).

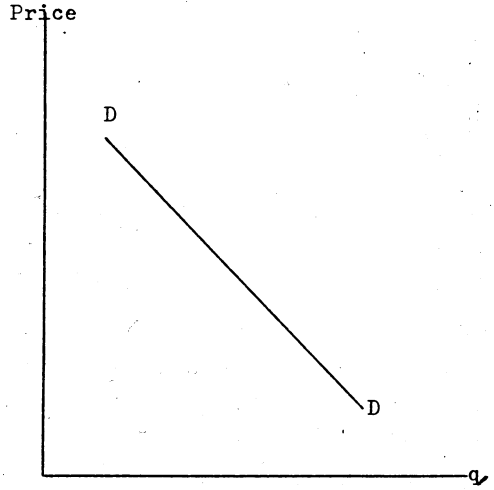

Fig.A.2(a) Individual Firm's Linear Demand Curve - Imperfect Competition

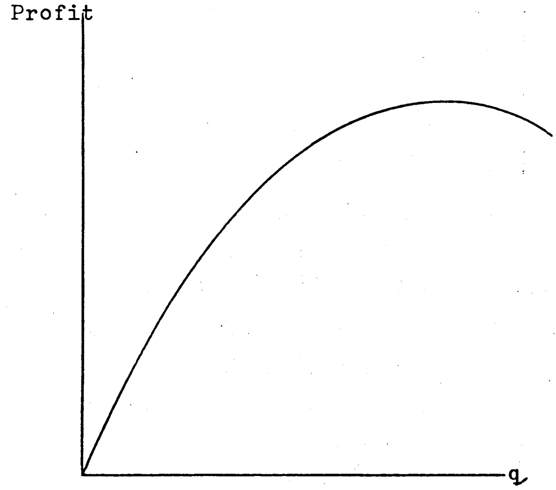

Fig.A.2(b) Individual Firm's Profit Function

Generalising, assume the firm can produce n commodities at levels x_j $(j = 1, \ldots, n)$, and these products may be sold at a price p_j, where -

$$(1) \qquad p_j = a_j - b_j x_j \quad (a_j > o,\ b_j \geq o \text{ for all } j).$$

Where b_j is zero, the firm can sell all it produces of product j at a fixed price. Should b_j be positive, however, an increase in the quantity of j produced must be accompanied by a reduction in price (p_j) if the entire output is to be sold.

To allow for the possibility that the firm may exert a monopsonistic influence in factor markets, the linear supply functions for the i factors $(i = 1, \ldots, m)$ may be written:

$$(2) \qquad p_i = a_i + b_i\, u_i,$$

where u_i is the quantity of factor i purchased. If b_i is zero, the firm could purchase any amount of factor i at a fixed price; if b_i is positive the price of factor i would increase as the quantity of the factor purchased was increased; and if b_i is negative, the factor supply curve would be downward sloping.

Therefore, the firm's net revenue function may be written as

Net revenue = total revenue - total variable costs

$$= \sum_{j=1}^{n} p_j x_j - \sum_{i=1}^{m} p_i u_i$$

$$= \sum_{j=1}^{n} (a_j - b_j x_j) x_j - \sum_{i=1}^{m} (a_i + b_i u_i) u_i$$

(3) $$= \sum_{j=1}^{n} a_j x_j - \sum_{i=1}^{m} a_i u_i - \sum_{j=1}^{n} b_j x_j^2 - \sum_{i=1}^{m} b_i u_i^2$$

The objective function (3) is therefore of quadratic form, and may be maximised subject to the usual linear restraints

(4) $$\sum_{j=1}^{n} r_{ij} x_j \leqslant d_i \quad (i = 1, \ldots, m)$$

$$x_j \geqslant 0 \quad (j = 1, \ldots, n)$$

where d_i is the supply of the i^{th} resource, and r_{ij} are fixed technical coefficients indicating the per unit requirement of x_j for d_i.

To present a graphical representation of the model, assume the firm produces only two products, x_1 and x_2. In Fig. A.3 resource supplies limit the production of x_2 to OA, and

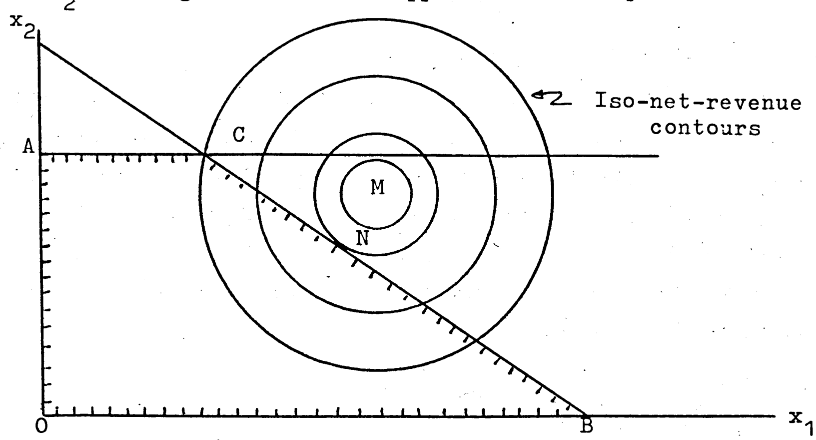

Fig.A.3 The Quadratic Model with Two Products and Two Restraints

production of x_1 to OB. Thus all feasible combinations of x_1 and x_2 are bounded by OACB. The iso-net-revenue contours represent a 'birds-eye' view of the quadratic net revenue surface, with net revenue reaching a maximum at M. Since M lies outside the feasible region, however, it does not give the solution to the problem. Rather, point N gives the net-revenue maximising combination of x_1 and x_2, since it is that feasible combination of x_1 and x_2 which lies on the highest possible iso-net-revenue contour.

A.3 AN EMPIRICAL APPLICATION OF THE MODEL [4]

The above type of quadratic programming model was constructed for a nursery firm. (A linear programming model was not appropriate since the nurseryman stated that any increase in output of his products may have to be accompanied by a reduction in price.)

Since the nursery produced well over 1,000 different plant types, the formulation of the optimum output of the nursery as a whole was beyond the capability of the available computing facilities. Data was therefore collected from the nurseryman relating only to a small number of plants (nineteen) propagated in glasshouses during the spring and summer.

The limiting resources included a heated glasshouse, an unheated glasshouse, a lath-house, labour, and an area of cropland. Other nursery resources such as a soil sterilisation unit, worksheds, stock plants to provide cutting material and outdoor frames were assumed to be non-limiting resources present in more than adequate supply, so did not require inclusion in the model.

The supplies of limiting resources were set equal to the total requirements of the nineteen plant types for the various nursery resources when produced at the levels of the past season. The end result was a nursery which although hypothetical, was based on 'real-life' data. Thus, the quadratic programme was formulated and solved, not to assist the nurseryman to increase his profits, but rather to illustrate how such production and marketing situations may be handled by quadratic programming.

A.3.1 THE OBJECTIVE FUNCTION

Although imperfectly competitive in some product markets, the nurseryman had no influence over the price paid for factors of production (i.e. he is a perfect competitor in factor markets). This, accompanied by the assumption of linear production functions for the n commodities, meant that average and marginal variable costs of production (c_j) would remain constant for all (positive) values of x_j ($j = 1, \ldots, n$).

The objective, to maximise total net revenue, may be written as

Maximise $TNR = TR_n - TVC_n$

$$
\begin{aligned}
&= \sum_{j=1}^{n} p_j x_j - \sum_{j=1}^{n} c_j x_j \\
(5) \qquad &= \sum_{j=1}^{n} \left[x_j (p_j - c_j) \right] \\
&= \sum_{j=1}^{n} \left[x_j (a_j - b_j x_j - c_j) \right] \\
&= \sum_{j=1}^{n} \left[(a_j - c_j) x_j - b_j x_j^2 \right]
\end{aligned}
$$

4. A full discussion of the model and its solution is given in Rae, A.N., op.cit., Ch.5.

It can be verified that (5) is equivalent to (3) when $b_i = 0$,

since $$\sum_{i=1}^{m} a_i u_i = \sum_{j=1}^{n} c_j x_j = TVC$$

The problem then is to maximise (5) subject to the linear restraints (4).

A.3.1.1 ESTIMATION OF DEMAND FUNCTIONS

A more reliable estimate of the linear demand functions could have been made had accurate elasticity coefficients been available[5].

The demand function

$$x_j = f(p_j, p_i)$$

could then be calculated from

$$\frac{x_j - \bar{x}_j}{\bar{x}_j} = -\varepsilon_j \frac{p_j - \bar{p}_j}{\bar{p}_j} + \sum_i \varepsilon_{ij} \frac{p_i - \bar{p}_i}{\bar{p}_i} \qquad (6)$$

where

ε_j is the demand elasticity of product j with respect to its own price,

ε_{ij} is the cross-elasticity of demand between product j and the price of product i,

and

$\bar{x}_j$, $\bar{p}_j$, and $\bar{p}_i$ are the average values observed during the last season.

The lack of such data meant that to obtain some indication of the slope of the demand curves, two sets of data had to be used - the past season's outputs and prices, and the nurseryman's estimate of by how much price would need to be reduced to sell (say) an extra 100 plants. The nurseryman was also asked if the quantity sold of some plants would affect the demand for others, but found this question difficult to answer. This measurement problem made it necessary to assume that cross-effects did not exist[6].

The nurseryman considered that he could sell as much as he produced of nine of the nineteen plant types, so the demand curves for those products would be horizontal.

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5. See Louwes, S.L., Boot, J.C.G., and Wage, S., "A Quadratic Programming Approach to the Optimal Use of Milk in the Netherlands", Journal of Farm Economics, vol.49, pp.309-317, 1963. **Regression techniques may also be used if adequate data is available.**

6. Such an assumption is probably unrealistic, since the wide range of ornamental trees and shrubs available suggests that significant substitution relationships would be present. This is also suggested by the high own-price elasticity coefficients mentioned in a note to Table A.1.

For the remaining ten products, the linear demand functions were estimated as

$$(7) \qquad x - x_1 = \frac{x_2 - x_1}{P_2 - P_1}(P - P_1)$$

where P_1 and x_1 is one set of price-quantity co-ordinates, and P_2 and x_2 is the other.

The linear demand functions are to be found in Table A.1 7/ . (Price is measured in $1 units, and quantities are measured in units of 100 plants).

A.3.1.2 Average variable costs

Although variable costs will not be tabulated here, they include the cost of such items as soil-fumigation materials, fungicides, insecticides, and weedicides, polythene film used for weed control, and packing and marketing materials.

A.3.1.3 The total net revenue objective function

The objective function is identical to that given by equation (5). Average variable costs (c_j) were subtracted from the a_j values of the demand functions, and then the (now average net revenue functions) were multiplied throughout by x_j to give the quadratic total net revenue function to be maximised.

A.3.2 The Solution to the Quadratic Programming Problem 8/

Since the solution to the quadratic programme will be given in terms of the x_j's, the price which the nurseryman can charge in order to sell the entire output of product j is found by substituting the appropriate value for x_j into the demand function for that product.

For example, the value of x_2 (the production level of Acacia) in the solution is 26.40 hundred plants. To obtain the maximum price which the nurseryman can charge, this value is substituted into the demand function for Acacia as follows:

$$\begin{aligned} P_2 &= 61.67 - 0.6667x_2 \\ &= 61.67 - 0.6667\ (26.40) \\ &= 44.07 \end{aligned}$$

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7. The functions are presented as inverse demand functions, since they express $p = f(x)$ rather than $x = f(p)$. Such transposition was necessary since the problem must be solved in terms of the x values (as in the objective function (5)) as some prices are assumed to remain constant and hence are already known.

8. Only production levels and prices will be presented. Other aspects of the solution such as resource requirements, shadow prices of scarce resources, and marginal opportunity costs of growing omitted products, may be found in Rae, A.N., op.cit., Ch. 5, section 5.8.

TABLE A.1 The Demand Function

Product	Demand Function ($p_j = a_j - b_j x_j$)
Telopea	$p_1 = 81.78 - 0.1220x_1$
Acacia	$p_2 = 61.67 - 0.6667x_2$
Passiflora	$p_3 = 55.00$
Banksia	$p_4 = 78.00$
Photinia	$p_5 = 60.00$
Eucalyptus	$p_6 = 56.13 - 0.5732x_6$
Stachyurus	$p_7 = 65.00$
Cistus	$p_8 = 60.25 - 0.9524x_8$
Protea	$p_9 = 100.00$
Tibouchina	$p_{10} = 55.00$
Azalea indica	$p_{11} = 71.96 - 0.5357x_{11}$
Viburnum	$p_{12} = 64.67 - 1.3333x_{12}$
Rhododendron	$p_{13} = 125.65 - 0.8696x_{13}$
Weigela	$p_{14} = 54.32 - 1.8000x_{14}$
Forsythia	$p_{15} = 73.01 - 1.3986x_{15}$
Azalea occidentalis	$p_{16} = 130.00$
Magnolia	$p_{17} = 110.00$
Callicarpa	$p_{18} = 60.00$
Hypericum	$p_{19} = 62.86 - 0.8276x_{19}$

Note: Own-price demand elasticities (E) may be derived from the linear demand functions as

$$E = \frac{dx.p}{dp.x}$$

Using the means of the two quantity-price coordinates, the estimated functions imply elasticity coefficients of between -3.42 and -7.16 for eight of the ten 'downward-sloping' functions. The two remaining coefficients are somewhat higher being -9.21 (for Viburnum) and -17.62 (for Telopea). Such high coefficients would appear reasonable a priori, at least those between -3 and -7. This is because purchasers are faced with a wide range of ornamental trees and shrubs from which to choose, and this type of plant is considered a luxury rather than a necessity, both these factors suggesting a highly elastic demand.

Therefore profit maximisation requires inter alia the production of 2,640 Acacia plants sold at a price of $44.07 per hundred (i.e. 44 cents each). The prices and production levels of all products in the solution are given in Table A.2, as well as the corresponding values for last season's production of the nineteen plant types.

TABLE A.2

Output, Prices and Total Net Revenue

Product	Value in Optimum Plan		Value in Past Year	
	Output (No. plants)	Price ($ per plant)	Output (No. plants)	Price ($ per plant)
Telopea	2,974	0.78	3,100	0.78
Acacia	2,640	0.44	1,000	0.55
Passiflora	16,000	0.55	4,000	0.55
Banksia	5,659	0.78	200	0.78
Photinia	-	-	5,100	0.60
Eucalyptus	1,405	0.48	1,070	0.50
Stachyurus	-	-	1,200	0.65
Cistus	595	0.55	550	0.55
Protea	-	-	220	1.00
Tibouchina	-	-	320	0.55
Azalea indica	-	-	1,300	0.65
Viburnum	-	-	350	0.60
Rhododendron	904	1.18	1,800	1.10
Weigela	-	-	240	0.50
Forsythia	-	-	930	0.60
Azalea occid.	3,769	1.30	300	1.30
Magnolia	3,933	1.10	1,275	1.10
Callicarpa	-	-	200	0.60
Hypericum	-	-	950	0.55
Total Net Revenue	$26,891		$15,769	

It can be seen from Table A.2 that prices to be charged for six of the nine products included in the optimum solution are similar to those of the past year. Four of these six products (Passiflora, Banksia, Azalea occid. and Magnolia) can be produced at any level with no effect on price, and the quantities of each of these plants to be produced in the optimum plan are considerably above those of the past year. Changes in the output levels of Telopea and Cistus from those of the past season are sufficiently small to have had a negligible effect on price. Profit maximisation requires a reduction in price for both the Acacia and Eucalyptus products, but allows Rhododendron prices to rise.

The maximum value of total net revenue from the existing supply of resources is $26,891, which is 70 percent greater than the total net revenue from the past season's output.

A.4 Some Comments on the Quadratic Programming Model

Since this application of quadratic programming was intended only to illustrate a method of solving a not uncommon problem in horticultural management, rather than to provide the manager with a plan for possible adoption, some comments can be made on additions to the model which may be necessary in practical applications.

Only plants propagated during Spring and Summer were considered, and as a result resources such as glasshouses were unoccupied for about five months of the year. In practice, plants are propagated all year round and glasshouses may always be occupied. Thus in a practical application additional products and restraints would be necessary to represent all-year-round production.

It may be necessary to differentiate between different types of labour, since some operations (e.g. making cuttings) are often carried out by women, whilst others (e.g. spraying and soil sterilisation) usually make use of male labour.

The quantity of cutting material available from existing stock plants is likely to form additional restraints on production. (For example, the above solution requires large increases in the outputs of Passiflora, Banksia, Azalea occid. and Magnolia, which may not be consistent with the quantities of cutting material available). Such restraints would simply take the form of an upper bound on the production level of the crop in question, and the values which could be inputed to such restraints would indicate to the nurseryman the extent to which profits are likely to rise if he were to increase the number of stock plants in the stoolbed.

Apart from the problem of estimating the demand functions as accurately as possible, the profit-maximising point of a linear demand curve may be so far removed from the quantity-price co-ordinate of the past year as to give an unrealistic price and quantity estimate. It is interesting to note, however, that for any given price, a linear function will usually underestimate the quantity demanded and hence revenue (see Fig. A.4). At a price of p_1, 'true' demand is q_2, whereas the linear demand curve approximates demand as q_1.

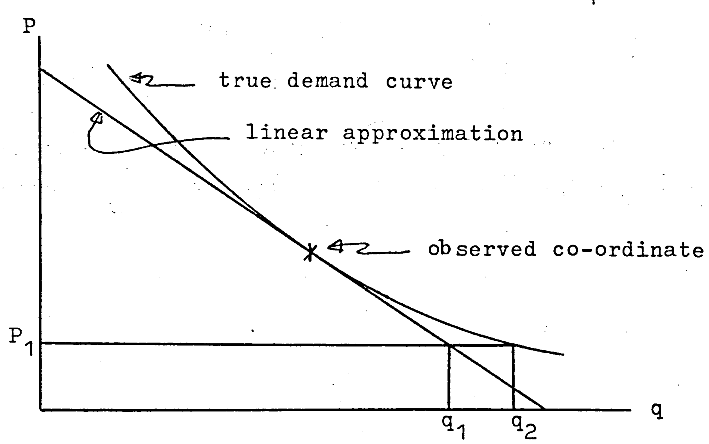

Fig. A.4 Linear Functions may Underestimate True Demand

A method of providing a better approximation of a non-linear demand function has been suggested[9]. The principle is analagous to a market demand curve being the sum of all individual's demand curves - the total demand curve for a product may be the sum of two or more separate linear functions.

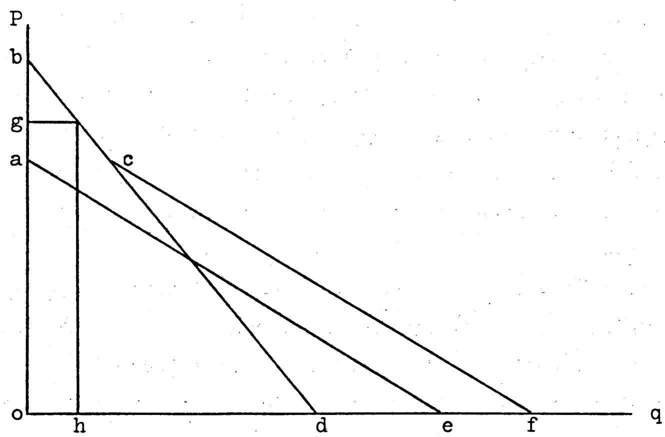

Fig. A.5 Approximation of Demand Curve with Several Linear Functions

In Fig.A.5, the separate linear demand curves are bd and ae. When summed horizontally they give a total demand curve for the particular product, of bcf.

The equations of bd and ae may be written

$$p_j - u_j = a_j - b_j x_j$$

and

$$p_i - u_i = a_i - b_i x_i$$

Three further restraints must be added to the model:

$$p_j = p_i$$

$$x_j u_j = 0 \qquad (x_j, u_j \geqslant o)$$

$$x_i u_i = 0 \qquad (x_i, u_i \geqslant o)$$

Thus, if price is equal to og, a total quantity oh of the product is produced (i.e. x_j = oh, p_j = og, and u_j = o). But since a zero quantity of x_i is produced,

$$x_i = o$$

$$p_i = og,$$

and

$$u_i = ag.$$

9. The author wishes to thank A.D.Woodland of the Department of Economics, University of New England, for suggesting this approach.

Part B

CROP DIVERSIFICATION FOR RISK AVERSION

B.1 The Stochastic Nature of Production

Both budgeting and linear programming assume that all information relating to the management problem, such as crop yields, prices and costs, resource supplies, and input-output data, is known with certainty. Often, production plans formulated under this assumption, may be acceptable to managers[1]. In other cases, however, farm managers may criticise linear programmed solutions, saying they cannot follow a pre-determined plan since values of many variables (such as prices) cannot be known with certainty, but may be considered to occur at random. In these situations, farm planning methods should take account of the random or "chance" variability inherent in the production data. Such chance, or stochastic, variation in data values may occur for many reasons, a few of the more important being :

(i) the price of produce sold on the auction floor may vary, both from day to day, and from one period of the year to the corresponding period of the following year;

(ii) yields may vary from season to season due to both climatic conditions and the incidence of disease (the latter may be partly influenced by the weather);

(iii) the time of planting may need to be altered due to bad weather at the scheduled planting time, or an unexpected frost may damage the young seedlings so that a second and consequently later planting is required;

(iv) the labour input for many operations such as land preparation, handweeding and spraying will vary, due mainly to climatic conditions; and

(v) the labour supply may be reduced through sickness, or difficulties may be experienced in obtaining labour when required.

If the above aspects of risk were not important, then the linear programming model (1) would be appropriate.

(1)
$$\begin{aligned} &\text{Maximise} \quad Z = cx \\ &\text{subject to } Ax \leq b \\ &\qquad\qquad x \geq 0 \end{aligned}$$

Should risk play an important role in a farm manager's future planning, though, model (1) may be generalised to a <u>stochastic</u> linear programming model (2) where parameters (e.g. prices) are represented, not as discrete values known with certainty, but as distributions of values.

(2)
$$\begin{aligned} &\text{Maximise} \quad Z = (c+\gamma)x \\ &\text{subject to } (A+\alpha)x \leq (b+\beta) \\ &\qquad\qquad x \geq 0 \end{aligned}$$

1. For the comments of two horticulturists on linear programmed cropping systems, see Rae, A.N., op.cit., sections 3.12 and 4.10.

where

Z = net revenue

x = vector of cropping levels

c = vector of gross margins

b = vector of resource supplies, and

A = matrix of input-output data.

That is, model (2) allows all coefficients to have associated error terms, $\mathcal{3}$, α and β, which correspond to random deviations between the real world value of the parameter and the estimated value.

Methods are available to allow the solution of stochastic problems[2], but unfortunately only trivially small problems have been solved in practice.

If, however, we are prepared to assume that stochastic errors are associated only with the components of the objective function (i.e. prices, costs and yields), and that other parameters of the problem (resource supplies and input-output data) are known with certainty, quadratic programming can provide solutions to practical problems. In other words, we assume that the only aspect of risk of importance to managers is variation in their incomes, and the procedure is to find a series of farm plans, each of which minimises this variation for various levels of average income[3].

B.2 Theoretical Background to Crop Combination under Risk

This section will introduce iso-income and iso-variance concepts, and will indicate the combination of crops which will produce some level of expected income with the minimum income variability, or minimum risk.

B.2.1 Iso-income curves

The expected income from two cropping activities produced at levels x_1 and x_2, may be written

(3) $$E(\text{income}) = p_1x_1 + p_2x_2,$$

where

p_1 and p_2 are the expected net incomes (gross margins) of the two activities.

A given level of income may be obtained from some acreage of x_1, some acreage of x_2, or from some combination of both x_1 and x_2.

In Fig.B.1 a level of income E_1 may be obtained from all combinations of x_1 and x_2 as given by all points on the iso-income line ab.

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2. See, for example, Cocks, K.D., "Discrete Stochastic Programming", Management Science vol.15, pp.72-79, 1968.

3. The procedure was first postulated by Markowitz in relation to selecting portfolios of securities which minimised income variations. See Markowitz, Harry M., "Portfolio Selection - Efficient Diversification of Investments", John Wiley and Sons, Inc., 1959.

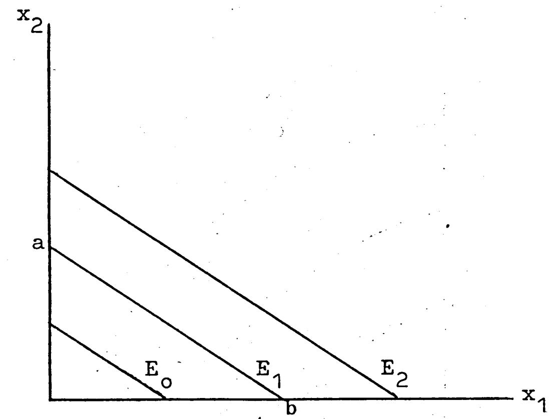

Fig.B.1 Iso-income Curves

B.2.2 Iso-variance curves

The variance of income[4] from a combination of the two crops x_1 and x_2 is equal to

(4) $$V(\text{Variance}) = x_1^2 q_{11} + x_2^2 q_{22} - 2x_1 x_2 q_{12}$$

where

q_{11} is the income variance of x_1,

q_{22} is the income variance of x_2, and

q_{12} is the income covariance between x_1 and x_2.[5]

The iso-variance curves, which indicate all levels of x_1 and x_2 which produce the same variance of income, are indicated by Fig.B.2.

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4. In the quadratic programming approach to risk aversion, the variance of incomes is taken as an index of the degree of risk attached to a particular plan.

An unbiased estimate of variance is given by

$$q_{ii} = \frac{\sum_k (c_i - \bar{c}_i)^2}{n-1},$$

and an unbiased estimate of covariance is given by

$$q_{ij} = \frac{\sum_k (c_i - \bar{c}_i)(c_j - \bar{c}_j)}{n-1}, \text{ where}$$

c_i and c_j are the observed net revenues from the i^{th} and j^{th} activities respectively, in each of the k years,

$\bar{c}_i$ and $\bar{c}_j$ are the average net revenues of the i^{th} and j^{th} activities respectively, and

n is the number of observations.

5. A measure of net revenue covariance between each pair of crops is necessary since, for example, crops A and B may both be subject to high net revenue variance and therefore high risk, while combinations of A and B may provide a net revenue subject to much less variance if the net revenues from both crops are inversely correlated - that is, a low return from A in one year will be offset to some degree by a high return from B. Generally, for anything less than perfect (positive) correlation between net revenues, a combination of the crops will result in some offsetting effects. See Heady, E.O., "Economics of Agricultural Production and Resource Use", Prentice-Hall, Inc., New Jersey, 1952, pp.510-524.

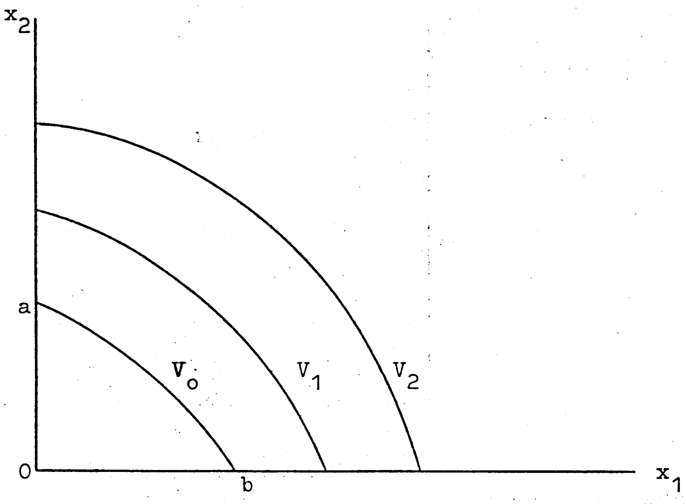

Fig.B.2 Iso-Variance Curves

For example oa of x_2, ob of x_1, or any combination of x_1 and x_2 given by the curve ab will produce a variance of income of Vo. (It may be noted that the iso-variance curves would be straight lines only if net incomes from x_1 and x_2 were perfectly correlated).

B.2.3 Choice of the preferred (Minimum Variance) level of crops

The above iso-income and iso-variance curves are drawn on a single graph in Fig.B.3. It then becomes apparent that tangency of the two types of curve indicates the levels of x_1 and x_2 which will produce some level of expected income at minimum variance. (Or conversely, such points of tangency indicate the crop combination which will produce a maximum level of income for a given level of income variance.

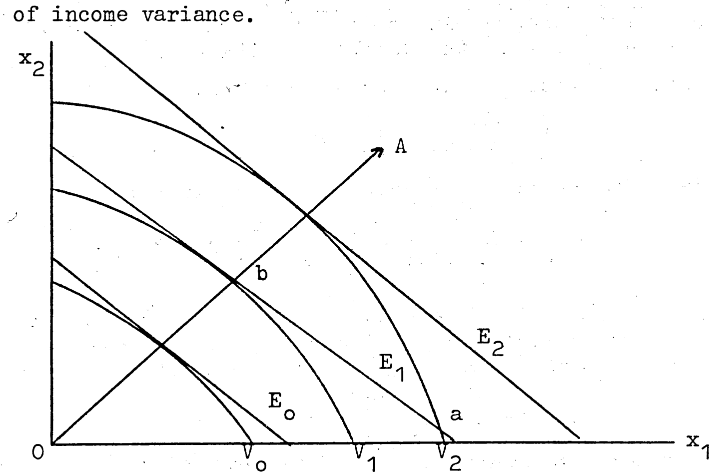

Fig.B.3 Preferred Crop Combinations

For example, the co-ordinates of point a are the levels of x_1 and x_2 which produce an income of E_1 with a variance of V_2. However, point b gives another combination of x_1 and x_2 which produces the same income (E_1) but has a smaller income variance, V_1. Should the farmer be averse to risk (as measured by income variability), he will prefer combination b to combination a. The line OA is the locus of all such preferred combinations of the two crops, which minimise the variance of income for any given level of income.

The preferred, minimum-risk crop combination (i.e. all points on OA), may be determined by minimising the variance function (4) subject to a given level of income (3). That is,

$$\text{(5)} \qquad \text{Minimise} \quad Z = V$$
$$\text{subject to } E_1 = p_1x_1 + p_2x_2$$

Forming the lagrangian function and setting its partial derivatives with respect to x_1, x_2 and the lagrangian multiplier (λ) equal to zero, gives equations (6), whose simultaneous solution will give the levels of x_1 and x_2 which will produce the income E_1 at minimum variance.

$$\text{(6)} \qquad \begin{aligned} x_1q_{11} + x_2q_{12} + \lambda p_1 &= 0 \\ x_1q_{12} + x_2q_{22} + \lambda p_2 &= 0 \\ x_1p_1 + x_2p_2 &= E \end{aligned}$$

B.2.4 <u>The preferred crop combinations when restraints on production exist</u>

The above analysis assumes that x_1 and x_2 may be produced at any (non-negative) level, that is, infinite supplies of resources are available. In practice, the farmer owns only so much land, labour, buildings etc., which prevents the production of crops from being expanded continuously.

Fig.B.4 is similar to Fig.B.3, except that upper bounds have been placed on the levels of x_1 and x_2.

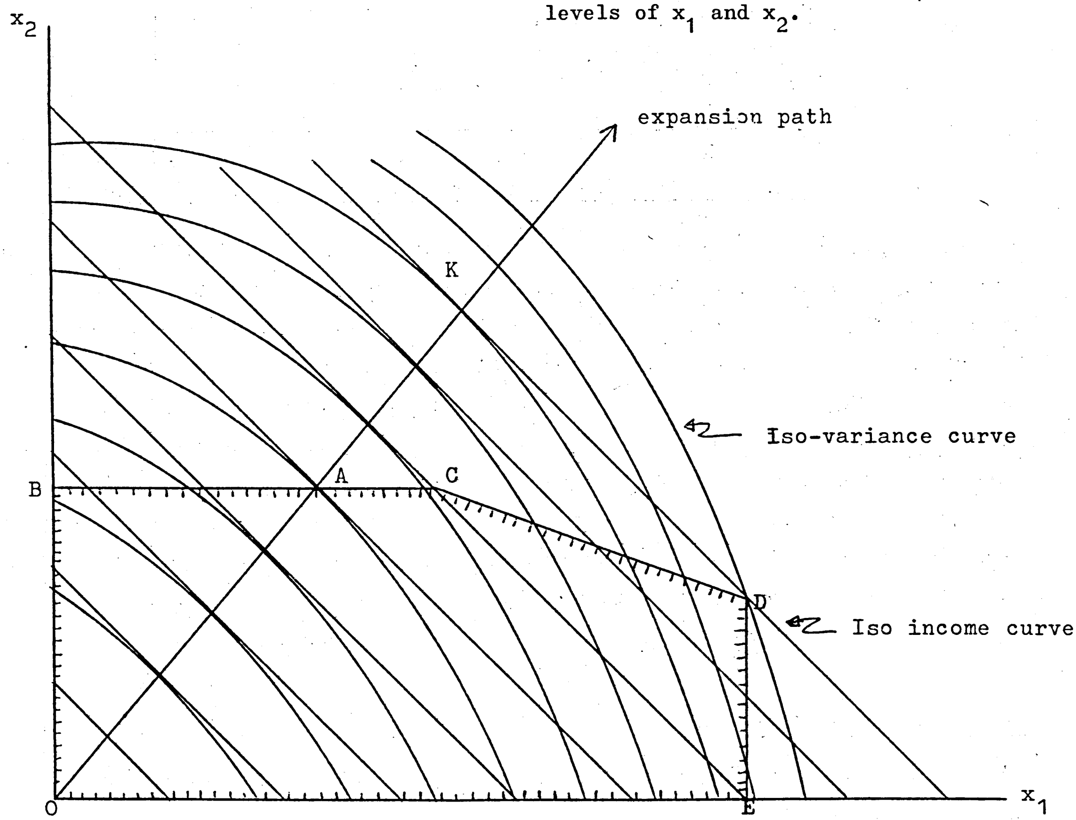

Fig.B.4 Preferred Crop Combinations with Restraints on Production

Resource supplies are such that all feasible combinations of x_1 and x_2 are bounded by OBCDE. As in the previous example, the expansion path OA joins all points of tangency between the iso-income and iso-variance curves. Thus, as production is expanded from O to A, variance of the resulting income will have beenminimised. Because of the production restraint it is not possible to move along the expansion path beyond A, so a new direction of expansion must be found within the feasible region, along which income can be increased with a minimum increase in variance. Such a path is from A to C, where another restraint becomes effective. Production may then be expanded to D, at which the maximum possible income is attained. Note that as production deviated further and further from the expansion path (i.e. to C and then to D), the increment in variance associated with a given increment in income became larger and larger.

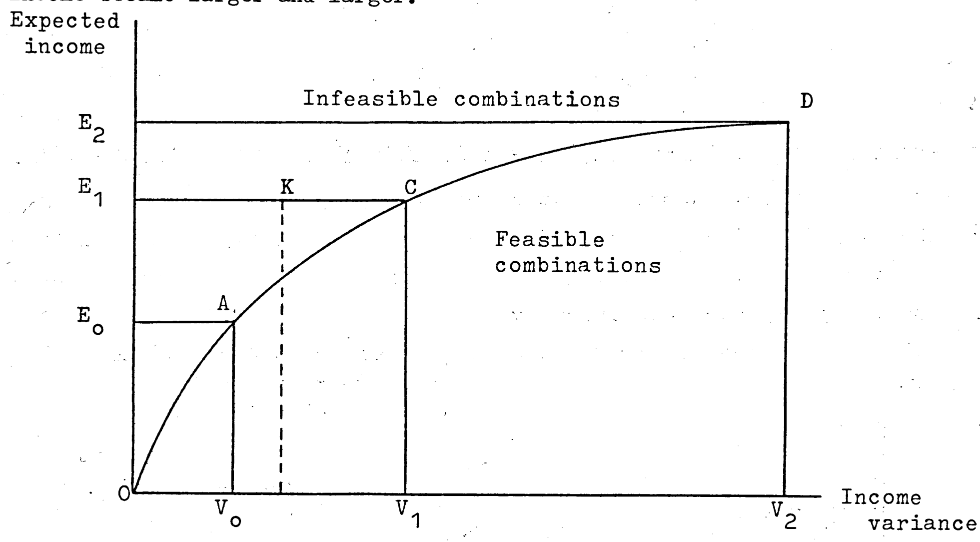

Fig. B.5 Preferred Combinations of Income and Variance

Letting income and variance associated with points A, C and D be E_oV_o, E_1V_1, and E_2V_2 respectively, the preferred feasible crop combinations from Fig. B.4 may be graphed in E-V space (Fig.B.5). All combinations below OD are feasible, whereas those above OD are infeasible. For example, K is preferred to C since it has the same income, but lower variance, but from Fig.B.4 it is seen that K lies outside the feasible region. It is also easily seen that for a given increase in variance, the increase in income becomes smaller, until the point of maximum income (D) is reached.

All points on OD are preferred to all points to the right of OD, since the points on OD represent plans with smaller income variance or greater income than any other feasible plan with the same income or variance. Thus the problem of finding all feasible variance-minimising plans (as in Fig.B.4) is equivalent to identifying all points on OD in Fig.B.5.

B.2.5 An E-V indiferrence system

The decision maker is assumed to behave in accordance with an E-V indifference system as in Fig.B.6.

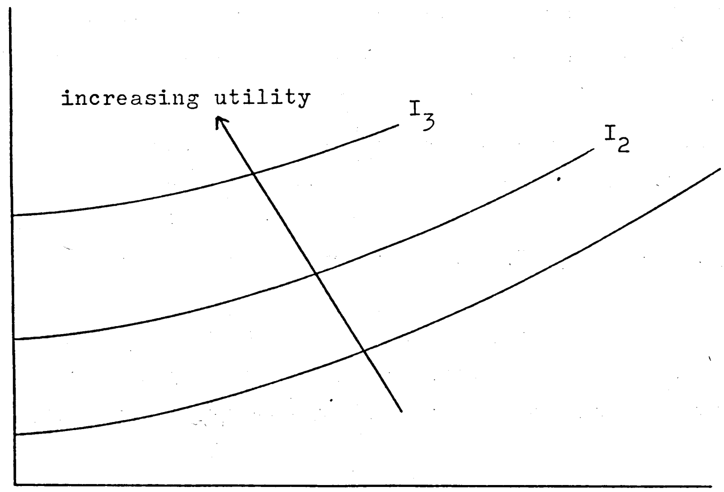

Fig.B.6 An E-V Indifference System

I_1, I_2 and I_3 represent successively higher indifference curves. The manager will be indifferent between all points on any one indifference curve - each indifference curve gives all possible combinations of income and variance which result in equal satisfaction to the decision maker.

Once all points along OD (Fig.B.5) have been found, the particular combination of income and variance which maximises the individual's satisfaction will be defined by tangency of an indifference curve to OD, as in Fig.B.7.

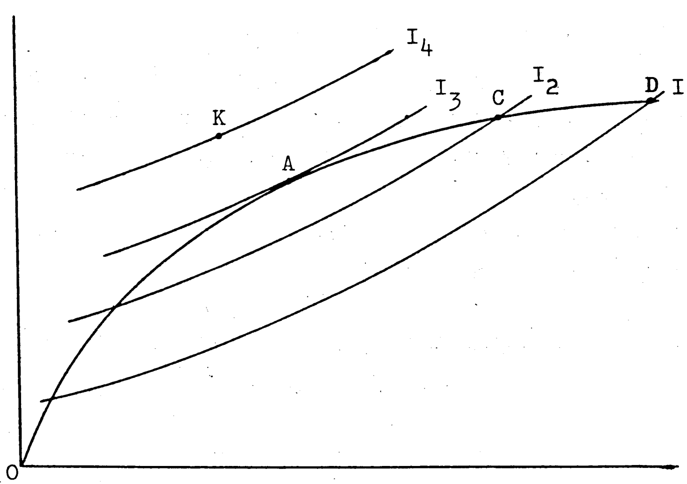

Fig.B.7 Maximisation of Utility

Thus A represents the crop combination which will maximise the decision maker's utility. Even though K would give him greater utility than A, it is an infeasible combination. Plans C and D are not preferred to Plan A since they would give levels of utility of I_2 and I_1, which are both less than the utility derived from Plan A.

Note that as the decision maker becomes <u>less</u> averse to risk, the indifference curves will become less steep. For a farmer who completely disregards risk, the indifference curves will be horizontal, and the plan giving maximum income (i.e. Plan D) will also give maximum utility.

In practice, the farmer may be able to make a choice amongst all preferred plans without having recourse to indifference curves. Thus the problem of defining the farmer's E-V indifference system is avoided, although the choice of a plan which the farmer believes will maximise his utility implies tangency of the preferred path OD and an indifference curve.

On the other hand, it may be possible to estimate the farmer's utility function[6]. If this is a quadratic, the E-V pairs for all preferred plans may be substituted into the expected utility function, so that the utility associated with each plan can be numerically specified.

B.2.6 Summary of the risk minimisation model

The model assumes (i) that the variance of crop returns and total income is an adequate measure of risk. That is, very high and very low returns are equally undesirable, and the model seeks to eliminate both these extremes. Although some farmers may be averse to only very low returns and regard high returns as being acceptable, the assumption is acceptable to others who wish to obtain a stable income;

(ii) that the variance of crop returns as observed in the past is a true indication of such variability in the future; and

(iii) the decision maker's indifference system is of an E-V type. Such a system appears to be satisfactory for farmers who consider expected income as desirable and income variance (risk) as undesirable.

Given the above assumptions, it has been shown that it is possible to define those crop combinations which are preferred to all others. Such combinations are those with maximum expected income for any given level of variance (or minimum variance for any given level of expected income).

The decision maker is then left to choose, from among all such preferred crop combinations, that which maximises his utility.

B.3 The Use of Quadratic Programming to Solve the Risk Minimisation Problem

A parametric quadratic programming technique can be employed to find all points

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6. Makeham, J.P., Halter, A.N., and Dillon, J.L., "Best-Bet Farm Decisions", Professional Farm Management Guidebook No.6, Department of Farm Management, University of New England, 1968, pp.31-69.

on the boundary OD (Fig.B.5)[7]. This model may be specified as [8]:

$$\text{(7)} \qquad \text{Maximise } Z = \theta cx - x'Qx$$
$$\text{subject to } Ax \leq b$$
$$x \geq 0$$

where C = a vector of crop average net incomes,

Q = an income variance-covariance matrix,

A = a matrix of input-output coefficients,

b = a vector of resource supplies.

First, the problem is solved for $\theta = 0$, and the algorithm then proceeds to trace out the preferred boundary (OD) by allowing θ to increase until the plan corresponding to maximum income is obtained (beyond which any further increase of θ will have no effect on the solution).

B.4 An Empirical Application of the Model[9]

B.4.1 Introduction

Market prices of fresh vegetables may be expected to fluctuate markedly even from day to day, and many producers may attempt to reduce the consequent fluctuations in their incomes by cultivating crops which may be expected to realise, on average, a rather low but stable income. However, other growers may be less averse to risk and may wish to cultivate crops whose prices may vary greatly, in the hope of receiving a high but fluctuating income.

Under such conditions, the quadratic programming risk-minimisation model is most appropriate to enable management plans to be formulated. An empirical model was constructed with data provided by a New Zealand producer of fresh vegetables, where both market auction prices and crop yields were subject to stochastic variability.

The farmer owned 90 acres of cropland and had the opportunity of leasing a further 19 acres of land especially suitable for some spring and summer crops. The labour supply consisted of the owner and two men.

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7. Wolfe, P., "The Simplex Model for Quadratic Programming", Econometrica, vol.27, pp.382-398, 1959.

8. The algorithm used to solve the empirical application of the following section minimised the objective function

$$Z' = \theta cx + x'Qx, \text{ which is equivalent to maximising } Z.$$

9. The formulation of this programming model, (e.g. discussion of activities, restraints, construction of the matrix and estimation of net revenues and the variance-covariance matrix) will be discussed only briefly here. For such details, the reader should refer to Rae, A.N., op.cit., Ch.6.

For other examples of quadratic risk programming, see:
Heady, Earl O., and Candler, Wilfred, "Linear Programming Methods", Iowa State University Press, 1958, Ch.17;
Sturgess, N.H., "Enterprise Combination Under Risk", M.Agr.Econ.thesis, University of New England, Australia, 1965;
Camm, B.M., "Risk in Vegetable Production on a Fen Farm", The Farm Economist, vol.10, p.89, 1962-65;
McFarquhar, A.M.M., "Rational Decision Making and Risk in Farm Planning - An Application of Quadratic Programming in British Arable Farming", Journal of Agricultural Economics, vol.14, p.552, 1961

Crops which may be grown include spring carrots, crown pumpkin, butternut pumpkin and buttercup pumpkin, parsnips and both winter and spring crops of cabbage, cauliflower and lettuce. Also, the carrot, parsnip and pumpkin crops may be grown on either land block.

Crop rotation, as well as the grower's past marketing experience, also imposed restraints on the maximum possible acreage of crops cultivated.

B.4.2 Formulation of the objective function

Since construction of the restraints is exactly the same as in linear programming models, attention will be concentrated on the objective function.

It is necessary to calculate the average net income from each crop, as well as the variance-covariance matrix of net income for all crops. From the grower's marketing accounts total quantities marketed and average prices received could be accurately derived over each of the three previous seasons. Thus the gross income earned by each crop in each year could be determined and by subtracting variable costs of production[10], the net income from each crop in each year was found. From this information it was possible to calculate the average net income and the variances and covariances of net income (using the formulae of footnote 4). Table B. 1 gives an example of the crop data.

TABLE B.1 Costs and Returns - Winter Cabbage

		1963-64	1964-65	1965-66
Average wholesale price	($/case)	1.062	1.238	1.362
Quantity sold	(cases/acre)	528	383	546
Gross income	($/acre)	560.74	474.16	743.66
Variable costs	($/acre)	88.92	81.68	89.82
Net income	($/acre)	471.82	392.48	653.84

Note: Wholesale price is the average of prices received in each fortnightly period of the marketing season, weighted by the quantity sold in each fortnightly period.

Annual net incomes and their averages, for all crops, are given in Table B.2.

10. Variable costs of production included such charges as seed, fertiliser and spray materials, tractor running expenses, contract labour, and marketing costs (containers and auctioneer's commission).

TABLE B.2 Crop Annual Net Incomes ($/acre)

Crop	1963-64	1964-65	1965-66	Average
;ring carrot	664.91	469.11	1047.81	727.26
arsnip	334.60	282.40	No crop	308.50
rown Pumpkin	328.88	66.80	44.38	146.68
3uttercup pumpkin	416.06	183.72	318.46	306.08
Butternut pumpkin	No crop	202.46	350.86	276.66
/inter cauliflower	457.60	546.68	712.78	572.36
Spring cauliflower	401.10	460.86	587.68	483.22
/inter cabbage	471.82	392.48	653.84	506.04
.pring cabbage	439.38	343.76	662.86	482.00
Vinter lettuce	931.18	No crop	2015.40	1473.30
Spring lettuce	1072.18	736.54	471.74	760.16

Once the data of Table B.2 had been assembled, the next task was to calculate all crop net income variance and covariance estimates[11]. These are presented in Table B.3.

Thus all the necessary data for the quadratic programme (equation (7)) had been colletted and the model could then be set up and solved.

B.4.3 Solution to the risk-minimisation model

The 24 preferred cropping programmes which comprise the solution to the risk minimisation model are given in Table B.4, along with the expected value of net income and the 90 percent confidence limits[12].

11. To allow completion of the variance-covariance matrix, an estimate was required for covariances involving the three crops which were grown in only two of the three years.

Since

$q_{ij} = r_{ij}\sqrt{q_{ii}q_{jj}}$, the estimate of covariance was obtained from

$$q_{ij}^{est} = \overline{r_{ij}}\sqrt{q_{ii}q_{jj}},$$

where

$\overline{r_{ij}}$ is the mean value of all correlation coefficients $(i \neq j)$ which could be calculated.

12. The 90 percent confidence limits are given by

$$(E \pm 2.92S),$$

where

E is expected income

S is the standard deviation of expected income, and

2.92 is the value of t for a probability of 0.10 and two degrees of freedom.

TABLE B.3 Net Income Variance-Covariance Matrix

Crop	Spring carrot	Parsnip	Crown pumpkin	Buttercup pumpkin	Butternut pumpkin	Winter cauliflower	Spring cauliflower	Winter cabbage	Spring cabbage	Winter lettuce	Spring lettuce
Spring carrot	86,618	2,108	-11,765	14,347	5,991	29,395	22,186	39,408	48,153	43,772	-52,900
Parsnip		1,362	1,133	836	752	928	682	960	1,173	5,491	2,155
Crown pumpkin			25,022	14,273	3,220	-16,611	-11,931	-6,142	-7,612	23,526	44,121
Buttercup pumpkin				13,610	2,375	-3,870	-2,501	5,981	7,233	17,351	16,818
Butternut pumpkin					11,011	2,637	1,940	2,728	3,334	15,608	6,126
Winter cauliflower						16,774	12,333	13,799	16,919	19,264	-37,850
Spring cauliflower							9,078	10,394	12,742	14,171	-27,612
Winter cabbage								17,956	21,944	19,930	-25,312
Spring cabbage									26,819	24,356	-31,098
Winter lettuce										587,767	44,756
Spring lettuce											90,550

Note: For the quadratic programme computations, variance and covariance estimates were calculated to two decimal places.

For example, if Plan 1 was adopted, the grower could expect, over a number of years, an average net income of $7,110. But in any single year, there is a 90 percent probability (i.e. a "9 in 10 chance") that his net income will fall somewhere between $6,099 and $8,121. Conversely, there is only a "1 in 10 chance" that his net income will fall outside these limits.

The preferred combinations of expected income and variance (the boundary separating the feasible and infeasible region, as in Fig.B.5) are graphed in Fig.B.8 and expected income and its confidence limits are presented in Fig.B.9.

It can be easily seen from the latter two figures that for expected incomes greater than about $23,000 any further increase in expected income is made only at the expense of very large increases in income variance - the confidence limits are relatively close together until an expected income of around $23,000 is reached, and beyond this point the limits tend to become very wide.

As an extreme, Plan 24 provides the highest possible average net income ($36,681), but net income in any one year may fall anywhere (with a "9 in 10 chance") between the very wide limits of $9,331 and $64,032.

When data was collected from the vegetable grower, the cropping programme which he was following at that time was noted for comparison with the preferred plans. This plan is summarised in Table B.5 and its expected income and variance are plotted in Fig.B.8 as point X. The producer could obtain the same level of expected income, but with a greatly reduced level of variance[13/] by moving to point Y on the preferred boundary in Fig.B.8. Or he could obtain the same income variance as from his present cropping system (i.e. run the same level of risk), but a higher level of expected income by moving to point Z on the preferred boundary. Thus, without any knowledge of the farmer's indifference system, it is at least known that he would obtain greater satisfaction from <u>any</u> plan corresponding to all points between Y and Z, as well as Y and Z themselves, than from his present cropping programme. This is because all such points correspond to an expected income not <u>lower</u> than, and an income variance not <u>higher</u> than, that of his present system (point X).

Attention will now be directed towards the levels of individual crop acreages in the preferred plans.

The initial plan (which, incidentally, does not cover the fixed costs of the holding of $7,725) includes only spring crops of cauliflower and lettuce. Although spring lettuce is a high income variance crop, returns from the two crops are negatively correlated, a low return from one tending to be accompanied by a high return from the other, thus stabilising income.

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13. The 90 percent confidence limits of the grower's present cropping system are $52,579 and $6,636. However, Plan 16 with an expected income similar to that of the grower's present plan, has the much narrower confidence limits of $43,874 and $15,012.

TABLE B.4 The Preferred Cropping Plans

Crop		Plan 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Spring carrot	(acres)															1.7	8.0	8.8	10.7	13.3	16.7	19.3	18.9	18.7	20.1
Parsnip	"					18.9	20.0	22.5	23.6	23.1	22.7	22.3	21.6	21.6	21.5	19.8	12.3	11.2	8.5	6.0					
Crown pumpkin	"				0.7	0.3	0.3	0.3				0.3	1.2	1.4	1.5	1.6	4.7	4.2	2.8	2.8	5.8	2.8	2.1	2.0	4.8
Buttercup pumpkin	"									0.4	1.1	1.2	2.1	2.0	2.3	2.1	1.9	2.8	4.9	4.6	3.1	5.4	8.3	13.9	8.6
Butternut pumpkin	"												1.8	1.8	1.6	1.9	6.4	6.6	7.2	7.5	10.6	6.7	5.2		
Winter cauliflower	"		5.0	6.9	10.0	10.0	9.9	9.3	7.1	6.1	5.0	5.0	5.0	5.0	5.0	5.0	7.9	8.5	10.0	10.0	10.0	10.0	10.0	10.0	9.8
Spring cauliflower	"	10.0	10.0	8.1	5.0	5.0	5.1	5.7	7.9	8.9	10.0	10.0	10.0	10.0	10.0	10.0	7.1	6.5	5.0	5.0	5.0	5.0	5.0	5.0	5.2
Winter cabbage	"						0.1	0.7	1.0	2.8	4.3	5.0	5.0	5.0	5.0	5.0	2.1	1.5							
Spring cabbage	"																								2.6
Winter lettuce	"													0.1	0.7	1.4	2.7	3.0	3.9	3.9	4.0	3.9	4.8	5.0	5.0
Spring lettuce	"	3.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	4.1	3.8	2.6
Upper limit	$	8,121	13,148	13,359	13,862	21,713	22,214	23,637	23,959	25,012	26,195	26,642	27,840	28,117	29,772	33,358	43,874	45,608	49,881	52,499	56,836	59,137	61,391	62,521	64,032
Expected income	$	7,110	11,495	11,661	12,038	17,824	18,194	19,247	19,484	20,226	21,009	21,293	22,001	22,156	23,017	24,738	29,443	30,171	31,950	33,010	34,712	35,544	36,251	36,574	36,681
Lower Limit	$	6,099	9,842	9,964	10,215	13,934	14,174	14,857	15,008	15,439	15,823	15,944	16,163	16,194	16,262	16,118	15,012	14,735	14,019	13,521	12,588	11,951	11,110	10,628	9,331

Note: A rotation restraint required the area of Spring carrot and parsnip crops grown in the freehold land to be no more than 20 acres. For those plans in which the combined acreage of these crops exceeds 20 acres, any balance is to be cultivated on the leased land block.

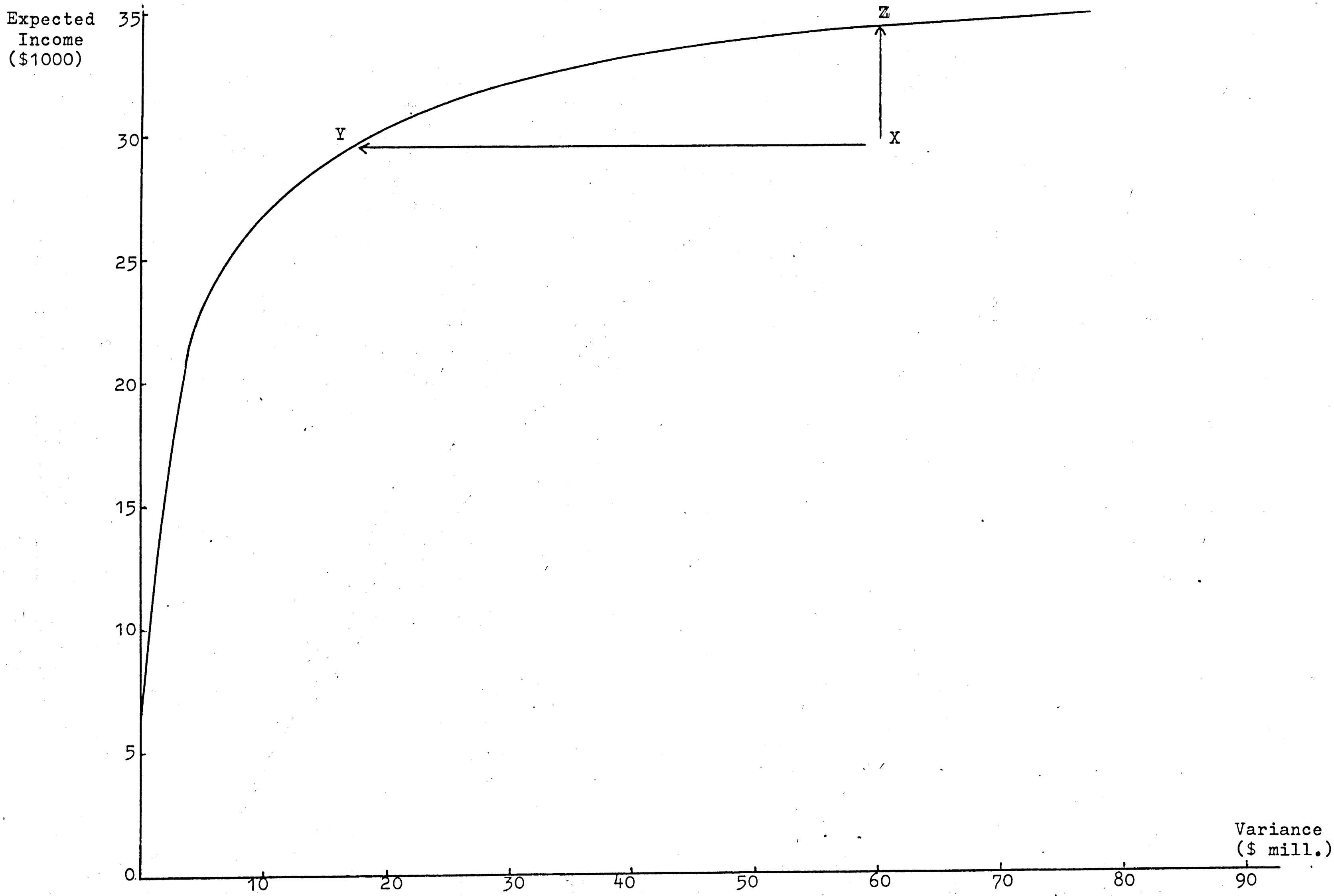

Fig. B.8 Preferred Combinations of Income and Variance

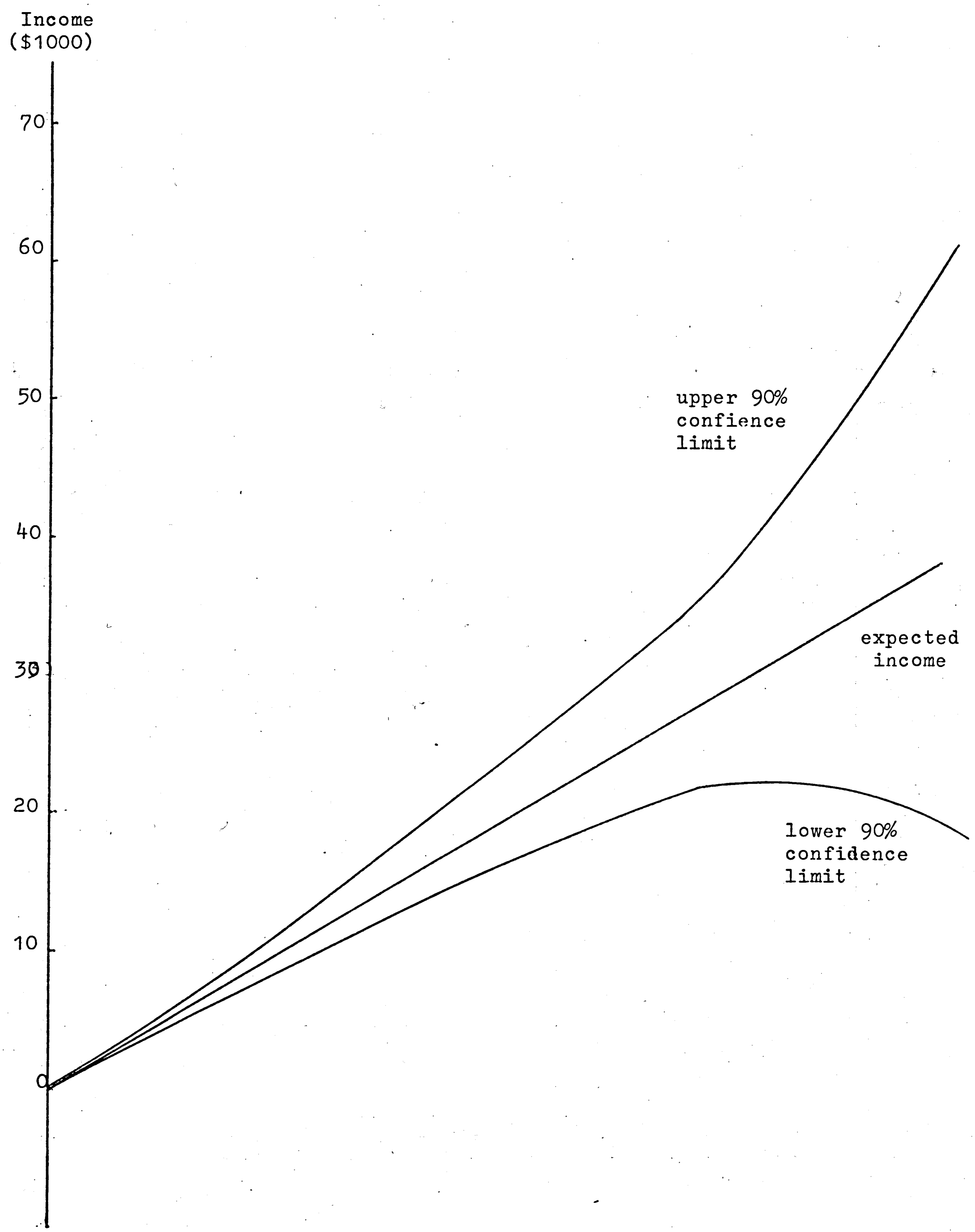

Fig.B.9 Expected Income and Confidence Limits

TABLE B.5 The Grower's Present Cropping System

Crop	Acreage
Spring carrot	18.0
Parsnip	2.0
Crown pumpkin	4.5
Buttercup pumpkin	7.0
Butternut pumpkin	2.0
Winter cauliflower	6.0
Spring cauliflower	4.0
Winter cabbage	4.0
Spring cabbage	3.0
Winter lettuce	2.0
Spring lettuce	1.0
Expected net income	$29,608
Income variance	61,887,300

Next, a Winter cauliflower crop is introduced, its net income also being negatively correlated with Spring lettuce incomes.

The next noticeable features of the preferred plans are the introduction of parsnip and Winter cabbage, in Plans 5 and 6 respectively. Although parsnip net incomes are positively correlated with those of all other crops, its net income variance is lowest of all. Winter cabbage net income is inversely correlated with net incomes of two other crops in Plan 6, parsnip and Spring lettuce, so the introduction of Winter cabbage will have some effect in reducing income variance.

The acreage of parsnip and Winter cabbage increases over the next several plans, and all three pumpkin crops are gradually introduced. Although the latter three crops have the lowest average net income of all crops, the variance of both buttercup pumpkin and butternut pumpkin net incomes is very low (only parsnip has a lower net income variance). Also, crown pumpkin net income is negatively correlated with both Winter and Spring cauliflower and Winter cabbage, and buttercup pumpkin net income is negatively correlated with that of both cauliflower crops.

Plans 13 and 15 see the introduction of two high average income and high income variance crops. Winter lettuce has the highest average income and variance of all crops, and Spring carrot, the third highest income and variance. Winter lettuce net income is positively correlated with all crops in the preferred plans at that stage, although Spring carrot net income is inversely correlated with the net incomes of crown pumpkin and spring lettuce.

Incidentally, Plans 15 to 17 exhibit the maximum diversification - all crops with the exception of Spring cabbage are included in these plans.

From this point (Plan 15) onwards, the acreages of Spring carrot, Winter lettuce and Winter cauliflower gradually increase while the area planted in parsnip, Spring cauliflower, Winter cabbage and Spring lettuce fall, as expected income is allowed to rise and less importance is placed on low income variance in the selection of preferred crop combinations. Winter cabbage is excluded from all preferred plans beyond Plan 17, parsnip from Plan 19, and the butternut pumpkin crop reaches a maximum in Plan 20, but is excluded from the two final plans.

Plan 24 represents the combination of crops which is expected to realise the highest possible net income from the fixed resources of the holding. This plan, in which income variance plays no role in the selection and combination of crops, is identical with the plan which would be obtained from the normal linear programming model, with the expected crop net incomes forming the coefficients of the objective function.

B.5 Summary and Conclusions of the Risk Minimisation Model

After describing those situations in horticultural management in which risk is likely to assume an important role when cropping programmes are formulated, attention was concentrated on only one aspect of risk, namely stochastic variability of income. The theoretical background to crop combination to reduce such variability was then presented, and the use of quadratic programming to solve such problems was indicated. Finally, an empirical application of the model to a New Zealand fresh vegetable holding was described.

It may be noted, however, that the maximum expected income plan (i.e. the normal linear programming solution) would also maximise the grower's utility <u>only</u> if his utility function was linear - that is, expected utility is equivalent to (a linear function of) expected income. For a grower who is averse to risk, however, utility will be a function of both expected income <u>and</u> income variance, and linear programming methods will <u>not</u> allow him to maximise his utility.

Thus, where horticultural producers are averse to risk, and such risk may be adequately measured by the variance of income, quadratic programming (rather than linear programming) becomes the appropriate tool.

<u>Note</u>: Unfortunately, since the author had left New Zealand by the time a solution to the problem was obtained, it was not possible to present the grower with the list of preferred plans.

Part C

CONCLUSIONS

Provided that perfect competition prevails in both product and factor markets (and, of course, that other linear programming assumptions are applicable) linear programming may be usefully employed to define optimising behaviour of producers. However, should producers consider future prices, yields and technical coefficients as "risky", that is, not known with certainty, then stochastic linear programming will generally provide more meaningful results. Since further research into such techniques is required before the general stochastic model can be employed on practical problems, though, it is necessary to assume that only the variables in the objective function (prices, costs and yields) may occur at random and that all other management data is known with certainty. Parametric quadratic programming can then be used to obtain solutions and, for producers who are averse to income variability, will be more useful than ordinary linear programming.

Also, once the assumption of perfect competition in product and factor markets is inapplicable to the situation under study, linear programming may only approximate the optimum behaviour of the firm. If the product demand curves and factor supply curves facing the firm are restricted to a linear form, then the profit-maximising model becomes a quadratic programming problem.

To summarise, then, the existence of imperfectly competitive elements in some horticultural product (or factor) markets could provide many instances where quadratic programming techniques would be more appropriate than a linear programming approach. Vegetable growers, as well as nurserymen, may find that marketing increasing quantities has a depressing effect upon prices, especially since large holdings are becoming more numerous and replacing, to some extent, the traditional small family unit.

It is hoped that the risk-minimisation model (and its empirical application) has indicated the likely role of this technique in management advisory work, especially for growers who sell their produce through an auction system. Many growers would prefer to avoid excessive price fluctuations so as to guard against the likelihood of low incomes. The risk model, although doing nothing to reduce such price fluctuations in the market, does provide producers with cropping plans for which the variability of income has been minimised. By estimating the grower's (quadratic) utility function, or simply by allowing him to make a choice amongst all such plans, the particular combination of income and risk (and the corresponding cropping programme) which maximises the grower's utility, may be identified.