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Part 3

PROCEEDINGS

Agricultural Economics Seminar

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USING LINEAR PROGRAMMING IN A
FARM CONSOLIDATION PROBLEM*

By
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Linear programming has been used to develop guides to assist in farm decision making for about twenty years, and for planning by the military and industry for only slightly longer. Today, computer facilities are available at most colleges and universities and throughout the business world. The necessary programs to routinely handle the calculations and output are available. The mechanical ingredients for the wide-spread use of linear programming in farm and industrial decisions making are available. Persons knowledgeable about farming or other businesses and who also understand the use of linear programming are not. The human ingredient is not plentiful.

Linear programming is ideally suited for problems in which some objective is maximized (or minimized) within a set of limitations on available resources. In farm planning, the maximization of net income, returns over variable costs, or increase in net worth are often the objectives to be maximized. In another farm problem determining the minimum cost ration for the producing a cwt. of beef, a dozen eggs, or a pound of broilers might be the objective. In a feed processing business, the objective might be to find the minimum cost blend of ingredients to meet all the requirements for a protein supplement.

The first major operational use of linear programming was to help determine the most efficient way to transport war materials from factories to the many places they were were used during the second World War. Many useful applications to farm planning were demonstrated in the early 1950's. With the advent of electronic computers with large capacities, high speed, and low cost per unit of output farm planners were able to experiment widely and use the tool in research and teaching. Its usefulness in farm decision making has been established. Its wide spread use has been delayed by the relatively high cost for the farm planner's time. It requires from 10 to 30 hours to properly develop an appropriate model for a farm business, and until more agriculturists trained in linear programming are available, this will remain relatively costly.

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The concept of linear programming is often covered in intermediate micro-economic theory courses, in relation to theory of the firm. At Wisconsin, the concept and application is also covered in some detail in a graduate level research methods course, and the application to farm planning has been taught as a two credit course for beginning graduate students. a/ While some effort is required to become experienced in its use, the concept can be covered briefly.

The information needed to develop a farm plan using linear programming is similar to that used in budgeting. The major farm resources that may limit production must be identified and their quantities determined, e.g., acres of cropland, hours of available labor, dollars of capital, etc. The production activities to be considered in the plan must be specified, such as crop rotations and livestock enterprises. The input-output relations for each activity must be specified, e.g., the labor, capital, land, and cash costs to produce an acre of corn or tobacco. Finally, the planner must formulate price expectations for the products.

The linear programming solution specified the optimal use of the resources given the production activities, input-output coefficients, and prices specified for the model. In addition, the solution provides insight into the effect of changing the quantities of resources available or changing product prices.

A Simple Crop Production Problem

To illustrate linear programming procedures and the information contained in the solution a simple crop production problem is presented.b/

The Problem--The problem is how to organize the farm business (i.e., what crops to raise) to maximize net returns over variable costs given the conditions described below.

Restfictions--

Land	12 acres
Labor	48 hours
Capital	$360

Activities-- The activities are organized in units of one acre, i.e., one acre of corn production, one acre of soybean production, and one acre of oats production. The coefficients and net prices are for these one acre units.

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a/ At Iowa State University, Professor Ray Beneke annually has over 100 advanced undergraduate students in his three credit course on the application of linear programming to farm planning.

b/ From "Linear Programming Applications to Farm Planning" by Raymond R. Beneke and William E. Saupe. Memorial Union Bookstore, Iowa State University, Ames, Iowa. Revised edition. 1967.

Coefficients--Corn production requares one acre of land, six hours of labor, and $36 of capital. Soybean production requires one acre of land, six hours of labor, and $24 of capital. Oat production requires one acre of land, two hours of labor, and $18 of capital.

Net Returns-- The "net return" of an activity is the value of gross sales minus variable costs of production for that activity. In our example, if a unit (one acre) of the corn production activity produces gross value of sales of (say) $75 and if variable costs of producing that acre of corn were (say) $35, the net price would be $40.

Net prices used in this example are $40 per acre for corn, $30 per acre for soybeans, and $20 per acre for oats.

Table 1. Crop production problem arranged in matrix format.

Restriction	Quantity	Production Activities			Disposal Activities		
		Corn (1 acre)	Soybeans (1 acre)	Oats (1 acre)	Land	Labor	Capital
Land	12 acres	1 acre	1 acre	1 acre	1 acre		
Labor	48 hours	6 hours	6 hours	2 hours		1 acre	
Capital	$360	$36	$24	$18			1 acre
(Gross Sales) minus (Variable costs)		$40	$30	$20			

For example, producing one unit (1 acre) of corn takes 1 acre of land, 6 hours of labor, and $36 of capital (reading down the column title "corn"). The net return (gross sales minus variable costs) is $40 per acre. The disposal activities are a device for adding realism to the model; they allow resources to remain unused.

The Solution-- The solution contains three classes of information useful to the farm planner. The "Value of the Program" is the total gross sales from all production activities included in the final plan, minus their variable costs. In the crop production example the value of the program turns out to be $360 in the final plan. There is no way that the farm business can be reorganized given the resource restraints and price relationships assumed in the problem to yield a value of the program greater than $360.

Our objective was to organize the farm business so that net returns over variable costs would be maximized, given the resources that were available and the net prices used. In the crop production problem, the "Final Plan" (optimal plan) included:

Corn production	6 acres
Oats production	6 acres
Unused capital	\$36

There is no way that the land, labor, and capital available can be recombined that will generate more income than the above plan.

"Shadow Prices" for production activities indicate how the value of the program would be changed (how much income would be penalized) if an additional unit of the activity were forced into the final plan. In our example problem, soybeans did not enter the final plan. The solution specifies that if an acre were for some reason "forced in" the final plan (replacing an acre of corn) the value of the program would be reduced by \$10, from \$360 to \$350.

"Shadow Prices" for the disposal activities provide information concerning the productivity of added resources. All of the land available in our problem was planted to corn and oats. If one acre of land were taken away, the value of the program would be reduced by \$10, according to information specified in the solution. An extra hour of labor would add \$5 to the value of the program, but an extra dollar of capital would add nothing since all of the original supply was not used in the final plan.

Algebraic Formulation

It is not necessary for the planner to understand the details of linear programming computations to build planning models. However, he may not be satisfied without knowing something about how they are made.

The data in Table 1 form a set of equations which are solved simultaneously using division, subtraction, and multiplication in a particular sequence (simplex algorithm). The details of solving are not discussed here.

The crop production problem can be stated algebraically. We will let:

X_1 = units (acres) of corn produced

X_2 = units (acres) of soybeans produced

X_3 = units (acres) of oats produced

$$\text{(Equation 1)} \quad 12 \geq 1X_1 + 1X_2 + 1X_3$$

$$\text{(Equation 2)} \quad 48 \geq 6X_1 + 6X_2 + 2X_3$$

$$\text{(Equation 3)} \quad 360 \geq 36X_1 + 24X_2 + 18X_3$$

$$\text{(Equation 4)} \quad X_1 \geq 0$$

$$\text{(Equation 5)} \quad X_2 \geq 0$$

$$\text{(Equation 6)} \quad X_3 \geq 0$$

We next change the system of inequalities to one of equalities by adding disposal activities. These activities provide for the possibility that any portion of the supply of any resource may go unused:

X_4 = the quantity of unused land

X_5 = the quantity of unused labor

X_6 = the quantity of unused capital

Adding one disposal activity for each of the three resources (labor, capital and land) to the system of inequalities in Equations (1), (2), and (3) above, we arrive at the following set of equations:

$$\text{(Equation 7)} \quad 12 = 1X_1 + 1X_2 + 1X_3 + 1X_4$$

$$\text{(Equation 8)} \quad 48 = 6X_1 + 6X_2 + 2X_3 + 1X_5$$

$$\text{(Equation 9)} \quad 360 = 36X_1 + 24X_2 + 18X_3 + 1X_6$$

Where C = net return over variable costs the problem then is to maximize C where:

$$\text{(Equation 10)} \quad C = 40X_1 + 30X_2 + 20X_3$$

subject to the conditions imposed by Equations (4) through (9).

Steps Toward a More Realistic Plan

This example is much too limited in both restrictions and activities to have practical application. However, even with the limited number of enterprises and resources the computations required would be substantial. Fortunately, computations can now be made rapidly at a low cost by electronic computers. Instead of the corn, oats and soybean activities used in the simple illustrations, a realistic application of the method would involve perhaps 25 or 30 cropping systems, including fertilization at different levels and as many livestock activities. Instead of only the three restrictions land, labor and capital, there might be 3 or 4 types of land, 4 or 5 labor restrictions, several capital restrictions plus others arising from feed supplies, government programs, buildings, and facilities, and management and risk considerations.

The difficult tasks in programming are:

a) defining meaningful restrictions,
b) estimating realistic input-output relationships,
c) developing accurate price expectations.

Once these judgments have been made and the data properly arranged, large complex programs can be processed quickly and accurately by modern computers.

In our example we referred only to maximum restrictions. This feature provides that activity cannot enter the solution above a specified level. Minimum restrictions provide for the opposite, i.e., that an activity be carried on at least at the minimum level or a higher level. Thus planning models can be constructed to provide that a dairy herd of at least 30 cows and/or soil conserving crops of 80 acres or more must appear in the plan, for example. The programming process would then attempt to find the optimum plan given those restrictions.

Next we will move to an example of linear programming in a real farm decision making situation. The location is southeastern Wisconsin, and the problem concerns adding more land and labor to an existing farm business.

Farm Consolidation Problem

Existing Situation and Alternatives

There are three farms and two farm operators involved in this farm consolidation problem. All are located in Racine County, in southeastern Wisconsin.

Farmer A currently operates 178 acres of cropland and has a 40 cow dairy herd. He owns part of his farm and rents the remainder. His labor supply is presently limited to his own and a relatively small amount of hired summer labor.

Farmer A's decisions concern selecting the crop and livestock systems for his own farm that will maximize his income.

Farmer B currently works in town but contemplates buying a nearby farm. The farm has 85 crop acres, room for 25 dairy cows, and farrowing space for 15 sows. He would plan originally to rent and use Farmer A's farm machinery.

A third farm could be rented by Farmer B if he buys the farm. It has 90 acres of cropland, a place to finish 210 butcher hogs, and no dairy facilities. It could also be farmed at first with machinery rented from Farmer A.

Aside from decisions concerning cropping systems and livestock programs, some decisions regarding farm consolidation must be faced. The alternatives regarding consolidation are as follows:

Alternative I. No changes from the present would be made. Farmer A would continue to operate his present farm. Farmer B will continue to work in town, and not buy or operate any farm land. Resources and income are only those for Farmer A.

Alternative II. Farmer B would buy the farm with 85 crop acres and rent the farm with 90 crop acres. He would rent machinery from Farmer A, but they would not operate the farms together. Resources and income are only those of Farmer B.

Alternative III. This would be the same as Alternative II, except that some form of consolidation would take place with the two men pooling their labor and capital and operating the three farms as one unit. This would involve some type of partnership, corporation, or other agreement. Resources and income are for the two farmers combined.

As a first step, an evaluation of the income generating capacity of each of the alternatives was made. The farm organization that generated the maximum amount of income for each alternative within the resource limits specified was determined. Comparisons can be made among the alternatives regarding income.

A second step would be the working out of equitable terms for providing resources and sharing income, expenses, etc., if the consolidation seems the best alternative to follow. The present analysis attempts only the evaluation of the income generating capacities of the three alternatives, and does not explore this second step.

Resources

Linear programming was used as the analytical tool, and we limited our considerations to the crop and livestock systems described in the following, and within the resource limits described. Data used are based on Farmer A's farm record analyses where possible and in general reflect conditions as they apply to these three farms.

Resources and restrictions were considerably more complex than in the simple crop production model. They include land, labor (by months), capital, borrowing capacity, building capacity, and feed and livestock transfer activities. They are reported in Table 2.

In this analysis, it was assumed that Farmer A had $31,000 of capital in the form of livestock and feed inventories. He could borrow $30,000 at 5½% interest and an additional $10,000 at 6% interest.

It was assumed that Farmer B would have a total of $15,000 capital available after buying the farm for investment in livestock, feed machinery, production expense, building additions or improvements, and so on. He could borrow $5,000 more at 6% interest.

Each man provides 250 hours of labor per month during January, February, March, November and December, and 350 per month throughout the rest of the year. Farmer A can hire a young boy during the summer, but no other hired labor is considered available.

Dominant soil types on the three farms include (in order of descending importance) Elliot, Morley, Blount, Ashkum, Beecher, Elba, Fox, and Arlington silt loam soils. All were considered to have about the same productivity in the analysis, so only one soil (land) restriction was needed.

There is a 40 cow stanchion barn on the Farmer A's farm, adequate crop and machinery storage and no hog facilities.

The farm Farmer B considers buying has a 25 cow stanchion barn, and space to farrow 15 sows twice a year, and adequate crop and machinery storage.

The third farm has facilities to finish 210 hogs twice a year, no dairy cow facilities, and adequate crop storage.

Table 2. Resources and Restrictions

	Alternative I (Farmer A)	Alternative II (Farmer B)	Alternative III (Farmers A & B)
R 1 Cropland	178	175	353
R 2 Labor-January	250	250	500
R 3 Labor-February	250	250	500
R 4 Labor-March	250	250	500
R 5 Labor-April	350	350	700
R 6 Labor-May	350	350	700
R 7 Labor-June	350	350	700
R 8 Labor-July	350	350	700
R 9 Labor-August	350	350	700
R10 Labor-September	350	350	700
R11 Labor-October	350	350	700
R12 Labor-November	250	250	500
R13 Labor-December	250	250	500
R14 Capital	31,000	15,000	46,000
R15 Borrowing at 5½%	30,000	0	30,000
R16 Borrowing at 6%	10,000	5,000	15,000
R17 Dairy cow space	40	25	65
R18 Hire labor-June	150	0	150
R19 Hire labor-July	150	0	150
R20 Hire labor-August	150	0	150
R21 Silo capacity (tons)	400	120	520
R22 Hay transfer	0	0	0
R23 Corn silage transfer	0	0	0
R24 Corn transfer	0	0	0
R25 Oats transfer	0	0	0
R26 Barley transfer	0	0	0
R27 Wheat transfer	0	0	0
R28 Soybeans transfer	0	0	0
R29 Milk transfer (cwt)	0	0	0
R30 Farrowing space	0	15	15
R31 Hog finishing space	0	210	210
R32 Pork transfer (cwt)	0	0	0
R33 Work off the farm in June	300	300	300
R34 Work off the farm in July	300	300	300
R35 Work off the farm in August	300	300	300
R36 Feeder pig transfer	0	0	0

There were four basic crop rotations considered:

a) Corn-Soybeans-Corn-Small Grain-Hay-Hay
b) Corn-Corn-Corn-Small Grain-Hay-Hay
c) Corn-Corn-Small Grain-Hay-Hay-Hay
d) Corn-Corn-Corn-Corn-Small Grain-Hay-Hay

The "small grain" in each of the four rotations could be either oats, barley, or wheat, making a total of twelve rotations that were compared.

The cash costs, labor input, and expected yields for the crops are reported in Table 3. These are based on farm records and past performance of farmer A.

Table 3. Crop Inputs and Production

	Corn Following Hay	Corn Following Soybeans	Corn Following Corn	Oats, Barley, or Wheat	Soybeans	Alfalfa
Seed	$3.00	$3.00	$3.00	$1.50a/	$1.50a/	$6.50 b/
Fertilizer	8.50	11.00	18.00	7.00	5.00	9.00
Chemicals	2.00	2.00	4.75	---	---	---
Chemicals c/	5.50	5.50	8.50	---	---	---
Power & Machinery variable costs d/	10.00	10.00	10.00	10.00	10.00	10.00
Total labor per acre (hours)	6	6	6	4	5	6
Yield per acre	95 Bu.	95 Bu.	95 Bu.	70 Oats 60 Barley 50 Wheat	30 Bu.	3.5 T.

Corn silage yields 17 T. per acre, takes 2 hours more labor per acre than corn for grain, and has $5.00 more variable costs per acre.

a/ Pro-rated cost of seed purchased about every 4 years.
b/ First year hay only.
c/ On farms operated by Farmer B, only.
d/ Average per acre variable costs for the rotation; does not include depreciation, taxes, insurance.

The input-output coefficients for a dairy cow activity and two swine activities are reported in Table 4.

Table 4. Feed, labor, and capital required for a dairy cow, sows and hogs.

	Dairy Cow Producing 12,800 #	Sow and Two Litters Sell 13.6 Feeder Pigs	Sow and Two Litters Sell 13.6 Butchers
Feed Required:			
Hay equivalent	3.9 T	.3T	.65 T.
Corn silage	9 T. a/		
Corn	29 Bu. b/	28 Bu. b/	162 Bu. b/
Oats	38	38	48
Labor Required: (Hours)			
January	6	.5	.9
February	6	5.	5.4
March	6	3.	5.5
April	6	.5	2.6
May	5.6	.5	2.6
June	5.6	.5	2.6
July	5.6	.5	2.6
August	5.6	5.	.9
September	5.6	3.	5.5
October	6	.5	2.6
November	6	.5	2.7
December	6	.5	2.8
Total	70.0 hours	20.0 hours	39.5 hours
Capital Investment:			
Livestock and purchased feed	$340.	$50.	$76.

a/ Provisions were included in the program for 1 T. of hay equivalent (as low moisture silage) to be substituted for 3T. of corn silage in the dairy cow ration.

b/ Oats could be substituted for corn at the rate of 2 bushels oats equals 1 bushel of corn.

Prices

Since the farm reorganizations considered here are long run plans, the plans should be evaluated in terms of expected long run product prices. Since these are not known, and since relative product prices are especially important, the average prices for some past period of years may be used as a first approximation. This has the advantage of keeping prices near their past relations to each other.

Many linear programming computing systems provide "price ranging" information in their solutions. That is, the range over which the price of milk (for example) can vary without changing the optimal farm plan will be given. Additionally, the linear programming model can be organized so that changing a product price involves replacing only one card, and then re-running the analysis to evaluate the effect of the changed price.

Table 5. Prices used in farm consolidation study.

	Unit	Buy	Sell
Corn	Bushel	$ 1.20	$ 1.15
Oats	Bushel	.70	.65
Hay	Ton	22.00	20.00
Barley	Bushel	--	1.00
Wheat	Bushel	--	1.65
Soybeans	Bushel	--	2.75
Feeder pigs	Each	--	12.00
Butcher hogs	Cwt.	--	16.00
Milk	Cwt.	--	3.71

Farm Plans and Net Income

The income maximizing farm plans for each alternative are reported in Table 6. Farmer A earned $11,022 by himself. Farmer B earned $12,588 by himself, and if they pooled their resources and consolidated their combined earnings would be $25,495. Any payment by Farmer B to Farmer A for use of his machinery would be added to the income of the latter and subtracted from the income of Farmer B.

The income from the combined unit was about $1900 more than could be earned farming the same land separately. Major reason for the increase was the more complete use of the total labor supply.

Table 6. Linear Programming Solutions

	Alternative I (Farmer A)	Alternative II (Farmer B)	Alternative III (Farmers A & B)
Crop Rotation (acres):			
Corn	60	100	118
Soybeans	30	--	59
Wheat	29	25	59
Hay	60	50	118
Livestock Activities:			
Dairy Cows	40	25	65
Sows & 2 Litters	--	15	15
Sell Butcher Hogs	--	480 Cwt.	480 Cwt.
Crop Activities:			
Sell Corn	4,116 Bu.	6,356 Bu.	6,870 Bu.
Sell Wheat	1,483 Bu.	1,250 Bu.	2,942 Bu.
Sell Soybeans	762 Bu.	--	1,677 Bu.
Sell Hay	--	40 T.	--
Buy Oats	1,520 Bu.	1,666 Bu.	3,186 Bu.
Buy Hay	--	--	34 T.
Make Corn Silage	65 T.	--	--
Make Haylage	85 T.	54 T.	194 T.
Labor Activities:			
Hire Labor-June	79 Hrs.	--	146 Hrs.
Hire Labor-July	32 Hrs.	--	25 Hrs.
Used all Available Labor	Sept.	June	Sept.
	Oct.	--	Oct.
	Nov.	--	--
Returns Above Variable Cost:	$20,172	$18,288	$40,345
Fixed Costs:			
Insurance	$ 100	$ 100	$ 200
Taxes	210	900	1,110
Depreciation			
Buildings	--	700	700
Machinery	2,800	--	2,800
Interest	700	2,000	2,700
Cash Rent	5,340	2,000	7,340
Total Fixed Costs	$9,150	$ 5,700	$14,850
Net Return:	$11,022	$12,588	$25,495

Value of Added Resources

The linear programming solution specifies the effect on income of adding (or reducing) units of the resources. These shadow prices (marginal value products) are calculated for the first unit of resource added (or subtracted), and would eventually change as more units were changed. Selected values from the solution are reported in Table 7.

Table 7. Change in farm income from adding one extra unit of resources which were in short supply.

	Alternative I (Farmer A)	Alternative II (Farmer B)	Alternative III (Farmers A & B)
Labor (per hours:			
June	$ .50	$33.13	$ .50
July	.50	---	.50
September	5.06	---	10.47
October	18.59	---	22.45
November	6.42	---	---
Farrowing space (per sow)	37.11	49.33	22.16
Hogs finishing space (per bucher hog)	1.58	1.45	1.04
Cropland (per acre Per year)	47.50	25.21	41.38

Stability of the Solutions

The linear programming solution is optimal given the resources, activities, input-output coefficients and prices specified by the planner. Should any of these be changed, the optimal plan may be altered. I relatively minor changes cause major shifts in the optimal plan, the solution can be said to be unstable.

A measure of the stability can be gained by observing the "price ranges" in the solution. They describe the range over which product prices can shift without affecting the optimal farm organization. The income generated will change as product prices are changed, but the kinds and numbers of livestock, cropping systems, etc. will not change over the range of prices specified.

Selected price ranges are reported in Table 8. For example the optimal farm plan will not be changed in Alternative I as long as milk is between \$3.50 - \$4.14 per cwt., between \$2.44 - \$4.09 in Alternative II, and between \$3.50 - \$3.88 in Alternative III, and all other prices, coefficients and resources remain the same.

Table 8. Ranges over which product selling prices could vary without altering the income-maximizing farm organization.

	Alternative I (Farmer A)	Alternative II (Farmer B)	Alternative III (Farmers A & B)
Milk (cwt.)	\$ 3.50 - \$ 4.14	\$ 2.44 - \$ 4.09	\$ 3.50 - \$ 3.88
Butcher hogs (cwt.)	15.31 or more	15.36 or more	15.54 or more
Feeder pigs (each)	9.50 - 13.95	8.70 - 14.53	11.01 - 13.95
Soybeans (bu.)	2.46 - 2.94	up to 3.16	2.46 - 2.94
Wheat (bu.)	1.20 or more	1.20 - 3.37	1.20 or more
Corn (bu.)	1.03 - 1.20	1.14 - 1.20	1.03 - 1.20
Oats (bu.)	up to\$ .70	up to \$.70	up to \$.70
Hay (ton)	up to \$22.00	11.44 - 20.18	up to \$22.00
Barley (bu.)	up to \$1.37	up to \$1.33	up to \$1.37