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Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation

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Abstract

I construct a putty-clay model of directed technical change and use it to analyze the effect of environmental policy on energy use in the United States. The model matches key data patterns that cannot be explained by the standard Cobb-Douglas approach used in climate change economics. My primary analysis examines the impact of new energy taxes. The new putty-clay model suggests that tax-inclusive energy prices need to be 290% higher than laissez-faire levels in 2055 in order to achieve policy goals consistent with the international agreements. By contrast, the Cobb-Douglas approach suggests that tax-inclusive energy prices need only be 150% higher. Similarly, the new model predicts that the energy expenditure share must rise to 5% – compared with 3.3% today – while the standard model assumes that rapid declines in energy use will leave the expenditure share unchanged. The putty-clay model also implies that final good consumption must fall by 2.8% to meet policy goals, which is double the prediction from the standard model. In a second analysis, I find that policy interventions cannot achieve long-run reductions in energy use without increasing prices, implying that, by themselves, energy efficiency mandates and R&D subsidies have limited potential as tools for climate change mitigation. Finally, I use the model to analyze the long-run sustainability of economic growth in a world with non-renewable resources. Using two definitions of sustainability, I find that the new putty-clay model delivers results that are more optimistic than the existing literature.

Keywords Energy, Climate Change, Directed Technical Change, Growth

JEL Classification Codes O30, O44, H23

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1 Introduction

To address global climate change, it is crucial to understand how carbon emissions will respond to policy interventions. Changes in energy efficiency will be an important component of this response. Indeed, rising energy efficiency – rather than the use of less carbon intensive energy sources – has been the major force behind the decline in the carbon intensity of output in the United States over the last 40 years (Nordhaus, 2013). Thus, energy efficiency will almost certainly be a critical factor in any future approach to mitigating climate change.

Integrated assessment models (IAMs) are the standard tool in climate change economics. They combine models of the economy and climate to calculate optimal carbon taxes. The leading models in this literature frequently treat energy as an input in a Cobb-Douglas aggregate production function (e.g., Nordhaus and Boyer, 2003; Golosov et al., 2014).[1] Despite the significant insights gained from the IAMs, there are two restrictive assumptions in this approach to modeling energy. First, in response to changes in energy prices, the Cobb-Douglas approach allows immediate substitution between capital and energy, which is at odds with short-run features of the U.S. data (Pindyck and Rotemberg, 1983; Hassler et al., 2012, 2016b). This suggests that the standard approach may not fully capture the effect of new environmental taxes, which will raise the effective price of energy. Second, technological change is exogenous and undirected in the standard model. A substantial literature, however, suggests that improvements in energy-specific technology will play a pivotal role in combating climate change and that environmentally-friendly research investments respond to economic incentives (e.g., Popp et al., 2010; Acemoglu et al., 2012).

In this paper, I construct a putty-clay model of directed technical change that matches several key features of the data on U.S. energy use. In particular, the model captures both the short- and long-run elasticity of substitution between energy and non-energy inputs, as well as trends in energy efficiency. In the model, each piece of capital requires a fixed amount of energy to operate at full potential. Technical change, however, can lower this input requirement in the next vintage of the capital good, or it can increase the ability of the next vintage to produce final output.[2,3] When energy prices rise unexpectedly, the energy expenditure share of output will increase in the

[1] This is particularly relevant to the literature building on the standard neoclassical growth model. Another strand of the climate change literature uses large computable general equilibrium (CGE) models. Of particular relevance to the current paper are analyses using the EPPA (Morris et al., 2012) or Imaclim (Crassous et al., 2006) models, each of which has elements of putty-clay production.

[2] Capital good producers turn raw capital, 'putty,' into a capital good with certain technological characteristics, including energy efficiency. While energy efficiency can be improved by research and development, there is no substitution between energy and non-energy inputs once the capital good in constructed, capturing the rigid 'clay' properties of installed capital.

[3] The literature on putty-clay production functions has a long history (e.g., Johansen, 1959; Solow, 1962; Cass and Stiglitz, 1969; Calvo, 1976). Of particular relevance is work by Atkeson and Kehoe (1999) who investigate the role of putty-clay production in explaining the patterns of substitution between energy and non-energy inputs in production. The older literature on putty-clay models focuses on choosing a type of capital from an existing distribution. The current paper focuses on how the cutting-edge of technology, which is embodied in capital goods, evolves over time. As discussed in the next section, this modeling approach draws insight from Hassler et al. (2012, 2016b), who provide econometric evidence that a putty-clay model of directed technical change could fit patterns of substitution in U.S. energy use and investigate the implication of these forces for long-run economic growth in a social planner's model with finite energy resources and an aggregate production function.

short run, but firms will have an increased incentive to improve the energy efficiency of new capital goods, driving the expenditure share back down.

Rather than importing the seminal directed technical change model developed by Acemoglu (1998, 2002, 2003), I take a new approach in which innovation occurs in different characteristics of capital goods, not in different sectors. In other words, energy efficiency occurs when capital goods require less energy to run, not when the energy sector becomes more efficient at turning primary energy (e.g., coal) into final-use energy (e.g., electricity). This modeling choice is motivated by data from the United States, where reductions in the carbon intensity of output have been driven by decreases in final-use energy intensity. Incentives for research and development in the new putty-clay model of directed technical change differ from those in the seminal directed technical change model.

The new putty-clay model allows for a simple and transparent calibration procedure. It resembles the standard neoclassical approach in several important ways, implying that many parameters are standard and can be taken from the existing literature. I calibrate the innovation and energy sectors to aggregate U.S. data on economic growth and fossil fuel energy use.[4] I then use the model to perform three exercises. First, I examine the effect of environmental taxes on energy use and compare the results with the standard Cobb-Douglas approach. Second, I analyze whether it is possible for policies, such as R&D subsidies or efficiency mandates, to reduce long-run energy use without raising the price of energy. Finally, I asses how the presence of non-renewable resources impacts the potential for economic growth to be sustained in the very long run.

In the absence of climate policy, the new model and the standard Cobb-Douglas approach have identical predictions for long-run energy use. The putty-clay model of directed technical change, however, predicts significantly different reactions to climate policy. The new model suggests that tax-inclusive energy prices need to be 290% higher than laissez-faire levels in 2055 in order to achieve policy goals consistent with the Paris Agreement.[5] By contrast, the standard Cobb-Douglas approach suggests that tax-inclusive energy prices need only be 150% higher. Similarly, the new model predicts that the energy expenditure share must rise to 5% – compared with 3.3% today – while the Cobb-Douglas model suggests that rapid declines in energy will leave the expenditure share essentially unchanged. The new model also suggests that final good consumption must fall by 2.8% to meet policy goals, which is double the prediction from the standard approach. Thus, compared to the standard approach, the new model predicts that greater taxation and more forgone consumption are necessary to achieve environmental policy goals. When applying the same

[4] A single parameter, measuring congestion in the R&D sector, is not identified from aggregate data. I show that the results of the paper are robust to a wide range of values, including the limiting case of no congestion, which minimizes the difference with the standard Cobb-Douglas approach.

[5] In particular, I simulate taxes needed to reduce energy use to 60% of 2005 levels by the year 2055. This is consistent with goals laid out in the Paris Agreement, which suggests that the United States adopt policies consistent with a 80% reduction in carbon emissions by 2050. Thus, I examine a case where half of the required reduction in carbon emissions comes from reductions in energy use. The goals are outlined in the Intended Nationally Determined Contribution (INDC) submitted by the United States to the United Nations Framework Convention on Climate Change (UNFCC), which is available at: http://www4.unfccc.int/submissions/INDC/Published%20Documents/United%20States%20of%20America/1/U.S.%20Cover%20Note%20INDC%20and%20Accompanying%20Information.pdf.

taxes to both models, the new putty-clay model of directed technical change predicts 16% greater cumulative energy use over the next century. This indicates that policy designed with the Cobb-Douglas model will yield significantly different environmental results in a world better represented by the new putty-clay model.

Research subsidies and efficiency mandates are commonly used in attempts mitigate climate change and achieve energy security (Gillingham et al., 2009; Allcott and Greenstone, 2012). Despite their popularity, these policies may be ineffective due to rebound effects. Rebound occurs when economic behavior lessens the reduction in energy use following efficiency improvements. A long existing literature attempts to evaluate the effectiveness of such policies by estimating the size of rebound effects, usually in partial equilibrium or static settings (Gillingham, 2014; Gillingham et al., 2015). The new putty-clay model, however, makes it possible to directly analyze the broader motivating question: can policies that improve energy efficiency achieve long-term reductions in energy use, even if they do not increase energy prices? I find several key results. First, one-off improvements in energy efficiency lead to short-run reductions in energy use, but lead to absolute increases in future energy use relative to world without policy, an extreme form of rebound known as 'backfire.' Permanent policy interventions, however, can overcome rebound effects to achieve long-run reductions in energy use relative to laissez-fair, even without raising energy prices. To achieve absolute decreases in energy use, however, would require constantly increasing subsidies that tend towards 100% of expenditure on energy efficient research. Together, these results suggest that, while rebound effects are not a first-order concern at the aggregate level, policies that do not raise the price of energy will be unable to meet long-run goals of decreasing energy use.

I also examine the sustainability of economic growth in a world with non-renewable resources. Using two different versions of sustainability, I find results that are more optimistic than the existing literature. The first, and more standard, definition ignores the dangers of climate change and is concerned with the ability of an economy to maintain current levels of consumption growth. Focusing on models with exhaustible resources, the existing DTC literature suggests that this form of sustainability is impossible because energy use is currently increasing, which is not possible in the long run (e.g., André and Smulders, 2014; Hassler et al., 2012, 2016b). I consider the case where resources are inexhaustible, but only accessible at increasing and unbounded extraction costs,[6] a formulation that captures the abundance of coal and the potential to exploit 'unconventional' sources of oil and natural gas (Rogner, 1997; Rogner et al., 2012). In this setting, I find that energy use will necessarily increase in the long run (in the absence of policy intervention), implying that the presence of non-renewable resources alone does not pose a threat to this form of sustainability. The second definition of sustainability asks whether environmental policy can keep the stock of pollution low enough to prevent an 'environmental disaster.' The existing literature suggests that this form of sustainability is impossible when polluting and non-polluting factors of production are complements (Acemoglu et al., 2012). By considering the ability of energy efficient technologies

[6] Models with increasing extraction costs have a long history in economics (e.g., Heal, 1976; Solow and Wan, 1976; Pindyck, 1978).

to reduce the use of fossil fuels, I show that it is possible for environmental policy to prevent an environmental disaster, even in the case of perfect complementarity.

The rest of this paper is structured as follows. Section 2 discusses the related literature. Section 3 discusses the empirical motivation underlying the theory. The model is presented in Section 4 and the calibration in Section 5. Section 6 reports the results of the quantitative analyses, and Section 7 concludes.

2 Related Literature

As described above, this paper contributes to the literature on climate change economics that takes a Cobb-Douglas approach to energy modeling in IAMs. This paper is also closely related to a growing literature demonstrating that directed technical change (DTC) has important implications for environmental policy. These studies generally focus on clean versus dirty sources of energy, rather than energy efficiency. Acemoglu et al. (2012) demonstrate the role that DTC can play in preventing environmental disasters and emphasize the elasticity of substitution between clean and dirty production methods. The model in this paper bears more resemblance to an 'alternate' approach they mention where firms can invest in quality improvements or carbon abatement, where the latter only occurs in the presence of carbon taxes. Several other studies also investigate the case where policy interventions affect how technological change is directed between production and abatement activities.[7] Lemoine (2015) demonstrates how the transition between sources of energy is affected by both innovation and increasing extraction costs in a world where new innovations are complementary to energy sources, but different energy sources are close substitutes. Aghion et al. (2016) provide a static DTC model of clean and dirty innovation in the automotive industry that includes an intra-product decision about energy efficiency. I build on these earlier works by constructing a new model of directed technical change, focusing on energy efficiency, quantitatively investigating the macroeconomic effects of prominent environmental policies, and comparing the results to the standard approach taken in IAMs.[8] I also provide evidence that 'environmental disasters' can be averted even when polluting and non-polluting inputs are perfect complements, a result that is more optimistic than those in the existing DTC literature (Acemoglu et al., 2012).

Two recent papers extend the standard DTC model to quantitative investigation of macroeconomic policy (Acemoglu et al., 2016; Fried, forthcoming). Both focus on the issue of clean versus dirty energy sources, rather than energy efficiency. Methodologically, this paper is closer to the approach taken by Fried (forthcoming), who accounts for energy efficiency by calibrating growth in clean energy to overall de-carbonization of the economy, which includes energy efficiency as well as shifts towards the clean energy sector. In this way, the current paper builds on her work by

[7]See, for example, Hart (2008), Peretto (2008), Grimaud and Rouge (2008), and Gans (2012). Hart (2004) considers the decision to investment in abatement technology in a model where technology is embodied in different vintages of capital.

[8]A related and influential literature looks at induced, but not directed, technical change and its implications for climate policy. These models tend to focus on social planner problems. Key contributions in this literature include Goulder and Schneider (1999), Goulder and Mathai (2000), Sue Wing (2003), and Popp (2004).

explicitly investigating energy efficiency as a separate source of innovation that is complementary with other inputs, using a new underlying model of DTC, and comparing the results to the standard approach taken in climate change economics.[9]

This paper is also related to the literature on DTC and energy use, which focuses on the efficiency of the energy transformation sector, rather than the energy requirements of capital goods. The literature begins with Smulders and De Nooij (2003) who apply the original DTC model directly to energy efficiency and use it to analyze the effects of exogenous changes in energy availability. Subsequent literature has focused on the relationship between DTC and the sustainability of long-run economic growth in the presence of exhaustible resources (e.g., Di Maria and Valente, 2008; André and Smulders, 2014; Hassler et al., 2012, 2016b). I build on the existing literature by constructing a new model of DTC that can recreate key data patterns, using it to quantitatively evaluate prominent environmental policies, and comparing the results to the Cobb-Douglas approach. In addition, I show how the prospects for long-run sustainability improve when considering the more empirically relevant case where resources are inexhaustible, but only accessible at increasing and unbounded extraction costs.[10]

The new putty-clay model of directed technical change builds on the aggregate social planner's model of innovation and exhaustible resources developed by Hassler et al. (2012, 2016b). In order to investigate the role of energy efficiency in climate change mitigation policy, the current model differs from their work in two key aspects. First, I construct a decentralized model with incentives for innovation, which is necessary to quantify the effects of policy and to account for externalities. Rather than importing the seminal directed technical change model developed by Acemoglu (1998, 2002, 2003), I take a new approach in which innovation occurs in different characteristics of capital goods, not in different sectors. The new approach is motivated by data on U.S. energy use. Second, I consider the case of infinite potential supplies of energy and increasing extraction costs. The potentially infinite supply of energy incorporates the role of coal in fossil fuel energy use[11] and the possibility for new methods of resource extraction to become feasible as costs rise (Rogner, 1997; Rogner et al., 2012). Moreover, the model of DTC and increasing extraction costs predicts that, in the absence of policy, energy use will increase in the long run. This is consistent with data and a first-order concern for climate policy, but contrary to the predictions of models with exhaustible

[9]It is also important to note that the DTC literature is supported by microeconomic studies that investigate the presence of directed technical change. Newell et al. (1999) and Jaffe et al. (2003) demonstrate that the energy efficiency of energy intensive consumer durables (air conditioners and gas water heaters) responds to changes in prices and government regulations, providing evidence for the existence of directed technical change. Similarly, Popp (2002) finds that energy efficiency innovation, as measured by patents, responds to changes in energy prices. He looks at both innovations in the energy sector and in the energy efficiency characteristics of other capital goods. More recently, Dechezleprêtre et al. (2011) and Calel and Dechezlepretre (2016) find that patents for 'low carbon' technologies, which include more energy efficient and less carbon intensive innovations, respond to both energy prices and public policies designed specifically to address climate change. Aghion et al. (2016) find that government policies have a strong effect on energy efficient research in the automotive sector.

[10]Peretto and Valente (2015) focus on another form of sustainability, the growth of population in a world with a fixed amount of land. Their model includes two types of innovation, horizontal and vertical, but not innovation that is directed towards different factors of production.

[11]Coal is predicted to to be the primary driver of global carbon emissions and is available in abundant supply (van der Ploeg and Withagen, 2012; Golosov et al., 2014; Hassler et al., 2016a).

resources. More generally, the goal of climate change policy is to avoid using all available fossil fuels, implying that the optimal management of exhaustible resources in not a primary concern in this context (Covert et al., 2016).

This study is also related to the literature on the rebound effect, which measures how energy use responds to increases in energy efficiency. Gillingham (2014) and Gillingham et al. (2015) provide extensive reviews of the literature, which can be thought of in two parts: a microeconomic literature that estimates rebound effects for specific goods[12] and a macroeconomic literature that investigates static general equilibrium effects.[13] By studying this question in the context of a growth model, I incorporate several factors that are generally excluded from the literature. Most importantly, the putty-clay model incorporates the effects of changes in energy efficiency on subsequent innovation, a neglected issue that Gillingham et al. (2015) describe as a 'wild card' in our understanding of the long-run effects of energy efficiency policies. The model also takes into account the direct cost of enacting energy policies,[14] which occurs because researchers must be reallocated in order to generate efficiency improvements. Moreover, the existing macroeconomic rebound literature focuses heavily on the elasticity of substitution between energy and non-energy inputs in production (e.g., Sorrell et al., 2007; Borenstein et al., 2015; Lemoine, 2016). The putty-clay model of directed technical change allows the elasticity to vary over time, matching key features of U.S. data on energy use. The model also accounts for long-run changes in energy extraction costs and capital accumulation, neither of which has received much attention in the existing quantitative literature.

3 Empirical Motivation

In this section, I discuss a number of patterns in the data that motivate the theoretical choices made in this paper. In particular, I present evidence that a) declines in the final-use energy intensity of output drive reductions in the carbon intensity of output, b) there is a very low short-run elasticity of substitution between energy and non-energy inputs, and c) there is no long-run trend in the energy expenditure share of final output.

To analyze the determinants of the carbon intensity of output, I consider the following decomposition:

$$\frac{CO_2}{Y} = \frac{CO_2}{E_p} \cdot \frac{E_p}{E_f} \cdot \frac{E_f}{Y}, \tag{1}$$

where CO_2 is yearly carbon emissions, Y is gross domestic product, E_p is primary energy use (e.g., coal, oil), and E_f is final-use energy consumption (e.g., electricity, gasoline). The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, captures substitution between clean and dirty sources of energy (e.g., coal versus solar). The efficiency of the energy sector, which transforms primary energy into

[12]See, for example, Allcott (2011) and Jessoe and Rapson (2014), amongst others.

[13]See Lemoine (2016) for a recent theoretical treatment of rebound. For quantitative results from CGE models, see Turner (2009) and Barker et al. (2009), amongst others.

[14]Fowlie et al. (2015) discuss the costs of achieving energy efficiency in a microeconomic setting.

final-use energy, is captured by $\frac{E_p}{E_f}$. For example, the ratio decreases when power plants become more efficient at transforming coal into electricity. The final-use energy intensity of output, $\frac{E_f}{Y}$, measures the quantity of final-use energy used in production and consumption. For example, the ratio decreases when manufacturing firms use less electricity to produce the same quantity of goods.

The results of this decomposition are presented in Figure 1, which plots the carbon intensity of output and each component from equation (1) for the United States from 1971-2011. Data are normalized to 1971 values. Energy and carbon dioxide data are from the International Energy Agency (IEA).[15] Real GDP data are from the U.S. Bureau of Economic Analysis (BEA).[16] The carbon intensity of output fell over 60% during this time period, and this decline is matched almost exactly by the decline in the final-use energy intensity of output. Thus, the results demonstrate the primary importance of $\frac{E_f}{Y}$ in understanding how the economy will react to climate change mitigation policy. The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, declined approximately 10% over this period. While this is a significant improvement for environmental outcomes, it is relatively small compared to the overall improvements in the carbon intensity of output. Finally, the efficiency of the energy transformation sector, as measured by the inverse of $\frac{E_p}{E_f}$, actually declined roughly 15% over this period, indicating that it offset the environmental benefits achieved elsewhere.[17] This result rejects the notion that improvements in the carbon or energy intensity of output have been driven by technological improvements in the energy transformation sector.

Motivated by this evidence, I construct a model that focuses on the final-use energy intensity of output. This creates a significant break with existing work. Existing macroeconomic research on directed technical change and climate change focuses on clean versus dirty sources of energy and abstracts from energy efficiency (e.g., Acemoglu et al., 2012, 2016; Fried, forthcoming). Transition to cleaner energy sources will undoubtedly be an important component of any approach to mitigate climate change, but the historical data strongly suggest that improved energy efficiency will be a pivotal aspect of any policy response. At the same time, applying the seminal DTC model of Acemoglu (1998, 2002, 2003) to the question of energy efficiency would require focusing on the efficiency of the energy sector (e.g., Smulders and De Nooij, 2003; André and Smulders, 2014). Thus, I construct a new model where energy efficiency is driven by the energy requirements of capital goods. This theoretical innovation significantly alters the underlying incentives for research and development.[18]

[15] See 'IEA Headline Energy Data' at http://www.iea.org/statistics/topics/energybalances/.

[16] See Section 1 of the NIPA tables at: https://www.bea.gov//national/nipaweb/DownSS2.asp.

[17] This result is driven by differences in the efficiency of transformation across different sources of primary energy, rather than technological regress.

[18] Of course, not all improvements in energy efficiency need to driven by technical change. In particular, sectoral reallocation could explain aggregate changes in energy use. Decomposition exercises suggest that improvements in intra-sectoral efficiency, rather than reallocation, have been the key driver of falling energy intensity over this period (Sue Wing, 2008; Metcalf, 2008). They also suggest that, prior to 1970, sectoral reallocation with the primary driver of falling energy intensity. This paper will focus on the post-1970 period. Existing work suggests that there was a significant regime shift in both energy prices and energy efficiency improvements after this period (e.g., Hassler et al., 2012, 2016b; Fried, forthcoming; Baumeister and Kilian, 2016).

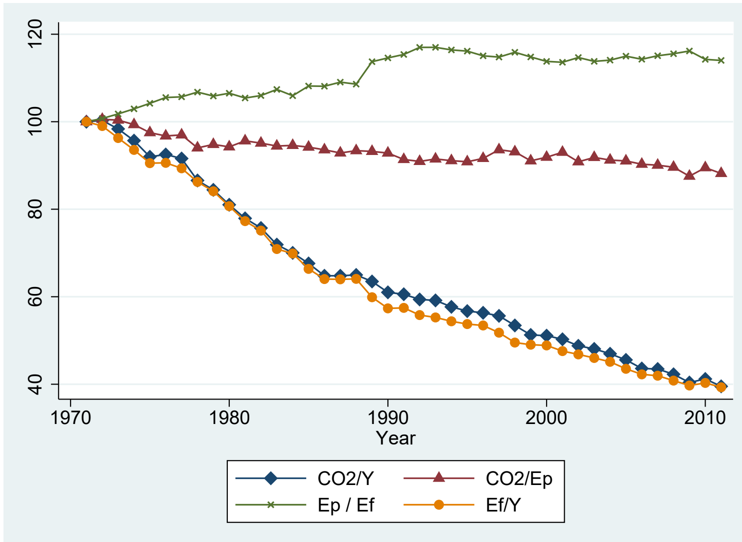


Figure 1: This figure decomposes the decline in the carbon intensity of output. CO_2 is yearly carbon emissions, Y is GDP, E_p is primary energy, and E_f is final-use energy. This figure demonstrates that the fall in the carbon intensity of output $\frac{CO_2}{Y}$ has been driven by decreases in final-use energy intensity of output $\frac{E_f}{Y}$, rather than the use of cleaner energy sources, $\frac{CO_2}{E_p}$, or a more efficient energy transformation sector, $\frac{E_p}{E_f}$. Data are from the International Energy Agency (IEA) and the Bureau of Economic Analysis (BEA). All values are normalized to 1971 levels.

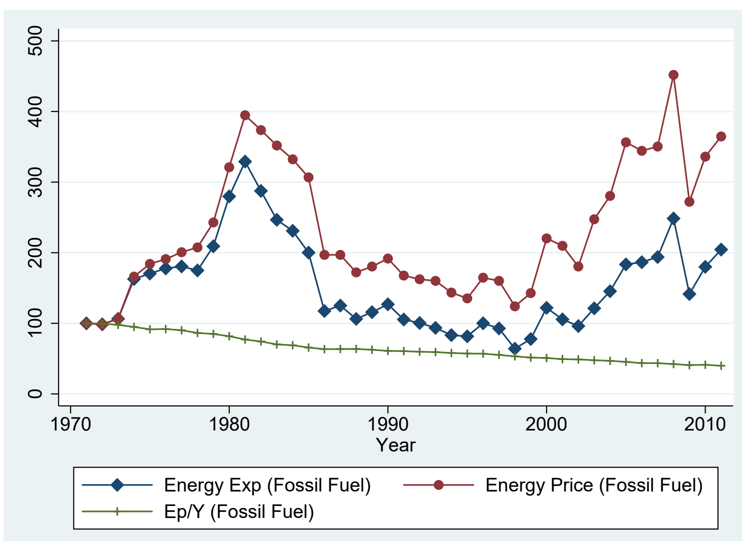


Figure 2: This figure demonstrates that short-run movements in energy prices affect short-run expenditures, but have very little affect on short-run energy use. At the same time, there is no trend in the energy expenditure share of output. Only fossil fuels are considered due to limitations in price data. Data are taken from the Bureau of Economic Analysis and the Energy Information Agency. All values are normalized to 1971 levels.

Figure 2 plots an index of real fossil fuel prices, the expenditure share of fossil fuel energy, and total fossil fuel energy use in the United States from 1971-2011. Energy use and price data are from the U.S. Energy Information Agency.[19] The sample is restricted to fossil fuels due to limitation on the price data, and a very similar graph serves as the motivation for Hassler et al. (2012). Output is again from the BEA. The data indicate that expenditure, but not total fossil fuel energy use, reacts to short-term price fluctuations, suggesting that there is very little short-run substitution between energy and non-energy inputs. At the same time, there is no trend in the energy expenditure share of output, suggesting a constant long-run level in the absence of fundamental changes in parameters or policy. The model in this paper will match both of these facts. Hassler et al. (2012, 2016b) provide a formal maximum likelihood estimate of the short-run elasticity of substitution between energy and non-energy inputs using this data. They find an elasticity of substitution very close to zero. For the purposes of this paper, I will treat the elasticity as exactly zero and use a Leontief production structure, which allows for the construction of a tractable putty-clay model. They also find that energy efficiency increases after prices rise, suggesting a DTC model of the type investigated here.[20]

The trendless expenditure share of energy in Figure 2 serves as the motivation for the Cobb-Douglas production function in IAMs (Golosov et al., 2014; Barrage, 2014). At the same time, the analysis by Hassler et al. (2012, 2016b) suggests that the long-run energy expenditure share – which will eventually be constant – must be significantly higher than the current level. The model developed in this paper will bridge the gap between these two approaches. It yields a constant energy expenditure share that matches the current level, while simultaneously replicating both short- and long-run patterns of substitution.

[19]See table 3.1 'Fossil fuel production prices, 1949-2011' and table 1.3 'Primary energy consumption estimates by source, 1949-2012' at http://www.eia.gov/totalenergy/data/annual/.

[20]As demonstrated in Figure 2, the price of energy in the United States had an upward trend from 1970-2011. Once again, this is a good match for post-1970 data, but not for U.S. data in the preceding two decades, where energy price actually declined. Consistent with the predictions of the model, decomposition exercises suggest that intra-sectoral energy efficiency declined during this period of falling prices (Sue Wing, 2008). In recent years, fossil fuel energy prices have declined substantially. The model suggests that this will slow the growth of energy efficiency. In this paper, I focus on the case where prices increase in the long run, though this is not central to any of the policy analysis. Increasing prices are consistent with theoretical work based on the Hotelling problem or increasing extraction costs (e.g., Hotelling, 1931; Heal, 1976; Pindyck, 1978), as well as empirical work suggesting a U-shaped pattern in long-run energy prices (e.g., Slade, 1982; Pindyck, 1999; Hamilton, 2012). The U.S. Energy Information Agency predicts the energy prices will increase across a wide range of sources and end-uses over the next several decades. See 'Table 3. Energy Prices by Sector and Source' at https://www.eia.gov/outlooks/aeo/. Given the general difficulty in predicting future energy prices, especially in the short to medium run, I focus on relative outcomes, where the comparison occurs between models or relative to a 'business as usual' case (Baumeister and Kilian, 2016).

4 Model

4.1 Structure

4.1.1 Final Good Production

The production structure of the model extends the standard endogenous growth production function to account for energy use. To match the extremely low short-run elasticity of substitution between energy and non-energy inputs, I will consider a Leontief structure

$$Q_t = \int_0^1 \min[A_{N,t}(i)X_t(i)^\alpha L_t^{1-\alpha}, A_{E,t}(i)E_t(i)]\, di, \tag{2}$$

$$s.t. \quad A_{E,t}(i)E_t(i) \leq A_{N,t}(i)X_t(i)^\alpha L_t^{1-\alpha}\ \forall i, \tag{3}$$

where Q_t is gross output at time t, L_t is the aggregate (and inelastic) labor supply, $A_{N,t}(i)$ is the the quality of capital good i, $X_t(i)$ is the quantity of capital good i, $A_{E,t}(i)$ is the energy efficiency of capital good i, and $E_t(i)$ is the amount of energy devoted to operating capital good i. Several components of the production function warrant further discussion. As in the standard endogenous growth production function, output is generated by a Cobb-Douglas combination of aggregate labor, L_t, and a series of production process, each of which uses a different capital good, indexed by i. Unlike the endogenous growth literature, each production process also requires energy to run. Thus, the usual capital-labor composite measures the potential output that can be created using each production process, and the actual level of output depends on the amount of energy devoted to each process, $E_t(i)$. The notion of potential output is captured by constraint (3). Each capital good i has two distinct technological characteristics. The quality of the capital good, $A_{N,t}(i)$, measures its ability to produce output, and the energy efficiency of the capital good, $A_{E,t}(i)$, lowers the amount of energy needed to operate the production process at full potential.[21]

4.1.2 Energy Sector

Energy is available in infinite supply, but is subject to to increasing extraction costs (see, e.g., Heal, 1976; Pindyck, 1978). Extraction costs are paid in final goods, and energy is provided by a perfectly competitive sector. The increasing extraction cost incorporates two main forces that govern long-run energy availability. First, it captures the increase in cost needed to extract conventional energy resources from harder-to-access areas.[22] Second, it captures the increase in cost that may occur when a particular energy source is exhausted, necessitating a switch to a type of energy which is

[21] Consistent with the econometric literature on energy use, energy requirements depend both on the amount of capital and the amount of labor being used in the production process (Van der Werf, 2008; Hassler et al., 2012, 2016b). Second, consistent with both the econometric and DTC literatures, improvements in non-energy technology, $A_N(i)$, raise energy requirements (e.g., Smulders and De Nooij, 2003; Van der Werf, 2008; Hassler et al., 2012, 2016b; Fried, forthcoming).

[22] For example, recent research suggests that most new oil production comes from the exploitation of new geographic areas, rather than improved technology applied to existing sources of energy (Hamilton, 2012).

more difficult to extract. In particular, the infinite supply of energy and increasing extraction costs capture the existence of 'unconventional' energy sources, which have high extraction costs, but are available in vast quantities (Rogner, 1997; Rogner et al., 2012).[23] As in Golosov et al. (2014), the treatment of energy sources as infinite in potential supply also incorporates the abundance of coal, which is predicted to be the major driver of climate change (van der Ploeg and Withagen, 2012; Hassler et al., 2016a).[24]

The marginal cost of extraction, which will also be equal to the price due to the perfect competition, is given by

$$p_{E,t} \;\; = \;\; \xi \bar{E}_{t-1}^{\iota}, \tag{4}$$

where $\bar{E}_{t-1}$ is total energy ever extracted at the start of the period. The law of motion for the stock of extracted energy is given by

$$\bar{E}_t = E_{t-1} + \bar{E}_{t-1}. \tag{5}$$

Intuitively, at the beginning of each period, energy producers search for new sources of energy to exploit, and the extraction cost of the new source is determined by total amount of energy ever extracted.[25,26]

[23] For example, Rogner et al. (2012) estimate a resource base of 4,900 – 13,700 exajoules (EJ) for conventional oil, compared with annual production of 416 EJ across all energy sources. Thus, constraints on availability of conventional oil sources may be binding. The ability to exhaust fossil fuel energy sources, however, appears much less likely when considering other options. The resource base for unconventional sources of oil is estimated to be an additional 3,750 – 20,400 EJ. Meanwhile, the resource base for coal and natural gas (conventional and unconventional) are 17,300– 435,000 EJ and 25,100 – 130,800 EJ, respectively. These estimates rely on projections regarding which resources will be profitable to extract from the environment. When considering the full range of energy sources that could become profitable to extract as resource prices tend towards infinity, the numbers grow even larger. In particular, such 'additional occurrences' are estimated to be larger than 1 million EJ for natural gas and 2.6 million EJ for uranium.

[24] Technically, Golosov et al. (2014) specify a finite amount of coal, but assume it is not fully depleted. Thus, it has no scarcity rent, although it does have an extraction cost. Oil, by contrast, is assumed to have no extraction cost, but does have a positive scarcity rent. Hart and Spiro (2011) survey the empirical literature and find little evidence that scarcity rents are a significant component of energy costs. They suggest that policy exercises focusing on scarcity rents will give misleading results.

[25] This is consistent, for example, with recent evidence from the oil industry, where drilling, but not within-well production, responds to changes in prices (Anderson et al., 2014).

[26] A primary goal of this paper is to compare the results of the putty-clay model to the standard Cobb-Douglas approach used in IAMs. Since IAMs examine worldwide outcomes, it is crucial to consider the equilibrium effect of policy on energy prices. Hence, the comparison between models is most accurate when considering endogenous prices. At the same time, I also use the model to investigate the affect of policies pursued in the United States. In this case, endogenous energy prices can be motivated in two ways. First, it is possible to think of the United States as a closed economy, which has obvious limitations considering the global nature of energy sector. Alternatively, one can imagine the policies being applied on a worldwide level with the United States making up a constant fraction of total energy. To ensure that the key qualitative results of the paper are not driven by this assumption, I also consider the opposite extreme of exogenous energy prices, which implicitly treats the United States as a small open economy taking unilateral policy actions. In this case, energy prices will increase at a constant exogenous rate.

4.1.3 Final Output

Final output is given by gross production less total energy extraction costs, which are equal to energy expenditures by the final good producer. As long as equation (3) holds with equality,[27] final output is given by

$$Y_t \quad = \quad L_t^{1-\alpha} \int_0^1 A_{N,t}(i)\big[1 - \frac{p_{E,t}}{A_{E,t}(i)}\big] X_t(i)^{\alpha} \; di. \tag{6}$$

This formulation further illuminates the continuity between the production function used here and the standard approach in endogenous growth models. Output has the classic Cobb-Douglas form with aggregate labor interacting with a continuum of perfectly substitutable types of capital. As in the endogenous growth literature, this structure maintains tractability in the putty-clay model, despite the Leontief nature of production.

Final output can either be consumed or saved for next period. In the empirical application, each period will be ten years. Following existing literature, I assume complete depreciation between periods (Golosov et al., 2014). Thus, market clearing in final goods implies

$$Y_t = C_t + K_{t+1} = L_t w_t + r_t K_t + \Pi_t + p_t^R + T_t, \tag{7}$$

where K_t is aggregate capital, Π_t is total profits, T_t is the net government budget, and p_t^R is total payments to R&D inputs (discussed in the next section). When considering environmental policy, I restrict attention to lump-sum taxes and transfers.

4.1.4 Capital Goods and Research

Each type of capital good is produced by a single profit-maximizing monopolist in each period. This monopolist also undertakes R&D activities to improve the characteristics of the machine, $A_{N,t}(i)$ and $A_{E,t}(i)$. The R&D production function is given by

$$A_{J,t}(i) \quad = \quad \big[1 + \eta_J R_{J,t}(i) R_{J,t}^{-\lambda}\big] A_{J,t-1}, \quad J \in \{N, E\}, \tag{8}$$

where $R_{J,t}(i)$ is R&D inputs assigned to characteristic J by firm i in period t, $R_{J,t} \equiv \int_0^1 R_{J,t}(i) di$, and $A_{J,t-1} \equiv \max\{A_{J,t-1}(i)\}$. In words, R&D builds on aggregate knowledge, $A_{J,t-1}$, and current period within-firm research allocations, $R_{J,t}(i)$, but is also subject to a congestion externality $R_{J,t}^{-\lambda}$ caused by duplicated research effort. When the period ends, patents expire and the best technology becomes available to all firms. Monopolists make decisions to maximize single period profits.[28]

[27] To ensure that equation (3) holds with equality, it is sufficient, but not necessary, to assume that capital fully depreciates after each period. If capital fully depreciates, then in equilibrium forward looking consumers will never 'over-invest' in capital and drive its return to zero. This assumption will be maintained in the empirical analysis, which uses a time period of ten years, and is also employed in Golosov et al. (2014).

[28] This can be motivated in several ways. Most directly, the identity of the firm producing capital good i could change after each period. Alternatively, it could be the case that firms are infinitely lives but myopic, which seems reasonable considering the ten year period length. The set-up presented here is isomorphic to one where firms are

There are a unit mass of R&D inputs, yielding[29]

$$R_{N,t} + R_{E,t} = 1 \ \forall t. \tag{9}$$

I assume that the investment price is fixed at unity. Thus, market clearing implies that

$$\int_0^1 X_t(i)di = K_t, \tag{10}$$

where K_t is aggregate capital.

4.1.5 Consumer Problem

The consumer side of the problem is standard. In particular, the representative household chooses a path of consumption such that

$$\{c_t\}_{t=0}^{\infty} = \text{argmax} \sum_{t=0}^{\infty} \beta^t L_t \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{11}$$

where $c_t = C_t/L_t$. Population growth is given exogenously by

$$L_{t+1} = (1+n)L_t. \tag{12}$$

I am interested in the decentralized equilibrium. Thus, I consider the case where the representative household takes prices and technology as given. In other words, the household's budget constraint is given by the second equality in (7).

4.2 Analysis

As demonstrated in Appendix Section A.1, the first order conditions for the final good producer yield the following inverse demand functions:

$$p_{X,t}(i) = \alpha A_{N,t}(i)\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big] L_t^{1-\alpha} X_t(i)^{\alpha-1}, \tag{13}$$

$$w_t = (1-\alpha) A_{N,t}(i)\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big] L_t^{-\alpha} X_t(i)^{\alpha}, \tag{14}$$

infinitely lived and the aggregate technology, $A_{J,t-1}$, is given by the average of the previous period's technology as in Fried (forthcoming). This would open up the possibility of technological regress, though it would not occur in equilibrium.

[29] This is consistent with both existing literature on DTC and the environment (Acemoglu et al., 2012; Fried, forthcoming) and the social planner model provided by (Hassler et al., 2012, 2016b). Often, models of directed technical change refer to the fixed set of research inputs as scientists (e.g., Acemoglu et al., 2012; Fried, forthcoming). This would be applicable here, though generating the standard Euler equation would require the representative household to ignore scientist welfare (in the environmental literature, directed technical change and capital are generally not included simultaneously). This would be a close approximation to a more inclusive utility function as long as scientists made up a small portion of the overall population. For simplicity, I refer to research inputs, which could be scientists, research labs, etc.

where $\tau_t \geq 1$ is a proportional tax on energy. The intuition for the result is straightforward. The final good producer demands capital goods until marginal revenue is equal to marginal cost. Unlike the usual endogenous growth model, marginal revenue is equal to marginal product minus the cost of energy needed to operate capital goods. Consider the case where the final good producer is already operating at a point where $A_{N,t}(i)L_t^{1-\alpha}X_t(i)^\alpha = A_{E,t}(i)E_t(i)$. If the final good producer purchases more capital, he receives no increase in output unless there is a corresponding increase in energy purchased. The final good producer realizes this when making optimal decisions and adjusts demand for capital accordingly. This iso-elastic form for inverse demand maintains the tractability of the model.

Monopolist providers of capital goods must decide on optimal production levels and optimal research allocations. See Appendix Section A.2 for a formal derivation of the monopolists' behavior. Given the iso-elastic inverse demand function, monopolists set price equal to a constant markup over unit costs. Since capital goods must be rented from consumers, the unit cost is given by the rental rate, r_t. Thus, monopolist optimization yields

$$p_{X,t}(i) = \frac{1}{\alpha} r_t, \tag{15}$$

$$X_t(i) = \alpha^{\frac{2}{1-\alpha}} r_t^{\frac{1}{\alpha-1}} A_{N,t}(i)^{\frac{1}{1-\alpha}} L_t \big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}, \tag{16}$$

$$\bar{\pi}_{X,t}(i) = (\frac{1}{\alpha} - 1)\alpha^{\frac{2}{1-\alpha}} r_t^{\frac{\alpha}{\alpha-1}} A_{N,t}(i)^{\frac{1}{1-\alpha}} L_t \big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}, \tag{17}$$

where $\bar{\pi}_{X,t}(i)$ is production profits (i.e., profits excluding research costs) of the monopolist.

To understand research dynamics, it is helpful to look at the relative prices for research inputs,

$$\frac{(1-\eta_t^S)p_{E,t}^R(i)}{p_{N,t}^R(i)} = \frac{\tau_t p_{E,t} A_{N,t}(i)}{A_{E,t}(i)^2\big[1 - (\frac{\tau_t p_{E,t}}{A_{E,t}(i)})\big]} \frac{\eta_E R_{E,t}^{-\lambda} A_{E,t-1}}{\eta_N R_{N,t}^{-\lambda} A_{N,t-1}}, \tag{18}$$

where $p_{J,t}^R(i)$ is the rent paid to research inputs used by firm i to improve technological characteristic J at time t and $\eta_t^S \in [0,1)$ is a subsidy for energy efficient research. There are several forces affecting the returns to R&D investment. First, increases in the tax-inclusive price of energy increase the relative return to investing in energy efficiency. Second, the return to investing in a particular type of R&D is increasing in the efficiency of research in that sector. Research efficiency, in turn, depends on inherent productivity, η_J, accumulated knowledge, $A_{J,t-1}$, and the amount of congestion, $R_{J,t}^{-\lambda}$. Since energy and non-energy inputs are complements in production, increases in $A_{N,t}(i)$ raise the return to investing in $A_{E,t}(i)$ and vice versa. These effects, however, are asymmetric. To maximize profits, monopolists balance two forces that drive demand for their products: 'output-increasing' technological progress, $A_{N,t}(i)$, and 'cost-saving' technological progress, $A_{E,t}(i)$. The asymmetry occurs because energy efficiency, $A_{E,t}(i)$, has a negative and convex effect on the cost of energy per unit of final output, $\frac{\tau_t p_{E,t}}{A_{E,t}(i)}$. Conversely, proportional increases in $A_{N,t}(i)$ lead to proportional increases in output.

In the usual DTC model, this analysis would demonstrate the role of market size and price effects in research incentives. As demonstrated in equation (18), however, aggregate inputs do not affect R&D decisions in this model. This is due to the short-run complementarity between energy and non-energy inputs. In other words, market size effects play no role in this model. Moreover, the price effects in this model differ from the standard approach. Since the price of the final good is the numeraire, $\frac{\tau_t p_{E,t}}{A_{E,t}(i)}$ is the cost of energy per unit of final good production, and $1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}$ is the cost of non-energy inputs in final good production. Thus, the relative input prices do affect research allocations, but the relative price is completely determined by the cost of energy extraction.

Given that all firms use common technology at the start of the period, they make identical R&D decisions and, as a result, they end the period with identical technology. Moreover, there is a unit mass of monopolists. Thus, $R_{J,t}(i) = R_{J,t} \ \forall i, J, t$. The optimal research allocations are given by the implicit solution to (19) and (20),

$$R_{E,t} = \frac{\sqrt{\frac{\tau_t p_{E,t}}{A_{E,t-1}}}\sqrt{\frac{1}{1-\eta_t^S}\left[\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N(1-R_{E,t})^{-\lambda}} + \eta_E R_{E,t}^{-\lambda} - \eta_E R_{E,t}^{1-\lambda}\right] + (1+\eta_E R_E^{1-\lambda})} - 1}{\eta_E R_{E,t}^{-\lambda}}, \tag{19}$$

$$R_{N,t} = 1 - R_{E,t}. \tag{20}$$

This formulation highlights the simple closed form solution in the special case where $\lambda = 0$ and $\eta_t^S = 0$. To analyze the determinants of research activity, it is instructive to consider multiplying both sides through by $\eta_E R_{E,t}^{-\lambda}$ so that the growth rate of energy efficiency technology is given as a function of the other parameters. Since $\eta_t^S \in [0, 1)$, the left-hand side is strictly increasing in $R_{E,t}$ in this formulation and the right-hand side is strictly decreasing in $R_{E,t}$. Thus, we can note a few important partial effects. First, the level of non-energy technology does not affect the research allocation. The perfect complementarity in final good productions drives this result. As expected, increases in the tax-inclusive price of energy lead to increases in the fraction of research inputs devoted to advancing energy efficient technology. More surprisingly, increases in past energy efficiency lead to decreases in the amount of research effort devoted to energy efficiency, even though the research productivity builds on past knowledge. As in the case of non-energy research, this improvement in research productivity is exactly balanced by the complementary nature of production. In the case of energy efficiency, however, the convex relationship between energy efficiency and the effective cost of energy, $\frac{\tau_t p_{E,t}}{A_{E,t}(i)}$, creates further disincentive to invest in energy research when energy efficiency is already high. As expected, the growth rate of energy efficiency is increasing in the size of the research subsidy, η_t^S.

Utility maximization yields

$$\left(\frac{c_t}{c_{t+1}}\right)^{-\sigma} = \frac{\beta r_{t+1}}{(1+n)}. \tag{21}$$

Noting that all monopolists make the same decisions and that there is a unit mass of monopolists, the real interest rate is given by

$$r_t \quad = \quad \alpha^2 A_{N,t}[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}]L_t^{1-\alpha} K_t^{\alpha-1}, \tag{22}$$

where the market clearing condition from equation (10) has been applied.

4.3 Equilibrium

Definition 1. *A* competitive equilibrium *is a sequence of prices,* $\{w_t, p_{X,t}, r_t, p_t^R, p_{E,t}\}_{t=0}^{\infty}$*, allocations,* $\{C_t, K_t, L_t, E_t, R_{N,t}, R_{E,t}\}_{t=0}^{\infty}$*, technology levels,* $\{A_{N,t}, A_{E,t}\}_{t=0}^{\infty}$*, and environmental policies,* $\{\tau_t, \eta_t^S\}_{t=0}^{\infty}$*, such that each of the following conditions holds* $\forall t$*:*

- *The economy obeys market clearing conditions for final goods, (7), and capital goods, (10).*
- *Optimal research allocations solve (19) and (20).*
- *The dynamics for technology follow (18), noting that all monopolists make identical decisions.*
- *Consumer behavior follows the Euler equation, (21).*
- *Factor prices are given by (4), (14), (15), and (22), noting that all monopolists make identical decisions and that the market for capital goods clears.*
- *The economy obeys laws of motion for total extracted energy, (5), and population, (12).*
- *Initial Conditions* $A_{J,-1}$ *for* $J \in [E, N]$*,* K_0*,* L_0*, and* $\bar{E}_{-1}$ *are given.*

4.4 Balanced Growth under Laissez-Faire

In this section, I examine long-run outcomes in the absence of environmental policy. To focus on empirically relevant cases, I maintain the following assumption for the remainder of the paper:

$$\eta_E > n, \tag{A.1}$$

which rules out extreme cases where all research activity is devoted to improving energy efficiency even in the absence of environmental policy. Section 5.2 shows that this assumption is satisfied by an order of magnitude in the data.

Definition 2. *A* laissez-faire equilibrium *is a* competitive equilibrium *without environmental policy. Formally,* $\tau_t = 1$ *and* $\eta_t^S = 0$ $\forall t$*.*

Definition 3. *A* balanced growth path *(BGP) occurs when final output, technology, and consumption grow at constant rates.*

On a balanced growth path (BGP), research allocations must remain fixed. Consider the laissez-faire case where there is no energy policy. From equations (19) and (20), it is immediate that $\frac{A_{E,t-1}}{p_{E,t}}$ is constant. Intuitively, this occurs because of the non-linear relationship between energy efficiency, $A_{E,t}$, and the cost of energy per unit of output, $\frac{p_{E,t}}{A_{E,t}}$. When energy prices increase, monopolists have greater incentive to invest in energy efficient technology, but this incentive dissipates as technology improves. As a result, both energy prices and energy efficient technology grow at the same constant rate, g_E^*, on the BGP.[30] Thus, the increasing price of energy is exactly offset by improvements in energy efficiency.

Definition 4. *The energy share of expenditure, denoted by θ_E, is the sum of resources paid to energy producers and energy taxes as a fraction of final output. Formally, $\theta_E \equiv \frac{\tau_t p_{E,t} E_t}{Y_t}$.*

Given that energy prices and energy efficient technology grow at the same rate on the BGP, it is straightforward to show that the energy share of expenditure is constant in a laissez-fair equilibrium. In particular,

$$\theta_{E,t} \quad = \quad \frac{p_{E,t}/A_{E,t}}{1 - p_{E,t}/A_{E,t}}, \tag{23}$$

which must be constant given that $\frac{p_{E,t}}{A_{E,t-1}}$ is fixed and the growth rate of energy efficient technology is constant. Thus, despite the Leontief nature of production, the model still delivers a constant long-run energy expenditure share. As demonstrated in Section 3, this is consistent with aggregate data on U.S. energy use. Importantly, the expenditure share is only constant on the BGP. The low short-run elasticity of substitution between energy and non-energy inputs implies that the expenditure share would increase one-for-one with an unexpected increase in the energy price, until research allocations had a chance to react to the change in prices. This creates a significant difference with the Cobb-Douglas model, where the energy expenditure share is constant even on the transition path following a price shock. The Cobb-Douglas model is discussed further in the Section 4.6.

The fact that energy efficient technology and the price of energy grow at the same rate yields the first of two key BGP relationships. In particular, noting the relationship between energy use and the price of energy, as given by (4) and (5), yields

$$(1 + g_M^*)^\iota = (1 + g_E^*), \tag{BGP-RD}$$

where g_M^* is the growth rate of energy use. This equation summarizes the conditions for a BGP on the research side of the economy.

I now move to considering the remainder of the economy, which for simplicity, I will refer to as the 'output-side'. Consider the growth rate of TFP in this model.

[30] For the price of energy to grow at a constant rate, energy use must also grow at a constant rate, which will occur on the BGP.

Definition 5. Total factor productivity *is defined as in the standard neoclassical growth model. Formally,* $TFP \equiv \frac{Y_t}{K_t^\alpha L_t^{1-\alpha}}$.

It is immediate that

$$TFP_t = A_{N,t}\big[1 - \frac{p_{E,t}}{A_{E,t}}\big]. \tag{24}$$

Since $\frac{p_{E,t}}{A_{E,t}}$ is constant on the BGP in the absence of policy, TFP grows at the rate of non-energy technology, g_N^*, which is also constant. Since the consumer problem is standard, the model now reduces to the neoclassical growth model with monopolistic competition, implying that the putty-clay model with directed technical change will have the usual BGP properties. In particular, both final and gross output will grow at rate $g_Y^* = (1+g_N^*)^{\frac{1}{1-\alpha}}(1+n) - 1$. Given equation (2), the growth rate of energy use is given by

$$1 + g_M^* = \frac{(1+g_N^*)^{\frac{1}{1-\alpha}}}{1+g_E^*}(1+n). \tag{BGP-QE}$$

Together, equations (BGP-RD) and (BGP-QE) determine the relative growth rates of technology on the unique BGP. Adding in market clearing for R&D inputs, (9), yields the optimal research allocations and applying the law of motion for technology, (8), gives the technology and energy use growth rates. The technology growth rates are then sufficient to characterize the output-side of the BGP, which behaves as in the standard model.

Remark. *In a* laissez-fair equilibrium*, energy use is strictly increasing on the* BGP*, i.e.,* $g_M^* > 0$.

Proof. The remark follows from equation (BGP-RD) and the proof to Proposition 2, which demonstrates that research allocations are interior on the BGP. □

Contrary to a world with only exhaustible energy sources, the current model predicts that energy use will be increasing in the long-run in the absence of environmental policy. Intuitively, this result holds because there is only incentive for energy efficient research when energy use (and, therefore, the price of energy) is increasing. But, in the absence of energy efficient research, energy use is necessarily increasing. Thus, there is no equilibrium with decreasing energy use. This has immediate implications for climate policy, which depends on limiting the use of fossil energy, and for the long-run sustainability of economic growth.

Definition 6. *An* environmental disaster *occurs when* $\bar{E}_t > \hat{E}$.

The concept of environmental disasters has gained attention in the recent literature on climate change and DTC (Acemoglu et al., 2012; Lemoine, 2015). Since the focus of this paper is fossil fuel energy sources, it is convenient to view an environmental disaster as being determined by total energy usage.

Proposition 1. *The* BGP *in a* laissez-faire equilibrium *always leads to an* environmental disaster*.*

Proof. The proof follows from the definition of an environmental disaster and the preceding remark. □

Section 6.1 discusses the concept of an environmental disaster in greater detail. Proposition 2 summarizes and extends the results from this section. In particular, it uses the relationship between equations (BGP-RD) and (BGP-QE) to explicitly characterize the balanced growth path.

Proposition 2. *In a* laissez-fair equilibrium*, there exists a* unique BGP *on which each of the following holds true:*

1. *The research allocations are implicitly given by:*
   $R_E^* = \left(\frac{(1+\eta_N(1-R_E^*)^{1-\lambda})^{\frac{1}{(1-\alpha)(1+1/\iota)}}(1+n)^{\frac{1}{1+1/\iota}}-1}{\eta_E} \right)^{\frac{1}{1-\lambda}}.$

2. *Technological growth rates are given by* $g_E^* = \eta_E(R_E^*)^{1-\lambda}$ *and* $g_N^* = \eta_N(1-R_E^*)^{1-\lambda}$*. The relationship between growth rates can be expressed as:*
   $(1+g_E^*)^{\frac{\iota+1}{\iota}} = (1+g_N^*)^{\frac{1}{1-\alpha}}(1+n).$

3. *Output per worker and consumption per worker grow at a constant rate,* $g_R^* = (1+g_N^*)^{\frac{1}{1-\alpha}}-1.$

4. *Total output and the capital stock grow at a constant rate,* $g_Y^* = (1+g_R^*)(1+n)-1$*, which implies that the capital-output ratio is fixed .*

5. *The real interest rate,* r_t*, is constant.*

6. *Energy use grows at rate* $g_M^* = \frac{1+g_R^*}{1+g_E^*}(1+n)-1 > 0.$

7. *The expenditure shares of energy, capital, labor, R&D inputs, and profits are all constant. In particular, the expenditure share of energy is implicitly given by:*
   $\frac{\theta_E^*}{1-\theta_E^*} = \frac{\left(1+\eta_E(R_E^*)^{1-\lambda}\right)^2}{\left(\frac{\eta_E(R_E^*)^{-\lambda}}{\eta_N(1-R_E^*)^{-\lambda}}+\eta_E(R_E^*)^{-\lambda}+1\right)}.$

Proof. The intuition is provided in the text, and a formal proof is provided in Appendix Section A.4. □

4.5 Balanced Growth with Environmental Policy

In this section, I consider long-run economic outcomes in the presence of environmental policy.[31]

Definition 7. *An* equilibrium with environmental policy *is a* competitive equilibrium *where* $\tau_t = \tau_0(1+g_\tau)^t,\ g_\tau, \tau_0 \geq 0$ *and* $\eta_t^S = \eta^S \geq 0\ \forall t.$

[31] In this definition, the laissez-faire equilibrium is a special case of an equilibrium with environmental policy. I restrict the formal analysis to the case of exponentially increasing taxes and a fixed research subsidy for analytic convenience. In particular, this restriction allows for the simple characterization of a balanced growth path, but does not drive any of the underlying intuition.

In a world with increasing energy taxes, equations (19) and (20) now imply that the growth rate of energy efficiency is equal to the product of growth in the energy price and the growth of the taxes. Thus, balanced growth on the research side of the economy requires

$$(1+g_M^*)^{\iota}(1+g_\tau) = (1+g_E^*), \tag{BGP-RD$'$}$$

which is equivalent to the laissez-faire condition if $g_\tau = 0$. This also implies that, on a BGP, $\lim_{t\to\infty} \frac{p_{E,t}}{A_{E,t}} = 0$. Thus, $\lim_{t\to\infty}[Q_t - Y_t] = 0$ and $\lim_{t\to\infty} \theta_{E,t} = \frac{\tau_t p_{E,t}}{A_{E,t}}$, which is constant. In the limit, the model again reduces to that of the standard neoclassical growth model with monopolistic competition. As a result, the BGP condition for the output side of the economy is unchanged:

$$1+g_M^* = \frac{(1+g_N^*)^{\frac{1}{1-\alpha}}}{1+g_E^*}(1+n). \tag{BGP-QE$'$}$$

The economy will not reach a BGP in finite time. Using the same steps as in Section 4.4, it is now possible to characterize the BGP. Noting the similarity between (BGP-RD′) and (BGP-QE′) on one hand and (BGP-RD) and (BGP-QE) on the other, it is immediate that the growth rate of technological progress is unaffected by the level of taxes or the research subsidy.

Remark. *In an* equilibrium with environmental policy*, changes in energy research subsidies and the level of energy taxes have no effect on the BGP growth rate of energy. Formally,* $\frac{dg_M^*}{d\tau_0} = \frac{dg_M^*}{d\eta^S} = 0$.

Proof. The intuition follows from the preceding discussion and, more formally, from Proposition 4. □

Noting that changes in the level of subsidies do not affect the long-run allocation of research inputs, examination of (19) indicates that research subsidies do affect the energy expenditure share and, therefore, the level of energy use. This creates another significant difference with the Cobb-Douglas model, where the energy expenditure share is virtually fixed in response to environmental policy.[32] This result is summarized in the following remark.

Remark. *In an* equilibrium with environmental policy*, increases in the research subsidy decrease the energy expenditure share on the BGP. Formally,* $\frac{d\theta_E^*}{d\eta^S} > 0$.

Proof. The remark follows from Proposition 4. The intuition is given in the preceding discussion. □

As demonstrated in equation (BGP-RD′), the existence of increasing energy taxes weakens the link between the cost of energy extraction, $p_{E,t}$, and energy efficient research. In particular, there can be incentives for energy efficient research even when the price of energy is decreasing, as long as the tax on energy is increasing quickly enough. Thus, it is possible to have an equilibrium with a declining energy price, which corresponds to decreasing energy use.

[32] Tax-inclusive energy expenditure is a constant share of gross output, but the rebate of taxes implies that the share in total output decreases slightly in response to an increase in taxes.

Remark. *In an* equilibrium with environmental policy*, energy use can be increasing or decreasing on the* BGP*. Formally,* $g_M^* \lesseqgtr 0$*. Moreover,* $\frac{dg_M^*}{dg_\tau} < 0$*.*

Proof. The remark follows from the proof to proposition 2. □

Proposition 3. *An* environmental disaster*can be averted in an* equilibrium with environmental policy *if and only if* $\bar{E}_{-1} + \frac{E_0}{1-\frac{1+n}{1+\eta_E}} < \hat{E}$*.*

Proof. As demonstrated in proposition 4, policy can push $R_{E,t}$ arbitrarily close to 1 by choosing a high enough tax rate. Thus, the minimum amount of energy that can by used is given by $\bar{E}_{-1} + \frac{E_0}{1-\frac{1+n}{1+\eta_E}}$. □

All of the results presented thus far are summarized and extended in Proposition 4. In particular, it uses the relationship between equations (BGP-RD′) and (BGP-QE′) to explicitly characterize the BGP in the presence of environmental policy.

Proposition 4. *In an* equilibrium with environmental policy*, there exists a* unique BGP *on which each of the following holds true:*

1. *The research allocations are implicitly given by:*
   $R_E^* = \left(\frac{(1+\eta_N(1-R_E^*)^{1-\lambda})^{\frac{1}{(1-\alpha)(1+1/\iota)}}[(1+n)(1+g_\tau)]^{\frac{1}{1+1/\iota}}-1}{\eta_E} \right)^{\frac{1}{1-\lambda}}$.

2. *Technological growth rates are given by* $g_E^* = \eta_E(R_E^*)^{1-\lambda}$ *and* $g_N^* = \eta_N(1-R_E^*)^{1-\lambda}$*. The relationship between growth rates can be expressed as:*
   $(1+g_E^*)^{\frac{\iota+1}{\iota}} = (1+g_N^*)^{\frac{1}{1-\alpha}}(1+n)(1+g_\tau)$.

3. *Output per worker and consumption per worker grow at a constant rate,* $g_R^* = (1+g_N^*)^{\frac{1}{1-\alpha}} - 1$.

4. *Total output and the capital stock grow at a constant rate,* $g_Y^* = (1+g_R^*)(1+n) - 1$*, which implies that the capital-output ratio is fixed .*

5. *The real interest rate,* r_t*, is constant.*

6. *Energy use grows at rate* $g_M^* = \frac{1+g_R^*}{1+g_E^*}(1+n) - 1 \lesseqgtr 0$*.*

7. *The expenditure shares of energy, capital, labor, R&D inputs, and profits are all constant. In particular, the expenditure share of energy is implicitly given by:*
   $\frac{\theta_E^*}{1-\theta_E^*} = \frac{\left(1+\eta_E(R_E^*)^{1-\lambda}\right)^2}{\sqrt{\frac{1}{1-\eta^S}\left[\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N(1-R_{E,t})^{-\lambda}} + \eta_E R_{E,t}^{-\lambda} - \eta_E R_{E,t}^{1-\lambda}\right] + (1+\eta_E R_E^{1-\lambda}) - 1}}$.

Proof. The intuition is provided in the text, and a formal proof is provided in Appendix Section A.4. □

4.6 Comparison to Cobb-Douglas

As mentioned in the introduction, the standard approach in climate change economics is to treat energy as a Cobb-Douglas component of the aggregate production function (Nordhaus and Boyer, 2003; Golosov et al., 2014). The standard Cobb-Douglas production function is given by:

$$Q_t^{CD} = A_t^{CD} K_t^{\alpha} E_t^{\nu} L_t^{1-\alpha-\nu},$$

where A_t^{CD} grows at an exogenous rate, g_{CD}. Since energy extraction costs $p_{E,t}$ units of the final good, final output is given by

$$Y_t^{CD} = (1 - \frac{\nu}{\tau}) A_t^{CD} K_t^{\alpha} E_t^{\nu} L_t^{1-\alpha-\nu}.$$

As a result, the energy expenditure share under Cobb-Douglas is given by

$$\theta_{E,t}^{CD} = \frac{\nu}{1 - \frac{\nu}{\tau_t}}.$$

In the absence of policy, therefore, the energy expenditure share is constant, matching the long-run elasticity of substitution between energy and non-energy inputs, but not the near-zero short-run elasticity of substitution. This has important implications for climate policy: in the Cobb-Douglas model a tax on energy use – no matter how large – generates declines in energy use that are sufficient to leave the expenditure share essentially unchanged.[33]

Since addressing climate change inherently involves long-run outcomes, it has been posited that the Cobb-Douglas approach may provide accurate predictions about the reaction of energy use to policy interventions over the relevant time frame, even it cannot match short-run responses (Golosov et al., 2014). The analytical results from Section 4.5, however, cast doubt on this assertion. The putty-clay model of directed technical change matches both the short- and long-run elasticities, suggesting that it will more accurately predict the effect of environmental taxes on energy use. This new model suggests that, in response to policy, energy use will not fall by enough to leave the expenditure share unchanged. In particular, the energy expenditure share will not be constant on the transition path and the balanced growth level of the energy expenditure share may increase permanently in response to policy. Thus, there is good reason to expect that the Cobb-Douglas approach overestimates the decline in energy use following an environmental policy intervention. Section 6.2 quantifies the difference in predictions between the models.[34]

[33] In response to new energy taxes, there is actually a slight *decrease* in the energy expenditure share, which is due purely to the tax rebate. This effect is quantitatively unimportant.

[34] In Appendix Section A.5, I explain the calibration procedure for Cobb-Douglas and describe the balanced growth path. I calibrate both models so that they have identical predictions for output and energy use in the absence of environmental taxes. Due to other differences between the models, especially the difference in market structure – monopolistic competition in the putty-clay model with directed technical change and perfect competition in the Cobb-Douglas model – predictions for interest rates and levels (though not growth rates) of consumption and capital differ between the models. Given that incentives for innovation are an important part of the difference between the two models, I maintain these differences in the quantitative analysis.

5 Calibration

5.1 External Parameters

The model is solved in 10 year periods. As discussed above, the consumer side of the problem is standard. Thus, I take several parameters from the existing literature. In particular, I follow Golosov et al. (2014) and set $\alpha = .35$, $\delta = 1$, $\sigma = 1$, and $\beta = .860$.[35] I assume that the economy starts without environmental policy. Thus, all taxes and subsidies can be thought of as relative to 'business as usual' case, which serves as the baseline.

I take trend growth rates and the average energy expenditure share from the data. As discussed above, I use data from 1971-2011. Energy use and energy price data are from the U.S. Energy Information Agency. Prices are only available for fossil fuel energy. Thus, I also only use fossil fuel energy in the analysis. As discussed above, this is a good fit the energy sector in the model, which is motivated by increasing extraction costs for non-nonrenewable resources. Gross Domestic Product data are from the BEA.

To calculate the BGP growth rates for technology and population, I use averages from the Penn World Tables version 8.0 (Feenstra et al., 2015). In the data, $n = 0.10$. Following the structure of the model, I calculate gross output, Q_t, as final output, Y_t, plus energy expenditure. I measure $A_{E,t} = E_t/Q_t$, yielding $g_E^* = 0.24$ on the BGP (2.2% annual growth). On the BGP, the growth rate of income per capita is given by $g_R^* = (1 + g_N^*)^{\frac{1}{1-\alpha}} - 1$. In the data, $g_R^* = 0.23$, which yields $g_N^* = 0.14$. The average energy expenditure share in the data is 3.3%, which I take to be the balanced growth level.

Below, I calibrate the R&D sector of the model to match key BGP moments. The BGP is uninformative about research congestion, λ, which measures the trade-off between advances in overall productivity and energy efficiency. As a base value, I take $\lambda = 0.21$ from Fried (forthcoming), who also captures the congestion of moving research inputs from energy-related research to general purpose research, making it a natural starting point for quantitative exercises presented here. I will also consider cases where $\lambda \in \{0, 0.105, 0.31\}$ for robustness.

5.2 R&D Calibration

The key R&D parameters remaining to be calibrated are the inherent efficiencies of each sector, η_N and η_E. To calibrate them, it is also necessary to solve for R_E^*. To start, I re-write the research arbitrage equation in terms of observables,

$$\frac{1 + g_E^*}{1 + g_N^*} = E_{share}^* \frac{\eta_E}{\eta_N} \left(\frac{R_E^*}{1 - R_E^*}\right)^{-\lambda}. \tag{25}$$

[35] I normalize $TFP_0 = E_0 = L_0 = 10$. This normalization simply sets the units of the analysis and has no effect on the quantitative results of the model. I also assume that the economy is on the BGP at time $t = 0$. Given the other parameters in the model, this yields $Y_0 = 87.16$, $K_0 = 6.75$, $p_{E,0} = 0.29$, $A_{E,0} = 9.00$, $A_{N,0} = 10.33$. These normalizations set the scale for energy sector parameters, ξ and $\bar{E}_1$.

This equation has a natural interpretation. Monopolists must trade-off the relevant benefits and costs of investing in the two types of technology. E_{share} is a summary measure of the incentive to invest in energy efficiency that fully captures the relative benefits of improving each type of technology. When the energy share of expenditure is higher, monopolists have greater incentive to invest in energy efficiency. The remaining terms capture the relative costs, i.e. relative research efficiencies, of investing in the two types of technology. The term $\frac{\eta_E}{\eta_N}$ captures the inherent productivities of the two sectors, while $\left(\frac{R_E^*}{1-R_E^*}\right)^{-\lambda}$ captures the differences in efficiencies due to the differing levels of congestion.[36]

In the data, $g_E^* > g_N^*$. Since the measured energy expenditure share is low (3.3%), the relative efficiency of energy efficiency research must be high. Moreover, a substantial fraction of this relative efficiency must come from the inherent productivities. This is true because total productivity growth in each type of technology is an increasing function of R&D inputs devoted to that sector. If the difference in marginal research efficiencies was due only to congestion, then the growth rate of energy efficiency technology, g_E^*, would have to be much smaller than the growth rate of output-increasing technology, g_N^*. Thus, the data strongly suggest that the inherent productivity of energy efficiency research is significantly higher than the efficiency of other types of research.[37]

To complete the R&D calibration, I add the following two equations,

$$g_E^* = \eta_E (R_E^*)^{1-\lambda}, \tag{26}$$

$$g_N^* = \eta_N (1 - R_E^*)^{1-\lambda}, \tag{27}$$

which ensure that levels of technological progress match their values in the data. As expected, η_E is significantly greater than η_N. The exact values for all of the parameters are provided in table 1.

5.3 Energy Sector Calibration

To calibrate the remaining parameters for the energy sector, I start by noting that, on the BGP, energy use grows at a constant rate, g_M^*. The most important parameter for the energy sector is ι, which captures the rate at which growth in energy use translates into growth in energy prices,

$$\iota = \frac{ln(1+g_E^*)}{ln(1+g_M^*)}. \tag{28}$$

In the model, energy taxes will lower energy use, which in turns lowers the price energy and the incentive for energy efficient research. The size of these effects depends directly on ι.

[36] Hassler et al. (2016b) identify a similar relationship between equilibrium growth rates and the expenditure share of energy when considering a social planner solution with a general CES production function and finite set of energy resources that can be extracted from the environmental without cost.

[37] From an environmental perspective, this seems like a very promising result – improvements in energy efficiency can occur with only small amounts of labor reallocation. Despite this optimistic result, the putty-clay model with directed technical change suggests that much less energy is saved in response to new taxes, when compared to the standard Cobb-Douglas approach.

Table 1: Parameters

Parameter	Value	Description	Source
α	.35	Capital share of income	Golosov *et al.* (2014)
δ	1	Depreciation	Golosov *et al.* (2014)
β	.860	Discount factor	Golosov *et al.* (2014)
σ	1	Inter-temporal substitution	Golosov *et al.* (2014)
n	0.10	Population growth	PWT
λ	0.21	Research congestion	Fried (2015)
η_E	2.66	Research efficiency	Calibrated
η_N	0.15	Research efficiency	Calibrated
ι	2.43	Energy cost growth	Calibrated
ξ	$3.47 \cdot 10^{-6}$	Energy cost scale	Calibrated
$\bar{E}_{-1}$	107	Initial extracted energy	Calibrated

Next, to ensure that the economy starts in a steady state, it must be the case the total extracted energy grows at a constant rate. Thus, I calculate the initial level of extracted energy as:

$$\bar{E}_{-1} \quad = \quad g_M^*/E_0, \tag{29}$$

where $\bar{E}_{-1}$ is the total energy used on the last period before the energy taxes are announced. As noted above, the specific level of $\bar{E}_{-1}$ is uninformative and simply reflects the scale chosen for E_0. Finally, ξ is a scale parameter calibrated to the starting price,

$$\xi \quad = \quad \frac{p_{E,0}}{\bar{E}_{-1}^{\iota}}. \tag{30}$$

5.4 Solving the Model

Conditional on the price of energy, the model can separated into three pieces: the R&D allocations, the standard consumer problem from the neoclassical growth model with monopolistic competition, and the energy sector. The fact that innovation occurs in different characteristics of capital goods, rather than in different sectors, facilitates the solution of the model. In particular, equations (19) and (20) demonstrate that, conditional on the price of energy, the R&D allocations and technology growth rates can be solved independently of the consumer problem. To find the equilibrium, then, I employ the following steps:[38]

1. Guess a vector of energy prices.

2. Solve for productivity paths and R&D allocations using equations (8), (19) and (20), noting that all monopolists make identical research decisions.

[38] In all quantitative applications, this procedure is sufficient to find a competitive equilibrium. I have not shown that such a procedure must converge to an equilibrium. In all cases, I use the BGP in the absence of energy taxes to generate the initial guess of energy prices.

3. Solve the neoclassical growth model conditional on the path of productivities using equations (A.30) - (A.36) in Appendix Section A.4.

4. Back out implied energy use and energy prices using equations (2), (4), and (5). This takes advantage of the fact that (3) holds with equality in all periods.

5. Check if the initial guess and resulting prices are the same. If they are, then consumers have made optimal decisions taking all future prices as given and the economy is in equilibrium.

6. If the economy is not in equilibrium, start from step 1 with a convex combination of initial guess and resulting prices.

6 Results

6.1 Long-Run Sustainability

Before using the calibrated model to investigate the impacts of policy, I briefly consider the implications for sustainable economic growth in the putty-clay model with directed technical change. I consider two different versions of sustainability and, in both cases, find results that are more optimistic than the existing literature. I first consider the more standard version of sustainability which asks whether consumption growth can continue at current level even as renewable resources are depleted. The existing DTC literature focuses on the case of exhaustible resources and suggests that it is not possible to maintain current consumption levels (André and Smulders, 2014; Hassler et al., 2012, 2016b). Energy use in the United States is currently increasing, which is not a long-run possibility when all resources are exhaustible. Thus, the models suggest that, in the long-run, energy use will eventually begin to decrease. Since energy and non-energy inputs are complements, this also implies that some research effort will be shifted towards energy efficiency, slowing growth in overall TFP and consumption.

In contrast, I consider the more empirically relevant case where the potential supply of energy is infinite, but can only be accessed at increasing and unbounded extraction costs. On the balanced growth path, energy efficiency fully offsets increases in the extraction cost, implying that there is no need for energy use to decrease in the long run. Indeed, the model suggests that energy use is necessarily growing in the long run. As a result, there is no reason to expect that consumption growth will decrease in the long-run, even as nonrenewable resources are depleted and the economy is forced to expend more resources for each unit of energy extracted.

The second notion of sustainability is more closely related to the question of climate change. In particular, it asks whether policy intervention can prevent an 'environmental disaster' (Acemoglu et al., 2012; Lemoine, 2015). In the context of climate change, an environmental disaster could be a very high degree of warming which causes significant hardship to human beings. Thus, we can think of energy as the polluting resource in this context. Existing work has focused on the substitution between clean and dirty sources of energy and found that disasters are inevitable when polluting and non-polluting are complements (Acemoglu et al., 2012). By contrast, Section 4.5 demonstrates that environmental disasters can be avoided even in the case of perfect complementarity. This difference occurs because the model accounts for the fact that energy-augmenting technology is also energy-saving. In other words, it distinguishes between the polluting resource and the augmenting technology, which contributes to output but does not itself pollute.[39] Acemoglu et al. (2012), by contrast, focus on the case where technological advances contribute to the production of the polluting good. Thus, their formulation captures technologies that aid in the extraction of fossil fuels, but not in the energy efficiency of capital goods.

6.2 Energy Taxes

In this section, I examine the effect of energy taxes in the putty-clay model of directed technical change and compare the results to those in the standard Cobb-Douglas model. The time period in the model is ten years. All future policies are announced in the initial period, which I take as 2005 to match the stated objectives of international climate agreements. All policies take effect in 2015. The gap between the announcement and implementation of the policy allows one round of endogenous and directed technical change to occur before comparing the outcomes across the two models. If the policy were unexpected, the final good producer in the Cobb-Douglas model could react, whereas there would be no adjustment in the putty-clay model with directed technical change due to the Leontief structure. Thus, this approach lessens the difference between the two models by not considering the very short run.

To best understand the quantitative impacts of the new model of energy use developed in this paper, it is necessary to consider a realistic path of future energy taxes. Under the Paris Agreement on climate change, the United States aims to adopt policies consistent with a 80% reduction in carbon emissions by the year 2050, when compared to 2005 levels. I apply taxes such that half of this gain, a 40% reduction, comes from lower energy use.[40] The evidence in Figure 1 suggests that energy efficiency has been responsible for well more than half of past decreases in the carbon intensity of output.

Recall that $\tau_t \geq 1$ is the energy tax, such that $\tau_t p_{E,t}$ becomes the tax-inclusive price of energy. As in Section 4.4, I consider a path of proportional energy taxes that grow at a constant rate,

$$\tau_t = 1 \cdot (1 + g_\tau)^{\frac{t-2005}{10}}. \tag{31}$$

To achieve the environmental goals given above, the putty-clay model with directed technical change requires $g_\tau = .38$, implying that heavy energy taxation is necessary. When taking into account the general equilibrium effect of energy use on extraction costs, this yields a tax-inclusive

[39] Acemoglu et al. (2012) note that an environmental disaster can be averted when pollution comes only from an exhaustible resource, and there is not enough of the resource to create a disaster. This is a statement about the potential for a disaster. As in the majority of their paper, I am concerned with whether policy can prevent a disaster than would occur under a laissez-faire approach.

[40] Since the model is solved in ten year periods, I choose taxes such that the 40% reduction occurs by 2055.

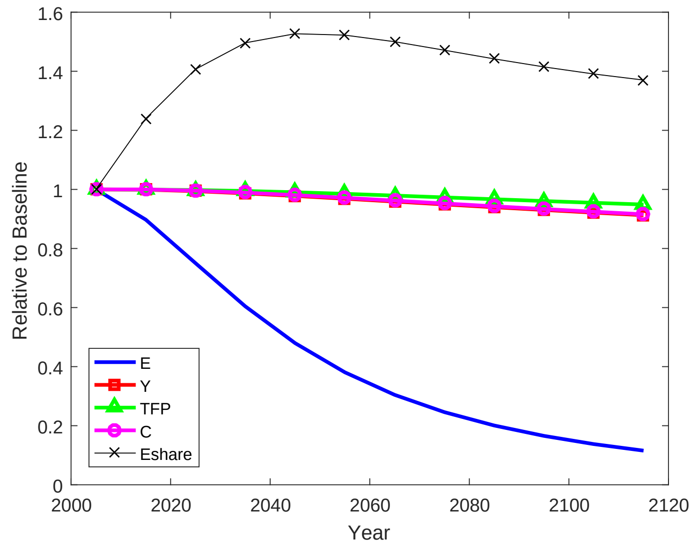


Figure 3: This figure demonstrates the effect of energy taxes in the putty-clay model with directed technical change. Energy taxes are proportional to the price of energy and grow at a constant rate: $\tau_t = 1 \cdot (1+g_\tau)^{\frac{t-2005}{10}}$, with $g_\tau = .38$. This level of taxation achieves a 40% reduction in energy in by 2055, compared to 2005 levels. All taxes are rebated to consumers in a lump sum fashion. All outcomes in the figure are given as a fraction of the outcomes in the baseline scenario, which has no energy taxation.

energy price that is 290% higher than laissez-faire rates in 2055. All taxes are rebated to consumers in a lump-sum fashion.

Figure 3 presents the outcomes of the model under the path of proportional energy taxes outlined above. In particular, it demonstrates the paths of energy use, output, TFP, consumption, and the energy expenditure share from 2005 to 2115. All outcomes are given as a fraction of the baseline scenario, which has zero energy taxation. As expected, energy taxes simultaneously increase the energy expenditure share and decrease energy use. In other words, capital good producers have increased incentive to invest in energy efficiency, but the resulting improvement in energy efficiency is insufficient to fully offset the increase in the price of energy. In this way, it is already apparent that the results will differ from those in the Cobb-Douglas model.

By 2055, the economy experiences a 2.8% decrease in consumption and 1.5% decrease in TFP relative to the baseline. Energy use plummets to 13.8% of baseline by 2115, one century after the policy is initially implemented. At the same time, consumption decreases by 7.5% and TFP is 5.5% lower than in the business as usual scenario. Discounted back 100 years, this lost consumption will have a very small impact of the current-day utility of the representative household. Within climate change economics, however, there is a spirited debate as to whether the discount rate held by individual consumers is appropriate for social welfare calculations (Nordhaus, 2007; Stern, 2013;

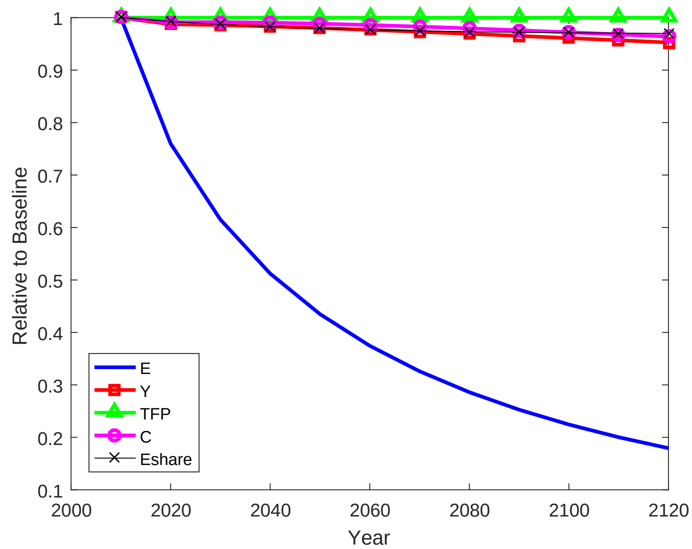


Figure 4: This figure demonstrates the effect of energy taxes in the standard Cobb-Douglas model with exogenous technological progress. Energy taxes are proportional to the price of energy and grow at a constant rate: $\tau_t = 1 \cdot (1+g_\tau)^{\frac{t-2005}{10}}$, with $g_\tau = 0.29$. This level of taxation achieves a 40% reduction in energy in by 2055, compared to 2005 levels. All taxes are rebated to consumers in a lump sum fashion. All outcomes in the figure are given as a fraction of the outcomes in the baseline scenario, which has zero energy taxation.

Barrage, 2016). Given that consumption losses are back-loaded, discount rate choices would have significant effects on welfare in this setting.[41]

Figure 4 repeats the analysis for the standard Cobb-Douglas model with exogenous technological progress. Once again, all outcomes are given relative to the business as usual case. The effect of policy in the Cobb-Douglas approach differs considerably from the putty-clay model with directed technical change. In this case, $g_\tau = 0.29$ is sufficient to achieve a 40% reduction in energy use by 2055. This leads to a tax-inclusive energy price that is 150% greater than baseline. To achieve the environmental policy priories, consumption decreases by 1.4% in 2055 and 3.2% by 2115, relative to a business as usual case without taxes. By 2115, energy use is 20% of baseline levels.

As expected, the energy share of expenditure is essentially unchanged in the Cobb-Douglas model.[42] Thus, energy use decreases by enough to fully offset the increase in energy prices. This can be seen in how quickly the Cobb-Douglas model responds to new taxes. In 2015, energy use decreases by almost 25% compared to the baseline, in comparison to a 10% decrease in the putty-clay model with directed technical change. This occurs even though the tax rate is lower in the Cobb-Douglas model.

[41] Adding climate damages, which are also back-loaded, would complicate the relationship between welfare and the discount rate.

[42] The slight decrease in the energy expenditure share is due to the lump sum tax rebates. The expenditure share of energy in gross output is constant, but after taxes are implemented, a proportion of energy expenditure is rebated to consumers.

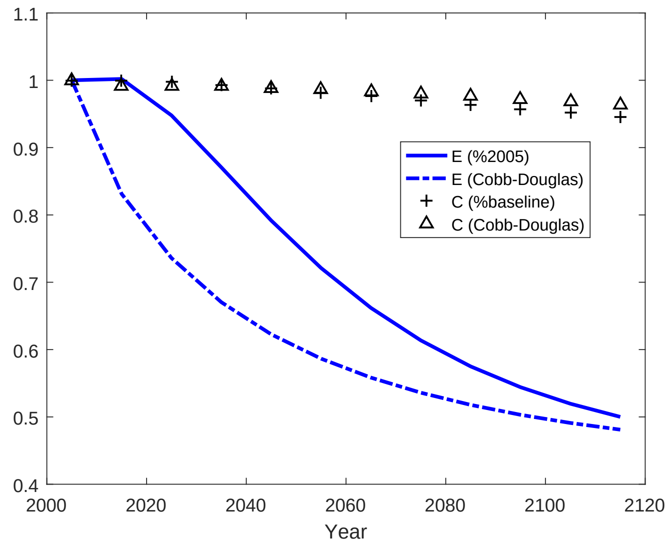


Figure 5: This figure demonstrates the difference between the putty-clay model of directed technical change and the standard Cobb-Douglas model with exogenous technological progress. Energy taxes are proportional to the price of energy and grow at a constant rate: $\tau_t = 1 \cdot (1+g_\tau)^{\frac{t-2015}{10}}$, with $g_\tau = 0.29$. In the Cobb-Douglas model with exogenous technical change, this level of taxation achieves a 40% reduction in energy use by 2055, compared to 2005 levels. All taxes are rebated to consumers in a lump sum fashion. Energy use is measured as a fraction of 2005 levels. Consumption is measured relative to the baseline, which does not include energy taxes. The baseline level of consumption differs in the two models.

Figure 5 provides a direct comparison of energy use and consumption in the two models when applying the same path of energy taxes, specifically those necessary to achieve environmental policy priorities in the Cobb-Douglas model. Thus, the analysis quantifies the error that would occur if policy was designed with the Cobb-Douglas model, but the true economy was putty-clay with directed technical change. Energy use is measured as a fraction of the 2005 level, and consumption is measured relative to the baseline.[43]

When applying the requisite taxes from the Cobb-Douglas model to the putty-clay model with directed technical change, energy use in 2055 declines by 26% when compared to 2005 levels, missing the environmental target by 14 percentage points. Forgone consumption is roughly the same as in the Cobb-Douglas model. Despite the stated goals of policy, what matters for overall environmental conditions is cumulative in energy use. The difference in cumulative energy use between the two models is given by the area between the two energy use curves. Over the course of the coming century, cumulative energy use is 16% higher in the putty-clay model with directed technical change. These results further illuminate the important differences between the two models and demonstrate that policy designed for the Cobb-Douglas model would yield drastically different outcomes in a world more closely resembling the putty-clay model with directed technical change.

[43]Given the difference in market structure, the baseline level of consumption, but not the growth rate of consumption, differs in the two models.

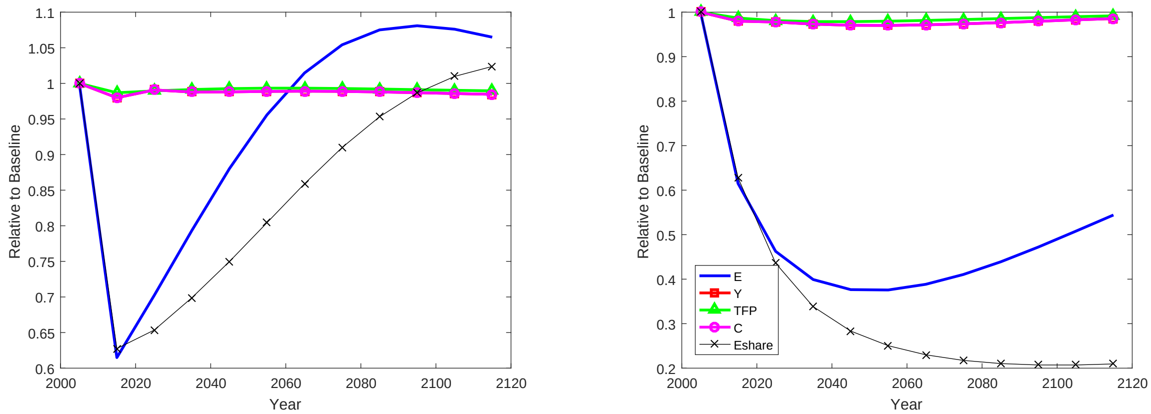
Figure A.1 in Appendix Section A.7 presents the results from several robustness exercises. As discussed in Section 5.2, the research congestion parameter, λ, is not identified on the BGP. So, I consider several alternate values. Most importantly, in panel (a), I consider the limiting case without congestion, i.e., $\lambda = 0$. This minimizes the difference between the two models by making research reallocation as effective as possible. The quantitative results still differ substantially between the two models. In particular, cumulative energy use with the putty-clay model is 10% greater by 2055 and 8% greater by 2115. Applying the taxes from the Cobb-Douglas model to the putty-clay model causes the economy to miss the policy goal by 4 percentage points. Panel (b) considers the case of $\lambda = .105$, which splits the difference between the baseline and most conservative estimates. Cumulative energy use is 11% greater by 2115 with the putty-clay model, and applying the Cobb-Douglas tax rates causes the model to miss the policy target by 8.6 percentage points in 2055. Naturally, the differences are magnified with considering greater values of λ. In particular, cumulative energy use is 20% higher by 2115 and the policy target is missed by 20 percentage points in the putty-clay model when $\lambda = 0.31$.

The model was calibrated to the United States. As noted in Section 4.1.2, the fact that energy prices are fully endogenous can be motivated in two ways. First, we can think of the U.S. as a closed economy. Second, we can think of policy being applied to the whole world, with the US making up a constant fraction of total energy use. To ensure that the assumption of fully endogenous energy prices is not driving the results, I consider the case where the price of energy is exogenous and equal to the baseline rate. This captures the scenario where the U.S. is a small open economy taking unilateral action to lessen energy use. The results are presented in panel (d). In this case, cumulative energy use is 24% higher in the putty-clay model by 2115 and the policy target is missed by almost 25 percentage points. Thus, taking energy prices as fully endogenous is a conservative approach that lessens the difference between the putty-clay and Cobb-Douglas models.

6.3 Research Subsidies

In this section, I use the putty-clay model of directed technical change to analyze the long-run impacts of R&D policies. Many policy makers are in favor of approaches, such as research subsidies or energy efficiency mandates, that try to achieve reductions in energy use without increasing prices (Gillingham et al., 2009; Allcott and Greenstone, 2012).[44] A large academic literature, however, suggests that rebound effects will undermine the effectiveness of such policies (Gillingham, 2014; Gillingham et al., 2015). Rebound occurs when the income and substitution effects following improvements in energy efficiency lead to increases in energy use, at least partially undoing the initial reduction. Existing work attempts to gauge the effectiveness of such policies indirectly by measuring the degree of rebound. Using the putty-clay model of directed technical change, however,

[44] In the putty-clay model of directed technical change, all innovation occurs in different characteristics of capital goods. Thus, research subsidies and efficiency mandates are equivalent. In particular, for any given subsidy, their is an equivalent energy efficiency mandate that yields the same research allocation.



(a) Single Period Subsidy

(b) Permanent Subsidy

Figure 6: The effects of research subsidies on energy use. *Panel A* demonstrates the effects of a single period research subsidy of 75%. *Panel B* demonstrates the effects of a permanent subsidy of 75%. This policy achieves a 40% reduction in energy use by 2055, compared to 2005 levels.

I can address the broader motivating question and directly analyze the impact of such policies on long-run energy use.

Figure 6 presents the results. Panel (a) considers a single period research subsidy of 75% in 2015. This is analogous to the setting in most of the existing literature, which examines the effect of one-off efficiency improvements on energy use. In the short-run, energy use decreases considerably, which is unsurprising considering the low short-run elasticity of substitution between energy and non-energy inputs. Over time, however, energy use catches back up with the baseline, and by 2060, energy use is actually higher than in the business as usual case. This extreme result is known as 'backfire' in the existing literature. In this case, backfire is driven by two forces. First, improvements in energy efficiency increase the relative return to future investments in energy-using technology. This is the force that drives the difference between the short- and long-run elasticities of substitution. The effect of the elasticity of substitution on the degree of rebound and potential for backfire is the central element of the existing macroeconomic literature (e.g., Saunders, 1992; Sorrell et al., 2007; Lemoine, 2016). The second major reason for backfire is the decline in the energy price, resulting from decreases in energy use. This important general equilibrium channel has received very little attention in the existing literature.[45] It is apparent from panel (a) that the economy has not yet returned to the BGP even 100 years after the policy intervention. In the very long run, cumulative energy use is essentially identical with the baseline case.

While the existing literature generally focuses on one-off shocks in order to estimate the degree of rebound, there is no particular reason why attempts to reduce long-run energy use would be constrained to single interventions. In this sense, the usual focus on rebound could give misleading implications about the effectiveness of these policies. Thus, in panel (b), I consider the effects of a

[45] In an important exception, Lemoine (2016) considers the theoretical implication of changes in resource prices in a static setting.

permanent subsidy of 75% to energy efficiency research. This subsidy is sufficient to achieve the 40% reduction in energy use discussed in the previous section. Despite the fact that the model predicts backfire in the case of a single period research subsidies, permanent interventions can meet goals consistent with the Paris Agreement and permanently reduce energy use relative to a business as usual scenario.[46] At the same time, however, Section 4.5 demonstrates that even permanent R&D subsidies can not lead to absolute decreases in energy use in the long-run. This would only be possible if the subsidy was constantly increasing towards 100% of energy efficiency R&D expenditure. Given the need to decrease total carbon emissions in order to avoid dangerous warming levels, it appears that increases in energy prices will be a necessary component of mitigation policies.

7 Conclusion

In this paper, I build a tractable putty-clay model of directed technical change and use it to analyze the effect of environmental policies on energy use in the United States. The model matches several key data patterns that cannot be explained by the standard Cobb-Douglas approach used in climate change economics. I use the model to perform three separate exercises. First, I analyze the effect of environmental taxes on energy use. Compared with the standard approach, the new model suggests that higher taxes and more forgone consumption are necessary to achieve policy goals consistent with the Paris Agreement. Second, I analyze the ability of policies, such as R&D subsidies or efficiency mandates, to lower energy use without raising prices and find that such policies are insufficient to generate absolute declines in long-run energy use. Finally, I examine the possibility for sustained economic growth in a world with non-renewable resources and find results that are more optimistic than the existing literature.

There are several possible extensions to the analyses presented here that would provide important insights into environmental policy questions. The most direct extension would entail adding a third margin of technological investment in clean versus dirty technology. In this case, it would be possible to gain a more complete understanding of the effect of carbon taxes on emissions. Combined with a model of the carbon cycle, such an analysis could yield important updates to existing estimates of optimal carbon taxes. It would also allow for the comparison of second-best policies. For example, it would be interesting to compare subsidies for renewable energy, which would limit the incentive to improve energy efficiency, and energy taxes, which provide no incentive to invest in clean energy sources.

Another extension would be to expand the geographic scope. The analyses presented here focus on a single economy, but there are important implications for a multi-region world. In particular, existing work with exogenous technological progress suggest that unilateral policy actions among rich countries will have small impacts on overall carbon emissions (Nordhaus, 2010). In a world with endogenous technological progress and diffusion or trade, however, unilateral policies would improve worldwide energy efficiency, leading to greater environmental benefit (Di Maria and Van der Werf,

[46] In this case, the relative energy use is 66% of baseline on the BGP.

2008; Hémous, 2016). This magnifies the difference with the standard Cobb-Douglas approach, where substitution of capital for energy in one country would have no direct impact on other countries. The positive implications of these international spillovers could potentially outweigh the more pessimistic conclusions about the reaction of energy use to taxation that result from considering the putty-clay model with directed technical change.

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A Appendix

A.1 Final Good Producer Problem

In this section, I show derive the inverse demand functions (13) and (14). Consider the maximization of (2) subject to (3) with $\upsilon_t(i)$ as the Lagrange multiplier attached to capital good i,

$$\mathcal{L} = \int_0^1 A_{E,t}(i)E_t(i)di - w_t L_t - \int_0^1 \; p_{X,t}(i)X_t(i)\; di - \tau_t p_{E,t}\int_0^1 \; E_t(i)\; di \\ - \int_0^1 \upsilon_t(i)\big[A_{E,t}(i)E_t(i) - A_{N,t}(i)X_t(i)^{\alpha}L_t^{1-\alpha}\big]di. \quad \text{(A.1)}$$

Complementary slackness implies

$$\upsilon_t(i)\big[A_{E,t}(i)E_t(i) - A_{N,t}(i)X_t(i)^{\alpha}L_t^{1-\alpha}\big] = 0 \;\forall i. \quad \text{(A.2)}$$

I focus on the case where the constraint is always binding. This will necessarily be true in the empirical exercise because $\delta = 1$ is a sufficient, but not necessary, condition for this to hold true. The first order conditions are given by

$$\begin{aligned}
\Big(\frac{\partial\mathcal{L}}{\partial E_t(i)}\Big): \quad \upsilon_t(i) &= 1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}, && \text{(A.3)}\\
\Big(\frac{\partial\mathcal{L}}{\partial X_t(i)}\Big): \quad \upsilon_t(i) &= \frac{p_{X,t}(i)}{\alpha A_{N,t}(i)L_t^{1-\alpha}X_t(i)^{\alpha-1}}, && \text{(A.4)}\\
\Big(\frac{\partial\mathcal{L}}{\partial L_t}\Big): \quad \upsilon_t(i) &= \frac{w_t}{(1-\alpha)A_{N,t}(i)L_t^{-\alpha}X_t(i)^{\alpha}}. && \text{(A.5)}
\end{aligned}$$

Substituting (A.4) and (A.5) into (A.3), respectively, and multiplying through yields

$$\begin{aligned}
p_{X,t}(i) &= \alpha A_{N,t}(i)\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]L_t^{1-\alpha}X_t(i)^{\alpha-1}, && \text{(A.6)}\\
w_t &= (1-\alpha)A_{N,t}(i)\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]L_t^{-\alpha}X_t(i)^{\alpha}. && \text{(A.7)}
\end{aligned}$$

Thus, we have arrived at equations (13) and (14) from the text. They key result here is that inverse demand is iso-elastic, which allows for the usual simple closed forms.

A.2 Monopolist Problem

The monopolist maximizes profits subject to demand and research productivity constraints:

$$\begin{aligned}
max\; \pi_{X,t}(i) &= p_{X,t}(i)X_t(i) - r_t X(i) - (1-\eta_t^S)p_{E,t}^R R_E(i) - p_{N,t}^R R_N(i), && \text{(A.8)}\\
& && \text{(A.9)}
\end{aligned}$$

subject to

$$
\begin{aligned}
p_{X,t}(i) &= \alpha A_{N,t}(i)\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]L_t^{1-\alpha}X_t(i)^{\alpha-1}, && (A.10)\\
A_{J,t}(i) &= \big[1+\eta_J R_{J,t}(i)R_{J,t}^{-\lambda}\big]A_{J,t-1}, \quad J\in\{N,E\}, && (A.11)\\
R_{J,t}(i) &\in [0,1], \quad J\in\{N,E\}. && (A.12)
\end{aligned}
$$

In equilibrium, the research allocation must be interior due to the congestion effects. Thus, I ignore the last constraint. First, substitute (A.10) into (A.8) and take the first order condition with respect to $X(i)$. Constraint (A.11) is independent of the production level, $X_t(i)$. Hence, the model yields the standard first order conditions and results,

$$
\begin{aligned}
r_t &= \alpha^2 A_{N,t}(i)\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]L_t^{1-\alpha}X_t(i)^{\alpha-1}, && (A.13)\\
X_t(i) &= \alpha^{\frac{2}{1-\alpha}}r^{\frac{-1}{1-\alpha}}A_{N,t}(i)^{\frac{1}{1-\alpha}}L_t\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}, && (A.14)\\
p_{X,t}(i) &= \frac{1}{\alpha}r_t. && (A.15)
\end{aligned}
$$

Next, to find optimal profits, we can re-write the monopolist problem after substituting in results we have found so far:

$$
max\ \pi_{X,t}(i) = \tilde{\alpha}r_t^{\frac{-\alpha}{1-\alpha}}A_{N,t}(i)^{\frac{1}{1-\alpha}}L_t\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}} - (1-\eta_t^S)p_{E,t}^R R_{E,t}(i) - p_{N,t}^R R_{N,t}(i) \quad (A.16)
$$

subject to

$$
\begin{aligned}
A_{J,t}(i) &= \big[1+\eta_J R_{J,t}(i)R_{J,t}^{-\lambda}\big]A_{J,t-1}, \quad J\in\{N,E\}, && (A.17)\\
R_{J,t}(i) &\in [0,1], \quad J\in\{N,E\}, && (A.18)
\end{aligned}
$$

where $\tilde{\alpha} = (\frac{1}{\alpha}-1)\alpha^{\frac{2}{1-\alpha}}$. Let κ_J be the lagrange multiplier for constraint (A.17). The first order conditions for technology levels and research scientist allocations yield

$$
\begin{aligned}
p_{N,t}^R &= \kappa_N A_{N,t-1}R_{N,t}^{-\lambda}, && (A.19)\\
(1-\eta_t^S)p_{E,t}^R &= \kappa_E A_{E,t-1}R_{E,t}^{-\lambda}, && (A.20)\\
\kappa_N &= \frac{1}{1-\alpha}\tilde{\alpha}r_t^{\frac{-\alpha}{1-\alpha}}L_t^{1-\alpha}A_{N,t}(i)^{\frac{1}{1-\alpha}-1}L_t\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}, && (A.21)\\
\kappa_E &= \frac{1}{1-\alpha}\tilde{\alpha}r_t^{\frac{-\alpha}{1-\alpha}}L_t^{1-\alpha}A_{N,t}(i)^{\frac{1}{1-\alpha}}L_t\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}-1}\tau_t p_{E,t}A_{E,t}^{-2}. && (A.22)
\end{aligned}
$$

Putting these together, we have

$$
\begin{aligned}
p_{N,t}^R &= \psi A_{N,t}^{\frac{\alpha}{1-\alpha}}\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}\eta_N R_{N,t}^{-\lambda}A_{N,t-1}, && (A.23)\\
(1-\eta_t^S)p_{E,t}^R &= \psi A_{N,t}^{\frac{1}{1-\alpha}}\tau_t p_{E,t}A_{E,t}(i)^{-2}\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{\alpha}{1-\alpha}}\eta_E R_{E,t}^{-\lambda}A_{E,t-1}, && (A.24)
\end{aligned}
$$

where $\psi = \frac{\tilde{\alpha}}{1-\alpha} r_t^{\frac{-\alpha}{1-\alpha}} L_t$ is common to both terms. In the next section, I shown the optimal research allocations resulting from these first order conditions. Taking ratios of these first order conditions yields (18) in the main text.

A.3 R&D Allocations

In this section, I derive the optimal research allocations given in equations (19) and (20). First, note that $R_{J,t}(i) = R_{J,t}\ \forall i, t, J$. This occurs because all monpolists make identical decisions and there is a unit mass of monopolists. This also implies that $A_{J,t}(i) = A_{J,t}\ \forall i, t, J$. Also, factor mobility ensures that $p^R_{E,t} = p^R_{N,t}\ \forall t$. Thus, equation (18) can then be re-written as

$$(1-\eta_t^S)\frac{A_{E,t}}{A_{E,t-1}}\big[\frac{A_{E,t}}{\tau_t p_{E,t}} - 1\big] \quad = \quad \frac{A_{N,t}}{A_{N,t-1}}\frac{\eta_E R_E^{-\lambda}}{\eta_N R_N^{-\lambda}}. \tag{A.25}$$

Replacing growth rates and technology levels with the values given by (8) and applying the resource constraint (9) yields

$$(1-\eta_t^S)(1+\eta_E R_{E,t}^{1-\lambda})\big[\frac{(1+\eta_E R_{E,t}^{1-\lambda})A_{E,t-1}}{\tau_t p_{E,t}} - 1\big] = (1+\eta_N(1-R_{E,t})^{1-\lambda})\frac{\eta_E R_E^{-\lambda}}{\eta_N(1-R_E)^{-\lambda}} \tag{A.26}$$

Dividing by $(1-\eta_t^S)$, then multiplying through on the left-hand side and isolating the term with energy prices yields:

$$(1+\eta_E R_E^{1-\lambda})^2\frac{A_{E,t-1}}{\tau_t p_{E,t}} = \frac{1}{1-\eta_t^S}\big[\frac{\eta_E R_E^{-\lambda}}{\eta_N(1-R_E)^{-\lambda}}\big(1+\eta_N(1-R_E)^{1-\lambda}\big)\big] + (1+\eta_E R_E^{1-\lambda}). \tag{A.27}$$

Distributing terms on the right-hand side leaves

$$(1+\eta_E R_E^{1-\lambda})^2\frac{A_{E,t-1}}{\tau_t p_{E,t}} = \frac{1}{1-\eta_t^S}\big[\frac{\eta_E R_E^{-\lambda}}{\eta_N(1-R_E)^{-\lambda}} + \eta_E R_E^{-\lambda} - \eta_E R_{E,t}^{1-\lambda}\big] + (1+\eta_E R_E^{1-\lambda}). \tag{A.28}$$

Now, (19) can be derived by multiplying through by $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$, taking the square root of both sides, subtracting one, and dividing by $\eta_E R_E^{-\lambda}$.

A.4 Solving the Model

In this section, I solve the consumer portion of the model in intensive form. This simultaneously demonstrates the conditions listed in Proposition 1 and demonstrates how to solve the model computationally as discussed in Section 5.4. As described in that section, I can take the path of productivities as given for portion of the solution. This portion of the model is almost equivalent to a standard neoclassical growth model. The only differences are a) the interest rate must be adjusted for monopolistic competition and taxes, and b) the growth rate of TFP may not be constant.

A.4.1 Intensive Form

Let τ_t be the proportional energy tax applied at time t. For any variable Z_t, I define:

$$z_t \equiv \frac{Z_t}{L_t A_{R,t}}, \tag{A.29}$$

where $A_{R,t} = TFP_t^{\frac{1}{1-\alpha}}$ and $TFP = A_{N,t}\big[1 - \frac{p_{E,t}}{A_{E,t}}\big]$. Applying (6), (7), and (10), this yields

$$y_t = k_t^{\alpha}, \tag{A.30}$$

$$k_{t+1} = \frac{y_t - c_t}{(1+g_{r,t+1})(1+n)}, \tag{A.31}$$

where $1 + g_{r,t} = \frac{A_{R,t}}{A_{R,t-1}} = (1 + g_{TFP,t})^{\frac{1}{1-\alpha}}$. Moreover, the Euler equation yields

$$\left(\frac{c_{t+1}}{c_t}\right) = \frac{\beta r_{t+1}}{(1+g_{r,t+1})(1+n)}, \tag{A.32}$$

where I have taken advantage of the fact that $\sigma = 1$.

Finally, when considering the interest rate, it is also important to keep track of the energy tax rate, τ_t. Let $\tilde{A}_{R,t} = A_{N,t}\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\big]$ be TFP adjusted for energy taxes. Then, from equation (16),

$$r_t = \alpha^2\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\big]A_{N,t}K_t^{\alpha-1}L_t^{1-\alpha} \tag{A.33}$$

$$= \alpha^2\left(\frac{K_t}{\tilde{A}_{R,t}L_t}\right)^{\alpha-1} \tag{A.34}$$

$$= \alpha^2\left(\frac{A_{R,t}}{\tilde{A}_{R,t}}\right)^{\alpha-1}\left(\frac{K_t}{A_{R,t}L_t}\right)^{\alpha-1} \tag{A.35}$$

$$= \tilde{\tau}_t\alpha^2 k_t^{\alpha-1}, \tag{A.36}$$

where $\tilde{\tau}_t \equiv \left(\frac{A_{R,t}}{\tilde{A}_{R,t}}\right)^{\alpha-1}$ is the interest rate wedge caused by the introduction of energy taxes.

Thus, the solution to the model is given by (A.30), (A.31) (A.32) and (A.36), noting that $g_{R,t}$ and $\tilde{\tau}_t$ are determined by the research allocations and can be taken as exogenous for this part of the solution. As described above, this is just the standard neoclassical growth model with a few additions. The α^2 term in (A.36) is the standard adjustment for monopolistic competition, $\tilde{\tau}_t$ is the wedge in the interest rate caused by carbon taxes, and $g_{R,t}$ may not be constant due to endogenous research allocations.

A.4.2 Proof to Propositions 1, 2 and 4.

To find the BGP, first note that $\tau_t = \bar{\tau}$, a constant. In the laissez-faire case, $\bar{\tau} = 1$. In the case of environmental policy (EP), $\bar{\tau} = \big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\big]$, which is also constant. In the EP case, the economy does not converge to the BGP in finite time. As discussed in the main text, $g_{TFP} = g_N^*$ on the BGP because $\big[1 - \frac{p_{E,t}}{A_{E,t}}\big]$ is fixed (at 1 in the case of EP). Thus, the growth rate of output per person is given by $g_R^* = (1+g_N^*)^{\frac{1}{1-\alpha}} - 1$. This yields

$$\bar{r} = \frac{(1+g_R^*)(1+n)}{\beta}, \tag{A.37}$$

$$\bar{k} = (\frac{\bar{r}}{\bar{\tau}\alpha^2})^{\frac{1}{\alpha-1}}, \tag{A.38}$$

$$\bar{y} = \bar{k}^\alpha, \tag{A.39}$$

$$\bar{c} = \bar{y} - (1+g_R^*)(1+n)\bar{k}. \tag{A.40}$$

Thus, r_t is constant, Y_t/L_t and C_t/L_t grow at rate g_R^*, and Y_t and K_t grow at rate $g_Y^* = (1+g_R^*)(1+n) - 1$.

At any point in time, energy use is given by

$$E_t = \frac{A_{N,t}}{A_{E,t}} K_t^\alpha L_t^{1-\alpha}. \tag{A.41}$$

On the BGP, therefore, the growth rate of energy is given by

$$g_M^* = \frac{(1+g_N^*)^{\frac{1}{1-\alpha}}}{(1+g_E^*)}(1+n) - 1. \tag{A.42}$$

We also know that energy efficiency grows at the rate of the energy price times the growth in energy taxes. Writing the price of energy is terms of energy use,

$$(1+g_M^*)^\iota (1+g_\tau) = (1+g_E^*). \tag{A.43}$$

Combining these two equations yields:

$$(1+g_E^*)^{1+1\iota}(1+g_\tau)^{-1} = (1+g_N^*)^{\frac{1}{1-\alpha}}(1+n). \tag{A.44}$$

Applying (8) and (9) to this equation yields items (1) – (6) in Propositions 2 and 4. To get the energy expenditure share in either case, simply rearrange equation (20).

All that remains to show for these two propositions is that expenditure share for the other variables are constant and that the research allocations are interior. To find expenditure shares, I apply all of the market clearing conditions to the factor price equations. To start, from equation (14) note that

$$w_t L_t = (1-\alpha) A_{N,t}[1 - \frac{p_{E,t}}{A_{E,t}}] K^\alpha L^{1-\alpha} = (1-\alpha) Y_t. \tag{A.45}$$

Next, from (22) and (16),

$$r_t K_t = \alpha^2 A_{N,t}[1 - \frac{p_{E,t}}{A_{E,t}}] K^\alpha L^{1-\alpha} = \alpha^2 Y_t. \tag{A.46}$$

The remaining share, $(1-\alpha-\alpha^2)Y_t$, is the production profits of the monopolists. This can be further divided into pure profits and payments to research inputs. All research inputs are hired at the same rate. By equation (A.23), total payments to research inputs is given by

$$p_t^R = (\frac{1}{\alpha}-1)\frac{r_t X_t}{A_{N,t}}\eta_N R_N^{-\lambda} A_{N,t-1} \tag{A.47}$$

$$= (\frac{1}{\alpha}-1)\cdot\frac{\eta_N (R_N^*)^{-\lambda}}{1+g_N^*}\cdot\alpha^2 Y_t, \tag{A.48}$$

noting that there is a unit mass of research inputs. The remaining share of final output is paid to monopolists as pure profits.

To note that research expenditure shares are interior, simply consider the case where $R_E=0$. This is ruled out by congestion effects, but more fundamentally, because energy use is increasing and energy efficiency constant when $R_E=0$. This contradicts equation (A.44). The opposite corner, where all R&D inputs are devoted to energy efficiency is ruled out by congestion effects and assumption (A.1) in the main text.

A.5 The Cobb-Douglas Model

In this section, I derive the BGP results for the Cobb-Douglas model and describe the calibration procedure. Let τ_t be the proportional energy tax applied at time t. To start, I note that, due to perfect competition, aggregate energy use is given by

$$E_t = (\frac{\nu}{\tau_t p_{E,t}})^{\frac{1}{1-\nu}}(A_t^{CD})^{\frac{1}{1-\nu}} K_t^{\frac{\alpha}{1-\nu}} L_t^{\frac{1-\alpha-\nu}{1-\nu}}. \tag{A.49}$$

This, in turn, yields

$$Q_t = (\frac{\nu}{p_{E,t}\cdot\tau_t})^{\frac{\nu}{1-\nu}}(A_t^{CD})^{\frac{1}{1-\nu}} K_t^{\frac{\alpha}{1-\nu}} L_t^{\frac{(1-\alpha-\nu)}{1-\nu}}, \tag{A.50}$$

$$Y_t = (1-\frac{\nu}{\tau})Q_t. \tag{A.51}$$

To find the BGP, I assume $\tau_t=1$ and consider the 'business as usual' scenario without any new energy taxes. The BGP exists for any constant growth rate of taxes. I define

$$z_t = \frac{Z_t}{L_t (A_t^{CD})^{\frac{1}{1-\alpha-\nu}}(\tau_t\cdot p_{E,t})^{\frac{-\nu}{1-\nu-\alpha}}}, \tag{A.52}$$

for any variable Z_t. This notation is specific to Appendix Section A.5.

The Euler equation is the same as in the putty-clay case. In intensive form,

$$\frac{c_{t+1}}{c_t} = \frac{\beta r_{t+1}}{(1+g_{CD})^{\frac{1}{1-\nu-\alpha}}(1+\tilde{g}_{P,t+1})^{\frac{-\nu}{1-\nu-\alpha}}(1+n)}, \tag{A.53}$$

where $1+\tilde{g}_{P,t+1} = (1+g_{\tau,t+1})(1+g_{P,t+1})$ and $1+g_{\tau,t} = \frac{\tau_t}{\tau_{t-1}}$. The rest of the dynamics are given by

$$k_{t+1} = \frac{y_t - c_t}{(1+g_{CD,t+1})^{\frac{1}{1-\nu-\alpha}}(1+\tilde{g}_{P,t+1})^{\frac{-\nu}{1-\nu-\alpha}}(1+n)}, \tag{A.54}$$

$$y_t = (1-\frac{\nu}{\tau})k_t^{\frac{\alpha}{1-\nu}}, \tag{A.55}$$

$$r_t = \alpha k_t^{\frac{\alpha-(1-\nu)}{1-\nu}}. \tag{A.56}$$

Thus, on the initial BGP, where energy prices grow at a constant rate, g_P^*, and energy taxes are constant,

$$\bar{r} = \frac{(1+g_{CD}^*)^{\frac{1}{1-\nu-\alpha}}(1+g_P^*)^{\frac{-\nu}{1-\nu-\alpha}}(1+n)}{\beta}, \tag{A.57}$$

$$\bar{k} = (\bar{r}/\alpha)^{\frac{1-\nu}{\alpha-(1-\nu)}}, \tag{A.58}$$

$$\bar{y} = (1-\nu)\bar{k}^{\frac{\alpha}{1-\nu}}, \tag{A.59}$$

$$\bar{c} = \bar{y} - (1+g_{CD}^*)^{\frac{1}{1-\nu-\alpha}}(1+g_P^*)^{\frac{-\nu}{1-\nu-\alpha}}(1+n)\bar{k}. \tag{A.60}$$

As a result, r_t is constant, Y_t/L_t and C_t/L_t grow at rate $(g_R^*)^{CD} = (1+g_{CD}^*)^{\frac{1}{1-\nu-\alpha}}(1+g_P^*)^{\frac{-\nu}{1-\nu-\alpha}} - 1$, and Y_t and K_t grow at rate $g_Y^{CD} = (1+g_R^*)^{CD}(1+n) - 1$.

I calibrate the model to the BGP using the same data as employed for the putty-clay model, leading to observationally equivalent paths for output and energy use. To match the energy expenditure share, I set

$$\frac{\nu}{1-\nu} = 3.3\% \;\Rightarrow \nu = .032. \tag{A.61}$$

All that remains is to ensure that total output grows at the same rate in the two models, which implies that energy use will also grow at the same rate. Since the energy sector is equivalent in the two models, this further implies that the price of energy will grow at the same rate. Thus, I set $(g_R^*)^{CD} = g_R^*$, where the later comes from the putty-clay model in section A.4. This implies that

$$g_R^* = (1+g_{CD}^*)^{\frac{1}{1-\nu-\alpha}}(1+g_P^*)^{\frac{-\nu}{1-\nu-\alpha}} - 1 \;\Rightarrow \tag{A.62}$$

$$g_{CD}^* = (1+g_R^*)^{1-\alpha-\nu}(1+g_E^*)^{\nu} - 1. \tag{A.63}$$

The calibration yields $g_{CD}^* = .42$, which corresponds to an annual growth rate of 3.5%. The growth rate of TFP is higher in the Cobb-Douglas case because it needs to overcome the drag of rising energy prices to achieve the same BGP rates of growth in consumption and output.

A.6 Microfoundation

In this section, I provide a simple microfoundation for the aggregate production function, (2), which further highlights the continuity with the existing DTC literature. Consider the following equation,

$$Y_t = L_t^{1-\alpha} \int_0^1 A_{N,t}(i) X_t(i)^\alpha \frac{E_t(i)}{R_t(i)} di \tag{A.64}$$

$$s.t. \quad E(i) \leq R(i), \tag{A.65}$$

where L_t is the aggregate (and inelastic) labor supply, $A_{N,t}(i)$ is the the quality of capital good i, $X_t(i)$ is the quantity of capital good i, $R_t(i)$ is the amount of energy required to run capital good i at full capacity, and $E_t(i)$ is the amount of actual energy used to run capital good i.

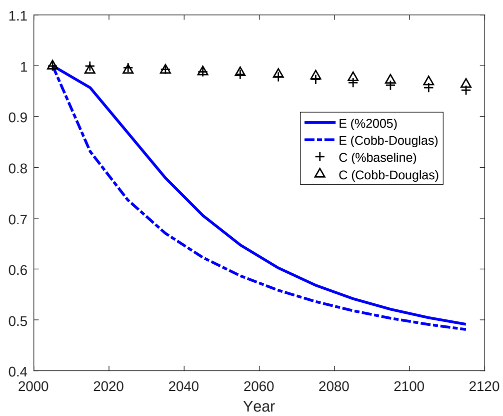
It is easiest to start by comparing this equation to the standard production function used in DTC models (and, more generally, in many endogenous growth models): $Y_t = L_t^{1-\alpha} \int_0^1 A_{N,t}(i) X_t(i)^\alpha di$. Here, final production is the combination of a set of processes, each of which combines aggregate labor, L_t, with a specific capital good, $X_t(i)$. The effectiveness of each process is determined by the quality of the capital good, $A_{N,t}(i)$. Each of these processes is perfectly substitutable with the others, though each is used in equilibrium because of diminishing returns. To this standard approach, I add energy requirements. In particular, I assume that each piece of capital requires a specific amount of energy, $R_t(i)$, to run at full capacity. If the amount of energy, $E_t(i)$, devoted to process i is less than $R_t(i)$, then the final goods producer receives less than the full benefit of that process. In particular, if the final good producer allocates, say, 80% of the required energy, i.e. $E_t(i)/R_t(i) = .8$, then it receives 80% of full capacity output.

To actually work with the model, it is necessary to assign a functional form to the energy requirement function, $R(i)$. Consider the following specification:

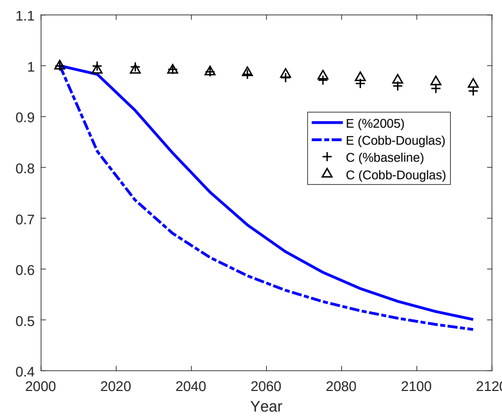
$$R_t(i) = A_{N,t}(i) L_t^{1-\alpha} X_t(i)^\alpha \frac{1}{A_{E,t}(i)}, \tag{A.66}$$

where $A_{E,t}(i)$ is a measure of energy efficiency. There are several key things to note about this function. First, consistent with the econometric literature on energy use, energy requirements depend both on the amount of capital and the amount of labor being used in the production process (Van der Werf, 2008; Hassler et al., 2012). Second, consistent with both the econometric and DTC literatures, improvements in non-energy technology, $A_{N,t}(i)$, raise energy requirements (Smulders and De Nooij, 2003; Van der Werf, 2008; Hassler et al., 2012, 2016b; Fried, forthcoming). Replacing (A.66) into (A.64) demonstrates that this set-up is identical to (2) and (3).

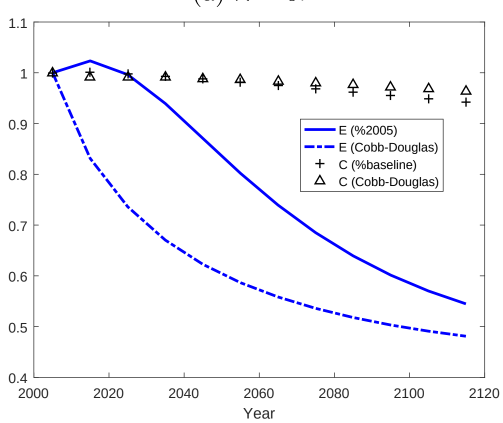
A.7 Robustness Exercises



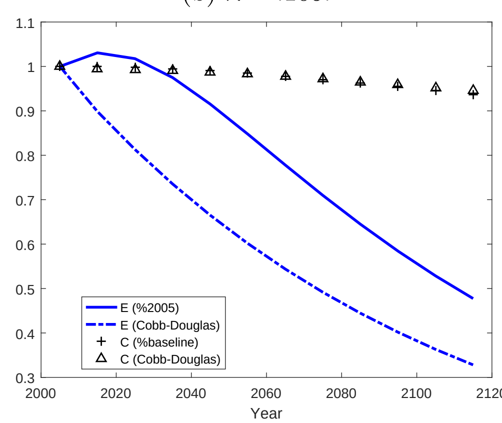
(a) $\lambda = 0$.



(b) $\lambda = .105$.



(c) $\lambda = .31$.



(d) Exogenous $p_{E,t}$.

Figure A.1: Robustness exercises. Panels (a) - (c) consider alternate values of research congestion, λ. In each case, $g_\tau = 0.29$, which is the tax rate requires to achieve policy goals with the Cobb-Douglas model. Panel (d) presents the results when energy prices grow exogenously at rate, $g_P = .23$. This matches the growth rate of energy prices on the BGP in the baseline scenarios. The tax rate is given by $g_\tau = 0.33$, which is the tax rate requires to achieve policy goals with the Cobb-Douglas model.