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Cattle Feeding

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## A NOTE ON FEED SUBSTITUTION IN MILK PRODUCTION

by

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and

J. A. LANGLEY

*Price Two Shillings and Sixpence*

I, COURTENAY PARK  
NEWTON ABBOT  
DEVON

UNIVERSITY OF EXETER

Department of Agricultural Economics

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## FOREWORD

Recent progress in science and technology has vastly increased the range of choices which face the farmer and resort to rule of thumb, a feature of practice in the past, is no longer adequate. To-day a much more rigorous definition of input/output relationships is needed and more precise tools to apply to the ever increasing number of complex economic problems of individual farms must be developed. The rôle of the economist in this development is that of pointing out the areas where data are needed and, what is possibly even more important, specifying the conceptual framework and the form in which the data are derived.

Livestock farmers are primarily concerned with the problem of how best to feed their stock so that they can obtain the highest possible return. In doing so they are critically concerned with levels of feeding, substitution rates between foods, reliance on home production in relation to purchased foods and the economic merits of different cropping policies. While judgement in these matters will continue to play a major rôle, increasingly, farmers and their advisers will look to the new tools of scientific management now being forged by economists and econometricians to aid their judgements.

The opinions of economists and those of other scientists will continue to differ on the question of approach, the kind and sources of data and their relative merits. Progress is more likely to follow fuller discussion

of the subject by research workers as well as farmers and their advisers. Already a start has been made to get greater collaboration between research economists and animal nutritionists. This short report represents an attempt by this Department to promote a greater awareness of and a keener interest in the subject of feed substitution in milk production.

S.T. Morris,

Provincial Agricultural Economist.

### A Note on Feed Substitution in Milk Production

The principles concerning relationships between input factors in agricultural production economics have received less attention in the past than those governing input/output and product/product relationships. Yet the general economic situation in the industry and some of the outstanding problems associated with the use of resources clearly demand greater study in this field. For example, some of the more vigorous discussions on the economics of milk production in recent years have revolved around the possibilities of substituting one feed for another. The fact that controversy has persisted over the years signifies that it is either impossible to resolve these issues or that some factors are being overlooked. The purpose of this note is to draw attention to certain implications of the experimental evidence on forage/concentrate substitution in milk production.

The conceptual framework illustrating how output of a product depends on two variable inputs can be set out graphically, where the two inputs are scaled on the axes of the graph and the different combinations of the two inputs that give rise to stated levels of output are shown by a series of isoquants (or iso-product contours). Price lines (iso-cost lines) and isoclines can be superimposed and these indicate expansion paths. Isoquants describe the production surface and when drawn in a

two-dimensional diagram they are assumed (a) to slope downwards to the right and (b) to be convex to the origin. This latter assumption implies diminishing marginal significance of one input in terms of the other. For inputs which are perfect or near perfect substitutes, and replace each other at constant or almost constant rates, the isoquants are straight or nearly straight lines. For inputs which combine most effectively in rather definite proportions the isoquants are likely to take on a shape closely resembling a right-angle. Other isoquants may show differing degrees of curvature, indicating different degrees of substitutability. Knowledge of the production surface and the shape of the isoquants is necessary for determining least-cost combinations of inputs.

### Experimental Results

Milk production is a process which is profoundly affected by the physiological characteristics of the dairy cow. For example, the ration must contain sufficient bulky-type foods to allow the digestive system to function normally but at the same time the physical capacity of the cow's stomach imposes a limit on the total food intake. There is a dearth of reliable information on this subject. The only basic references available to date appear to be reports of controlled experiments carried out by Heady et al at Iowa, U.S.A. in 1953/54<sup>1</sup> and in 1956/59<sup>2</sup>.

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<sup>1</sup> See "Milk production Functions, Hay/grain Substitution Rates and Economic Optima in Dairy Cow Rations" by Earl O. Heady, John A. Schnittker, N.L. Jacobson and Solomon Bloom, Agricultural Experiment

These experiments were specifically designed to study the problem of forage/concentrate substitution in milk production.

Heady and his colleagues fitted a number of different types of equations (linear, logarithmic and quadratic) to the experimental data in an attempt to describe the production surface. When the data from the two experiments were pooled, some 1368 weekly observations of feed input and milk output for 72 cow lactations were available and the regression equation finally selected was of quadratic form with 26 terms<sup>3</sup>. The independent

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Station, Iowa State College, Bulletin 444, October 1956.

The 1953/54 experiment with 36 Holstein cows at the Iowa Experiment Station was designed to allow prediction of the milk production surface and specification of least-cost rations and optimum levels of feeding. Cows were randomly assigned to grain/hay rations varying from 15 to 75 per cent. hay and were kept on these rations for a period of 182 days. They were fed at three levels of intake for each ration and animals representing low, average and high ability were assigned to each of the twelve ration level treatments. Cows were fed in dry-lot. Production functions were derived from the experimental data to predict the milk production surface. In addition to milk and feed, the ability of the cows and time (of lactation) were also taken as variable in the functions.

<sup>2</sup> See "Milk Production Functions Incorporating Variables for Cow Characteristics and Environment" by E.O. Heady, J.P. Madden, N.L. Jacobson and A.E. Freeman. Journal of Farm Economics Vol. 46 No. 1. February 1964. Although the 1956/59 experiment differed from the earlier one in certain respects, both experiments had the same general design and purpose, which allowed the results to be pooled.

<sup>3</sup> Of the 27  $t$  values for the variables and the constant, 21 were acceptable at the .01 level, 5 at the .05 and 1 at the .1 level of probability. Moreover the authors claim that the sign of each term in the equation is consistent with established principles and facts in dairy nutrition, animal physiology and production economics.



variables were for feed input (hay and grain), cow characteristics (stage of lactation, index of ability, coefficient of inbreeding, body weight and index of maturity) and environment (weekly temperature) and the function explained 83.6% of the variance in milk production.

When the variables representing cow characteristics and environment are set at their mean the equation reduces to:-

$$M = 1.046539H - 0.001088H^2 + 2.556283G - 0.005047G^2 \\ - 0.003521GH - 25.959018$$

Where M = pounds of milk (4% fat corrected) produced in one week  
H = pounds of hay consumed by a cow during one week  
G = pounds of grain consumed by a cow during one week

The mean for the stage of lactation was the 11th week of the experiment or the 20th week of the lactation. Obviously the exact position and shape of the isoquants derived from the function will depend on such factors as stage of lactation, cow ability and the characteristics of the food input. However, the important conclusions are that isoquants drawn on the basis of these experiments (a) show only slight curvature and (b) indicate that the two foods substitute for each other over a considerable range.

Blaxter<sup>1</sup> states that ".... work at the Hannah Institute suggests that the substitution value of foods when they are oxidised to meet the

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<sup>1</sup> "Economics and Animal Husbandry" by Dr. K.L. Blaxter. Journal of Agricultural Economics Vol. XIV. No. 2. May 1961.

energy demands of maintenance and muscular work, differs from the substitution values obtained when the foods serve as a source of energy for synthesis of body constituents. The difference is a very major one especially in ruminant nutrition. The experimental data at present available suggest that the iso-quantal lines are certainly not simple curves but are linear up to the maintenance datum and then show a marked change in slope and possibly some curvilinearity."

Neither corroborative nor conflicting evidence is available from the Iowa data since information on substitution rates for maintenance are not given separately. It would seem, therefore, that the joint effect of linearity for maintenance and of the marked change in slope together with curvilinearity for production may determine the substitution rates in the Iowa experiment. Far from detracting from the American experiments, work at the Hannah can be interpreted as giving further meaning to them.

#### The Implication of the Isoquants

For the purpose of this note the Iowa data are assumed to have relevance in this country and the equation has been used to calculate the combinations of hay and concentrates which would yield 800 lb. of milk (4% fat-corrected) in 28 days<sup>1</sup> from a cow of average ability. This level of milk yield about mid-point in the lactation may well portray the position

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<sup>1</sup> The formula with stage of lactation set at the mean has been assumed to apply to a 28 day period.

during winter for many dairy cows in British herds. The quantities of each food together with the marginal substitution rates are set out below.

Table 1. Combinations of foods and marginal substitution rates for 800 lb. of milk (4% fat-corrected) in 28 days from a cow of average ability

Concentrates lb.	Hay lb.	$\frac{dH^*}{dC}$
0	1308	4.19
40	1152	3.75
80	1008	3.43
120	876	3.19
160	752	3.00
200	636	2.84
240	524	2.71
280	416	2.59
320	316	2.48
360	216	2.38
400	124	2.29

\* pounds of hay replaced by one additional pound of concentrates along isoquant

The information shows hay and concentrates substituting for one another over a wide range but it should be pointed out that at both extremes the confidence limits are fairly wide particularly for the input of hay. Nevertheless, it does emphasise the scope for decision making in varying the food combinations for milk production. This is a far cry from reality because most farmers do not regard hay as a substitute for concentrates, and continue to use hay and other roughage crops for maintenance and rely

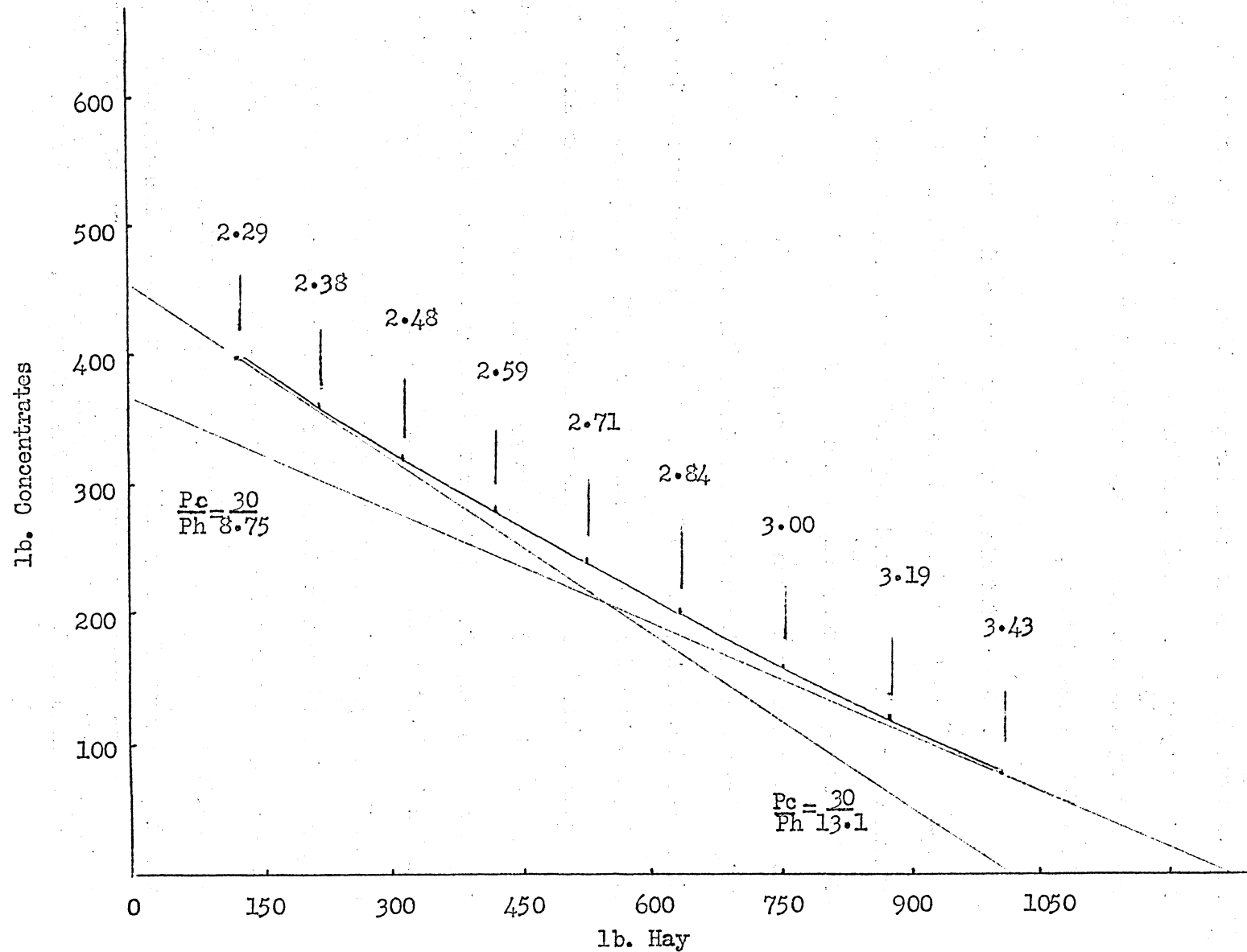
almost entirely on concentrates for production. Recent work on the feeding of barley to beef animals and the feeding of good quality hay and silage to dairy cows has gone a long way towards undermining the foundations of the old recommendations.

In order to determine the least-cost combination of foods, the data contained in the table (with the omission of the first two extreme combinations) are illustrated graphically as an isoquant in Figure 1 and the two price lines whose slopes are equal to the slopes at both ends of the isoquant are superimposed.<sup>1</sup> When hay is £13.10 and concentrates £30.00 per ton then the most profitable combination of feed is at the point where 2.29 lb. of hay are replaced by 1 lb. of concentrates - the combination in this case is 400 lb. of concentrates and 124 lb. of hay. A change in the price of hay to £8.75 per ton with concentrate price remaining the same would shift the least cost or most profitable combination to the other extreme where 3.43 lb. of hay are replaced by 1 lb. of concentrates - the combination being 80 lb. of concentrates and 1008 lb. of hay. If it is assumed that the entire feasible range of hay/concentrate combinations has been considered here then it can be said that the area of choice is restricted to those occasions when hay prices vary between £8.75 and £13.10 per ton, concentrate price remaining the same at £30.00. In maximising profit per cow the farmer should feed maximum quantities of hay when prices

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<sup>1</sup> The least-cost combination of inputs is depicted where  $\frac{dH}{dC}$  is equal to the inverse ratio of the prices of the two inputs.

Figure 1. Feed Inputs for 800 lb. milk isoquant (28 days) with marginal substitution rates and price lines tangential to the extreme points



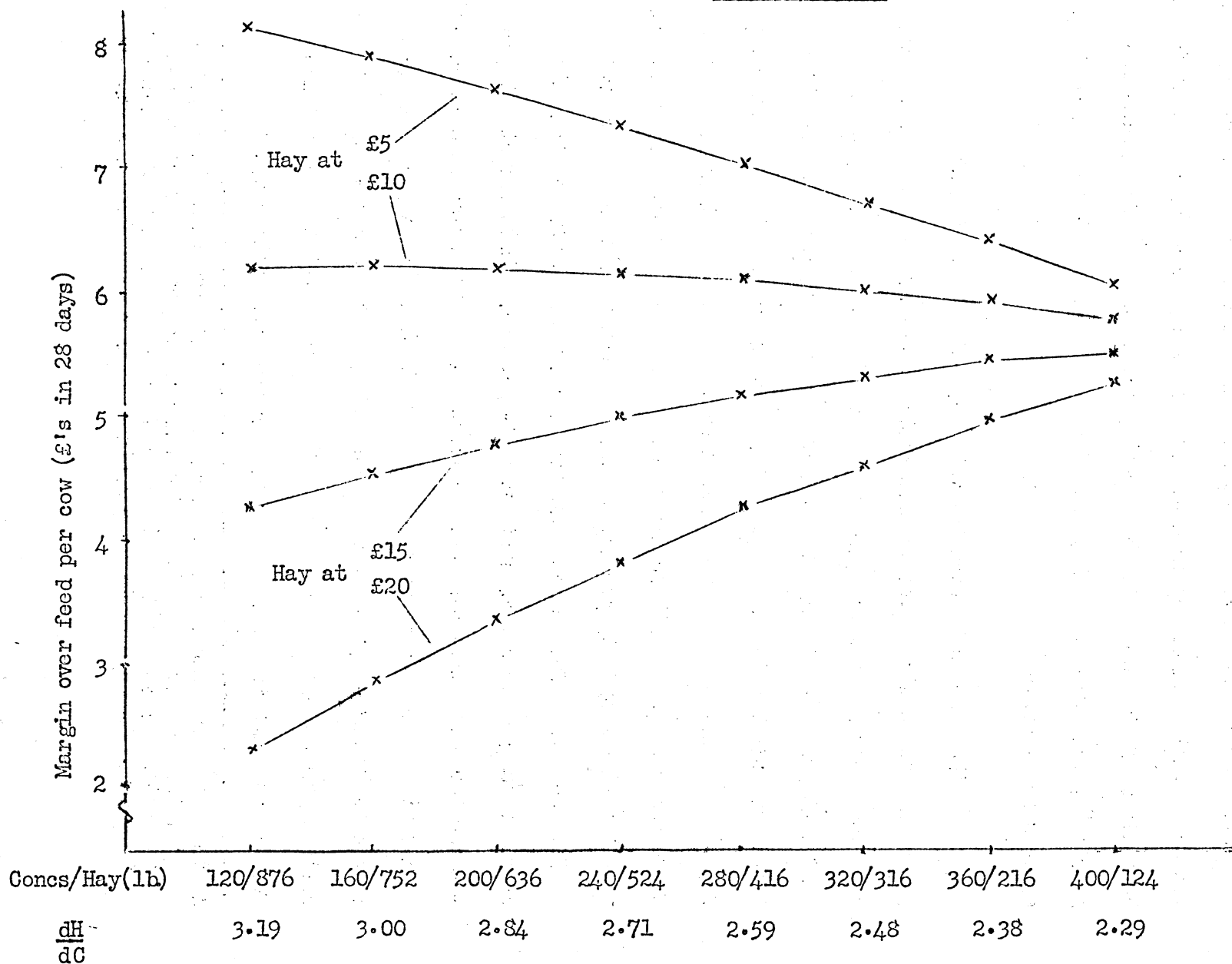
are £8.75 per ton or less. He should feed minimum quantities of hay with the appropriate quantities of concentrates when prices are £13.10 or more. Variations of this order in the market price of hay are frequently encountered, and farmers should, therefore, be prepared to vary their feeding systems accordingly.<sup>1</sup>

The margins between the value of 800 lb. of milk at 3/- per gallon and the costs of the various combinations of hay and concentrates have been calculated with concentrates at £30 per ton and hay at four different prices - £5, £10, £15 and £20 per ton and the data are illustrated graphically in Figure 2. Where the isoquant and price lines have similar slopes then even wide variations in the combinations of the two foods can take place with no marked effect on the margins. This happens with hay at £10 and £15 per ton giving a concentrate to hay price ratio of 3:1 and 2:1. For instance in the case of hay at £10 per ton, the range in margin per cow is just over 8/- per cow. With hay at £15 per ton the range is £1.25 per cow. However, where the price line is markedly different, as for hay at £5 and £20 per ton, then the ranges in margins become £2.08 and £2.93 per cow respectively. It seems then that at certain price levels there may be a large zone of relative profit indifference in which the range in margin per cow between high and low

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<sup>1</sup> There are problems involved in the choice of a suitable basis on which to value home-grown foods. For a discussion on this subject see "Economics of Forage Evaluation" by Johnson and Hardin, Bulletin 623, Agricultural Experiment Station, Purdue University, U.S.A. April 1955.

Figure 2. Margin over feed cost per cow in 28 days at various concentrate-hay ratios  
with concentrates at £30 per ton and with hay at four different  
prices per ton.



concentrate feeding is quite narrow. At other price levels margins vary significantly.

### Herd Considerations

These data have obvious implications as far as individual cow feeding is concerned. What about their implications with regard to the optimum numbers of cows that can be kept? Size of herd may be limited by a number of factors such as housing, labour availability and the amount of food including hay. It is probable that hay has set a fairly rigorous limit on the number of cows kept, and it is still common practice to think in terms of a minimum quantity of perhaps a ton or more per cow of hay or hay equivalent to satisfy the needs of the cow during winter. In deriving the data set out in Table 2, it has been assumed that the number of cows is limited to those that can be fed on 10 tons of hay during a 28 day period with different combinations of hay and concentrates. Obviously in such circumstances the low forage rations allow more dairy cows to be kept and with the limited combinations set out in this table, the numbers vary from 30 to just over 60. With hay charged at £5 per ton, then margin over feed cost per cow falls as more and more concentrates are used to replace hay because at this price hay is cheaper than concentrates at £30 for the level of milk output under discussion. However, the margin per cow does not fall off sufficiently rapidly to outweigh the effect of additional cows, and profit per herd continues to rise.



Table 2. Total feed costs, milk returns and margins over feed costs for different numbers of cows kept on a fixed amount of hay

Feed combination per Cow		Marginal Substi- tution Rates $\frac{dH}{dC}$	Cows kept on 10 tons Hay	Feed Costs per Herd			Value of Milk per Herd*	Margin over feed cost	
Concen- trates	Hay			Concen- trates at £30 per ton	Hay at £5 per ton	Total		Per Cow	Total
lb.	lb.		No.	£	£	£	£	£	£
160	752	3.00	29.8	63.8	50.0	113.8	348.7	7.88	234.9
180	692	2.92	32.4	78.1	50.0	128.1	379.1	7.75	251.0
200	636	2.84	35.2	94.3	50.0	144.3	411.8	7.60	267.5
220	580	2.77	38.6	113.9	50.0	163.9	451.6	7.45	287.7
240	524	2.71	42.7	137.1	50.0	187.1	499.6	7.32	312.5
260	472	2.65	47.5	165.3	50.0	215.3	555.8	7.17	340.5
280	416	2.59	53.8	201.8	50.0	251.8	629.5	7.02	377.7
300	364	2.53	61.5	247.2	50.0	297.2	719.5	6.87	422.3

\* 800 lb. of milk per cow at 3/- per gallon.

The table does, of course, relate to the production of 800 lb. of milk in 28 days from Friesian cows of average ability in the 20th week of lactation. The range in the marginal rates of substitution varies from 2.53 to 3.00 for the food combinations considered. The hay used in the experiment was of a very good quality but the concentrates were of similar quality to those generally used in this country. On many British farms hay is likely to be of poorer quality than that fed in the Iowa experiment, and hence 1 lb. of concentrates is likely to replace more hay. In this case it would be even more attractive to substitute concentrates for hay. It seems, therefore, that the general conclusion suggested on the basis of the Iowa data has particular application in the United Kingdom. The substitution of concentrates for hay and consequently the possibility of keeping more cows, rather than pursuing a policy of self sufficiency, has important economic implications for many dairy farmers in this country.

It is generally appreciated that milk yields vary according to a number of factors, including ability of the cows, stage of lactation and level of feeding, three factors which are of considerable importance. However, for convenience the equation set out earlier refers to cows of average ability in the 20th week of lactation. Nevertheless, even under these conditions milk yield will vary according to the general level of feed intake. The data show clearly that at higher levels of milk yield the range in marginal substitution rates are wider, but again, if these rates

are similar to the price ratios of the two feeds, then there will still be an area of profit indifference.

Readers may question the relevance of feed substitution data concerning short periods of time, because in planning farmers have to consider at least the period necessary to complete one lactation. Gossling<sup>1</sup>, using data from the first Iowa experiments, summated a series of equations in an attempt to derive equations for quarterly and annual periods for dairy herds under East Anglian conditions. This method has been applied by the authors to obtain an equation for a three-monthly winter period applicable to a herd with a calving pattern similar to that of the national dairy herd in England and Wales. Results from this equation suggest that the ranges for the marginal rates of feed substitution for the herd (made up of cows in different stages of lactation including dry periods) were similar to those considered earlier in this note. It would seem, therefore, that an extension of the reasoning outlined above may well be applicable on a herd basis under commercial conditions.

It should be stressed that this note refers to feed substitution at stated levels of milk production and is not concerned with optimum levels of feeding, although it is realised that this latter aspect of input/output relationships is a vital issue.

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<sup>1</sup> "The Economics of the Holstein Friesian Cow" by W.F. Gossling.  
Journal of Agricultural Economics, Vol. XV. No. 4. December 1963.

This analysis, briefly outlining the framework for viewing problems of feed substitution, highlights the important considerations in the forage versus concentrates controversy in milk production. With the marginal rates of substitution set out in this article and concentrates at £30 per ton, then hay is relatively cheap at £5 and relatively expensive at £20. With hay at £5 per ton, margin over feed costs per cow continues to fall as more concentrates are used to replace the relatively cheap hay. However, the fall in margin is not sufficiently steep to outweigh the effect of scale. With the assumptions made and the experimental data used there are still distinct opportunities of making additional profits by using concentrates and increasing stocking rates. Finally it should be emphasised that the experimental data quoted relate to Iowa conditions. Similar experimental work under British conditions and with traditional British crops used in milk production needs to be undertaken. Only then will it be possible to advise farmers confidently on the economics of feeding dairy cows.