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Variable and Fixed Rate Loans:

Determinants of Borrower Demand

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Abstract

Farmers and lenders continually make decisions about fixed and variable rate financing. Conditions are derived and illustrated under which risk-averse farmers will choose to borrow more under each type of financing and under which they will prefer each type of financing when debt is unconstrained or predetermined.

Variable and Fixed Rate Loans: Determinants of Borrower Demand

Agricultural borrowers have faced the choice between fixed and variable rate loans at least since 1957 when the Farm Credit Service first began making variable rate, intermediate-term loans. By 1963, Production Credit Associations were offering variable rate operating loans and Federal Land Banks had a long standing policy of adjusting loan rates to reflect their cost of funds (up to a contractual maximum). Various researchers have documented the overwhelming shift in agricultural lending to variable rate instruments by the early 1980s (LaDue and Leatham, and Barry). There is some reason to believe this trend has been reversed in recent years (the farm credit system began offering fixed rate loans again in 1986 for the first time since 1969). Nonetheless, variable rate lending is an important feature of agricultural loan markets about which a number of issues remain unresolved.

Researchers in agricultural finance have long been aware of the qualitative effects of variable and fixed rate loans both borrowers and lenders. LaDue and Leatham suggest that widespread use of variable rate loan instruments reduces debt-carrying capacity, the efficiency of interest rates in allocating capital, and the level of farm investment while increasing default risk and the cyclicality of agricultural production and investment. Barry and Baker point out that the effect of variable rate loans on the riskiness of returns to equity is inversely related to the covariance of interest costs and the rate of return on farm assets. Barry, Baker and Sanint model the leverage decision when interest rates are stochastic and compare optimal debt under stochastic and fixed rates. More recently, Leatham and Baker used discrete stochastic programming model to investigate the choice of fixed and variable rate loans for a midwestern crop-hog farm. Under the specific assumptions of their simulation, they found that farmers would always choose fixed rate debt when

the premium for fixed rate debt was less than one percentage point. A greater premium induced risk-averse farmers to switch first to some combination of fixed and variable rate debt and then to all variable rate debt.

The predominant concern of this paper is to describe farmers' preferences for fixed and variable rate debt and to determine simultaneously the levels of debt demanded. Using a popular mean-variance formulation of farmer behavior, the choice between fixed and variable rate loans is analyzed. Results relate to previous research in several ways. First, explanations for shifts in the proportion of farm loans that carry variable interest rates are suggested by the analysis. Second, the analysis suggests qualitative results that can be compared to those obtained by LaDue and Leatham and suggests some caveats to their conclusions. Third, the analysis brings into question the generality of the results obtained by Leatham and Baker.

The paper proceeds as follows. In the next section, the standard mean-variance, expected utility model is introduced and previously published results concerning farm (asset) size and demand for debt are summarized and extended. Next, the choice between fixed and variable rate financing is explored, first in the context of mutually exclusive financing decisions and then when a portfolio of fixed and variable rate financing is possible. Finally, the major results are summarized and conclusions drawn.

The Model

The model used in this paper follows closely the predominant model used in the literature [e.g. Barry and Baker; Barry, Baker and Sanint; Collins; and Merton]. Assume a risk-averse, expected-utility-of-wealth-maximizing farmer wishes to determine whether he should incur fixed or floating rate debt. Assume also that this farmer has a predetermined amount of equity, E-bar,

(there being no market for equity), a coefficient of absolute risk aversion, ϕ, and a one period planning horizon. Merton has shown that the solutions to the one period model are equivalent to the solutions to an infinite horizon stochastic control model when the stochastic component follows a Wiener process. The discussion here is limited to the one period problem. If the farmer's risk preferences can be adequately captured in a negative exponential utility function and the rate of return on assets follows a joint normal distribution, the farmers objective function can be specified in terms of the certainty equivalent of profit as (1) if interest rates on debt are fixed or (2) if they are variable.

(1) $\max_{D} CE(\pi) = A\mu - Dk - (\phi A^2 \sigma_a^2)/2$,

(2) $\max_{D} CE(\pi) = A\mu - D\overline{k} - (\phi/2) * (A^2 \sigma_a^2 + D^2 \sigma_k^2 - 2AD\sigma_{ak})$,

where A is total assets, D is total debt, μ is the expected rate of return on assets, k (kbar) is the (expected effective) interest rate paid on debt, and σ_a^2, σ_k^2, and σ_{ak} are the variance of the rate of return on assets, the variance of the effective interest rate on debt and the covariance of the rate of return on assets and the interest cost of debt. This objective function can also be considered a second-order Taylor series approximation to the certainty equivalent of profit if the distributional or utility function assumptions are violated (Pratt). Note that by choosing D, the level of debt, the farmer is also choosing A, the level of assets since A = D + E-bar, where E-bar is fixed.

Barry, Baker and Sanint show the first order conditions for (1) and (2) are, respectively,

(1a) $D1^* = (\mu - k)/(\phi\sigma_a^2) - \text{E-bar}$,

(2a) $D2^* = (\mu - \text{k-bar})/(\phi\sigma_e^2) - \text{E-bar} * (\sigma_a^2 - \sigma_{ak})/\sigma_e^2$,

where $\sigma_e^2 = \sigma_a^2 + \sigma_k^2 - 2\sigma_{ak}$. Dividing (1a) by $A^* = (\mu - k)/(\phi\sigma_a^2)$ yields a result similar to Collins' δ^*, the optimal debt/asset or leverage ratio. The difference, a factor of E-bar in the second term arises because here it is assumed that the equity endowment is a binding constraint. If D^* is negative, the farmer will choose to lend rather than increase his farm assets.

From (1a), it is clear that the demand for fixed rate debt depends on the expectation and variance of the rate of return on assets, the effective interest rate, and the borrower's level of risk aversion and equity. From (2a), the demand for variable rate debt depends on these factors as well as the expectation and variance of the effective interest rate and its covariance with the rate of return on assets. It is also clear from the comparative statics presented in Table 1 that the demand for debt will be less sensitive under variable rate financing to changes in all relevant exogenous factors than under fixed rate financing. This is true because the absolute value of the derivatives of (1a) with respect to μ, k, σ_a^2, and E-bar are greater than the absolute value of the corresponding derivatives of (2a) when σ_{ak} is negative.

It is clear from the first order conditions that the expected utility maximizing level of debt and assets will differ under fixed and variable rate financing. This can be demonstrated by setting (1a) equal to (2a). Setting (1a) equal to (2a), it is clear that the expected utility of wealth maximizing level of debt (and assets) will be the same only under the condition that

(3) $(k - \text{kbar}) = [(\mu - \text{kbar})*(\sigma_k^2 - 2\sigma_{ak}) - \text{E-bar} * \phi * \sigma_a^2 * (\sigma_k^2 - \sigma_{ak})]/\sigma_e^2$

which, when k is equal to kbar, reduces to

(3a) $\mu - k = $ E-bar $* \phi\sigma_a^2 * (\sigma_k^2 - \sigma_{ak})/(\sigma_k^2 - 2\sigma_{ak})$.

When (3) or (3a) holds the optimal debt can be expressed, respectively, as

(4) $D1* - D2* = [(\mu - \text{kbar}) * \phi - \text{E-bar} * (\sigma_a^2 - \sigma_{ak})]/\sigma_e^2$ or

(4a) $D1* - D2* = \text{E-bar} * \sigma_{ak}/(\sigma_k^2 - 2\sigma_{ak})$.

Condition (3) can be interpreted as saying that a farmer will choose the same level of fixed or variable rate financing if the discount on variable over fixed rate loans is just adequate to compensate for the net risk-adjusted disadvantage of variable rate borrowing. If this condition is not met, then D1* (A1*) will be greater or less than D2* (A2*) as the left hand side of (3) is less or greater than the right hand side.

Choice Between Mutually Exclusive Fixed and Variable Rate Debt

The previous section addressed the question of when a farmer would choose to borrow more under fixed or variable rate financing. The reader should note that D1* > D2* does not always imply CE(D1*) > CE(D2*). Thus, the question of relative desirability of fixed and variable rate borrowing has not yet been addressed. An important aspect of the current research is to address this issue and to clarify factors affecting the choice between fixed and variable rate debt. There are two reasonable scenarios under which this choice could be made. The first is the unconstrained case, i.e., when the borrower solves (1a) and (2a) independently and chooses the fixed (variable) rate debt only if CE(D1*) > (<) CE(D2*). The second case is when, for exogenous reasons, the choice variable is not the level of debt but whether the debt carries a fixed

or variable rate. The latter case is referred to as the debt-constrained case for the remainder of the paper.

The Unconstrained Case

In general, the farmer will not be indifferent between fixed and variable rate instruments. Setting (1) equal to (2) and substituting from (1a) and (2a) implies the farmer will be indifferent to fixed or variable rate financing if and only if

$$(5) \quad (\mu - k) * [(\mu - k)/\phi\sigma_a^2 - (\mu - k)/2\phi\sigma_a^2 - \text{E-bar}] =$$
$$(\mu - \text{kbar}) * [(\mu - \text{kbar})/2\phi\sigma_e^2 - (\mu - \text{kbar})/2\phi\sigma_e^2 - \text{E-bar} * (\sigma_a^2 - \sigma_{ak})/\sigma_e^2]$$
$$- [\text{E-bar}^2/\sigma_e^4][(\sigma_a^2 - \sigma_{ak})^2\sigma_k^2 + 2(\sigma_a^2 - \sigma_{ak})(\sigma_k^2 - \sigma_{ak})\sigma_{ak} + (\sigma_k^2 - \sigma_{ak})^2\sigma_a^2].$$

Condition (5) is a difficult one to interpret. The left hand side is the expected advantage to leverage times the optimal asset level, A1*, less the risk premium and initial equity. The first term on the right hand side is directly analogous to the left hand side while the second term is a further adjustment for financial risk associated with variable interest rates. Some intuition can be gained by considering the degenerate case where σ_k is zero. In this case the second term on the right hand side is zero and the condition becomes simply k = kbar. A sufficient condition for preferring fixed to variable rate loans can be derived by term-wise comparison of the right and left hand sides of (5). This condition is

$$(5a) \quad \sigma_e/\sigma_a > (\mu - \text{kbar})/(\mu - k) > \sigma_e^2/(\sigma_a^2 - \sigma_{ak}).$$

Condition (5a) can be interpreted as saying that fixed rate loans will always be preferred regardless of risk preference if the ratio of the advantage to variable rate debt, $\mu - \text{kbar}$, to the advantage to fixed rate debt, $\mu - k$, lies between the ratios of the standard deviations under each type of financing and the ratio of total variance under variable rate financing to non-diversifiable variance under fixed rate financing.

Through these results the question of how changes in the financial and business environment affect the relative desirability of fixed and variable rate loans may be addressed. Such changes directly effect such factors as the expectation and variance of the rate of return on assets, the effective cost of variable rate debt, the covariance of these two factors and the wealth (and absolute risk aversion level) of business owners.

By rearranging the terms in (5), the following condition for indifference between fixed and variable rate loans is derived:

(5b) $(\mu - k) * [(\mu - k)/2\phi\sigma_a^2 - \text{E-bar}] -$

$(\mu - \text{kbar}) * [(\mu - \text{kbar})/2\phi\sigma_e^2 - \text{E-bar} * (\sigma_a^2 - \sigma_{ak})/\sigma_e^2]$

$+ [\text{E-bar}^2/\sigma_e^4][(\sigma_a^2 - \sigma_{ak})^2\sigma_k^2 + 2(\sigma_a^2 - \sigma_{ak})(\sigma_k^2 - \sigma_{ak})\sigma_{ak} + (\sigma_k^2 - \sigma_{ak})^2\sigma_a^2]$

$= 0.$

Table 2a provides comparative static results on (5b), and Table 2b provides information useful for interpreting these results. Only changes in σ_k^2 have an unambiguous effect on the desirability of fixed relative to variable rate loans. Consistent with intuition, an increase (decrease) in σ_k^2 makes fixed rate loans more (less) attractive. Another intuitive result, that the attractiveness of variable rate loans increases as the covariance of interest rates and returns on assets increases, does not hold universally except when

initial wealth is zero. At other wealth levels the impact of changes in the covariance depends on the magnitudes of such factors as absolute risk aversion, the variances of the return on assets, of the cost of debt, and of the return on equity.

Many of the other results depend on the ratio of reward to risk of fixed versus variable rate borrowing, where reward is measured as the expected gain from borrowing (rate of return on assets less interest cost) and the risk measure varies from variance to standard deviation to non-diversifiable risk (variance). For example, an increase (decrease) in the level of absolute risk aversion makes fixed (variable) rate loans relatively more attractive if and only if the reward to risk ratio of fixed rate leverage is greater than that of variable rate leverage where risk is measured by standard deviation. When E-bar is zero, an increase (decrease) in either the expectation or variance of the rate of return on assets makes fixed (variable) rate loans more attractive if and only if the reward to risk ratio of fixed rate leverage is greater than that of variable rate leverage where risk is measured by total variance. Similarly, when E-bar is zero, an increase (decrease) in E-bar makes fixed (variable) rate loans more attractive if and only if the reward to risk ratio of fixed rate leverage is greater than that of variable rate leverage where risk is measure by non-diversifiable variance. If initial wealth is large, an increase in μ, σ_a^2, or E-bar causes fixed rate debt to be unambiguously preferred to variable rate debt. This last result is an artifact of the implicit assumption of constant absolute risk aversion.

The Debt-Constrained Case

If the level of debt is determined exogenously, that is the same amount will be borrowed independent of the type of financing, then the fixed rate loan will be preferred if and only if

(6) $k < kbar + \phi/2 * [D\sigma_k^2 - 2A\sigma_{ak}]$.

This condition simply says borrowers will prefer fixed rate loans if and only if the expected effective variable rate plus an appropriate risk premium is greater than the effective fixed rate. Another way to interpret this condition is that fixed rate loans will be preferred if the expected difference in costs is less than the risk premium or

(6a) $k - kbar < \phi/2 * [D\sigma_k^2 - 2A\sigma_{ak}]$.

Condition (6a) will hold in the unconstrained case if D1* happens to equal D2*. It is clear from (6a) that variable rate loans will be preferred only under two conditions. Either the expected interest costs of variable rate loans must be lower than the fixed rate, or, if they are greater, the covariance must be sufficiently positive. The latter condition, a sufficiently positive covariance between rates of return on assets and interest costs is not supported by data presented in the appendix.

Choice When Fixed and Variable Rate Debt Instruments Are Not Mutually Exclusive

When the farmer can choose to allocate debt between both fixed and variable instruments, his objective function can be written as

(7) $\max_{Df, Dv} CE(\pi) = A\mu - Dfk - Dvkbar - (\phi/2)*(A^2\sigma_a^2 + D_v^2\sigma_k^2 - 2AD\sigma_{ak})$,

where Df and Dv refer to the level of fixed and variable rate debt,
respectively, and all other variables are as defined earlier. The first order
conditions for (7) are

(7a) Df* = $(\mu - k)/(\phi\sigma_a^2)$ - E-bar - Dv*, and

(7b) Dv* = $[(\mu - k)*(\sigma_{ak}/\sigma_a^2) - \phi * (k - kbar)]/[\phi(\sigma_k^2 - 2\sigma_{ak})$.

Comparing (7a) and (7b) to (1a), it is clear that Df* + Dv* = D1*. Thus,
optimal total debt/assets in the case of continuous choice is equal to optimal
debt/assets when only fixed rate loans are available. This is true because the
fixed rate debt acts like a risk free asset in determining portfolio weights.
From (7b) and (1a) conditions can be derived for an interior solution to (7),
i.e., when both types of loans will be used, and for specialization in
financing. An interior solution will obtain if

(8)   0 < (k - kbar) < $(\mu - k) * [\sigma_k^2 - 3\sigma_{ak}]/\sigma_a^2 - \phi E\text{-bar} * [\sigma_k^2 - 2\sigma_{ak}]$;

and the expected utility maximizing level of assets/debt will be the same as in
the case where only fixed rate loans are available. Thus, in the case where
both types of financing may be used simultaneously, (8) places a restriction on
the premium charged for fixed rate lending if both types of financing are to be
used. Specialization in variable rate instruments will occur when the second
inequality in (8) is violated; specialization in fixed rate instruments will
occur when

(9) $-(\mu - k) * \sigma_{ak}/\sigma_a^2 \geq (k - kbar)$.

Table 3 presents comparative static results for this model. Although the sum of variable and fixed rate debt behaves as fixed rate debt behaved in the previous model, the demand for each type of debt may be more or less responsive to changes in exogenous variables because of the possibility of refinancing existing debt with the alternative type of instrument. Thus, fixed (variable) rate borrowing is responsive to kbar (k) in this model where it wasn't in the discrete choice version. Simultaneously, the demand for variable rate debt is now independent of wealth where it wasn't before. These results are important because they offer testable hypotheses as to which representation of demand for debt is most appropriate. Indirect evidence from Federal Reserve Bank surveys of agricultural lenders on the refinancing experiences of bankers during the interest rate plunge of 1986 indicates this kind of shifting may be important.

Using the Results

To better understand the implications of the theoretical results developed thus far three simulations are presented. The illustrated results include conditions under which a farmer would choose (1) to borrow more under fixed or variable rate regimes, (2) would prefer each type of financing when debt is not exogenously constrained, and (3) would prefer each type of financing when debt is constrained. To perform these simulations, it was necessary to choose levels for the exogenous variables. The appendix discusses assumptions underlying the simulations presented in all figures. SASGRAPH PROC GCONTOUR was used to create the figures discussed in this section.

The panels of Figure 1 show iso-premium combinations of (μ - k) and (a) σ_a^2, (b) ϕ, and (c) the correlation coefficient (ρ) between the rate of return on assets and the effective cost of variable rate financing. For example, the

iso-premium (k - kbar) line marked zero maps all combinations of (μ - k) and other parameters for which a farmer with the assumed levels of wealth and risk aversion would borrow the same amount under both fixed and variable rates when the premium on fixed rate loans is set at zero. For all panels a movement to the right (left) from an iso-premium line, holding the premium charged constant, implies the farmer will choose to borrow more under fixed (variable) rate financing.

It is clear from panels 1a and 1b, starting on an iso-premium line and holding all other factors constant, the expected reward from debt financing must increase as σ_a^2 or ϕ increase if the choice of financing is to remain inconsequential to the amount borrowed. If the expected reward does not increase, the borrower will choose to borrow more under fixed than under variable rate financing. If the expected reward increases and other factors remains unchanged, the borrower will choose to borrow more under variable than under fixed rate financing. Panel 1c shows that the response to changes in σ_{ak} depends on its initial value. When ρ is negative the responsive tends to be quite small, but as ρ becomes more positive, any change in (kbar - k) will be inadequate to maintain equal levels of borrowing under both types of financing.

The panels of Figure 2 illustrate expected-utility preference regions for fixed and variable rate debt for different combinations when the level of indebtedness is not externally constrained. The shaded areas in the panels of Figure 2 show the regions where variable rate financing will be preferred under the assumptions listed in the appendix. The panels of Figure 2 show regions of preference between fixed and variable rate financing for combinations of (μ - k) and (a) (μ - kbar), (b) ϕ, and (c) ρ.

Variable rate financing, in general, is preferred when (1) the premium for fixed rate financing is sufficiently large, (2) ϕ and $(\mu - k)$ are sufficiently low, or (3) ρ is sufficiently positive. Of these conditions, available evidence indicates that (1) varies with the term structure of interest rates and competition among lenders, (2) probably holds for some portion of farmers and (3) is unrealistic.

The panels of Figure 3 show iso-premium $(k - \bar{k})$ combinations of $(\mu - k)$ and (a) σ_a^2, (b) ϕ, and (c) ρ. These figures are very similar to the panels of Figure 1. Again, for all panels a movement to the right (left) from an iso-premium line, holding the premium charged constant, implies the farmer will prefer to borrow a pre-determined amount at fixed (variable) rate financing. Note the contours of the iso-premia maps is quite different here from those of Figure 1, adding support to the conclusion that choosing to borrow more under a particular type of financing is not the same as higher expected utility from that type of borrowing.

Concluding Comments

Conditions have been derived for four important aspects of the choice between fixed and variable rate financing. These conditions include (1) when the optimal debt (asset) level will be greater under each type of financing, (2) when each type of financing will be preferred if the choice is mutually exclusive and the level of debt is not otherwise constrained, (3) when each type of financing will be preferred if the choice is mutually exclusive and there exists an exogenous constraint on the level of indebtedness and (4) when each type of financing would be preferred/used simultaneously when the choice is not mutually exclusive.

The results enhance the understanding gained from previous research of the fixed/variable rate financing decision. In particular, variable rate financing

does not universally lower debt-carrying capacity or the level of investment as LaDue and Leatham suggest, nor are there specific (positive) premium levels for fixed rate over variable rate financing loans below which all farmers will prefer fixed rate loans as Leatham and Baker imply.

Results also suggest several explanations for observed changes in the relative shares of fixed and variable rate lending. These hypotheses are summarized by the comparative statics presented in Tables 2a, 2b, and 3. The desirability of fixed rate lending relative to total lending should increase as the variability of interest rates increase, and, among well-off farmers, if the profitability of investment increases, wealth increases, or the variability of the return on assets increases. Among poorer farmers, the desirability of fixed rate financing should depend on various measures of the reward to risk ratios.

These results have important implications for lenders and policy makers. Use of variable rate loans is one strategy available to lenders to control interest rate risk. Knowing the nature of the demand curves they are facing for different types of loans, can help lenders maximize profit while efficiently managing interest rate risk. In doing so, they must set the premia on fixed rate loans and forgo some level of expected profit. Frequently, however, these premia are negative for short-term loans of equal quality originated by the Farm Credit System (Collender). In light of the results of this research, such pricing appears anomalous, but may indicate nonprice rationing of fixed rate loans to customers based on quality or other considerations. This last observation should be of interest to policy makers as it may be an indicator of market power or other market imperfection.

It should be noted that the results presented here solve only half the problem. As in any market, observed prices and quantities of fixed and

variable rate loans are determined by both supply and demand. This paper has focused on the demand side of the market. The supply of loans either in aggregate or to any particular borrower is not perfectly elastic. Thus, observed variation in the proportions of loans made at fixed and variable rates will, of course, not be entirely explained by induced shifts in demand. Another caveat worth noting relates to the calculation of the expected effective rate on variable rate loans. This should not be confused with the initial, often artificially low, 'teaser' rate. The expected effective rate on a new variable rate loan is a function of the initial rate, the term structure of interest rates, the expected holding period (often less than the contractual maturity), and any expected costs that may be incurred because of liquidity problems from rising interest rates. At least some of these elements are difficult or impossible to observe directly.

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Appendix: Base Assumptions for Simulations and Supporting Data

The simulations presented in Figures 1, 2, and 3 are based on the following assumptions. The initial level of equity E-bar is 1. The variance of the annual rate of return on assets is .003. The variance of the effective annual interest rate on debt is .002. The correlation coefficient between the annual rate of return on assets and the interest rate is -.5. The expected difference between the rate of return on assets and the fixed rate debt is 1 percent. The premium charged for fixed rate debt is 0.4 percent. Finally, the risk aversion coefficient is 1. Since initial wealth is one, the risk aversion coefficient can be interpreted as relative risk aversion. Most of these figures (risk aversion being the exception) are based on historical data as presented in Table A1.

Table A1: Historical Means, Variances and Correlations of Relevant Variables

	cost of debt		rate of return on assets	
	nominal	real	income	total
Mean:	.066	.035	.041	.040
Variance:	.0002	.002	.0004	.003

Correlations:	nominal	real
total ROA	-.44	-.49
income ROA	-.31	-.56

Source: Computed from Melichar, 1987.

TABLE 1: FACTORS AFFECTING DEMAND FOR FIXED AND VARIABLE RATE DEBT - DISCRETE CHOICE MODEL

derivative with respect to factor of:

factor	D_1^*	D_2^*
μ	$1/\phi\sigma_a^2$	$1/\phi\sigma_e^2$
k (k-bar)	$-1/\phi\sigma_a^2$	$-1/\phi\sigma_e^2$
σ_a^2	$\dfrac{-(\mu - k)}{\phi\sigma_a^4}$	$\dfrac{-(\mu - \text{k-bar})}{\phi\sigma_e^4} - \dfrac{\text{E-bar}(\sigma_k^2 - \sigma_{ak})}{\sigma_e^4}$
σ_k^2	0	$\dfrac{-(\mu - \text{k-bar})}{\phi\sigma_e^4} + \dfrac{\text{E-bar}(\sigma_a^2 - \sigma_{ak})}{\sigma_e^4}$
σ_{ak}	0	$\dfrac{\text{E-bar}}{\sigma_e^2}$
E-bar	-1	$\dfrac{-(\sigma_a^2 - \sigma_{ak})}{\sigma_e^2}$
ϕ	$\dfrac{-(\mu - k)}{\sigma_a^2 \phi^2}$	$\dfrac{-(\mu - \text{k-bar})}{\sigma_e^2 \phi^2}$

TABLE 2: Factors Affecting Preference for Fixed and Variable Rate Loans --
Discrete Choice Model

Factor	Change in (5b) with Respect to Factor $[\partial(5b)/\partial factor]$

μ

$$\frac{(\mu - k)}{\phi\sigma_a^2} - \frac{(\mu - kbar)}{\phi\sigma_e^2} + \text{E-bar} \left[\frac{\sigma_k^2 - \sigma_{ak}}{\sigma_e^2} \right]$$

E-bar

$$\frac{(\mu - kbar)(\sigma_a^2 - \sigma_{ak})}{\sigma_e^2} - (\mu - k)$$

$$+ \frac{2\text{E-bar}}{\sigma_e^4} [(\sigma_a^2 - \sigma_{ak})^2\sigma_k^2 + 2(\sigma_a^2 - \sigma_{ak})(\sigma_k^2 - \sigma_{ak})\sigma_{ak} + (\sigma_k^2 - \sigma_{ak})^2\sigma_a^2]$$

ϕ

$$- \frac{(\mu - k)^2}{2\phi^2\sigma_a^2} + \frac{(\mu - kbar)^2}{2\phi^2\sigma_e^2}$$

σ_a^2

$$- \frac{(\mu - k)^2}{2\phi\sigma_a^4} + \frac{(\mu - kbar)^2}{2\phi\sigma_e^4} - \text{E-bar} \frac{(\sigma_k^2 - \sigma_{ak})}{\sigma_e^2}$$

$$+ \frac{\text{E-bar}^2}{\sigma_e^6} (\sigma_k^2 - \sigma_{ak}) \left[(\sigma_k^2 - \sigma_{ak})^2 + (\sigma_a^2 - \sigma_{ak})(\sigma_k^2 + \sigma_{ak}) \right]$$

$$\sigma_k^2 \qquad \frac{(\mu - \text{kbar})^2}{2\phi\sigma_e^4} + \text{E-bar} \; \frac{(\sigma_a^2 - \sigma_{ak})}{\sigma_e^2}$$

$$+ \; \frac{\text{E-bar}^2}{\sigma_e^6} (\sigma_a^2 - \sigma_{ak}) \left[(\sigma_a^2 - \sigma_{ak})^2 + (\sigma_k^2 - \sigma_{ak})(\sigma_a^2 + \sigma_{ak}) \right]$$

$$\sigma_{ak} \qquad - \frac{(\mu - \text{kbar})^2}{\phi\sigma_e^4} + \frac{\text{E-bar}(\mu - \text{kbar})(\sigma_a^2 - \sigma_k^2)}{\sigma_e^4} - \frac{2\text{E-bar}^2\sigma_{ak}}{\sigma_e^2}$$

Table 2a: Signs of Changes in (5b) with Respect to Exogenous Factors[1]

Factor	Unconditional	Conditioned on: E-bar = 0	E-bar large
μ	?	$\begin{smallmatrix}<\\=\\>\end{smallmatrix}\ 0$ as $\dfrac{(\mu - k)}{\sigma_a^2} \begin{smallmatrix}<\\=\\>\end{smallmatrix} \dfrac{(\mu - \bar{k})}{\sigma_e^2}$	> 0
E-bar	?	$\begin{smallmatrix}<\\=\\>\end{smallmatrix}\ 0$ as $\dfrac{(\mu - k)}{\sigma_a^2 - \sigma_{ak}} \begin{smallmatrix}<\\=\\>\end{smallmatrix} \dfrac{(\mu - \bar{k})}{\sigma_e^2}$	> 0
ϕ	$\begin{smallmatrix}<\\=\\>\end{smallmatrix}\ 0$ as $\dfrac{(\mu - k)}{\sigma_a} \begin{smallmatrix}<\\=\\>\end{smallmatrix} \dfrac{(\mu - \bar{k})}{\sigma_e}$		
σ_a^2	?	$\begin{smallmatrix}<\\=\\>\end{smallmatrix}\ 0$ as $\dfrac{(\mu - k)}{\sigma_a^2} \begin{smallmatrix}<\\=\\>\end{smallmatrix} \dfrac{(\mu - \bar{k})}{\sigma_e^2}$	> 0
σ_k^2	> 0		
σ_{ak}	< 0 if $- \dfrac{\phi(\sigma_a^2 - \sigma_k^2)^2}{4\sigma_e^2} > \sigma_{ak}$,	< 0	?
	otherwise ambiguous		

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[1]A positive partial derivative indicates that as the exogenous factor increases, fixed rate loans become relatively more attractive. A negative partial derivative indicates that as the exogenous factor increases, variable rate loans become relatively more attractive.

TABLE 3:   FACTORS AFFECTING DEMAND FOR FIXED AND VARIABLE RATE DEBT - CONTINUOUS CHOICE MODEL

derivative with respect to factor of:[1]

factor	D^*	D_v^*
μ	$1/\phi\sigma_a^2$	$\dfrac{\sigma_{ak}}{\phi\sigma_a^2(\sigma_k^2 - 2\sigma_{ak})}$
k	$1/\phi\sigma_a^2$	$\dfrac{\sigma_a^2 - \sigma_{ak}}{\phi\sigma_a^2(\sigma_k^2 - 2\sigma_{ak})}$
k-bar	0	$\dfrac{-1}{\phi(\sigma_k^2 - 2\sigma_{ak})}$
σ_a^2	$\dfrac{-(\mu - k)}{\phi\sigma_a^4}$	$\dfrac{-(\mu - k)\sigma_{ak}}{\phi\sigma_a^4(\sigma_k^2 - 2\sigma_{ak})}$
σ_k^2	0	$\dfrac{-[(\mu - k)\sigma_{ak} + (k - \text{k-bar})\sigma_a^2]}{\phi\sigma_a^2(\sigma_k^2 - 2\sigma_{ak})^2}$
σ_{ak}	0	$\dfrac{(\mu - k)\sigma_k^2}{\phi\sigma_a^2(\sigma_k^2 - 2\sigma_{ak})^2}$
E-bar	-1	0
ϕ	$\dfrac{-(\mu - k)}{\sigma_a^2\,\phi^2}$	$\dfrac{-[(\mu - k)\sigma_{ak} + (k - \text{k-bar})]}{\phi^2\sigma_a^2(\sigma_k^2 - 2\sigma_{ak})}$

[1]The derivative of D_f^* with respect to each factor is simply the derivative of D^* less the derivative of D_v^*.

Figure 1a: Iso − premium contours for equality in debt demanded −−− expected return from leverage versus σ_a^2.
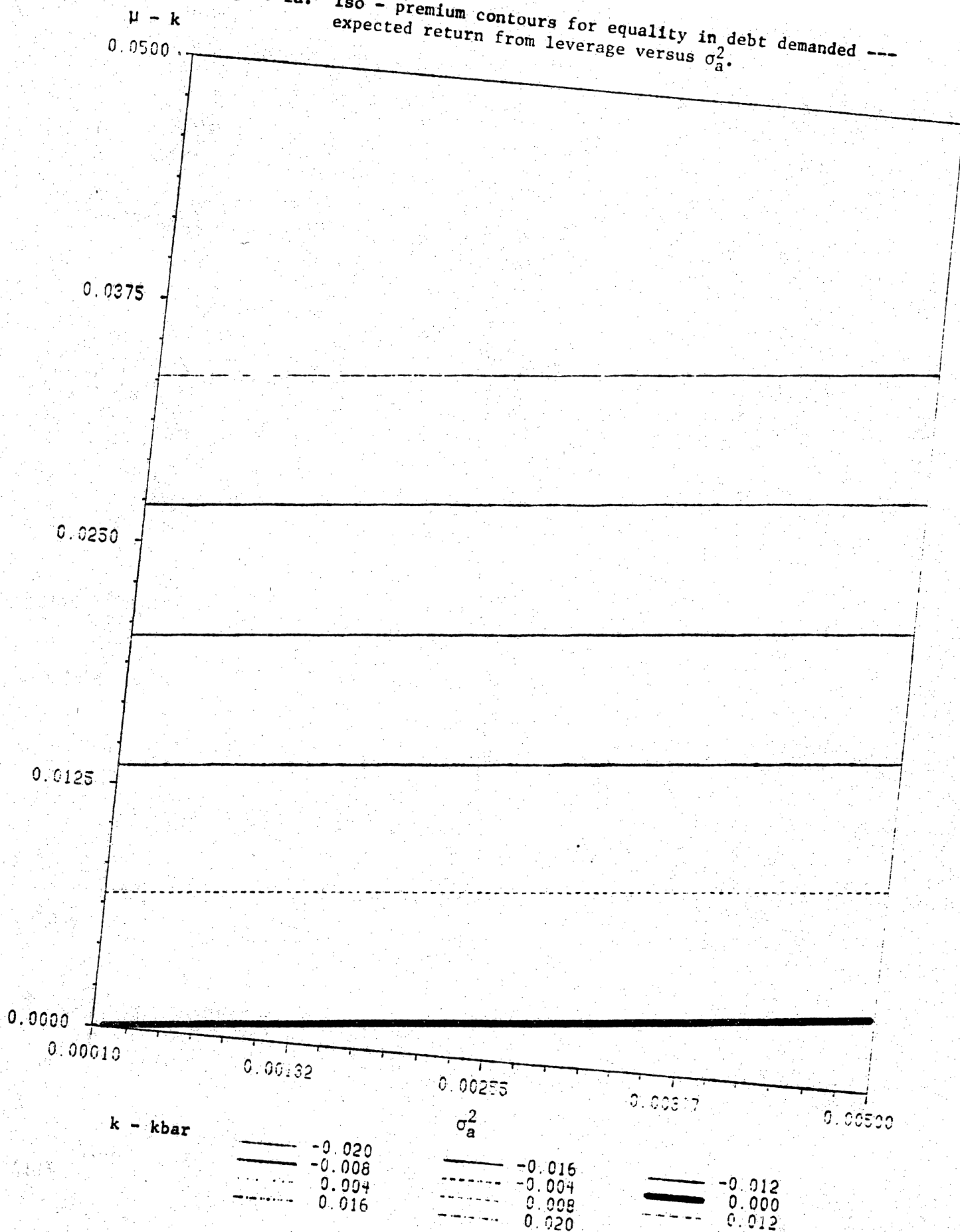
Figure 1b: Iso – premium contours for equality in debt demanded ---
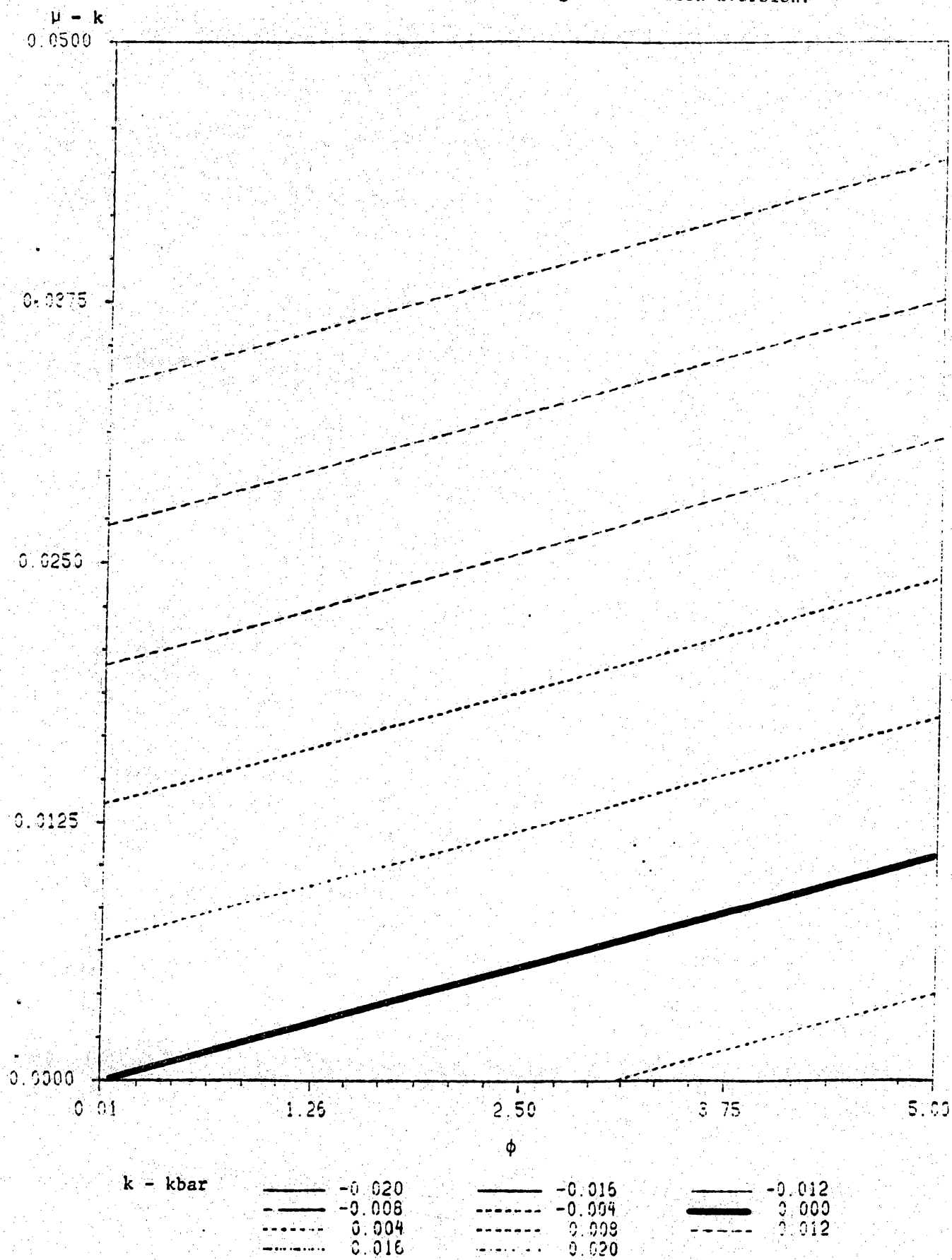expected return from leverage versus risk aversion.

Figure 1c: Iso - premium contours for equality in debt demanded ---
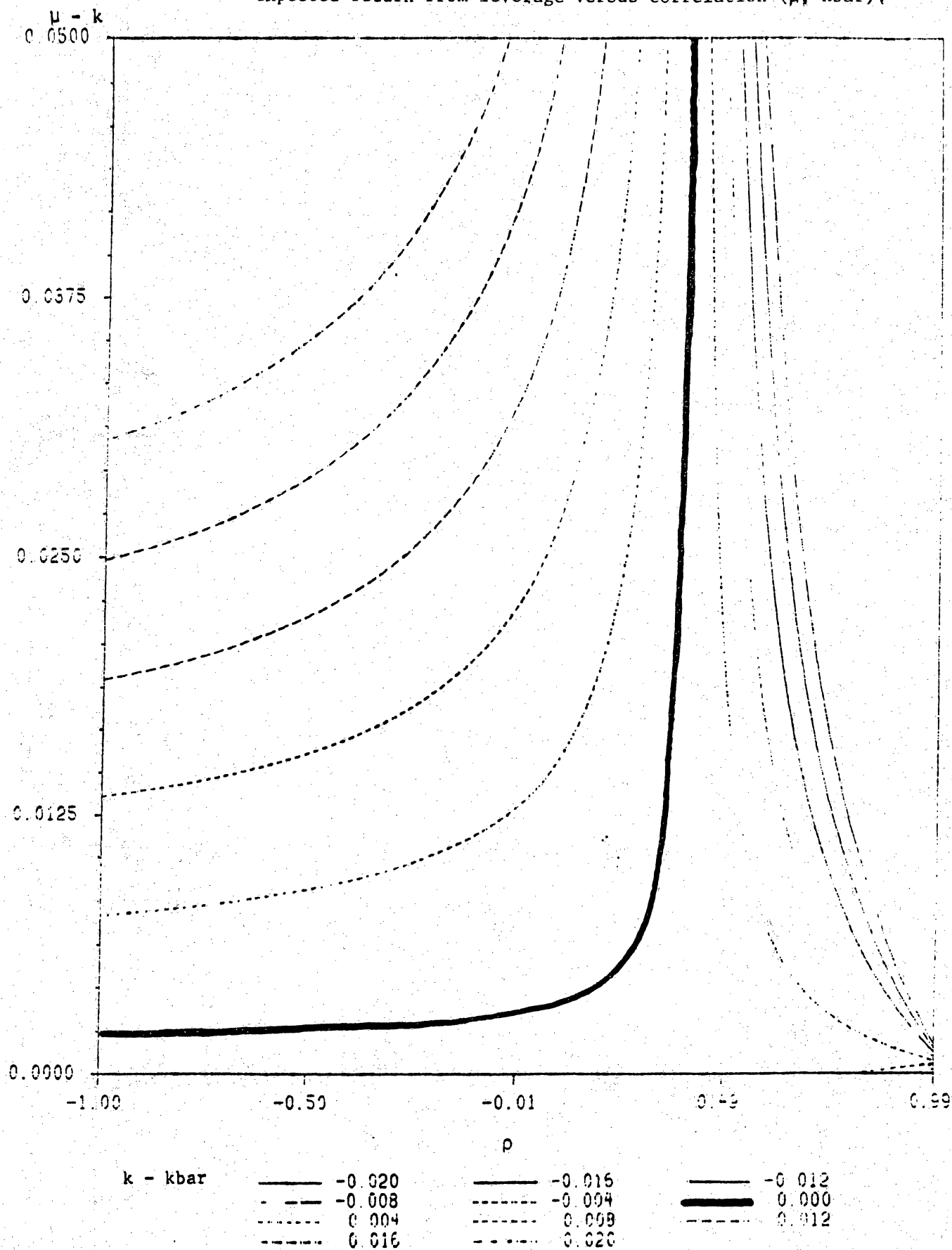expected return from leverage versus correlation (μ, kbar).

μ - k

0.0500

0.0375

0.0250

0.0125

0.0000

-1.00   -0.50   -0.01   0.49   0.99

ρ

k - kbar						
————	-0.020	————	-0.016	————	-0.012	
· ——	-0.008	········	-0.004	▬▬▬▬	0.000	
········	0.004	········	0.008	— — —	0.012	
—··—	0.016	— ·· —	0.020			

Figure 2a: Region of preference for variable rate loans (shaded area) --- expected return for fixed versus variable rate average.
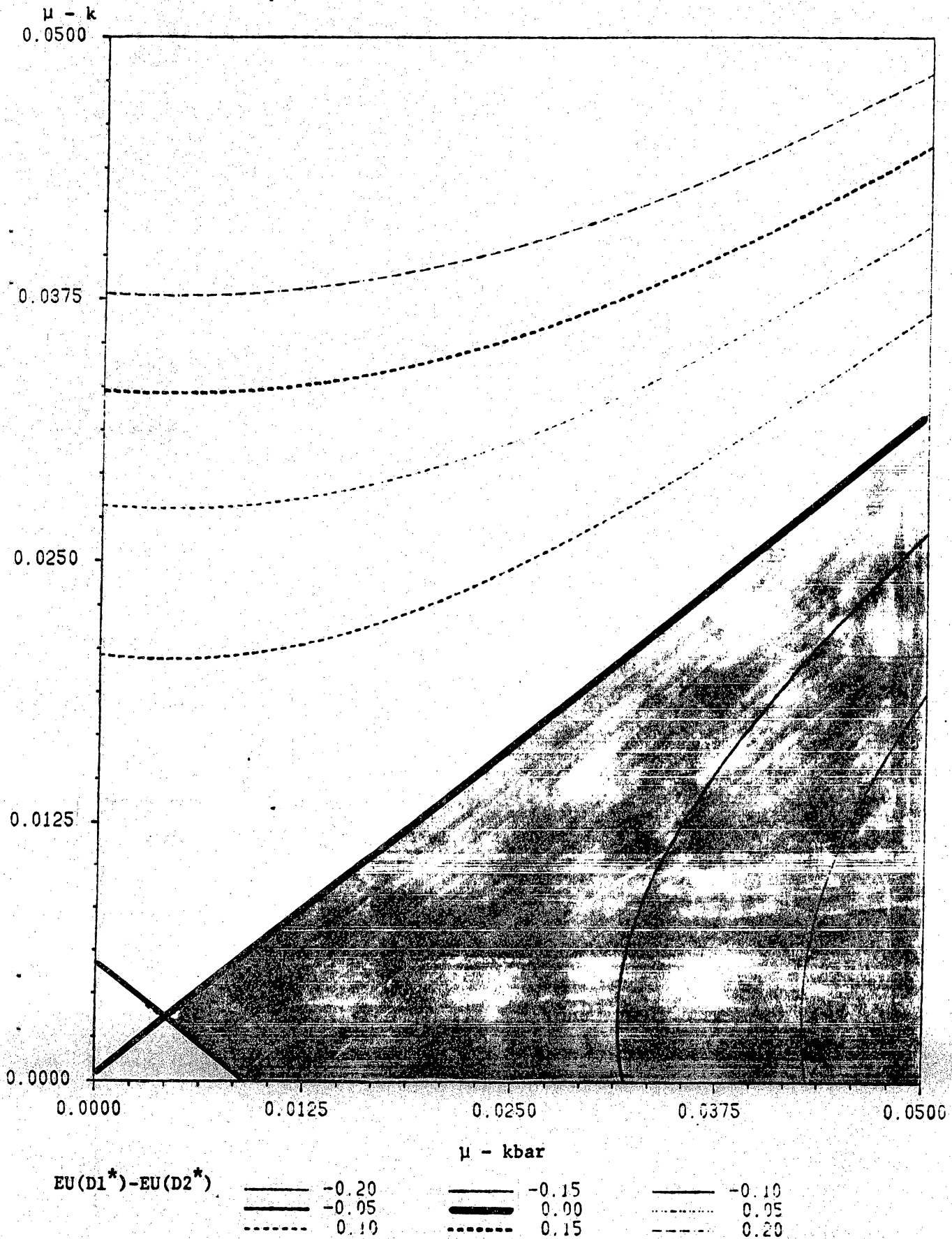
Figure 2b: Region of preference for variable rate loans (shaded area) ---
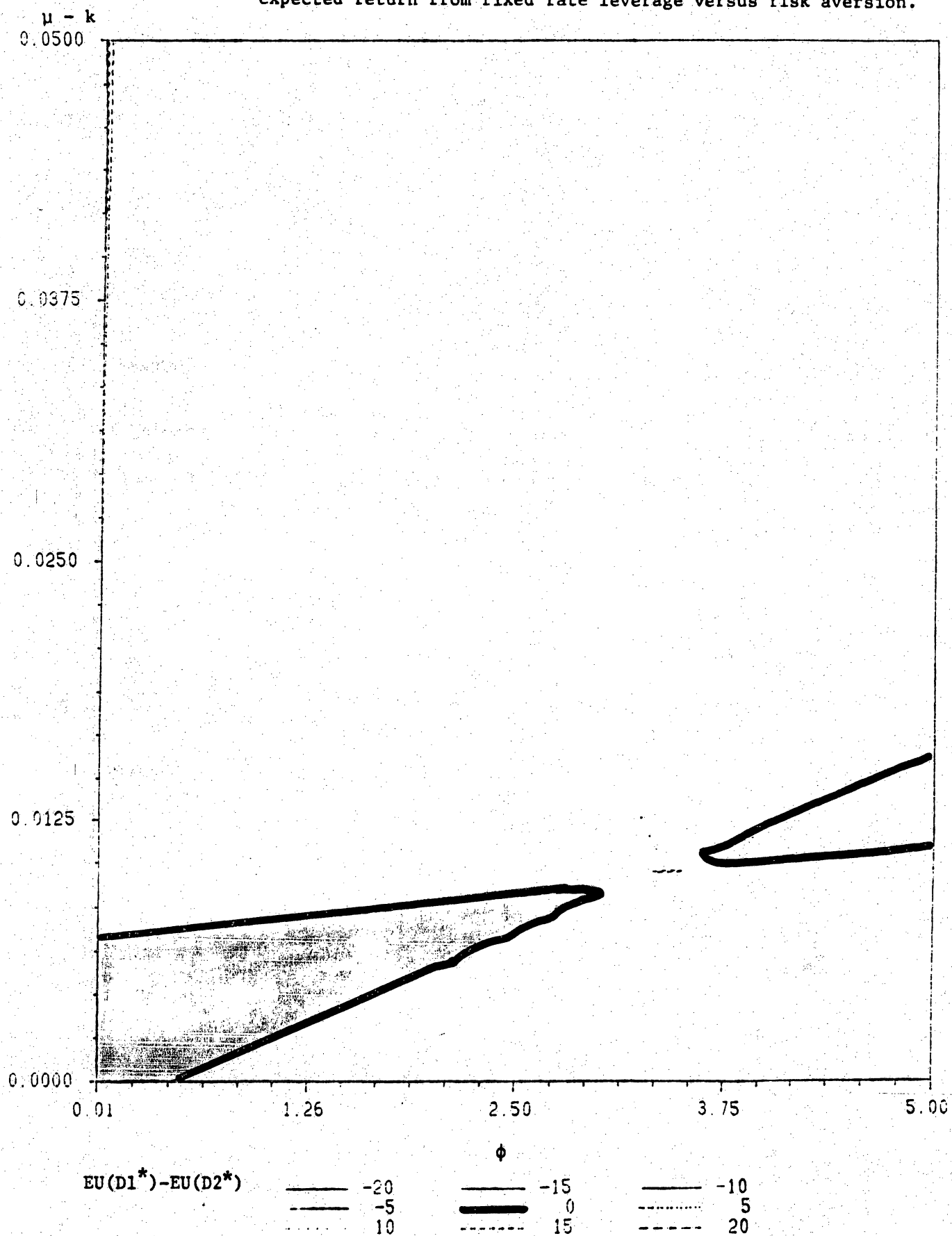expected return from fixed rate leverage versus risk aversion.

Figure 2c: Region of preference for variable rate loans (shaded area)—expected return on fixed rate leverage versus correlation (μ, kbar).
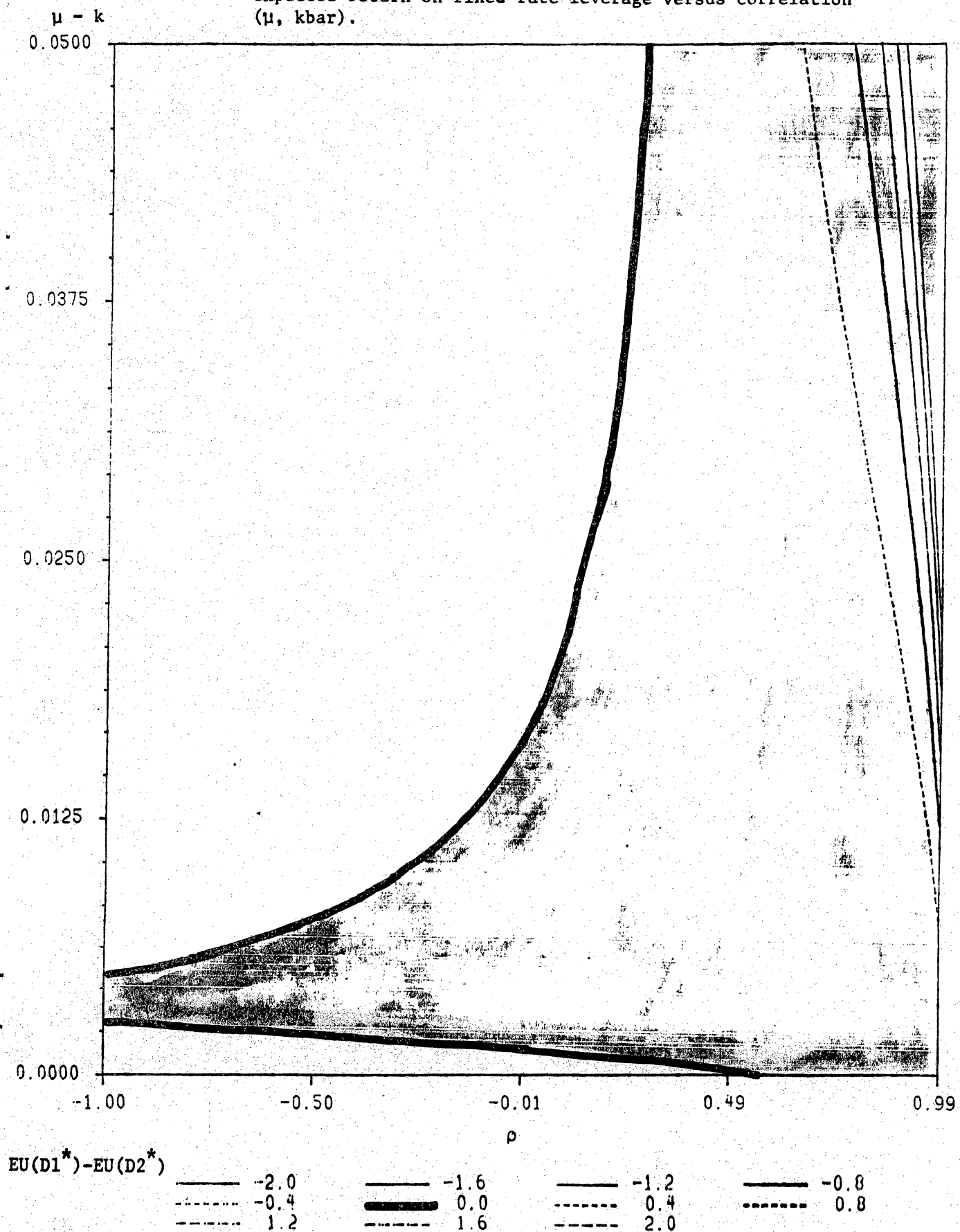
Figure 3a: Iso – premium contours for indifference in financing when debt is constrained --- expected return to leverage versus σ_a^2.
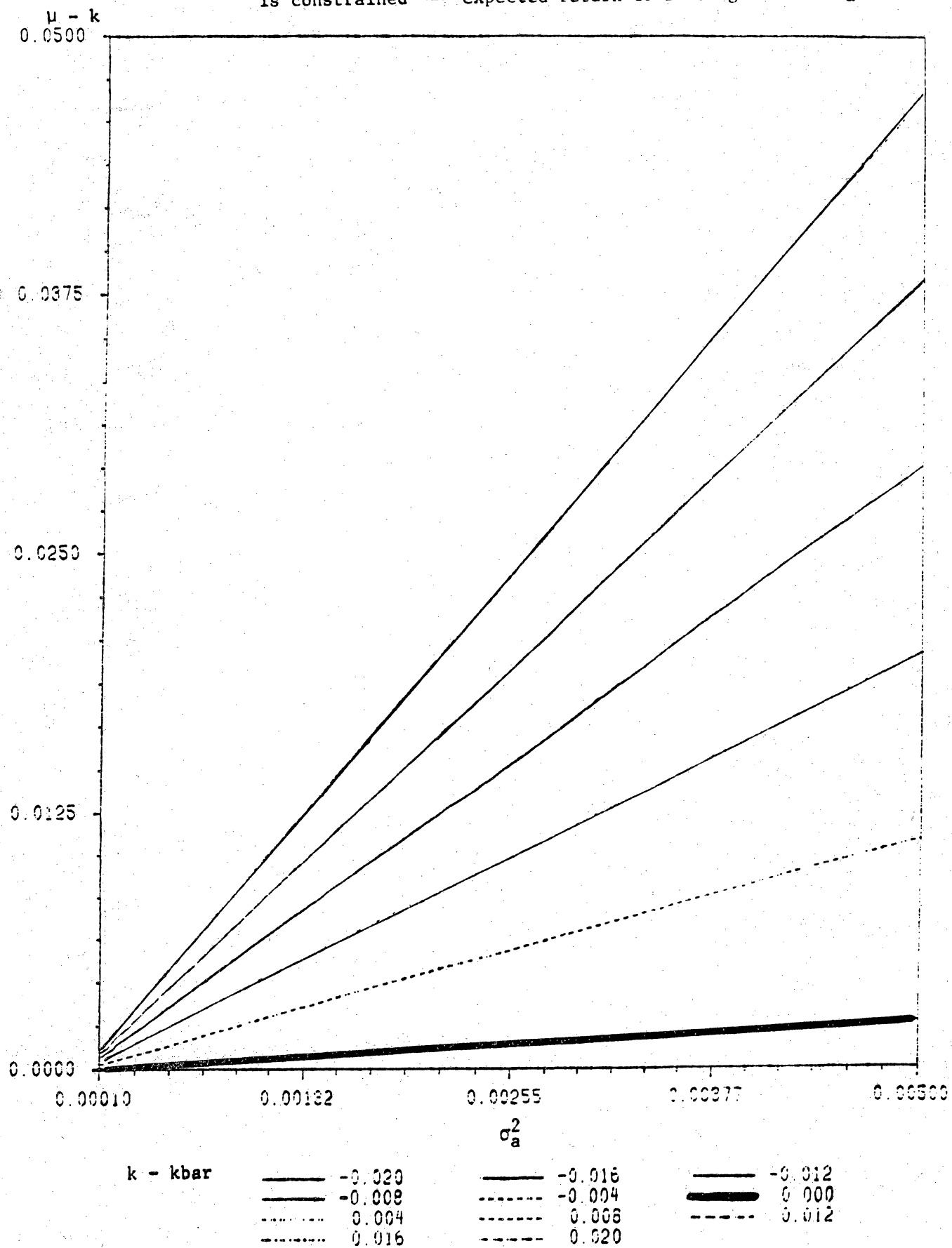
Figure 3b: Iso - premium contours for indifference in financing when debt is constrained --- expected return to leverage versus risk aversion.
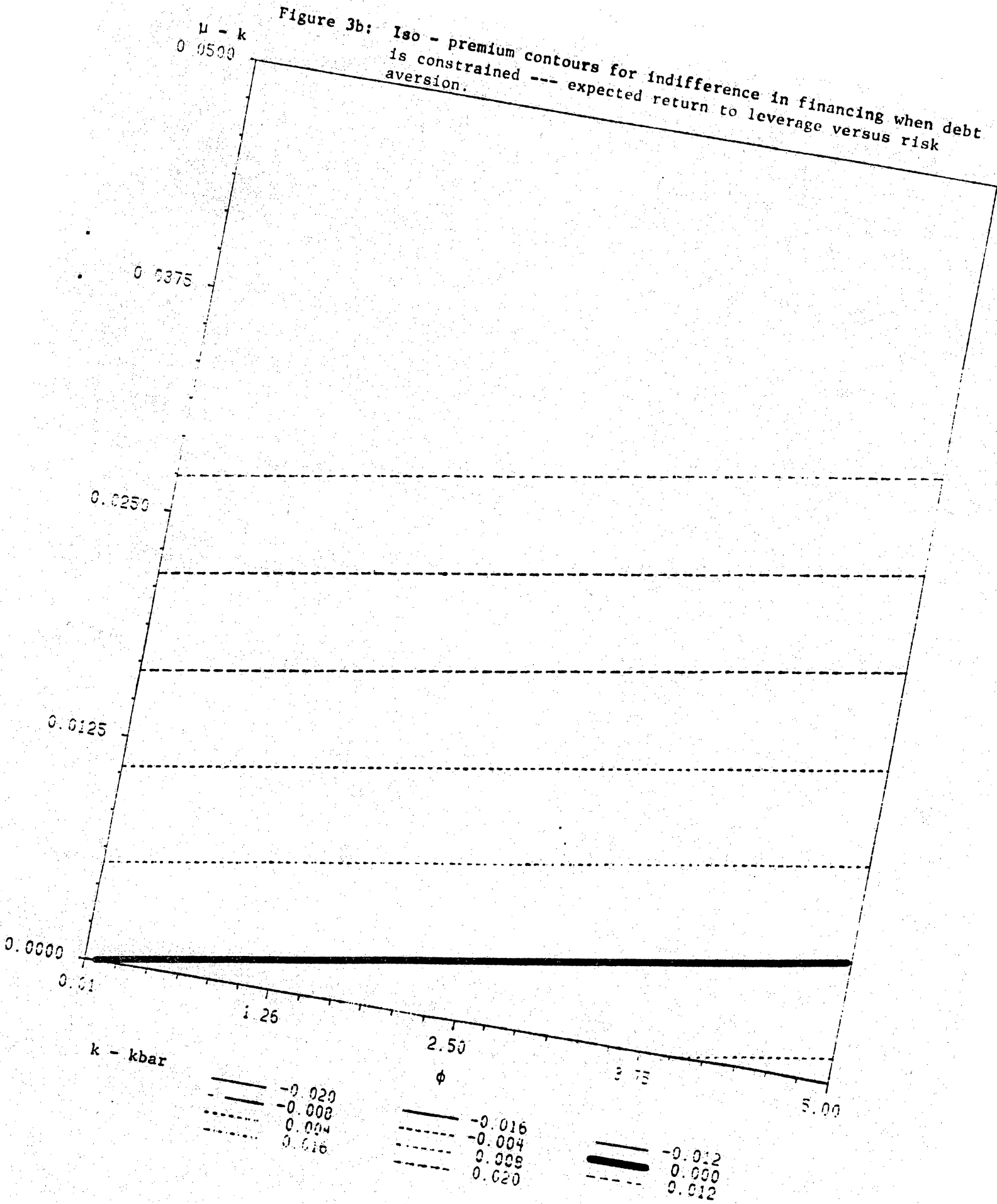
Figure 3c: Iso – premium contours for indifference in financing when debt is constrained ─── expected return to leverage versus correlation (μ, kbar).