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Agriculture - Econ. Aspects

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PROCEEDINGS

Agricultural Economics Seminar

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THE USE OF ECONOMIC THEORY AND RELATED STATISTICAL TOOLS
IN ANALYZING ECONOMIC PROBLEMS
IN AGRICULTURAL PRODUCTION ECONOMICS*

**Howard C. Williams

Economics is the science that deals with the choice between alternatives. Problems of choice arise because resources are limited and have alternative uses. The economic problem then is allocating scarce or limited resources among competing ends to maximize net goal achievement.

Problems of choice arise at every level of decision making. At the extremes there are the firm and the State. In between are various groups of individuals that enter into the decision making process-- the resource owner, the consumer and the Community or other economic units. Each economic unit must make certain decisions which in turn may influence the choices of other groups which make up the socio-economic structure of society. So there is interconnectedness between and among choices made. There is, therefore, a need for integrating the choice-making processes of individual firms, households, communities and other social aggregates in order to more fully maximize the welfare of society.

There are two basic problems in economics, which must be solved if the level of living is to be the highest possible from given resources. These are (1) resource allocation and (2) income allocation or income distribution, both of which are decided in the market place based on input-output values. Alternatively, these might be categorized as the organization of production and the organization of consumption. You will immediately recognize that each of these has many sub sets of problems. However, the significant thing is to note that in this context, economics is not as difficult as many of us make it appear in that it involves a very few basic principles, laws or relationships. The individual who has knowledge of those which apply to production also has knowledge of those that apply to consumption for each relationship in one of these problem areas has its counterpart in the other area. Knowledge of a few basic concepts provides tools for handling a wide range of problems.

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How We Define Problems:

Economic problems are almost always defined relative to ends or goals. The conditions of economics which specify a maximum or minimum can be used to express problems. Given the end which is to be maximized a problem exists if the optimum or maximum conditions have not been met. Here the problem is defined as a deviation from the optimum. Economists do define conceptual problems in this manner. On the other hand, entrepreneurs and consumers define and express problems in relation to ends or objectives. When they feel that the ends they deem important in production (resource use) or in maximizing satisfactions (consumption) are not being attained to the extent possible, appeal is made to research workers, educators, legislators or to others for advice to put the felt difficulties, doubts or confusions into perspective in order to resolve the problematic situation faced.

Both of these procedures are and should be used in defining problems. The latter leads to the resolution of current or practical day to day problems faced by decision makers. The former leads to the estimation of underlying basis relationships that permit the determination of the actual deviation in resource use from the optimum as defined by the stability conditions for economic maxima.

From the foregoing, it should be clearly evident that at any point in time or in space a wide range of problems either exist or loom on the horizon. Economic dynamics necessarily means that there must be continual change--continual adjustment to minimize doubts, confusions and felt difficulties; to bring about a more nearly full attainment of ends sought. The farm problem, the poverty problem are direct outgrowths of the failure to effect a sufficient rate of adjustment to change.

Agricultural Production Economics:

I have chosen the field of Agricultural Production Economics to illustrate the use of Economic Theory and Related Statistical Tools in analyzing economic problems. I have chosen this area because it is relatively easy to demonstrate the use of the few basic relationships in economics and they have their counterparts in all other areas dealing with the choice between alternatives because means are scarce relative to ends and have alternative uses.

Agricultural Production Economics is that branch which deals with the application of the principles of choice to the use of the factors of production (land, labor, capital and management) in the agricultural industry. It is concerned with the use and productivity of resources intra-firm inter-firm within a region between regions, at a point in time as well as over time. It is concerned not only with relationships in proportionality (short-run) but in scale relationships (long run) as well. (2)

The individual entrepreneur considering the problem of size and resource productivity has at least the following three broad questions:

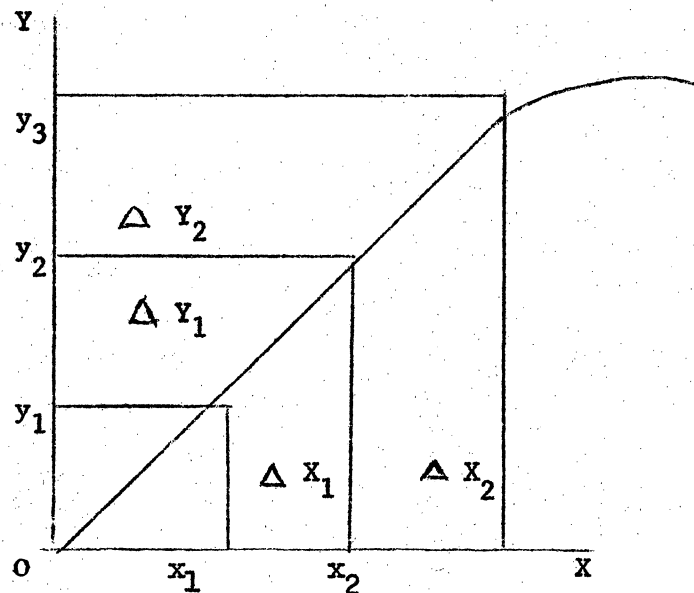
1. What should I produce?
2. How should I produce these products?
3. How much should I produce of these products?

The three broad questions are treated in production economics in the following three basic relationships, however, in the reverse order.

- (1) Factor Product
- (2) Factor-Factor
- (3) Product-Product

Let us briefly consider each of these.

Factor-Product. The factor-product relationship expresses the change in some output (y) in response to change in some factor input with one or more factors held constant. This can be illustrated graphically as follows:



When X changes from x_1 to x_2 (X_1) Y changes from y_1 to y_2 (Y_1). For every value of X, there exists a corresponding value for Y. This diagram illustrates a functional relationship between X and Y. It defines the possible levels of output of Y in response to changes in X. Even though all combinations are possible, they are not all equally profitable. The problem is finding the most profitable level of X to use in producing Y given this technical relationship. This level is defined by the economic conditions for an optimum. These conditions are:

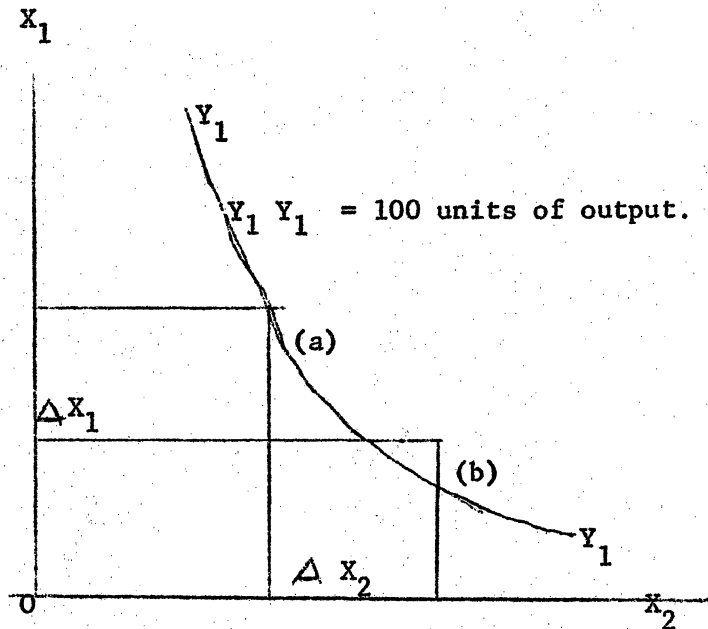
- (1) The marginal rate of transformation of factor X into product Y must be equal to the inverse of their price ratio and
- (2) The marginal rate of transformation must be decreasing.

Stated mathematically:

$$(1) \frac{\Delta Y}{\Delta X} = \frac{P_x}{P_y} \quad \text{and}$$

$$(2) \frac{\Delta^2 Y}{\Delta X^2} \quad \text{decreasing}$$

Factor-Factor. The factor-factor relationship deals with resource combinations and substitution. This can be illustrated graphically as follows:

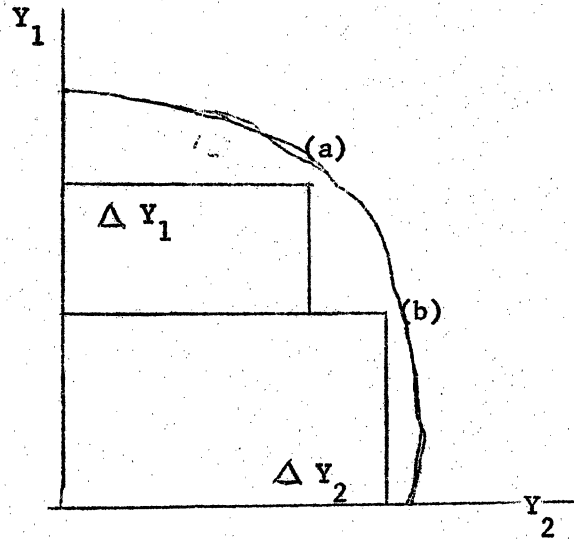


With output (y) constant at 100 units a reduction of X_1 in factor X_1 requires an increase of X_2 in factor X_1 , i.e., X_2 substitutes for X_1 in moving from factor combination (a) to factor combination (b) in producing 100 units of Y. Other points such as (a) and (b) exist on line $Y_1 Y_1$ indicating various combinations. The producer in combining X_1 and X_2 to produce a given output of Y, in general, is not indifferent to the combination used for costs are not constant for the various combinations. He is interested in the least-cost combination. The economic conditions for an optimum state that the marginal rate of substitution of X_2 for X_1 must be equal to the inverse of their price ratio and the marginal rate of substitution must be diminishing. Stated mathematically:

$$\frac{\Delta X_1}{\Delta X_2} = \frac{P_{X2}}{P_{X1}} \quad \text{and}$$

$$\left(\frac{\Delta X_1}{\Delta X_2} \right)_1 \rightarrow \left(\frac{\Delta X_1}{\Delta X_2} \right)_2 \rightarrow \dots \left(\frac{\Delta X_1}{\Delta X_2} \right)_N$$

Product-Product. The product-product relationship deals with output combinations and substitution of one or more products for one or more other products in the output mix. It is often referred to as the production possibility or opportunity curve. It can be illustrated graphically as follows:



With total resources held constant, an increase of Y_2 in output of Y_2 is accompanied by a decrease of Y_1 in the output of Y_1 ; i.e.,

Y_2 of Y_2 substitutes for Y_1 of Y_1 . Other points such as (a) and (b) exist on the production possibility curve indicating possible combinations of Y_1 and Y_2 that could be produced with a fixed bundle of resources. However, the relevant question is what combination of Y_1 and Y_2 should be produced in order to maximize net returns? The economic conditions for an optimum state that the marginal rate of substitution of Y_2 for Y_1 must be equal to the inverse of their price ratio and the marginal rate of substitution must be increasing.

Stated mathematically:

$$\frac{\Delta Y_1}{\Delta Y_2} = \frac{P_{Y2}}{P_{Y1}} \quad \text{and}$$

$$\left(\frac{\Delta Y_1}{\Delta Y_2} \right)_1 < \left(\frac{\Delta Y_1}{\Delta Y_2} \right)_2 \cdot \cdot \cdot < \left(\frac{\Delta Y_1}{\Delta Y_2} \right)_N$$

You will note that in the above relationships we have drawn smooth curves to estimate the relationship. These curves can be defined precisely by mathematical functions. Several functional forms are possible. The more common for one variable resource input are: (1, 4, 6)

$$(1) \quad Y = M - ar^X \quad (\text{Spillman or Mitscherlich})$$

$$(2) \quad Y = a X^b \quad (\text{Cobb-Douglas})$$

$$(3) \quad Y = a X b^X + c X^2 \quad (\text{Quadriatic})$$

$$(4) \quad Y = a - bX + c \sqrt{X} \quad (\text{Square root})$$

All of these forms can be expanded to include more than one variable resource input.

These functional forms differ in their implicit assumptions regarding the underlying relationships. In the choice of a specific function one should select the form that is consistent with a priori knowledge unless convenience and simplicity are overriding considerations. Even here one should be cognizant of the bias introduced through incorrect specification. Both Heady and McPherson suggest criteria of (1) "goodness of fit," a posteriori and (2) a priori logic based on the physiology of biological growth, other technical principles and principles of economic behavior (2, 6).

Use of Linear Programming.

Linear programming was developed largely during World War II as a method for specifying routes that would minimize travel distance in the use of the limited shipping facilities available to the Allies and in determining the best method of allocating scarce resources in the production of war goods. Since the early fifties, economists have used it as a research tool. Agricultural economists have used the technique to (1) specify the optimum organization of resources on farms, (2) to specify profit maximizing mixes of products produced by marketing firms, (3) to specify cost minimizing method in the production of processing products, (4) specify spatial equilibrium patterns in the flows of agricultural products, (5) to indicate the optimum inter-regional patterns of resource use and product specialization, and (6) to solve other related types of problems. It should be pointed out that linear programming is not a technique for estimating production functions. The input-output coefficients must be known or estimated before the technique can be used.

A linear programming problem has four components or major requirements. They are (1) an objective to be achieved, (2) alternative methods or courses of action, one of which will achieve the objective, (3) resources are limited in supply, and (4) the objective and limitations can be expressed as mathematical equations or inequalities and these must be expressed as linear equations or inequalities. Thus, a linear programming problem involves the maximization or minimization of a linear function subject to linear inequalities. (3, 5).

Let us look at a simple example to illustrate the technique. The example is taken from Thiel, et. al. (7).

A manufacturer produces two types of radio-- a standard model (r_1) and a luxury model (r_2). These radios are sold at a profit of \$20 and \$30, respectively. If the manufacturer makes r_1 standard radios and r_2 luxury radios each day, his daily profit (P_D) in dollars is $P_D = 20r_1 + 30r_2$. This total profit function is what he wants to maximize. Looking only at profit for each radio, it is obvious that

the manufacturer would produce only luxury radios. But this is impossible for he has capacity limitations (a restriction).

Suppose that the production of these two types of radios are produced on two conveyer belts--one for r_1 and one for r_2 . The capacity of these belts is limited. No more than 7 radios can be produced on belt 1 so that $r_1 \leq 7$.

The other belt has a capacity of 5, hence $r_2 \leq 5$.

There is also a labor restraint for if this were not the case the manufacturer would produce 12 radios ($r_1 = 7$; $r_2 = 5$) each day. Imagine that there are only 12 employees, so that available labor = 12 man days. Imagine further that it requires 1 man day to produce a standard radio and 2 man days to produce a luxury radio. Therefore, the labor constraint expressed mathematically is

$$r_1 + 2 r_2 \leq 12 \text{ (a linear inequality)}$$

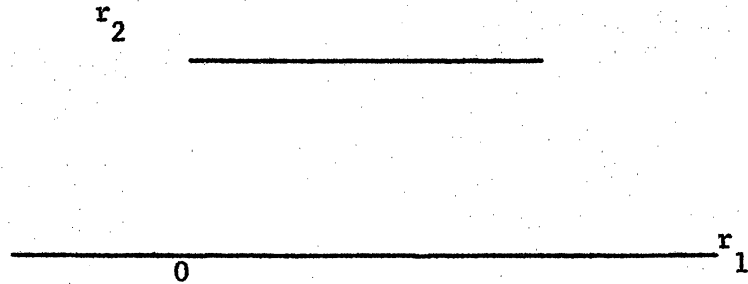
From the foregoing, it is easy to verify that we have a linear programming problem.

We have two decision variables r_1 and r_2 .

Decisions can be made with respect to these within certain limits. These limits are determined by $r_1 \leq 7$, $r_2 \leq 5$ (capacity of belts) $r_1 + 2 r_2 \leq 12$ (available labor supply) and the added restraints of $r_1 \geq 0$ and $r_2 \geq 0$. It is obvious that we cannot produce negative outputs of radios, however, it is necessary to specify these conditions because mathematical apparatus is not concerned with the question of whether restrictions are obvious or not.

We also have $20 r_1 + 30 r_2$ an objective function which we wish to maximize. Thus, we have a linear programming problem.

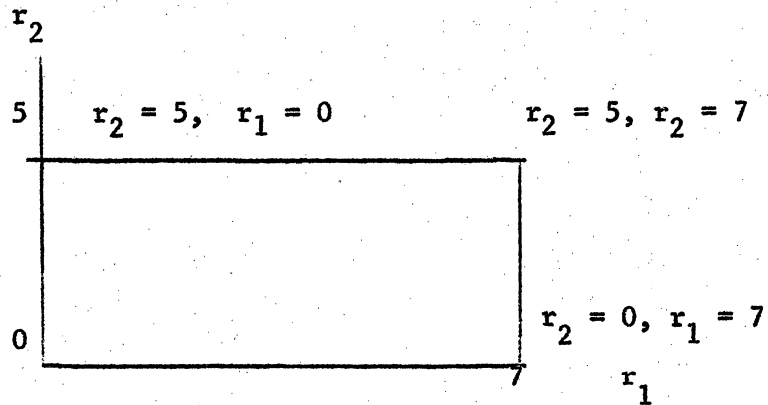
Since we have only two decision variable, it is easy to illustrate the problem graphically. Such a presentation makes it possible to see more clearly what is involved in linear programming and how a feasible solution is obtained.



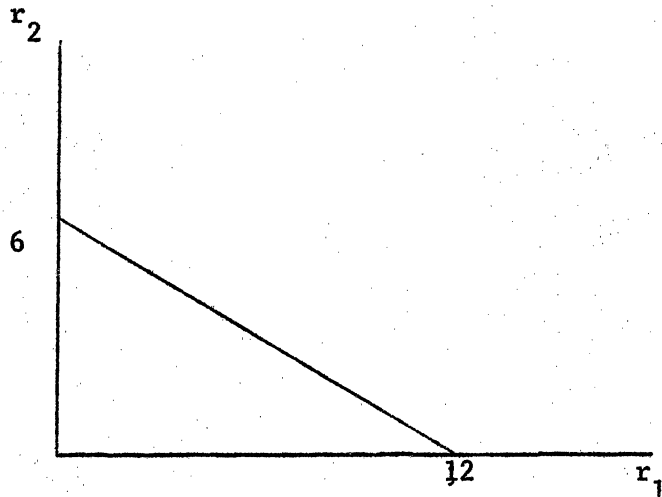
The non negative conditions limits the area for solution to the first Quadrant.

Within the first Quadrant the belt capacities indicate the number of radios of each type that can be produced up to a maximum of

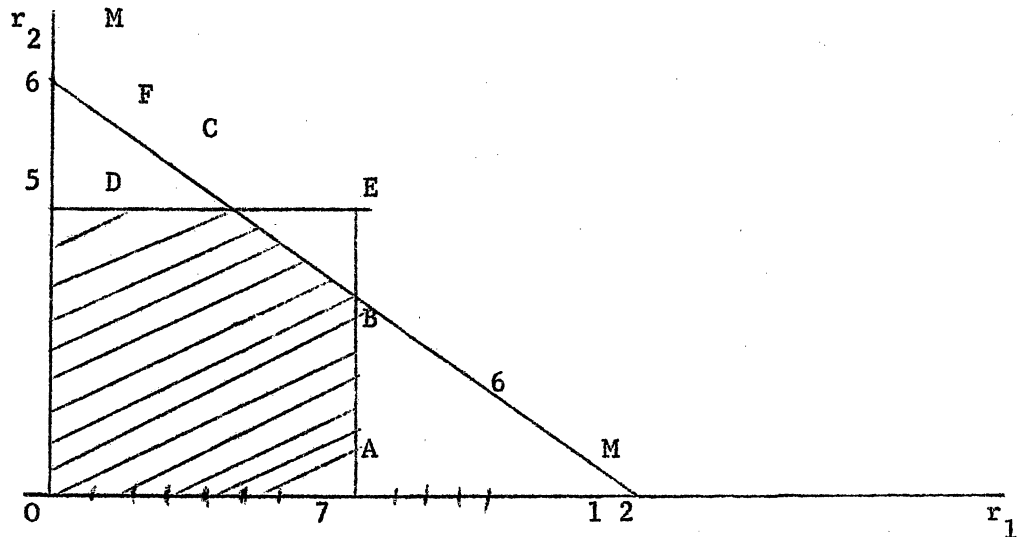
$$r_1 = 7; \quad r_2 = 5; \quad r_1 + r_2 = 12$$



The labor restraint ($r_1 + 2 r_2 = 12$) can also be plotted.

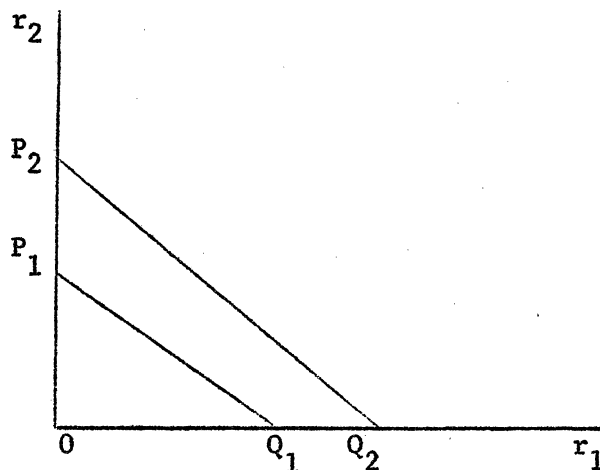


Now if we plot the belt capacity and available labor restraints in the same graph, we obtain the following:



The graph indicates that the areas DMC and BMA are inadmissible due to belt capacity limitation and the area CEB due to the labor restraint. The admissible area is found in the pentagon OA BCD.

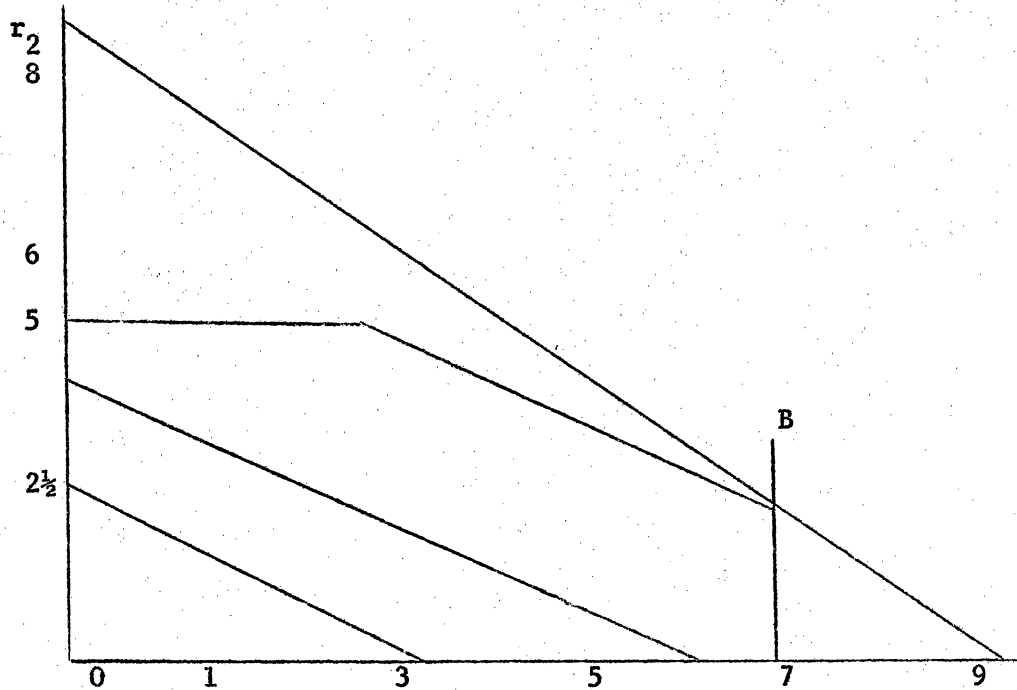
Now the problem is to select that combination of r_1 and r_2 that provides maximum profit. If we take the profit function $P_D = 20 r_1 + 30 r_2$ and assign dollar values to P_D we can determine r_1 and r_2 . For example, if $P_D = 60$ and $r_1 = 0$ then $r_2 = 2$. $P_D = 60$ and $r_2 = 0$ then $r_1 = 3$. We can then draw a straight line between $r_1 = 3$ and $r_2 = 2$ on the horizontal and vertical axes, respectively.



This is an iso-profit line. We can draw in others. They will all be parallel.

As we move out from the origin, we move to higher iso-profit curves. To maximize profits, we must reach the highest curve that has at least one common point with the feasible region.

Given the level of profits for r_1 and r_2 the line $\bar{P} \bar{Q}$ defines the point of profit maximization.



You will find that at Point B where 7 standard and $2\frac{1}{2}$ luxury radios are produced profits are $(= 7 \times 20 + 2\frac{1}{2} \times 30 = \$215)$ the maximum attainable.

The above looks at a problem where there are only two decision variables. With the technique of linear programming we are not limited in the number of decision variables that might be included. We are limited only by the capacity of computers used to obtain solutions. If the problematic situation contains the requirements of a linear programming problem, the technique is an efficient means of obtaining a solution.

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