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1. INTRODUCTION

Over the years it has become almost a dictum that a stable demand function for money is a necessary condition for money to exert a predictable influence on the economy. It has also become accepted, rather optimistically really, that this self-same money-demand function should have as arguments a small set of variables themselves representing significant links to spending and economic activity in the real sectors of the economy. While there are numerous assertions in the money-demand literature that such a "stable function of a few key variables" does (or did) exist (as in Laidler, 1971; Goldfeld, 1973), it is now perfectly clear that this is not the case, at least since Goldfeld's (1976) famous "missing money" paper in which it was shown that the then standard--and presumably stable--demand-for-money equation was forecasting badly. This has continued to be the case, as described, for example, in Judd and Scadding (1982) and Laidler (1985), and as numerous new variables or transformations of existing series have been applied to the problem. Whatever one might think of this activity, it is not unreasonable to claim that the standard demand for money specification in any of its variants was never stable in the way it was thought to be, and that any (temporary) impression of stability has been created by econometric approaches that are inadequate to support that conclusion (see Cooley and Leroy, 1981).

What is proposed in this paper are some empirical results from an important "microfoundations" approach to money demand. This approach proceeds from microfoundations to macro-estimation, employing an integrated "aggregation-theoretic" and "demand-systems" approach to the estimation of the demand for a set of financial (and non-financial) assets. The first step involves the employment of a set of monetary assets--currency, demand deposits, etc.--that have been shown to satisfy a test of the General Axiom of Revealed Preference (GARP). Satisfaction of GARP would imply that an aggregate utility function can be defined across the set of entities so selected. Because assets must be summed for both policy and estimation purposes, the next step involves checking for separable groupings and then forming sub-aggregates by employing Divisia quantity and price indices. The Divisia is chosen for this task because it is solidly based on microfoundations; in particular, the Divisia index is the transformed first-order conditions for a standard constrained optimization problem. In the last step, a system of share equations is estimated using the Fourier flexible functional form for the indirect utility function. This method of estimation is capable of approximating the unknown indirect utility function to any desired degree of approximation and will return--asymptotically and to an arbitrary level of accuracy--the true values of the income and price elasticities and elasticities of substitution for the aggregate consumer of financial services. These numbers and their estimated standard errors are available at every point in the data (which in this case is quarterly U.S. from 1970.1 to 1985.2). These time series are then inspected for clues as to why money demand might be hard to pin down.

The material presented in this paper is an indirect assault on the conventional money-demand studies. Those studies feature models that, strictly speaking, have no underlying utility function (and, concomitantly, no substitution elasticities). The argument here, quite simply, is that if it can be shown that income and substitution elasticities do fluctuate significantly over time, a demand technology that internalizes these fluctuations (and their presumed cause, the fluctuation of relative interest rates) may well offer superior predictions of money demand for policy purposes. The principle disadvantage of the systems approach, as we shall see, is in its rather un-Spartan use of degrees of freedom. In any case, the point of the present paper is not to compare the predictive performance of the two broad technologies but to illustrate the characteristics of the new approach as applied to a standard monetary data set and to emphasize that the aforementioned elasticities do, indeed, appear to change dramatically over recent U.S. experience in a particularly interesting way.

2. THE AGGREGATION OF MONETARY ASSETS

The present practice in most countries is to classify certain bank and public sector financial liabilities arbitrarily as either "narrow" money (M1)--that is, balances that can be used for transactions purposes--or "broad" money (M2, M3), where the latter usually include M1 plus savings (and time) deposits; they also include accounts on which drafts cannot be written directly. All of these measures (M1, M2, etc.) are frequently rearranged by the authorities, usually because the collection shows--or is expected to show--erratic behavior in the face of institutional or technological changes (such as the introduction of automatic transfers among accounts). Indeed, the dominant reason for such changes in recent discussions in the United States has been the presence of financial innovations that have, it is usually argued, changed the basic characteristics of many liquid financial commodities and (quite possibly) increased substitutability among many of these entities.

The aggregates just described are usually simple sums of their components (*e.g.*, M1 = C + D) and are constructed without benefit of index-number theory, essentially. While this practice might serve policy makers well when interest rate fluctuations are relatively mild, it is at a disadvantage when the relative interest rates on the monetary components fluctuate significantly. A Divisia index is an alternative (for example) that is based directly on economic theory; it will return the simple-sum index if the conditions for the simple sum exist, but the converse never holds if *any* relative price changes occur. The Divisia, indeed, is designed to internalize changes in substitution that arise from relative price changes. It might be hoped, of course, that for a certain time and place the simple sum works as well (for whatever purpose) as the alternatives, but for the United States, at least, the recent literature seems to suggest that no single measure of money works for all purposes (see Barnett (1987), Serletis, (1987,1988)) and Belongia and Chalfant (1989)).

3. THEORETICAL ISSUES

The issue at hand is substitutability among the potential monetary assets. In aggregation theory a quantity index should measure the income effects of a relative price change but should be unresponsive to substitution effects, which the index should internalize. In fact, the simple-sum index cannot effect this decomposition unless the component assets are indistinguishably perfect substitutes. In the face of what appear to be significant changes in the relative prices of financial assets in the period we are considering and with increasing numbers of apparently imperfect substitutes in the set of relevant financial assets, it is not surprising that considerable suspicion exists that part of the problem with the monetary aggregates is related to aggregation bias. This, clearly, could therefore be a substantial part of the problem with the aforementioned studies of money demand.

The Divisia Index provides a theoretically satisfactory (second degree) non-parametric means of aggregation with weights that are variable (they are price shares in the case of monetary quantity indices); it closely tracks the unknown true index implied by economic theory. Having a satisfactory procedure such as the Divisia does not, however, tell us exactly what set of assets to consider or how to group the subsets of the data for efficient estimation. A procedure that is available is the linear NONPAR program of Varian (1982, 1983), which is based directly on the General Axiom of Revealed Preference (GARP). Satisfaction of GARP on a set of data implies that there exists a nonsatiated, concave, monotonic utility function across that particular set. Such a set of data, if it exists, can be examined for logical groupings, again using the program NONPAR. If such groupings can be established--that is, if weak separability holds --then, according to the Leontief-Sono definition of separability, the marginal rates of substitution between any two prices (user costs in our case) in the monetary index are independent of changes in prices outside the monetary group. This group is then available for (Divisia) aggregation. It should be emphasized, though, that the NONPAR procedure sets too high a standard and, in particular, rejects separable groupings that are valid (in Monte Carlo simulations) as discussed by Barnett and Choi (1989). We will have cause to be concerned about this below, in our empirical discussion.

Swofford and Whitney (1987) have done the most thorough work with the NONPAR procedure on monetary data. On the quarterly U.S. data from 1970.1 to 1985.2, they have constructed a set of real per-capita measures of monetary quantities and a set of related nominal user costs (as defined below) to represent the prices of these quantities. With M1 denoting narrow money (excluding the deposits of businesses); OCD, other checkable deposits; SD, savings deposits in financial institutions; and STD, small time deposits in financial institutions, they found that both of the following arrangements passed the necessary conditions for the General Axiom of Revealed Preference.[2]

U[V(DUR, NONDUR, SERV, LEIS), W(M1, OCD, SD)]
U[V(DUR, NONDUR, SERV, LEIS), M1, OCD, SD, STD]

Here the first three items in each equation refer to components of total consumption, while LEIS refers to leisure (evaluated at the wage rate). Note that SD and STD describe vectors of the liabilities of the various financial institutions (e.g., SD = small time deposits in commercial banks, S&Ls, etc.). In what follows, M1 and OCD will be aggregated to economize on degrees of freedom; their user costs are, in any event, highly correlated (at .994).

The first of these two functions shows an apparently well-defined utility function across consumption and certain financial items, with two separable sub-groupings in V(·) and W(·), but this arrangement passes only the necessary and not the sufficient conditions for an optimum. The second arrangement just listed, with the consumption and leisure activities separable from the financial assets but not the converse, passed both necessary and sufficient conditions. We will, accordingly, work with this second set in our empirical investigations, since passing GARP in this way implies the existence of an aggregate utility function defined across these entities (for this time and place). Indeed, it is important to note that no collection was found by Swofford and Whitney for which financial items were separable from the real spending categories (in the sense of satisfying both the necessary and sufficient conditions for separability). Many possibilities were investigated, involving all of the 27 individual financial entities available from the Federal Reserve Board in the Farr and Johnson studies.

4. THE DEMAND SYSTEMS APPROACH

There is a literature on demand-systems for monetary analysis that runs as follows. Following the usual temporal substitution assumptions, the consumer can be assumed to maximize the current period utility function

$$u_t = U_t(c_1, \ldots, c_n; x_1, \ldots, x_m) \tag{1}$$

where c and x define consumer goods and financial commodities respectively. For this current period problem, the consumer will also be faced with

$$\sum_{j=1}^{n} p_j c_j + \sum_{i=1}^{m} \pi_i x_i = M_t' \tag{2}$$

where M_t' defines current resources and π_i is the user cost associated with the i^{th} financial commodity. This last is based on the yield of a representative non-money asset and, for the i^{th} monetary asset is

$$\pi_{it} = \frac{r_t^* - r_{it}}{1 + r_t^*} \qquad (3)$$

where r_t^* identifies the yield on a benchmark asset. This is the concept employed by Swofford and Whitney to measure the price of financial commodities; it was originally defined in Barnett (1978). This approach provides a way of getting at the implicit yield on a monetary asset by comparing it with an alternative asset that is assumed to provide no pecuniary services.[3]

In addition to the elasticities of substitution, the "systems approach" generally produces income elasticities, cross- and own-price elasticities, and elasticities of substitution. In the short empirical literature on these topics, we note, first that Chetty's (1969) early and important paper is almost unique in finding high substitutability among financial assets; he used the CES utility function. Donovan (1978), who uses a Gorman-polar-form representation of the indirect utility function, finds low substitutability between money and near money, while Offenbacher (1979), employing a flexible functional form (in this case the translog), finds that the substitutability between currency and time deposits is larger--in magnitude--than that between currency and demand deposits. Ewis and Fisher (1984, 1985) perform experiments with the basic translog and the "semi-nonparametric" Fourier flexible form providing evidence of low substitutability or even complementarity among the monetary assets. Finally, Fayyad (1986) uses the variable-parameter Rotterdam model to study the demands for five assets: food, nondurables, services, M1, and "All-but-M1" financial assets (included in the Federal Reserve's collection of 27 assets). The Divisia procedure is employed to calculate the indices and the results show that it is possible to specify a model of money demand that closely tracks the flow of M1's monetary services despite its considerable variability over the period. It is, further, calculated in a context in which it is apparently non-separable from real consumption decisions.

5. THE FOURIER FLEXIBLE FORM AND MONEY DEMAND

A way to employ flexibility--and gain some generality at the same time--is to estimate the elasticities of substitution among monetary assets by employing the classical Fourier sine/cosine series expansion of the indirect utility function. This method produces an expenditure system (and estimates of the elasticities of substitution) with the asymptotic property that the average prediction bias may be made arbitrarily small by increasing the number of terms in the expansion. Following Gallant (1981), the Fourier flexible form approximation of an indirect utility function h(v) may be written as

$$h_k(v) = a_o + b'v + \frac{1}{2}v'Cv + \sum_{\alpha=1}^{A} \sum_{j=-J}^{J} a_{j\alpha} e^{ijk'_\alpha v} \qquad \textbf{(4)}$$

Here

$$C = - \sum_{\alpha=1}^{A} a_{o\alpha} k_\alpha k'_\alpha \quad \textit{and} \quad a_j\alpha = \bar{a}_{-j\alpha}$$

and v is a vector of the expenditure-normalized user costs of the particular assets involved in the exercise. In an empirical investigation it is actually more convenient to work with a sine/cosine representation than with the exponential just written, so the following form is generally employed.[4]

$$h_k(\boldsymbol{v}) = u_o + \boldsymbol{b'v} + \frac{1}{2}\boldsymbol{v'Cv} + \sum_{\alpha=1}^{A}\left(u_{o\alpha} + 2\sum_{j=1}^{J} [u_{j\alpha}\cos(j\boldsymbol{k}'_\alpha\boldsymbol{v}) - w_{j\alpha}\sin(j\boldsymbol{k}'_\alpha\boldsymbol{v})]\right) \qquad (5)$$

in which

$$C = - \sum_{\alpha=1}^{A} u_{0\alpha}\boldsymbol{k}_\alpha\boldsymbol{k}'_\alpha.$$

The empirical problem is to choose A (the number of terms) and J (the degree of the approximation). The choice made is very much like that employed with the Almon lag, where one selects both degree and lag length for the approximating function. In practice, low numbers of J are chosen to balance the need to get an adequate approximation with the significant loss of degrees of freedom that results from higher values of J. In any case, a value of J = 1 actually enables one to capture the influence of changes in relative prices; in contrast, values of J greater than unity have no obvious intuition, although, of course, they fine-tune the approximation. A, in turn, has to be increased by the number of assets each time, which is expensive in degrees of freedom.

After differentiating Equation (4) and applying Roy's Identity, we arrive at the following set of equations.

$$f_i(v,\theta) = \frac{\boldsymbol{v_i b_i} - \sum_{\alpha=1}^{A} (u_{o\alpha}\boldsymbol{v'k_\alpha} + 2\sum_{j=1}^{J} j[u_{j\alpha}\sin(j\boldsymbol{k'_\alpha v}) + w_{j\alpha}\cos(j\boldsymbol{k'_\alpha})])k_{i\alpha}v_i}{\boldsymbol{b'v} - \sum_{\alpha=1}^{A} (u_{o\alpha}\boldsymbol{v'k_\alpha} + 2\sum_{j=1}^{J} j[u_{j\alpha}\sin(j\boldsymbol{k'_\alpha}) + w_{j\alpha}\cos(j\boldsymbol{k'_\alpha v})])\boldsymbol{k'_\alpha v}} \qquad \textbf{(5)}$$

These expressions are expenditure shares for the monetary assets; this system is what is estimated.[5] Note that while the Fourier procedure allows as close an estimate of the true Allen partial elasticity of substitution as the available degrees of freedom will permit, there is a potential problem of overfitting (see El Badawi, Gallant, and Souza (1982)). The practical problem is that the estimates of the elasticities may tend to oscillate--possibly wildly--as the degree of the approximating function increases beyond an unknown number. We will illustrate what might well seem to be an example of this phenomenon below.

7. THE ESTIMATION OF THE FOURIER FLEXIBLE FORM

In the discussion above we listed two collections of assets proposed by Swofford and Whitney on the basis of their GARP tests with Varian's NONPAR software. To be specific, these are

M1, OCD	A1
SDCB, SDSL, SDSB, SDCU	A2
STDCB, STDTH, STDCU	A3
DUR, NONDUR, SERV, LEIS	A4

Here SDCB, etc. are savings deposits at commercial banks, S&Ls, mutual savings banks and credit unions, while STDCB, etc. are small time deposits at commercial banks, thrifts, and credit unions.[6] Since prior summation is necessary to produce a reasonably small number of assets for effective parameter estimation, the above must be grouped; that is the reason for the arrangement just given. To attempt to preserve the characteristics of this set of data up to a third-order remainder term, Divisia index numbers are constructed from the individual quantities and their associated user costs; these are designated as A1, ..., A4. Note, again, that the highly correlated M1 and OCD are aggregated for convenience.

Putting all the pieces together, then, we have monetary data (and prices) that appear to satisfy an empirical test for revealed preference, we have summed them in a way designed to preserve their economic characteristics in the face of changes in relative prices and, finally, we propose to estimate the elasticities using a model that can come arbitrarily close to the elasticities implied by the true (but unknown) aggregate indirect utility function known to be defined (by the GARP test) over these entities.

As we have suggested above, the data set we are using does not show separability between consumption (plus leisure) and asset demands, so it is appropriate to fit a model where this separability is not imposed a priori.[7] When we use the four-commodity structure, we are dealing with a set of data that actually does pass the GARP test (necessary and sufficient); even so, there are reasons to be cautious about these results, as we shall see. In any event, quite a few versions of the test were attempted. The one that had the best fit of the basic model had $\acute{A} = 10$ and $J = 1$ and produced 33 coefficients in all; 18 of them were significant. Setting aside the uninformative estimated coefficients, the following table presents the results for the income and substitution elasticities. Since the Fourier procedure generates standard errors for each of these estimates, those parameters that are statistically significant are indicated in the table with an asterisk.

Table 2
Income and Substitution Elasticities (33 Parameter Model)

	S12	S13	S14	S23	S24	S34	E1	E2	E3	E4	Eigenvalue	
1970-1	0.809	4.738*	1.004	4.844*	1.414	-1.030	-0.105	-0.557	4.117*	-0.228	.0005	
-2	0.080	5.394*	1.281	5.344*	1.291	-1.114	-0.207	-1.398	5.047*	-0.174	.0005	
-3	-0.506	4.675*	0.839	4.087*	1.582	-1.152	-0.168	-0.398	3.743*	0.198	.0001	
-4	-0.619	4.222*	0.543	2.874*	1.637	-0.988	0.284	0.280	2.614*	0.356	.0003	
1971-1	-1.156	3.966*	0.645	1.599*	1.605*	-0.717*	0.944	0.250	1.762*	0.738	-.0001	
-2	-1.518	3.256*	0.206	1.715*	1.416*	-0.628*	0.560	0.621	1.894*	0.652	.3875	
-3	-1.423	2.673*	-0.054	1.638*	1.227*	-0.538*	0.631	1.044	1.716*	0.457	.6398	
-4	-1.312	2.350*	0.161	1.258*	1.023	-0.351*	1.097	0.836	1.453	0.529	.2643	
1972-1	-1.666	2.696*	0.346	1.299*	1.219*	-0.370	0.824	0.432	1.651	0.823	.3567	
-2	-1.570	1.922*	0.112	1.184*	0.847	-0.204	0.963	0.682	1.549	0.638	.6412	
-3	-1.309	1.289*	0.070	1.079*	0.522	-0.068	1.254	0.812	1.407	0.467	.5843	
-4	-0.761	0.777*	0.430	0.879*	0.195	0.037	1.868*	0.720	1.099	0.360	.0000	
1973-1	-0.879	0.443	-0.183	1.121*	0.047	0.013	1.509	1.238	1.321	0.018	.8684	
-2	0.575	0.523	0.805	0.742*	-0.229	-0.079	2.890*	0.942	0.606	-0.176	.0001	
-3	2.078	-0.013	0.329	1.080*	-0.234	-0.198*	2.217*	1.824*	0.586	-0.664	.2339	
-4	3.211*	0.215	0.705	0.640*	-0.612	-0.218*	3.162*	1.153	0.302	-0.685	.2314	
1974-1	3.344*	-0.858	1.332	0.767*	-1.099	0.144	3.232*	0.252	1.190	-0.545	.5132	
-2	2.096	0.345*	-0.168	1.193*	0.237	-0.180	1.719*	2.594*	0.564	-0.670	.2973	
-3	-0.912	-0.405	-0.769	2.289*	1.334	-0.174	-0.380	3.507*	1.025	-0.341	2.7904	
-4	2.290	1.358*	-0.122	0.871*	0.366	-0.332*	2.399*	2.353*	0.367	-0.554	.3907	
1975-1	-1.020	1.397*	-0.423	1.776*	0.968	-0.489*	0.437	2.232*	1.381	-0.106	1.0275	
-2	-2.185	0.915*	-0.556	1.940*	0.978	-0.533	-0.211	1.975*	1.785*	0.058	2.7374	
-3	-1.833	0.865*	-0.613	1.751*	0.831	-0.409	0.086	2.100*	1.600	-0.077	2.3601	
-4	-2.203	0.657	-0.706	1.767*	0.812	-0.390	-0.122	2.019*	1.719*	-0.028	3.2518	
1976-1	-2.151	0.772	-0.720	1.674*	0.717	-0.357	-0.013	1.875*	1.763*	0.003	3.0940	
-2	-1.770	0.849*	-0.660	1.472*	0.522	-0.268	0.395	1.772*	1.645	-0.046	2.4048	
-3	-1.407	0.834*	-0.542	1.298*	0.339	-0.206	0.797	1.670*	1.506	-0.090	1.7661	
-4	-0.633	0.758*	-0.152	1.011*	0.022	-0.144	1.643	1.426	1.173	-0.151	.4465	
1977-1	-0.952	0.549	-0.307	1.059*	-0.028	-0.085	1.381	1.372	1.347	-0.096	1.0528	
-2	-0.701	0.587*	-0.147	0.979*	-0.086	-0.091	1.646	1.303	1.220	-0.104	.4644	
-3	0.214	0.614*	0.448	0.743*	-0.311	-0.130	2.546*	1.076	0.769	-0.209	.1968	
-4	0.411	0.406	0.472	0.762*	-0.435	-0.115	2.623*	1.084	0.728	-0.293	.3096	
1978-1	-0.089	0.334	0.080	0.935*	-0.302	-0.090	2.098*	1.322	0.991	-0.294	.4580	
-2	-0.107	0.455	-0.005	0.989*	-0.188	-0.129	1.989*	1.484	0.998	-0.346	.4405	
-3	2.869*	0.574*	1.275	0.421*	-0.784	-0.289*	3.959*	0.746	0.143	-0.712	.1137	
-4	4.169*	-0.051	0.718	0.463*	-0.789	-0.115	3.030*	0.684	0.446	-0.516	.1934	
1979-1	4.496*	-0.444	0.568	0.419	-0.937	0.032	2.998*	0.275	0.883	-0.457	.4157	
-2	4.388*	-0.814	0.401	0.561	-0.933	0.125	2.922*	0.294	1.189	-0.482	.8218	
-3	5.280*	-0.028	0.272	0.046	-0.733	0.002	3.105*	0.697	0.420	-0.436	.6420	
-4	4.097*	0.100	0.373	-0.320	-0.076	0.077	1.722*	1.063	0.619	0.273	.1845	
1980-1	0.234	0.590*	0.116	-0.420	1.152	-0.485	1.670	1.717*	0.044	0.194	.8078	
-2	5.557*	0.354	-0.447	-0.027	-0.080	0.339*	0.899	3.103*	0.246	0.374	.5122	
-3	5.332*	0.617*	-0.650	0.354	0.203	0.100	1.448	3.001*	0.183	-0.122	.6574	
-4	-1.355	1.117*	0.402	-0.683	1.123	-0.545*	1.648	2.320*	-0.577	0.603	3.0628	
1981-1	-9.875	0.954*	-1.040	-0.143	3.069	-0.978*	1.453	3.479*	-1.220	0.503	19.5923*	F
-2	-7.508	1.551*	0.081	-0.447	1.875	-0.775*	1.957*	3.074*	-1.387	0.785	14.9958*	
-3	-26.722*	-2.381*	-5.598	-0.032	5.057	0.696	-2.207*	4.350*	0.525	3.114*	57.4774*	F
-4	-20.475*	-1.613*	-5.113	1.560	6.540	-0.615	-1.314	7.482*	-0.496	0.661	44.0585*	F
1982-1	-11.533*	0.537	-2.211	0.396	3.866	-0.906*	1.035	4.883*	-1.442	0.386	23.4302*	
-2	-5.608	1.073*	-1.246	0.008	2.306	-0.565*	1.571	4.414*	-1.457	0.473	11.4545	
-3	5.329	0.211	-0.067	0.561	-0.757	0.567*	0.332	3.714*	0.574	0.472	.1317	
-4	1.763	-0.040	-0.429	2.146*	1.117	-0.045	-0.052	3.996*	0.798	-0.247	.0001	
1983-1	6.860	0.934	-0.021	2.516*	-0.112	0.216	0.572	3.881*	0.930	-0.277	.0003	
-2	10.786	1.812*	-1.126	1.956*	0.006	0.199	1.761	4.525*	0.321	-0.491	1.2718	
-3	8.254	1.618*	-2.057	2.330*	1.114	-0.009	2.074*	6.434*	-0.228	-0.682	2.5122	
-4	9.415	1.650*	-1.616	1.835*	0.292	0.151	1.891	5.477*	-0.059	-0.462	2.0246	
1984-1	8.883	1.577*	-1.669	1.758*	0.253	0.172	1.872	5.605*	-0.142	-0.413	2.1546	
-2	7.155	1.404*	-0.846	1.906*	-1.163	0.522*	1.959	5.467*	-0.175	-0.512	1.3138	
-3	-7.282	0.809	-3.765	4.612*	5.035	-0.598	1.852	11.808*	-1.481	-0.989	11.8016	F
-4	8.602	1.449*	-1.976	1.180*	0.450	0.146	1.690	5.306*	-0.209	-0.166	2.6077	F
1985-1	9.403	1.383*	-0.453	2.092*	-0.484	0.268	0.938	4.356*	0.661	-0.226	.5474	
-2	8.508	1.428*	-0.012	2.969*	-0.157	0.135	0.567	3.941*	1.136	-0.393	.0000	

All of the substitution relations here show at least some significance, with σ_{23}--the elasticity of substitution between savings deposits and time deposits--exhibiting a particularly well-defined relation. The substitution elasticities also are often well in excess of unity, showing relatively strong substitution in those cases. The expenditure elasticities in the table, though, are not nearly as satisfying in view of a few cases of statistically significant negative numbers as well as the lack of any significance for the expenditure elasticities for A4, the composite consumption good.

One way of gathering in the information in Table (2) is to calculate the elasticities at the point of the means; this produces the following estimates.

Table 3
Elasticities at the Mean
Fourier Model (33 Parameters)

	Income	A1	A2	A3	A4
A1	.994	-3.838			
A2	2.796	4.396	-6.760		
A3	.399	.474	.439	-.720	
A4	.133	-.194	.342	.140	-.348

This is a set of results at only one data point, of course, and certainly does not capture the variability in Table (2), but the presence of substitutability among the financial assets and the negative own-rates of substitution for the same assets at the mean is reassuring. On the other hand, no real comfort can be gained from the results for A4, which are not well-determined.

While the calculation just given ruthlessly suppresses the interesting information in Table (2), it does afford a quick comparison with the popular translog model, which is estimated at a single point. For the same set of data, and estimating translog share equations, the comparable elasticities (at the point of approximation) are given in Table (4).

Table 4
Estimated Elasticities
Translog Model

	Income	A1	A2	A3	A4
A1	.688	-1.141			
A2	.865	.766	-.474		
A3	1.450	.023	.144	-.196	
A4	.822	.636	-.661	.136	-.393

These again show own-elasticities of substitution that are negative for all four assets, and a set of reasonable expenditure elasticities; the elasticity of .822 for A4, the composite consumption good, is particularly notable. The financial assets, further, demonstrate substitutability, as did the Fourier when that was estimated at the point of the means.

Table (2) also presents a column labelled "eigenvalues" and a column where the letter "F" has been set against certain data points. These reflect several efforts made to determine whether there was model failure at any points. Three things were done in this respect. First, it was verified that each of the predicted shares was positive at each point; this satisfies the monotonicity condition. Second, the own-elasticities of substitution were checked for cases in which the value of this parameter was significantly positive; for a t-value greater than 2, there were five such cases, as marked with an F. Finally, the eigenvalues associated with the matrix of Allen elasticities of substitution (AES) were examined at each point for evidence of large positive values.[8] The last column of the table gives the values of the largest eigenvalues for the AES at each point. By a simple t-test, the values marked with an asterisk produced eigenvalues significantly greater than zero. This, too, would indicate model failure.

What is evident immediately is that the data points at which relatively large elasticities are produced are those that also tend to show signs of model failure. To attempt to get a grip on the situation, both a finer approximation, with results listed in Tables (5) and (6) and many lesser approximations were attempted. The lesser approximations generally either failed to converge or produced no significant coefficients; these were not pursued further. For a slightly larger model, there were 37 parameters in all, but only eight were significant; this is, though, an interesting case and the results are given in Table (5).

Table 5
Income and Substitution Elasticites (37 Parameter Model)

	S12	S13	S14	S23	S24	S34	E1	E2	E3	E4	Eigenvalues
1970-1	4.939*	0.362	1.664	5.815*	0.128	0.550	2.495*	-3.743*	2.951*	1.362	.0003
-2	4.642*	0.358	1.641	6.132*	0.292	0.540	2.900*	-4.769*	3.843*	1.402	-.0001
-3	3.256*	0.307	0.900	5.023*	0.407	0.664	2.092*	-3.252*	3.165*	1.126	-.0003
-4	2.711*	0.477	0.668	3.938*	0.423	0.773*	1.873	-2.135*	2.470*	0.955	-.0005
1971-1	2.147*	0.466	0.656	2.549*	0.580	1.124*	1.709	-1.637	1.664	1.504	.0000
-2	1.164	0.176	0.031	2.367*	0.457	0.836*	1.092	-0.624	1.994*	0.922	-.0004
-3	0.789	0.058	-0.144	2.087*	0.358	0.644*	0.762	0.395	1.893*	0.589	-.0002
-4	0.747	-0.028	-0.001	1.666*	0.344	0.710*	0.722	0.593	1.527	0.919	.0299
1972-1	0.846	-0.018	-0.045	1.759*	0.417	0.903*	0.882	-0.338	1.650	1.248	-.0149
-2	0.350	-0.211	-0.239	1.399*	0.208	0.634*	0.492	0.654	1.659	0.920	.4546
-3	0.100	-0.376	-0.257	1.151*	0.017	0.476*	0.222	1.426	1.530	0.813	.9451
-4	0.190	-0.447	0.029	0.909*	-0.169	0.418*	0.190	1.888*	1.102	1.049	.5886
1973-1	-0.445	-0.642*	-0.292	0.979*	-0.326	0.137	-0.353	2.728*	1.583	0.280	2.0165
-2	0.693	-0.316	0.809	0.827*	-0.567	0.245	0.288	2.942*	0.415	0.918	.0395
-3	0.917	-0.062	1.450*	0.897*	-1.079	-0.304*	-0.038	4.143*	0.318	-0.389	1.1263
-4	2.262*	0.069	2.416*	0.820*	-1.536*	-0.252*	0.358	4.319*	-0.380	-0.013	.6501
1974-1	5.477*	0.052	4.766*	0.075	-2.330*	0.013	0.446	3.134*	-0.697	0.741	-.0004
-2	1.271	0.159	1.096	1.285*	-0.866	-0.116	-0.081	4.710*	0.523	-0.755	.3695
-3	-0.315	-0.527	-0.790	1.313*	0.540	0.016	-1.423	4.340*	1.749*	-1.518	2.2776
-4	2.108*	0.438*	1.229*	1.552*	-0.619	0.021	0.623	4.151*	0.402	-0.283	-.0004
1975-1	0.214	0.036	-0.387	1.591*	0.346	0.098	0.058	2.436*	1.897*	-0.536	.4586
-2	-0.621	-0.650	-0.905	1.347*	0.500	-0.053	-0.680	2.300*	2.412*	-0.787	3.0735
-3	-0.579	-0.489	-0.721	1.270*	0.290	0.567	-0.672	2.663*	2.263*	-0.847	2.6010
-4	-0.904	-0.803	-0.865	1.198*	0.281	-0.090	-1.034	2.684*	2.427*	-0.961	3.6691
1976-1	-0.886	-0.765	-0.835	1.186*	0.192	-0.069	-0.933	2.537*	2.445*	-0.839	3.5360
-2	-0.747	-0.583	-0.642	1.130*	0.004	-0.048	-0.710	2.576*	2.294*	-0.662	2.7500
-3	-0.627	-0.470	-0.455	1.055*	-0.147	-0.057	-0.538	2.655*	2.118*	-0.497	2.1406
-4	-0.282	-0.285	-0.060	0.909*	-0.357	-0.060	-0.143	2.782*	1.638	-0.077	.9996
1977-1	-0.610	-0.529*	-0.270	0.882*	-0.377	-0.120	-0.447	2.757*	1.881*	-0.155	2.0286
-2	-0.424	-0.414*	-0.130	0.844*	-0.401	-0.099	-0.250	2.751*	1.694*	-0.003	1.4652
-3	0.204	-0.143	0.454	0.707*	-0.531	-0.054	0.262	2.854*	0.970	0.422	.6396
-4	0.156	-0.197	0.540	0.662*	-0.633*	-0.120	0.186	3.073*	0.872	0.381	.9329
1978-1	-0.299	-0.380*	0.186	0.776*	-0.583	-0.171	-0.186	3.147*	1.335	0.036	1.4276
-2	-0.257	-0.304	0.199*	0.844*	-0.553	-0.158	-0.188	3.217*	1.377	-0.132	1.2616
-3	2.280*	0.192	2.377*	0.636*	-1.242*	-0.142	0.733	3.649*	-0.451	0.473	.0516
-4	3.998*	0.026	3.633*	0.652*	-2.306*	-0.292*	0.262	4.482*	-0.715	-0.059	.6147
1979-1	6.338*	-0.156	4.747*	0.315	-3.204*	-0.114	0.025	4.510*	-0.548	0.039	.2932
-2	7.235*	-0.150	5.031*	0.092	-3.361*	-0.025	0.050	4.312*	-0.395	0.133	.2318
-3	6.173*	0.357	4.542*	0.506	-2.783*	-0.457*	0.488	4.821*	-0.572	-0.402	.3754
-4	8.987*	0.056	6.236*	-0.140	-5.321*	-0.215	-0.383	5.120*	-0.180	-0.545	1.9246
1980-1	11.070*	-0.144	7.721*	-0.961*	-7.565*	0.003	-0.832	4.989*	0.423	-0.422	3.9090
-2	7.825*	0.891*	3.520*	1.281*	-3.733*	-0.166	0.355	6.115*	-0.213	-0.789	.4840
-3	5.496*	0.956*	2.680*	1.537*	-1.996*	-0.292*	0.646	5.685*	-0.165	-0.842	.0003
-4	11.702*	0.136	8.064*	-0.856*	-8.606	-0.100	-0.475	4.820*	0.479	-0.537	4.6908
1981-1	11.524*	-0.221	8.032*	-0.517	-8.766*	-0.050	-0.413	5.022*	0.254	-0.375	4.1378
-2	11.876*	0.123	8.461*	-0.700	-9.258*	-0.156	-0.117	4.465*	0.537	-0.570	4.8282
-3	10.380	-0.954	7.219*	2.202*	-7.848*	0.034	-0.458	7.382*	-1.481	-0.247	2.1773
-4	12.417	-0.681	6.133*	1.235	-8.100*	0.290	-0.030	6.214*	-0.998	0.357	1.5089
1982-1	12.623*	-0.303	7.089*	-0.234	-8.713*	0.061	-0.062	4.876*	0.184	-0.053	3.1960
-2	13.895*	-0.043	6.899*	-0.594	-9.121*	0.036	0.162	4.229*	0.616	-0.092	3.6183
-3	6.478*	1.098*	1.994*	1.522*	-3.206*	0.272	0.511	5.550*	0.067	-0.647	.2377
-4	1.636	0.410	-0.112	1.485*	0.235	0.077	-0.305	4.676*	1.032	-1.151	.0004
1983-1	7.206*	0.873	1.565*	3.710*	-1.598	0.136	1.628	2.772*	0.383	0.047	-.0002
-2	24.161*	1.134*	5.158*	3.243*	-7.410*	0.632*	3.558*	-0.109	-0.322	0.969	-.0001
-3	31.882*	0.968	6.477*	1.270	-10.658*	0.944*	3.854*	-1.638	-0.409	1.538	.0001
-4	27.188*	0.922*	5.538*	2.201*	-9.297*	0.735*	3.224*	-0.204	-0.238	1.156	.0001
1984-1	26.580*	0.867*	5.430*	2.012*	-9.385*	0.704*	3.024*	0.016	-0.154	1.100	-.0001
-2	23.570*	0.922*	5.332*	1.609*	-9.569*	0.611*	2.836*	-0.103	0.181	0.797	.0001
-3	26.633*	0.373	6.489*	-0.446	-11.466*	0.915*	2.682*	-0.916	-0.078	1.680	1.6057
-4	23.526*	0.765*	4.802*	2.015*	-8.206*	0.544*	2.479*	1.453	-0.099	0.863	-.0268
1985-1	13.797*	0.720*	2.876*	4.590*	-4.554*	0.428	1.815	2.840*	0.064	0.558	-.0005
-2	10.689*	0.616	2.288*	5.044*	-2.951*	0.355	1.739	2.191*	0.277	0.572	-.0005

Here we find a considerably larger number of significant elasticities than in Table (2); they are again indicated by asterisks. The substitution relations defined between A1 and A3 and A2 and A3 also appear to be well-defined. Again, in Table (6) we look at the values of the elasticities estimated at the mean. These show substitution among the financial assets, as did Tables (3) and (4); on the other hand, the income elasticities are somewhat less believable.

Table 6
Elasticities at the Mean
Fourier Model (37 Parameters)

	Income	A1	A2	A3	A4
A1	.873	-7.874			
A2	4.709	4.181	-4.623		
A3	.003	1.691	.738	-1.441	
A4	-1.219	2.754	-1.627	-.540	-.868

The composite consumption commodity (A4), further, continues to produce unexpected results.

As before, the predicted shares were examined as were the diagonals of the AES (for significantly positive own-substitution elasticities); there were no violations. Further, in the last column, the largest eigenvalues obtained from the AES-matrix are relatively small and never, in fact, significantly different from zero by a simple t-test. What could well be a problem, however, is the evidence of a poorer fit (only eight of 37 parameters were significant) along with larger oscillations in the elasticities over time. The latter is what might be anticipated from an overfitting of the model, although, as noted, there only seemed to be a narrow range (at a fairly high level of parameterization) where the Fourier model worked satisfactorily. In any case, while larger models were attempted, the oscillations of the elasticities increased dramatically, rendering further efforts in that direction pointless.[9]

The issue here, in any case, is whether one can claim that the elasticities of substitution among financial assets vary significantly over the sample period. It can be argued that this is a valid claim, on the whole, partly because of the actual patterns (to be discussed in a moment) and partly on the basis of other tests on different configurations of the same or other data (Ewis and Fisher (1985), Klonicki (1988), Fisher (1989)). In all such runs, the elasticities have varied over time, with the largest variations coming at or near cyclical turning points. As we shall see, this is what is shown here, as well, so we will proceed with an interpretation of the results.

8. INTERPRETATION OF RESULTS

The results of the study of the data by Varian's NONPAR procedure establish that the Federal Reserve has not constructed its aggregates from those primary entities that can be shown to be the result of a demand exercise on the part of the retail users of monetary services. That is, M1, etc., do not satisfy the General Axiom of Revealed Preference (as discussed by Swofford and

Whitney (1987)), while an alternative collection, unfortunately not separable from consumer spending, does. While the NONPAR procedure used for this purpose may well be setting too high a standard for separability, this result may be correct; if so, it provides one explanation why money demand functions have not performed adequately in recent years. This is not the contribution of the current paper, of course, but provides the starting point for a further dissection, in this case employing the data recommended in the earlier work of Swofford and Whitney instead of the official aggregates.

The detailed results from the application of the Fourier model provide a perspective on the inability of conventional money-demand functions to forecast effectively; most of the story is contained in Figures (1) and (2). In Figure (1) we exhibit the time series of substitution elasticities among the monetary quantities taken from Table (5), and in Figure (2) that of their income elasticities from the same table.

Figure 1
Substitution Elasticities

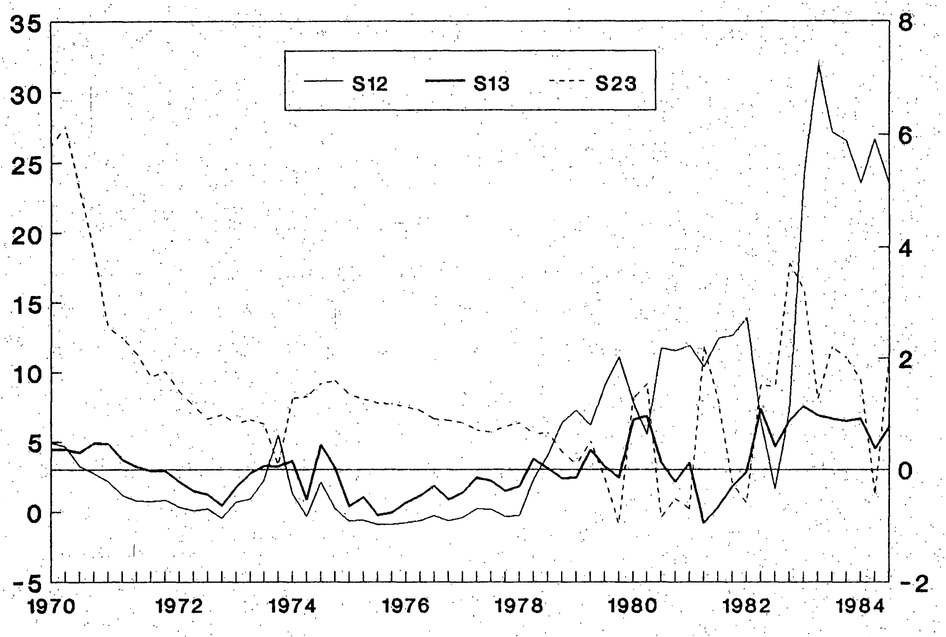
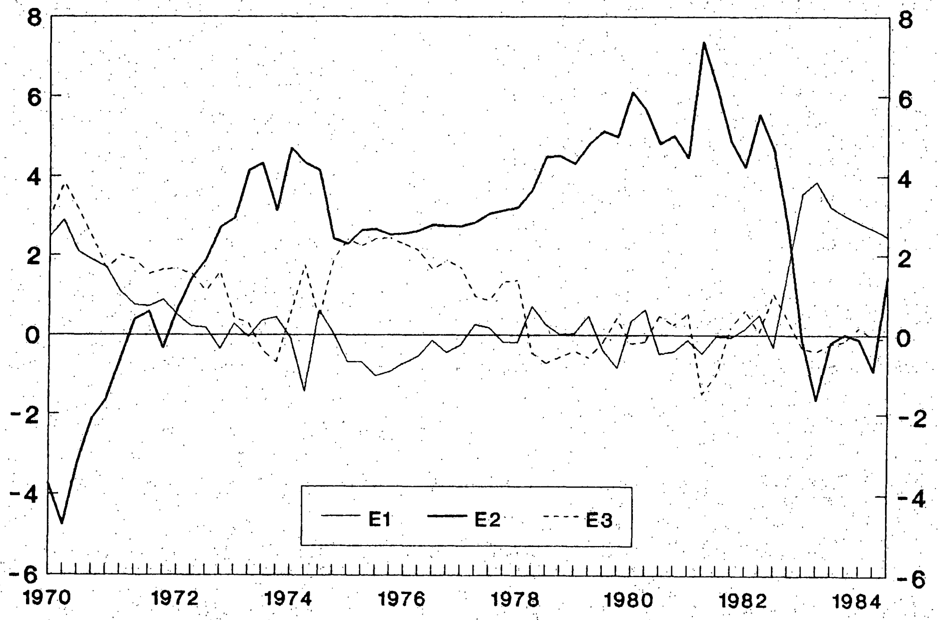


Figure 2
Income Elasticities



These numbers are highly variable, especially in and around the recessions during the period (and through the 1980s). One direct implication is that monetary policy may well be improved significantly if money demand is modelled in a way that permits internalization of this variation, as with a system of demand equations featuring variable elasticities of substitution over time. Another is that linear, nonintegrable money demand models may well be at a disadvantage in the face of fluctuating elasticities, since there is no direct way to capture their influence inherent in these models. The ultimate confrontation is empirical, of course; this is not the task of the current study.

Additional questions arise because of the apparent non-separability of financial asset-holding from consumption. In particular, this non-separability, if it should hold up in other tests, introduces an element of risk into a monetary policy that relies on the construction of purely financial measures of moneyness (such as is invariably the official practice). In particular, the relations between financial assets and the composite consumption commodity (A4) are often apparently quite close, as Table (5) shows. Furthermore, while A1 and A4 may well be substitutes in these tests, A2/A4 and A3/A4 are (significant) complements at times (and at the mean). In addition, the same cyclically instability in the parameters shows up in these numbers. Broadening the index to include consumption is really not a feasible policy alternative, at least if the purpose of such a policy is to fine-tune real activity, partly because the spending numbers are only available quarterly and partly because the model would be very hard to pin down. Nevertheless, redefining the basic data--and fine tuning the notion of user costs employed in such studies--might well provide an avenue for improvement in the conduct of monetary policy, although current official practice in the United States appears to be to move away from such calculations. This is not really a satisfactory way to leave the subject, but it is, formally, as far as the results of this study appear to take us.

9. CONCLUSIONS

It is important, first, to note that the model fitted in this paper was never totally satisfactory and that we were forced to balance concerns about the goodness of fit with the possible danger of overfitting the model. Even so, the claim can be made that there is significant volatility in all of the basic substitution and income elasticities among the monetary quantities and that the cyclical sensitivity of these numbers is especially obvious. It is possibly sufficient to argue that relative price changes among the financial assets (and consumption) are large enough to undermine the usual approaches to monetary aggregation, but even so, the swings in the elasticities of substitution estimated here suggest that the poor performance of money demand equations may also be produced by inappropriate functional forms quite apart from any other ways this can be achieved (such as using subsets of non-separable collections of assets). Divisia numbers of the entire set of financial assets used here would certainly internalize much of the diversity, but there are still questions of functional form (of the demand for money) and problems because of the possible violation of the assumption of the separability of financial assets from consumer goods. Even the official Divisia numbers once supplied by the Federal Reserve are vulnerable to these concerns, if only because the use of the Divisia does not solve the problem of an incorrect selection of assets from the point of view of consumer theory. In any case, these results, taken together, provide a possible explanation of one important aspect of the stability puzzles that have dominated the money-demand literature for the past dozen-odd years.

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Footnotes

1. I wish to thank the referees and an associate editor of this journal for their helpful comments on an earlier draft of this paper.

2. The original variables were supplied by the Federal Reserve and appear in several publications by Farr and Johnson (1985a, 1985b). In this study the monetary data are employed in per capita real form (where the latter is achieved by deflation with the CPI). SD represents savings deposits in commercial banks, S&Ls, mutual savings banks, and credit unions, while STD represents the small time deposits of the same institutions. OCD is other checkable deposits and includes NOW accounts. See Swofford and Whitney's two papers for more details on the construction of the data.

3. Note that the user cost is not an arbitrary opportunity cost but a theoretically-inspired concept that can be derived from an intertemporal optimization problem with money, consumer goods, leisure, and bonds in it. See Barnett (1978, 1981).

4. To go from Equation (4) to Equation (5), let $a_{0\alpha} = u_{0\alpha}$, $a_{j\alpha} = u_{ja} + iw_{j\alpha}$, and $a_{-j\alpha} = u_{j\alpha} - iw_{j\alpha}$ for $\alpha = 1, \ldots, A$ and $j = 1, \ldots, J$. Here i is the imaginary unit.

5. The Fourier procedure in the SAS library was employed. It generates income and price elasticities and the elasticities of substitution; the latter are calculated using Diewert's method (1974).

6. As discussed in Swofford and Whitney (1987, 1988), the data were prepared as follows. Each monetary asset is deflated by the consumer price index for urban areas. OCD includes super NOW accounts. Each user cost is defined as above. For leisure, the quantity is 98 hours less average weekly hours worked during the quarter (times 52). The wage rate measures the opportunity cost of time. The consumption figures are taken from Department of Commerce data that also provides the implicit deflator for each category. A 10 percent depreciation rate is used in calculating the one-period holding cost of a durable good.

7. The Fourier model was estimated on the collection {A1, A2, A3}, but the results proved unsatisfactory in terms of goodness of fit parameters.

8. The AES should be negative semi-definite.

9. Two other versions of the test were attempted. In one, a different set of assets, not involving time deposits, was aggregated into A1, ..., A4 using simple sum aggregation. In a second, involving the same entities that appear in the present study, this aggregation was effected using a Divisia index for quantities and (as suggested by Barnett (1987)), Fisher's "factor price reversal theorem" to back out the implicit user cost index for each observation. Both tests were reasonably well-determined and both show roughly the same pattern underscored in this paper.