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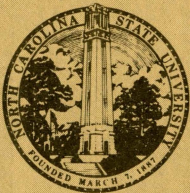
TESTING THE LIFE-CYCLE HYPOTHESIS ON PANEL DATA
USING DETAILED CONSUMPTION DIARIES AND INCOME BASED ON TAX RECORDS

Knut Anton Mork
Owen Graduate School of Management
Vanderbilt University

V. Kerry Smith
Department of Economics and Business
North Carolina State University

Faculty Working Paper No. 111

Revised October 1987



DEPARTMENT OF ECONOMICS AND BUSINESS
NORTH CAROLINA STATE UNIVERSITY
RALEIGH, NORTH CAROLINA

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1. Introduction

Introduced in the pioneering research of Friedman [1957] and Modigliani (Ando and Modigliani [1963]), the permanent income/life-cycle hypothesis of consumption is well known. Today, most economists have joined this hypothesis with that of rational expectations to describe how households consider their future income prospects. Despite its longstanding presence, however, support for the hypothesis is mixed, and, as a consequence, tests of its implications abound in the literature. We can trace this renewal in attention to the issues involved in modeling consumptions' responsiveness to income to Hall [1978] and Sargent's [1978] tests of the joint hypotheses (i.e., life-cycle/rational expectations) using aggregate time series data.

For the most part, the conflicting empirical results arise in macro time series studies (Flavin [1981], Mankiw [1982], Hayashi [1982], Bernanke [1985], Blinder and Deaton [1985], Christiano, Eichenbaum and Marshall [1987] are a few examples) where aggregation obscures the predicted relationships between consumption and permanent income. This follows because the central hypotheses are based on micro models of the intertemporal allocation decisions of households (Deaton [1986]). Consequently, some important recent studies have sought to use data at the household level. Unfortunately, tests of the hypothesis require a temporal history of households' consumption choices and their incomes. Complete panels with these records are extremely limited.

The majority of studies to date have not had access to such complete records. They have relied instead on a few available panels, primarily the University of Michigan's Panel Study of Income Dynamics (PSID). Hall and Mishkin [1982], Zeldes [1985], and Altonji and his collaborators (Altonji

and Siow [1987], Altonji, Martins and Siow [1986]) have all worked with these data. Based on interview responses, the PSID panel includes only food consumption, income, household characteristics, and some factors important to the determination of income (Bernanke's [1984] study is also based on a part of the Michigan survey but a different panel associated with expenditures on automobiles).

There are two problems with these studies: incomplete expenditure records, so that food expenditures (or automobile expenditures in the case of Bernanke) must serve as the indicator for total consumption's responsiveness to permanent and transitory income, and measurement errors in the income statistics (Duncan and Hill [1984], Altonji, Martin and Siow [1986]). Of course, if we are willing to impose structure on the error process, corrections can be reflected in the analysis. Similarly, incomplete expenditure records can be overcome with restrictive, implicit (Hall and Mishkin) or explicit (Browning [1987]) maintained hypotheses. But these decisions simply expand the set of conditions maintained as part of the tests and must be recognized in evaluating the conclusions derived from them.

The only exception to this pattern was Hayashi's [1985] study of a panel of Japanese households. Based on complete expenditure records, this study did have data for 11 commodity groups, disposable income, and one-quarter forward expectations for these variables. Unfortunately, the length of the panel is quite short--four quarters. While this allowed Hayashi to use a simple treatment of relative price effects, it does raise the prospect of problems induced by households' inventory holding decisions for some of the goods involved.

Thus, there remain questions even with the most detailed of the household level studies conducted to date. As a rule, these questions arise because incomplete data have required additional assumptions that are then a part of maintained conditions that must be recognized in interpreting the tests' findings on the responsiveness of consumption to income.

The purpose of this paper is to explore the implications of these types of assumptions using what we believe are the best data yet available for testing the permanent income hypothesis. They come from a panel constructed by the Central Bureau of Statistics of Norway and combine detailed expenditure information (i.e., expenditures and prices for 28 commodity categories) with income and wealth data from the Government's tax files for the households involved. The consumption data were collected from diaries with followup interviews. The expenditure data form the basis for construction of Norway's Consumer Price Indices. A previous attempt to use this type of data for tests of the life-cycle hypothesis was made by Bjørn [1980] but without use of the stochastic implications derived by Hall.

Our analysis of the hypothesis incorporates a progressive refinement in the level of detail of the information used in testing the model. There are four aspects of these refinements. They examine several of the key issues that would be expected to influence tests of the hypothesis, including: the effects of aggregation of all expenditures comprising consumption at the household level; description of the temporal character of the income process in relation to the available income records; adjustment for the influence of relative price changes in the measurement of consumption; and, finally, recognition of the effects of commodity durability on the static demand models used to account for the influence of relative price changes.

Our findings help to resolve the conflicts in the literature to date. With progressive refinement, we find reductions in the evidence of violations to the life-cycle model. Indeed, the results from the final model do not reject the hypothesis and, thereby, highlight the importance of assumptions that had to be made in earlier studies to implement tests with incomplete data.

2. The Norwegian Consumer Expenditure Survey Data

The data used in the analysis were made available to us by the Central Bureau of Statistics of Norway. They contain information on income, consumption expenditures, prices, and some demographic factors for a sample of 418 Norwegian households (see Biørn and Jansen [1980, 1982] for more detail). The sample is a combination of two panels with an equal number of households in each. The first covers 1975 and 1976 and the second 1976 and 1977. In addition to information on expenditures, prices, and demographics in each of two years of each panel, the data include income information for the year preceding the outset of each panel (i.e., 1974 for the 1975-76 panel and 1975 for the 1976-77 group).

The 1975-77 Consumer Expenditure Surveys in Norway were done with rotating samples, so that some of the households observed in 1975 were re-observed in 1976, and similarly for 1976 and 1977. No household was observed for all three years.

All consumption expenditures are recorded and classified into 28 expenditure categories with corresponding prices. This detailed breakdown permits construction of variable weight aggregate price indexes. For the present analysis we first studied consumption as a whole and then considered

two different five-commodity disaggregations constructed from the original 28.

The Norwegian Consumer Survey data differ from other data sets in the way they have been collected. The primary source of expenditure data is a detailed diary kept by the household itself for an "accounting period" of two weeks. The resulting figures are "blown up" by a factor of 26 to be expressed in units of annual spending. Different households kept these diaries for different accounting periods, but each household's accounting period remains invariant over the two years of the panel. The specific time periods are known for each household.

These diary data were supplemented with information obtained in a follow-up interview at the end of each accounting period. This interview was the source for the available demographic information. It also contained questions about major purchases over the 12 months preceding the interview. Unfortunately, the answers to these questions were used to modify the diary-based expenditure figures before the panel survey was made available to us. Although we are unable to disaggregate the two sources of expenditure data, we know the average share (across households) of the total expenditure for each of the 28 categories that came from the followup interview (and therefore that relate to expenditures for a 12-month period). Knowledge of these shares allowed expenditures to be treated as a weighted average of a two-week "snapshot" and an unweighted average of spending over the last 26 accounting periods.

The data for income, wealth, and taxes are annual averages taken directly from the government's tax file. This file is maintained by the Central Bureau of Statistics and was merged with the expenditure data

using the official person identification numbers (corresponding to Social Security numbers in the United States). Household income (or wealth, etc.) was defined as the sum of the incomes of all the members of the households.

Two income concepts are available. The first is taxable income, which includes both labor and capital income but excludes deductible expenses, notably interest expenses. It also excludes incomes too low to be taxed (Biørn [1976]). The second income concept is the sum of wage income, used as a basis for social insurance taxes and business income. This concept is not subject to personal deductions or exemptions, has no lower limit, and excludes capital income except for the return on investments in unincorporated business. This definition is our preferred income measure net of actual tax payments. The previous version of this paper referred to a definitional upper limit on this income concept. A careful investigation of our data for high-income households and discussions with colleagues in Oslo has convinced us that this limit is nonexistent or, at worst, empirically unimportant in our data.

Prices are taken directly from the Consumer Price Index, since the expenditure categories of our data match exactly with corresponding subcategories of the CPI. We use the two-week specific indices computed by Biørn and Jansen [1980] as interpolations of the monthly CPI components. (Regional price indices are not constructed in Norway.) Even though our panel covers a relatively short period, relative price variation was not trivial. For example, based on fixed-weight aggregate price indexes for the five broad categories we used in one of the models, the percentage changes from 1975 to 1977 relative to the overall fixed-weight CPI indicate reasonably large changes in relative prices among several categories:

Food	+0.2%
Clothing and footwear	-0.2%
Housing, household appliances, heating, and utilities	-1.3%
Transportation and recreation	+0.7%
Other expenditures	+0.8%

Our models reflecting the effects of relative price change are likely to incorporate even greater price variation at the household level because variable weights were used for the price indexes and are household specific.

These data are superior to those of all previous studies in detail and in their prospects for measurement errors. Nonetheless, there are potential problems. Underreporting may arise with the income data, especially the business income portion, which is the basis for income taxation. For this portion, allocation of income to calendar years also may be at the discretion of the taxpayer to some extent, although colleagues in Oslo suggested to us that this problem would have been minimal during our sample period. Second, the income measure and interview-based durable expenditures will not coincide because the question relates to the preceding 12 months. We seek to compensate for this discrepancy by careful modeling of the timing of income and consumption.

Third, the time-series dimension of our panel is short. This limitation affects our modeling of the income process, whereas one first difference is sufficient for estimation of the consumption responses. Finally, neither of the two available income measures includes nontaxable income, primarily from social insurance and other government transfers. Since the Norwegian system of social insurance is quite comprehensive, this

omission is likely to exaggerate the volatility of disposable income.

3. The Model

The basic model used to structure each refinement in the detail used in testing the hypothesis is cast within the same basic structure as the Hall-Mishkin framework. Our model is based on the following observables: the first difference in consumption (overall and the five subcategories in the refined versions of the model), the first difference in income, the first difference in income lagged, and the level of income lagged twice.

Permanent Income With a Consumption Aggregate

The Hall-Mishkin framework assumes the household maximizes an additive separable, dynamic utility function

$$V_t = E_t \sum_{k=t}^T (1+\rho)^{t-k} u(c_k), \quad (1)$$

where c_k is real consumption at time k , T is the household's time horizon (e.g. the known time of death), ρ is the subjective discount rate, and E_t is the expectations operator, conditional on information available at time t . Hall and Mishkin assume the instantaneous utility function is the quadratic

$$u(c_t) = -(1/2)(\bar{c} - c_t)^2. \quad (2)$$

In this case the first-order conditions for maximization of V_t subject to a standard intertemporal budget constraint together with the assumption that the subjective rate of time preference equals a constant real interest rate ($\rho=r$) imply that consumption in each period equals permanent income. Hall and Mishkin model the stochastic part of income as a two-component process. The first component, "permanent income," is a random walk with innovations ϵ_t . The second component, "transitory income," is a moving average of

finite order q with innovations η_t and parameters $\lambda_0, \dots, \lambda_q (\lambda_0=1)$. Under this specification, the innovation in permanent income is $\epsilon_t + \beta_t \eta_t$, where

$$\beta_t = \frac{\sum_{j=0}^q (1+r)^{-j} \lambda_j}{\sum_{k=t}^T (1+r)^{t-k}} \quad (3)$$

Clearly, $0 < \beta_t < 1$.

Hall and Mishkin equate the first difference of consumption to this formula, with three modifications. First they treat β_t as a constant across observations, which amounts to little more than an assumption that the distribution of household age is independent of transitory income. Second, since as we noted earlier they have data for food consumption only, they relate the change in this variable to α times the innovation in permanent income. This scaling factor, α , was interpreted as the slope of the Engel curve for food (relative-price and demographic effects were filtered out in a preliminary regression). Third, they allow for transitory consumption, possibly stemming from preference shocks and assumed independent of permanent consumption, denoted e_t , and derive (4).

$$\Delta c_t = \alpha(\epsilon_t + \beta \eta_t) + e_t \quad (4)$$

The Extension to Many Goods

To develop the permanent income framework for commodity choices in response to relative prices, we replace the instantaneous utility function expressed in terms of c_t with the instantaneous indirect utility function, which we assume has the form

$$v(m_t, p_t) = -(1/2) [c - (m_t - g \cdot p_t) / P_t]^2, \quad (5)$$

where m_t denotes the nominal value of consumption, p_t is an s vector of individual nominal prices, g is an s vector of constants that can be

interpreted as defining the subsistence consumption levels, and P_t a normalizing price factor. The latter is defined such that

$$\ln P_t = \sum_{j=1}^s b_j \ln \tilde{p}_j, \quad \sum_{j=1}^s b_j = 1,$$

where \tilde{p}_j are elements in p for each time period and the constant b_j are marginal budget shares. This model implies an equation with the same form as (4) for overall consumption (a derivation is presented in a technical appendix available from the authors) properly deflated (the appropriate deflator turns out to be a combination of P_t and an overall price index satisfying the Fisher equation). Transitory consumption now arises as the result of time variation in the real cost of subsistence consumption. Using the Hall-Mishkin interpretation, we would assume that $\alpha = 1$ because the model incorporates overall consumption. However, our empirical model allows α to differ from unity as a reflection of the prospect of an asymmetry in the information available to the analyst relative to that available to the households. Households are likely to know more about their future income streams than the econometrician can infer from the available income data by treating them as arising from simple time processes. This interpretation implies that $1 - \alpha$ of what the model identifies as the innovation in permanent income is actually old news to the household. Only the remaining fraction (α) is a true innovation. This formation is permitted by simply retaining (4) as the basis for estimation.

While the distinction between the households' and the econometrician's information is important in general, it is especially relevant to our analysis because it provides one means of reflecting our income measure's

exclusion of transfer payments. This omission should tend to make income appear more volatile than it really is.

The extension of equation (4) for the case of multiple commodities relies on the demand system corresponding to (5) and purges the effects of relative prices. Each demand function offers an "estimate" of consumption. Equations (6) and (7) illustrate; (6) is the linear expenditure equation derived from (5).

$$m_{jt} = g_j p_{jt} + b_j (m_t - g \cdot p_t), \quad j = 1, \dots, s, \quad (6)$$

where m_{jt} is nominal expenditure on good j . Each of these equations implies we can estimate m_t from each expenditure type as in equation (7):

$$(m_{jt} - g_j p_{jt}) / b_j + g \cdot p_t = m_t, \quad j = 1, \dots, s, \quad (7)$$

Thus each estimate should respond in the same way to innovations in permanent income. Letting c_{jt} denote purged components defined by (7) and deflated and allowing for good-specific transitory consumption (e_{jt}), we then obtain (8)

$$\Delta c_{jt} = \alpha(\epsilon_t + \beta \eta_t) + e_{jt}, \quad j = 1, \dots, s. \quad (8)$$

The Timing of Consumption and Income

The timing of the observations on income and spending should also be reflected in the model. Two alternative models of the timing of income information were considered. The first assumes news about income arrivals on each January 1. The second maintains that news arrives continuously at the beginning of each respondent's two-week accounting period.

For the first model, the income process becomes essentially identical to that of Hall and Mishkin. Since our data allow identification of only

one moving-average parameter for transitory income, we set $q = 1$. Our observable income variables then obey (9):

$$\Delta y_t = \epsilon_t + \eta_t - (1-\lambda) \eta_{t-1} - \lambda \eta_{t-2} \quad (9a)$$

$$\Delta y_{t-1} = \epsilon_{t-1} + \eta_{t-1} - (1-\lambda) \eta_{t-2} - \lambda \eta_{t-3} \quad (9b)$$

$$y_{t-2} = \sum_{k=0}^{t-2} \epsilon_k + \eta_{t-2} + \lambda \eta_{t-3} \quad (9c)$$

Observed consumption is a weighted average of a fraction $1-f$ derived from the diary kept during accounting period a in year t and a fraction f from the followup interview covering major purchases during the preceding 26 accounting periods. The first difference of the first component is determined by the change in permanent income from the previous to the current calendar year. The second component is an unweighted average of spending during a accounting periods of this calendar year and $26-a$ of the last. Thus, the change in observed overall consumption can be written as:

$$\begin{aligned} \Delta c_t = & \alpha \{ (1-f)(\epsilon_t + \beta \eta_t) + f[(a/26)(\epsilon_t + \beta \eta_t) \\ & + (1-a/26)(\epsilon_{t-1} + \beta \eta_{t-1})] \} + e_t. \end{aligned} \quad (10)$$

A similar expression can be obtained for each individual good, with commodity-specific f -values and allowance for the commodity-specific transitory-consumption term.

Assume that the accounting periods a are distributed uniformly and independently of the other variables. Assume f is distributed independently as well and redefine the symbol f to denote the mean of this distribution. Then the population covariance matrix for the vector of observables $(\Delta c_t, \Delta y_t, \Delta y_{t-1}, y_{t-2})$ is approximately given in equation (11) (the exact formulae used in the estimation are given in the technical Appendix available from the authors):

$$\Omega = \begin{bmatrix} s^2 & \cdot & \cdot & \cdot \\ \alpha\{(1-f/2)\sigma_\epsilon^2 + [1-f(1-\lambda/2)]\beta^2\eta\} \sigma_\epsilon^2 + 2(1+\lambda^2-\lambda)\sigma_\eta^2 & \cdot & \cdot & \cdot \\ \alpha(f/2)(\sigma_\epsilon^2 + \beta\sigma_\eta^2) & -(1-\lambda)^2\sigma_\eta^2 & \cdot & \cdot \\ 0 & -\lambda\sigma_\eta^2 & \cdot & \sigma^2 \end{bmatrix} \quad (11)$$

The third column is not stated explicitly because, by covariance stationarity, $V(\Delta y_{t-1}) = V(\Delta y_t)$ and $\text{cov}(\Delta y_{t-1}, y_{t-2}) = \text{cov}(\Delta y_t, \Delta y_{t-1}) + \text{cov}(\Delta y_t, y_{t-2})$. Since $V(\Delta c_t)$ and $V(y_{t-2})$ do not help identify important parameters, they are labeled as s^2 and σ^2 , respectively.

Formula (11) contains the familiar orthogonality constraint $\text{cov}(\Delta c_t, y_{t-2}) = 0$. No zero constraint applies to $\text{cov}(\Delta c_t, \Delta y_{t-1})$ as long as $f > 0$. Non-zero f implies that some of the consumption change recorded in the followup interview depends on last year's income news. Of course, this covariance is constrained by the parameters of the model, and the model remains testable via nonlinear constraints.

The information assumptions behind this model are rather extreme. This problem is overcome by the refinement underlying our second model. In this case the model has no zero restrictions as long as $f > 0$. Let ϵ_{at} , η_{at} denote the innovations to the lifetime and transitory income components for accounting period a in year t , respectively. As before, assume the lifetime component is a random walk but now with innovations arriving each accounting period. As before, we can identify only one parameter for the MA process of the transitory component. Consequently, we constrain the MA parameters to

be along a straight line starting at 1 at lag 0 and declining to λ , an estimable parameter, at lag q . After some experimentation, q was fixed at 39 accounting periods (1 and 1/2 years).

Given this income process, we matched the observed consumption changes for each observation period with the respective current and lagged income innovations and computed the theoretical population moments for the same observable as that in the first model. We define $\sigma_\epsilon^2 = 11,726 V(\epsilon_{at})$ and $\sigma_y^2 = (6,419,902/39^2) V(\eta_{at})$, respectively, for easy comparison with the parameters of the first model. These numbers arise directly from the specification of the accounting period and the way "news" enters the income process (e.g., $11,726 = 1^2 + 2^2 + \dots + 25^2 + 26^2 + 25^2 + \dots + 2^2 + 1^2$). The matrix corresponding to (11) becomes, approximately,

$$\Omega = \begin{bmatrix} s^2 & \cdot & \cdot & \cdot \\ \alpha\{(1-0.30f)\sigma_\epsilon^2 + [1.43 + 1.28\lambda - (1.43 + 0.04\lambda)f]\beta\sigma_\eta^2\} & \sigma_\epsilon^2 + 2(1 + \lambda^2 + 0.35\lambda)\sigma_\eta^2 & \cdot & \cdot \\ \alpha\{(0.25 + 0.42f)\sigma_\epsilon^2 + [0.59 + 0.11\lambda + (0.71 + 0.46\lambda)f]\beta\sigma_\eta^2\} & 0.25\sigma_\epsilon^2 - [0.52(1 + \lambda^2) - 0.54\lambda]\sigma_\eta^2 & \cdot & \cdot \\ \alpha f[0.06\sigma_\epsilon^2 + (0.14 + 0.02\lambda)\beta\sigma_\eta^2] & -[0.50(1 + \lambda^2) + 1.30\lambda]\sigma_\eta^2 & \cdot & \sigma^2 \end{bmatrix} \quad (12)$$

Comparing (12) with (11), we find a larger cov ($\Delta c_t, \Delta y_{t-1}$), even if $f = 0$, and cov ($\Delta c_t, y_{t-2}$) > 0 , although small, which is consistent with Christiano et al. [1987]. As found by Working [1960], the random walk

component of income follows an IMA(1,1) in annual averages, with an MA parameter of 0.25. Since $\text{cov}(\Delta y_t, \Delta y_{t-1})$ is slightly negative in our data, we expect this difference to lead to a lower estimate of σ_ϵ^2 and hence, a lower expected $\text{cov}(\Delta c_t, \Delta y_t)$.

To disaggregate this form to a five-commodity framework, the first row of Ω then is replaced by an 8 x 5 block, in which the top five rows are uninteresting and the last three rows have elements of the form:

$$\text{cov}(\Delta c_{jt}, \Delta y_t) \approx \alpha \{ (1 - 0.30f_j)\sigma_\epsilon^2 + [1.43 + 1.28\lambda - (1.43 + 0.04\lambda)f_j]\beta\sigma_\eta^2 \}, \quad j = 1, \dots, 5 \quad (13a)$$

$$\text{cov}(\Delta c_{jt}, \Delta y_{t-1}) \approx \alpha \{ (0.25 + 0.42f_j)\sigma_\epsilon^2 + [0.59 + 0.11\lambda + (0.71 + 0.46\lambda)f_j]\beta\sigma_\eta^2 \}, \quad j = 1, \dots, 5 \quad (13b)$$

$$\text{cov}(\Delta c_{jt}, \Delta y_{t-2}) \approx \alpha f_j [0.06\sigma_\epsilon^2 + (0.14 + 0.02\lambda)\beta\sigma_\eta^2], \quad j = 1, \dots, 5. \quad (13c)$$

Except for the possibility of differing fractions f_j of spending information derived from the followup interview, these covariances are constrained to be equal across goods.

4. Estimation Issues

Our estimation and testing of the permanent income hypothesis is a sequence of refinements on the basic model in equation 4. These begin using aggregate consumption under the assumption that income news arrives annually. This is followed by improvements in our treatment of the timing of income innovations in relation to consumption; then disaggregation of consumption to allow consistent treatment of relative price effects with the

refinements on our representation of income innovations; and finally, the elimination of durables to avoid the problems posed by product durability in our model.

To implement this sequence of refinements requires two distinct types of estimation. The first involves estimating the static demand parameters under alternative commodity definitions and the second requires estimating the covariance structures relevant to each description of consumption and income.

Consider the first of these tasks. Here we follow conventional practice. We estimated a set of expenditure share equations with the nonlinear restricted seemingly unrelated regressions estimator, normalizing each of the functions in (6) with the relevant definition of total expenditures. Demographic effects were included in the demand system by allowing for translating, so that subsistence parameters (g) were specified to be functions of household size, the number of children in the household, the age of the household head, and whether the household had only one wage earner. Details are in the Appendix (available from the authors upon request).

Efficient estimation of the covariance matrix Ω , based on the computed c_{jt} values requires consistent, though not necessarily efficient, estimates of the demand parameters (MaCurdy [1981]).

The two observations for each household's expenditure choices were used as independent observations in these estimates. Thus the analysis does not allow for stochastic individual effects as in Biørn [1981] and Biørn and Jansen [1983]. Nonetheless, our approach clearly is consistent.

The second estimation task involves the covariance matrix Ω for the consumption and income variables $\Delta c_{1t}, \dots, \Delta c_{st}, \Delta y_t, \Delta y_{t-1}$, and y_{t-1} . The sample moment matrix M is a sufficient statistic for this procedure. We compute these moments around means. These means were allowed to vary according to observation year (i.e., to vary across the two subpanels observed in 1974-76 and 1975-77, respectively) and according to the age of the household head. Three age groups were used (based on the first observation year for consumption): young, 39 years and younger; middle-aged, 40-64 years; and old, 65 years and older. Allowance for time variation in the means was important because the time dimension of the combined panel is too short to permit estimation of the covariance matrix for consumption and income movements arising from aggregate stocks. This point was first observed by Zeldes [1986] and is further analyzed in Mork [1987]. The age-group variation provides an added safeguard against including predetermined components in our estimate of Ω . We did not adjust the income variables for other demographic effects. Although the number of adults in a household clearly affects its earnings potential, we decided not to adjust for it because such an adjustment ex post might mean removal of some important income innovations, such as those associated with the sudden departure of a breadwinner.

To account for measurement errors in consumption, we assumed they are uncorrelated with the income variables and can therefore be included with transitory consumption in the e-terms in (4) and (8). For income, measurement errors are definitional rather than stochastic. Thus, the first differences of income are measured without error. We do allow for an error

in the level measurement of y_{t-2} but assume that it is uncorrelated with Δy_t and Δy_{t-1} . This implies the formulae for $\text{cov}(\Delta y_t, y_{t-2})$ and $\text{cov}(\Delta y_{t-1}, y_{t-2})$ are unaffected. The assumption that changes in income are measured without error is necessary for identification of the three parameters of the income process, σ_ϵ^2 , σ_y^2 , and λ .

We estimate the parameter vector underlying the matrix Ω by maximizing the log-likelihood function

$$L = - (n/2) [\ln |\Omega(\theta)| + \text{tr} \Omega(\theta)^{-1} M]. \quad (14)$$

Computationally, this maximization was carried out with a quasi-Newton method, with recalculation of the information matrix for each interaction using a simple stepsize search. This approach has been detailed in the appendix of Bound, Griliches, and Hall [1986].

The likelihood function in (14) is valid only if the underlying stochastic variables are distributed normally. Otherwise, maximization of (14) can be interpreted as quasi-maximum likelihood. This procedure is consistent under general conditions, and a robust estimate of the variance-covariance matrix of $\hat{\theta}$, computed from the information matrix and the mean of the squares of the score for each observation, has been derived by MaCurdy [1981]. We computed this robust estimate for some of the specifications as a check on the validity of the normality assumption. We found that the standard errors derived from the information matrix did not exhibit systematic differences between the two estimators. Given the considerable additional computation time needed for robust estimation, we worked with maximum likelihood estimator based on normality.

Implementation of the specific models required some adjustments to reflect the progressive relaxation of the assumptions inherent in earlier

tests of the permanent income hypotheses. Our first three models (i.e., the two-based aggregate consumption and one with five disaggregated components) treat all goods as nondurable. The final model removes durable goods (i.e., appliances, furniture, automobiles, bicycles and recreational vehicles) from the relevant aggregate categories. Housing remains in the expenditures in all cases. The housing expenditures had been converted to a service equivalent before we received the data.

Since our last specification implies a separability assumption between nondurables and the durables we identified, the budget shares used in estimating the demand system were expressed in terms of the share of the total of nondurable expenditures. Consequently, in this case computation of Δc_{jt} requires an additional normalization, rescaling by the budget share of nondurables in the total expenditures. The sample mean was used for this adjustment.

While both treatments of the income process were used in the disaggregated models, the results we present here are limited to the time-aggregated income model (rather than the annual announcement model) in these cases. This gives a total of four models: aggregate consumption with both income models; disaggregated consumption including durables with time-aggregated income; and disaggregated consumption excluding durables with time-aggregated income.

For each specification, we estimate Ω in three ways: (1) an unconstrained estimate imposing only covariance stationarity so the parameter vector consists of all the independent elements of Ω ; (2) constrained estimates in terms of the parameters of the model:

α , β , λ , σ_ϵ^2 , σ_η^2 , σ^2 , and s^2 (in the disaggregated cases, s^2 is replaced by the independent elements of the variance covariance matrix S of Δc_{jt}), using the f -values fixed at the average fractions for the sample (taken from Biørn and Jansen [1980, 1982]); and (3) restricting $\hat{\beta}$ to be consistent with its theoretical value implied by equation (3).

Comparison of the likelihood value for either of the restricted models with the unrestricted $\hat{\Omega}$ (except for stationarity) provides a test of each model. Our specification of the restrictions on β imposed equation (3), assuming an expected remaining lifetime ($T-t$) of 20 years (our average household head is 50) and an annual real interest rate of 2%. While the actual ex post real interest rates during this period tended to be negative, a negative real interest rate would be inappropriate for long-term planning. In any case, this particular specification was not influential. The results were not sensitive to reasonable variations in the real interest rate.

The length of the MA process for transitory income in the time-aggregated case was determined as follows. First, we noted a nontrivial negative covariance between Δy_t and y_{t-2} . Thus, even though we could identify only one MA parameter in addition to σ_ϵ^2 and σ_η^2 from the three independent moments $V(\Delta y_t)$, $\text{cov}(\Delta y_t, \Delta y_{t-1})$, and $\text{cov}(\Delta y_t, y_{t-2})$, a lag of only one two-week accounting period clearly would be too short to fit the data. Consequently we imposed the linear lag structure and computed the implied values for λ at breakoff points q after 26, 39, and 52 accounting periods. For $q = 52$, λ became slightly negative. For $q = 26$, λ was extremely difficult to identify empirically because the equations relating σ_ϵ^2 , σ_η^2 , and λ to the sample moments resulted in a quadratic equation with

complex roots for λ . For $q = 39$, λ could be solved for as a small, positive value and this was selected.

The formulae in the second column of Ω in (12) indicate the presence of a second root for λ equal to the reciprocal of the one just derived. We consistently specified initial values for λ so as to arrive at the stable root.

5. Estimation Results

Table 1 reports the unrestricted estimate of the Ω matrix for the simple aggregate of consumption. The variables are measured in units of 1974 NKr 10,000 (at the time, NKr1 \approx US \$0.18). These units are convenient because they imply a variance estimate for Δy_t of slightly above unity. Thus, the covariances between Δy_t and the other variables resemble regression coefficients.

As a preliminary observation, we note that the estimated $\text{cov}(\Delta y_t, \Delta y_{t-1})$ is negative but very close to zero. This observation contrasts with the PSID data, where the corresponding covariance has been a large negative number. Since a negative autocovariance is implied by white-noise measurement errors for levels, we take this finding as an indication that our income data have little white-noise measurement errors. In contrast, the substantial negative covariance between Δy_t and y_{t-2} is consistent with an MA process for transitory income.

It is a little curious that the mean spending level exceeds the sample mean for disposable income, implying a negative mean savings rate, which is somewhat at odds with the corresponding figures in the National Income Accounts. However, it seems easily explainable by the exclusion of transfer

income, capital income (other than the return to unincorporated capital), and possible systematic underreporting of income.

The variance of the change in consumption is five times as large as that of the income change. This is hardly surprising, since the transitory consumption component must be expected to be substantial when spending is observed for two weeks only.

The contemporaneous covariance between consumption and income changes is positive as expected and significantly different from zero. However, it is not large, indicating either that a large portion of the observed income change is transitory or that households have substantial advance information about income. Certainly there would be no excess sensitivity to transitory income based on this covariance. In comparison, the covariance with the lagged income change appears to be a little large. However, the only disturbing finding is the significantly negative estimate of $\text{cov}(\Delta c_t, y_{t-2})$, which is inconsistent with both income models.

Table 2 presents the parameter estimates for this model variant under the assumption that income is announced at the beginning of each calendar year. With f fixed at 0.30, this model satisfies the order condition for identification of α , β , λ , σ_ϵ^2 , σ_η^2 , s^2 and σ^2 . Since the model is nonlinear, identification is a local property. In practice, we were unable to estimate α and β independently. This problem seems to arise because identification of these parameters relies on $\text{cov}(\Delta c_t, \Delta y_{t-1})$. This covariance is forced to take on a low value because $f/2$ is as low as 0.15 (cf. equation 11). To overcome this problem, we constrained β to its theoretical value in (3). (The standard error reported for $\hat{\beta}$ stems from the dependence of $\hat{\beta}$ on $\hat{\lambda}$.)

The resulting estimates of σ_ϵ^2 and σ_η^2 indicate that a little over half the variations in income changes are permanent. This greatly exceeds the estimates of Hall and Mishkin and others' with the PSID data. This finding underscores the importance of improving the measures of income. Measurement errors may well masquerade as transitory income. The estimate of λ indicates that half the transitory income changes last for a year beyond the year they first occur. The estimate of α suggests that about a third of what appears to the analyst as innovations in permanent income were expected by the households. This value implies an estimate of $\text{cov}(\Delta c_t, \Delta y_t)$ close to its unconstrained value, while the model's estimate of $\text{cov}(\Delta c_t, \Delta y_{t-1})$ is a good deal smaller than suggested by the raw data. Of course, $\text{cov}(\Delta c_t, y_{t-2})$ is constrained to be zero, while \hat{s}^2 and $\hat{\sigma}^2$ are very close to their unconstrained values. In spite of the significantly negative unconstrained estimate of $\text{cov}(\Delta c_t, y_{t-2})$, the model is not rejected; though the significance level of 0.08 makes this a close decision.

Parameter estimates for the time aggregated model, with income still treated as a single aggregate, are presented in Table 3. When β is constrained, this model does perform slightly better (being not rejected at a probability level of 0.11). Moreover, it implies that a smaller portion of the variation in income changes -- about one-third -- is permanent. As a result, the predicted contemporaneous covariance between income and consumption changes is smaller. This model also provides a better fit for $\text{cov}(\Delta c_t, \Delta y_{t-1})$ because it recognizes that a good deal of the change in annual income is last year's news.

Finally, it allows independent identification of α and β . The unconstrained estimate of β lies comfortably within the unit interval. Its point

estimate is, perhaps, on the large side, but the standard error is even larger. As a result, the theoretical constraint for β is not rejected (with the probability = 0.75).

Unfortunately, the findings are not uniformly good. With β unconstrained, the model is rejected with probability = 0.04. This test has only one degree of freedom. Heuristically, this means that all the sample moments except $\text{cov}(\Delta c_t, y_{t-1})$ are used to identify the parameters. Since we already know that the unconstrained estimate of this covariance contradicts the model significantly, this rejection was not surprising. Consequently, it motivates our disaggregation of consumption, recognition of relative price and demographic effects, and focus on the time-aggregated income measure. Table 4 reports the unconstrained estimates of Ω with a five-good disaggregation of all consumption including durables. Only the last three rows of this matrix are of real interest.

The contemporaneous covariances between consumption and income changes are positive except for the "other" category. However, it is significant only for transportation and recreation. In the case of food, it is quite small. To see this covariance in the right perspective, note that the Δc_{jt} already have been normalized for differences in the slopes of their respective Engel curves. Thus, the low covariance for food cannot be explained in terms of a low total expenditure elasticity. Our estimates were quite stable with $b_j = .19$ for food, implying an income elasticity of .64.) The estimates of the contemporaneous covariances for food and other expenditures become even more disturbing in view of (13a), which indicates that these covariances should be larger if the spending data are compiled mainly from the two-week diaries (i.e., the f_j small). Since all the food expenditures

and virtually all the "other" expenditures are compiled this way, we would have expected their covariances to be larger than those for the other categories. One implication of this finding is that food consumption is far from representative for overall consumption. Of course, this implies that when data availability has required exclusive focus on food consumption this may not be an adequate basis for gauging the sensitivity of consumption to income.

The covariances between the contemporaneous consumption changes and the lagged income changes also seem puzzling. The significantly positive estimate for transportation and recreation is consistent with the theory, but the slightly negative estimate for housing is not. Given its conversion to a service flow and the method of data collection, we expected this to be the largest covariance.

The covariances with income lagged twice have varying signs and large absolute values. The negative covariance for transportation and recreation is significant and appears to be the main culprit for the large negative covariance for aggregate consumption. The estimates of the interesting parameters for this model variant are presented in Table 5. As expected, the parameters of the income process (σ_ϵ^2 , σ_η^2 , and λ) are virtually unchanged from the aggregate model. However, the estimates of α and β are somewhat different. As we noted earlier, the disaggregated model provides five estimates of total expenditures. Our estimation constrains $\hat{\alpha}$ and $\hat{\beta}$ to be constant across all categories. If we selected any one of these estimates and used it exclusively, the approach would parallel that used in earlier studies focusing on one component of consumption. When they both

unconstrained, $\hat{\alpha}$ exceeds unity, while a large negative value of $\hat{\beta}$ keeps the estimated values of $\text{cov}(\Delta c_{jt}, \Delta y_t)$ from getting too large. When β is constrained, $\hat{\alpha}$ drops to half its value for aggregate consumption, reflecting the fact that $\text{cov}(\Delta c_{jt}, \Delta y_t)$ is low for the "wrong" goods, namely those with low values of f_j . This mismatch also contributes to the poorer fit of the model, which is rejected at 4% and 2% with β unconstrained and constrained, respectively. The β constraint is not rejected, but again this is a marginal decision with a significance level of 8%.

Disaggregation appears to have uncovered some problems that did not show up in the aggregate. At the same time, it allows us to pinpoint the source of the negative $\text{cov}(\Delta c_t, y_{t-2})$ in the aggregate. Since this problem appears to be associated with durability, the final model version offers one method for dealing with this issue by assuming separability and removing expenditures on durable goods from the relevant categories. Unfortunately, this does not solve the problem completely. As can be seen from the last row in Table 6, $\text{cov}(\Delta c_{jt}, y_{t-2})$ remains significantly negative for the remaining nondurable component of transportation and recreation. In an attempt to identify the source of the problem we computed $\text{cov}(\Delta c_{jt}, y_{t-1})$ for each of the subcategories of this spending group. Since estimation of a demand system for these subcategories would have been prohibitively complicated, the covariances for these are not corrected for relative price effects and thus are not quite comparable to those in Table 6. Keeping this caveat in mind, we nevertheless were convinced that the main source of the problem was the category "operating expenses for private vehicles." Two characteristics of this category come to mind. First, it is quite likely to be complementary with private vehicles, which is one of the excluded durable

goods, so the implicit separability assumption may not be appropriate in this case. Second, the relative price of gasoline fluctuated substantially during our sample period, and the linear expenditure system may not capture completely the responses to these fluctuations. Both these factors imply that further refinement in the model is necessary to deal adequately with the instantaneous choice of commodities and would not necessarily be a violation of the life-cycle hypothesis.

Another feature of Table 6 is that the contemporaneous covariances between consumption and income changes have shrunk for those spending categories from which durable components were removed. This change is not surprising, as stock adjustments strengthen the contemporaneous responses to income changes. It eases the strain on the cross-equation constraint on these covariances. However, it also drastically reduces the estimates of α as shown in Table 7. Reconciling these data with the life-cycle hypothesis would require one to believe (with β constrained) that households had advance information about 88% ($1-\alpha=0.88$) of what an external analyst would classify as permanent income changes based on the income data alone. We find this percentage to be high, even recognizing the exclusion of transfer income from our data. This finding is similar to the failures of the variance tests of Campbell and Deaton [1987] and West [1987] on aggregate U.S. data.

If we accept $\hat{\alpha}$, then the overall fit of the model now is much more acceptable. Likelihood-ratio tests of the model result in significance levels of 13%, whether or not β is constrained; and the constraint of β is not rejected at the 26% level when tested individually. Thus, with the most refined model incorporating relative price effects, assuming separability of

durables and using time-aggregated income, our data are compatible with the life-cycle hypothesis under rational expectations.

6. Summary and Conclusion

Empirical tests of hypotheses are only as good as the data used for the testing. Our analysis has been based on the best data available to date for testing the permanent/income life-cycle hypothesis. We have demonstrated how this improved data can be used to gauge the implications of progressive refinements in our model for the conclusions of tests of the life cycle framework. As one would hope, when we have the ability to measure and incorporate the effects of relative prices, demographic effects, the timing of income innovations and durability of commodities at the household level with good income measures, the data are consistent with the hypothesis.

The methods of data collection appear to have reduced measurement errors substantially. Disaggregation into spending components provided additional testing power, revealed some weaknesses concealed by aggregation, and permitted identification of the source of the problem for $\text{cov}(\Delta c_t, y_{t-2})$. The distinction between durable and nondurable goods, even in the ad hoc fashion employed in this paper, improved the fit of the model considerably. At the same time, it revealed a covariance between contemporaneous changes in income and nondurable spending that is somewhat on the low side for reconciliation with the life-cycle hypothesis.

Of course, the failure to reject a hypothesis is not synonymous with acceptance of this view of behavior. Indeed, our tests do not fully take advantage of the rich detail that is possible with complete micro data. In this sense, then, they are not as powerful as they could be. Consequently,

* full confirmation of the life cycle/rational expectations hypothesis requires better understanding of the determinants of the levels of consumption over time, of the behavioral processes households at different stages in life-cycle use in forming their expectations of permanent income, and an appreciation of the role of real and perceived constraints on their behavior. Some of these issues have been explored by Shefrin and Thaler [1987]. With the available data, these areas are feasible next steps for future research.

Table 1

Estimated Ω - matrix with consumption treated as a single aggregate

Unconstrained estimates except for stationarity.

Units = NKr 10,000 (1974: NKr1 \approx US\$0.18).

Standard errors in parentheses, Z-statistics in brackets.

	Δc_t	Δy_t	Δy_{t-1}	y_{t-2}
Δc_t	7.180 (0.497) [14.457]			
Δy_t	0.378 (0.151) [2.507]	1.325 (0.065) [20.409]		
Δy_{t-1}	0.286 (0.150) [1.905]	-0.083 (0.062) [-1.344]	1.325 (0.065) [20.409]	
y_{t-2}	-0.630 (0.306) [-2.060]	-0.241 (0.099) [-2.424]	-0.324 (0.091) [-3.541]	5.386 (0.372) [14.465]

$$\bar{c} = 4.443$$

$$\bar{y} = 3.812$$

$$f\text{-value: } 0.30$$

Table 2

Model estimation results
 Consumption treated as a single aggregate.
 Income announced at the beginning of each calendar year.
 Standard errors in parentheses, Z-statistics in brackets,
 β constrained to its theoretical value.

Parameter estimates:

σ_{ϵ}^2	σ_{η}^2	λ	α	β	s^2	σ_2
0.703	0.414	0.545	0.625	0.091	7.165	5.388
(0.182)	(0.120)	(0.154)	(0.265)	(0.009)	(0.496)	(0.372)
[3.875]	[3.445]	[3.404]	[2.355]	[10.017]	[14.458]	[14.466]

LR test of model: $p = 0.080$, d.f. = 2

Estimated Ω matrix:

	Δc_t	Δy_t	Δy_{t-1}	y_{t-2}
Δc_t	7.165			
Δy_t	0.394	1.324		
Δy_{t-1}	0.067	-0.093	1.324	
y_{t-2}	0.000	-0.217	-0.310	5.388

Table 3

Model estimation results
 Consumption treated as a single aggregate
 Income announced biweekly
 Standard errors in parentheses, Z-statistics in brackets
 Fixed f-value: 0.30.

Parameter estimates:

	σ_ϵ^2	σ_η^2	λ	α	β	s^2	σ_2
β unconstrained	0.499 (0.100) [4.977]	0.409 (0.053) [7.719]	0.017 (0.189) [0.092]	0.528 (1.170) [0.451]	0.606 (3.335) [0.182]	7.176 (0.496) [14.457]	5.388 (0.372) [14.465]
β constrained	0.405 (0.100) [4.959]	0.411 (0.053) [7.782]	0.017 (0.188) [0.092]	0.820 (0.307) [2.672]	0.047 (0.009) [5.478]	7.171 (0.496) [14.458]	5.388 (0.372) [14.466]

LR test results:

Test of model, β unconstrained: p=0.037, d.f.=1
 Test of constraint for β : p=0.751, d.f.=1
 Test of model, β constrained: p=0.109, d.f.=2

Estimated Ω matrix:

	β unconstrained				β constrained			
	Δc_t	Δy_t	Δy_{t-1}	y_{t-2}	Δc_t	Δy_t	Δy_{t-1}	y_{t-2}
Δc_t	7.176				7.171			
Δy_t	0.374	1.323			0.386	1.323		
Δy_{t-1}	0.204	-0.086	1.323		0.165	-0.088	1.323	
y_{t-2}	0.010	-0.214	-0.300	5.388	0.008	-0.215	-0.303	5.388

Table 4

Results with five expenditure groups, all treated as nondurable.
Unconstrained (except for stationarity) estimate of the Ω matrix
Standard errors in parentheses, Z-statistics in brackets.

 Δc_t

Δc_t	food	clothing	housing	transp. & rec.	other	Δy_t	Δy_{t-1}	y_{t-2}
food	9.991 (0.691) [14.457]							
clothing	1.324 (0.914) [1.449]	34.756 (2.404) [14.457]						
housing	-0.696 (0.688) [-1.011]	1.175 (1.284) [0.915]	19.778 (1.368) [14.457]					
transp. & rec.	0.409 (0.848) [0.482]	4.991 (1.601) [3.118]	-0.305 (1.194) [-0.256]	30.103 (2.082) [14.457]				
other	2.636 (1.548) [1.703]	5.880 (2.981) [2.034]	-12.250 (2.251) [-5.442]	4.068 (2.684) [1.516]	99.515 (6.884) [14.457]			
Δy_t	0.057 (0.178) [0.321]	0.237 (0.332) [0.715]	0.326 (0.250) [1.304]	0.875 (0.308) [2.839]	-0.681 (0.562) [-1.213]	1.325 (0.065) [20.409]		
Δy_{t-1}	-0.161 (0.178) [-0.906]	0.197 (0.332) [0.593]	-0.070 (0.250) [-0.281]	0.717 (0.307) [2.340]	0.448 (0.562) [0.798]	-0.083 (0.062) [-1.344]	1.325 (0.065) [20.409]	
y_{t-2}	-0.327 (0.359) [-0.912]	0.513 (0.670) [0.766]	0.705 (0.505) [1.394]	-1.664 (0.628) [-2.649]	-1.307 (1.132) [-1.155]	-0.241 (0.099) [-2.424]	-0.324 (0.091) [-3.541]	5.386 (0.372) [14.465]
f-values:	0.00	0.16	0.79	0.28	0.06			

Table 5

Model estimation results
 Five expenditure groups, all treated as nondurables
 Income announced biweekly
 Standard errors in parentheses, Z-statistics in brackets.

Parameter estimates:

	σ_{ϵ}^2	σ_{η}^2	λ	α	β
β unconstrained	0.472 (0.099) [4.755]	0.417 (0.055) [7.559]	0.057 (0.190) [0.303]	1.482 (0.851) [1.741]	-0.620 (0.242) [-2.561]
β constrained	0.485 (0.100) [4.856]	0.411 (0.055) [7.523]	0.052 (0.192) [0.269]	0.386 (0.235) [1.640]	0.048 (0.009) [5.543]

LR test results:

Test of model, β unconstrained: $p=0.038$, d.f.=13

Test of constraint for β : $p=0.78$, d.f.=1

Test of model, β constrained: $p=0.023$, d.f.=14

Estimated Ω matrix, last three rows:

	food	clothing	housing	transp. & rec.	other	Δy_t	Δy_{t-1}	y_{t-2}
β unconstrained								
Δy_t	0.124	0.178	0.421	0.342	0.137	1.325		
Δy_{t-1}	-0.053	-0.050	-0.039	-0.043	-0.052	-0.088	1.325	
y_{t-2}	0.000	-0.002	-0.012	-0.009	-0.001	-0.240	-0.328	5.391
β constrained								
Δy_t	0.199	0.188	0.141	0.156	0.196	1.324		
Δy_{t-1}	0.051	0.065	0.125	0.106	0.055	-0.083	1.324	
y_{t-2}	0.000	0.002	0.010	0.008	0.000	-0.233	-0.317	5.385
f-values:	0.00	0.16	0.79	0.28	0.06			

Table 6

Results with five expenditure groups, durables ingnored.

Unconstrained (except for stationarity)

Estimate of the Ω matrix

Standard errors in parentheses, Z-statistics in brackets.

 Δc_t

Δc_t	food	clothing	housing	transp. & rec.	other	Δy_t	Δy_{t-1}	y_{t-2}
food	4.754 (0.329) [14.457]							
clothing	0.662 (0.466) [1.419]	19.033 (1.317) [14.457]						
housing	-0.117 (0.341) [-0.344]	1.026 (0.684) [1.500]	10.224 (0.707) [14.457]					
transp. & rec.	0.970 (0.552) [1.757]	2.903 (1.110) [2.614]	0.028 (0.807) [0.035]	26.638 (1.843) [14.457]				
other	1.452 (0.852) [1.704]	3.386 (1.707) [1.984]	-7.351 (1.296) [-5.674]	4.497 (2.021) [2.225]	63.357 (4.382) [14.457]			
Δy_t	0.050 (0.123) [0.409]	0.183 (0.245) [0.746]	0.135 (0.180) [0.751]	0.466 (0.290) [1.604]	-0.525 (0.448) [-1.171]	1.325 (0.065) [20.409]		
Δy_{t-1}	-0.117 (0.123) [-0.953]	0.135 (0.246) [0.550]	-0.197 (0.180) [-1.093]	0.624 (0.289) [2.158]	0.361 (0.448) [0.805]	-0.083 (0.062) [-1.344]	1.325 (0.065) [20.409]	
y_{t-2}	-0.266 (0.248) [-1.073]	0.326 (0.496) [0.659]	0.318 (0.363) [0.876]	-1.520 (0.590) [-2.575]	-1.094 (0.903) [-1.212]	-0.241 (0.099) [-2.424]	-0.324 (0.091) [-3.541]	5.386 (0.372) [14.465]
f-values:	0.00	0.16	0.87	0.02	0.06			

Table 7

Model estimation results
 Five expenditure groups, ignoring durables
 Income announced biweekly
 Standard errors in parentheses, Z-statistics in brackets.

Parameter estimates:

	σ_{ϵ}^2	σ_{η}^2	λ	α	β
β unconstrained	0.479 (0.100) [4.800]	0.413 (0.055) [7.489]	0.059 (0.192) [0.305]	0.758 (0.699) [1.084]	-0.713 (0.332) [-2.148]
β constrained	0.482 (0.100) [4.819]	0.411 (0.055) [7.440]	0.061 (0.193) [0.315]	0.121 (0.210) [0.575]	0.049 (0.009) [5.551]

LR test results:

Test of model, β unconstrained: $p=0.129$, d.f.=13

Test of constraint for β : $p=0.264$, d.f.=1

Test of model, β constrained: $p=0.128$, d.f.=14

Estimated Ω matrix, last three rows:

	food	clothing	housing	transp. & rec.	other	Δy_t	Δy_{t-1}	y_{t-2}
β unconstrained								
Δy_t	0.027	0.061	0.212	0.163	0.036	1.325		
Δy_{t-1}	-0.042	-0.043	-0.050	-0.048	-0.042	-0.085	1.325	
y_{t-2}	0.000	-0.002	-0.009	-0.007	-0.000	-0.238	-0.325	5.385
β constrained								
Δy_t	0.062	0.059	0.044	0.049	0.061	1.325		
Δy_{t-1}	0.016	0.020	0.039	0.033	0.017	-0.082	1.325	
y_{t-2}	0.000	0.001	0.003	0.002	0.000	-0.238	-0.321	5.386
f-values:	0.00	0.16	0.87	0.02	0.06			

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APPENDIX

A. STATIC AND DYNAMIC OPTIMIZATION WITH MANY GOODS

Our data contain information on a complete set of $M = 28$ expenditure categories. The categories and their average budget shares are listed in Table A1. We assume that the instantaneous preferences for household i at time t can be described by the separable utility function.

$$u_{it}[\phi_{lit}(z_{1lit}, \dots, z_{M_1lit}), \dots, \phi_{sit}(z_{1sit}, \dots, z_{M_ssit})].$$

This assumption allows us to define $s < M$ aggregate goods x_{lit}, \dots, x_{sit} . We construct a price index p_{jit} for each aggregate by an unchained divisia procedure. The values of the price indices vary across households because of budget variations in the shares. We assume $s = 5$, but the components going into each aggregate vary from model to model. A complete list of the aggregation for each model is given in Table A2.

Suppose the instantaneous utility function has the form

$$u_{it} = -1/2 (\bar{c}_i - \sum_{j=1}^s [(x_{jit} - g_{jit})/b_j]^{b_j})^2, \quad (A1)$$

where $\sum_{j=1}^s b_j = 1$. The parameters g_{jit} are indexed by household and observation period because they may depend on demographic characteristics.

The static demand function implied by (A1) from the Stone-Geary linear expenditure system:

$$m_{jit} = g_{jit} p_{jit} + b_j (m_{it} - g_{it} \cdot p_{it}) \quad j = 1, \dots, m \quad (A2)$$

where

$$m_{jit} = p_{jit} x_{jit}, \quad m_{it} = \sum_{j=1}^s m_{jit}, \quad g_{it} = (g_{lit}, \dots, g_{sit})', \quad \text{and}$$

$$P_{it} = (P_{lit}, \dots, P_{sit})'.$$

The instantaneous indirect utility function is

$$v_{it}(m_{it}, P_{it}) = - (1/2) [\bar{c}_i - (m_{it} - g_{it} \cdot P_{it}) / P_{it}]^2, \quad (A3)$$

$$\text{where } P_{it} = \sum_{j=1}^s p_{jit}^{b_j}.$$

By a two-step optimization argument, we embed v_{it} in the household's intertemporal optimization problem and assume that the household maximizes

$$V_{it} = E_t \sum_{k=t}^T (1+\rho)^{t-k} v_{ik}(m_{ik}, P_{ik}), \quad (A4)$$

where E_t denotes expectation conditional on information available at time t , ρ the subjective discount rate, and $T-t$ the household's expected remaining lifetime subject to the budget constraint

$$\sum_{k=t}^T (1+R_{kt})^{-1} (\tilde{y}_{ik} - m_{ik}) + \tilde{A}_{it} = 0. \quad (A5)$$

Here, \tilde{y}_{ik} denotes nominal labor income at time k , \tilde{A}_{it} , the nominal value of

assets at time t , and R_{kt} , the $(k-t)$ -period nominal interest rate at time t ($R_{tt} = 0$).

Consider a variation around the optimal path that subtracts x from m_{it} and adds $(1+R_{kt})x$ to m_{ik} for some $k > t$. Optimization with respect to x yields the first-order condition

$$\begin{aligned} (1+R_{kt})(1+\rho)^{t-k} E_t[\bar{c}_i/P_{ik} - (m_{ik} - g_{ik} \cdot P_{ik})/P_{ik}^2] \\ = \bar{c}_i/P_{it} - (m_{it} - g_{it} \cdot P_{it})/P_{it}^2. \end{aligned} \quad (A6)$$

Suppose that some overall price index P_t^* , common to all households, satisfies the Fisher equation

$$1+R_{kt} = (1+r)^{k-t} E_t P_k^*/P_t^*, \quad (A7)$$

where r is the real interest rate. We assume $r = \rho$. Empirically, we use a fixed-weight index for P_t^* ; when durables are included, it coincides with the overall CPI.

Substituting (A1) into (A6), using the formula for the expectation of a product and solving, we obtain

$$\begin{aligned} E_t m_{is} = m_{it} (P_t^*/E_t P_k^*) / (P_{it}^2 E_t P_{ik}^{-2}) \\ - [g_{it} \cdot P_{it} (P_t^*/E_t P_k^*) / (P_{it}^2 E_t P_{ik}^{-2}) - E_t g_{ik} \cdot P_{ik}] \\ - \text{cov} (m_{ik} - g_{ik} \cdot P_{ik}, P_{ik}^{-2}/E_t P_{ik}^{-2}) \\ - \bar{c}_i [(P_t^*/E_t P_k^*) / (P_{it} E_t P_{ik}^{-2}) - E_t P_{ik}^{-1}/E_t P_{ik}^{-2}]. \end{aligned} \quad (A8)$$

Next, substitute this formula into the expected value of the budget constraint and rearrange to obtain

$$m_{it} - g_{it} \cdot p_{it} = \gamma_{it} (\tilde{H}_{it} + \tilde{A}_{it} - \tilde{G}_{it} + \tilde{K}_{it}), \quad (A9)$$

where

$$\gamma_{it} = 1 / \left[\sum_{k=t}^T (1+r)^{t-k} (P_t^* / E_t P_k^*)^2 / (P_{it}^2 E_t P_{ik}^{-2}) \right]$$

$$\tilde{H}_{it} = \sum_{k=t}^T (1+r)^{t-k} (P_t^* / E_t P_k^*) E_t y_{ik}$$

$$\tilde{G}_{it} = \sum_{k=t}^T (1+r)^{t-k} E_t (g_{ik} \cdot p_{ik}) P_t^* / E_t P_k^*$$

$$\begin{aligned} \tilde{K}_{it} = & \sum_{k=t}^T (1+r)^{t-k} \text{cov}[(m_{ik} - p_{ik} \cdot g_{ik}) P_t^* / E_t P_k^*, P_{ik}^{-2} / E_t P_{ik}^{-2}] \\ & + \bar{c}_i \sum_{k=t}^T (1+r)^{t-k} [(P_t^* / E_t P_k^*)^2 / (P_{it} E_t P_{ik}^{-2}) - (P_t^* / E_t P_k^*) E_t P_k^{-1} / E_t P_k^{-2}]. \end{aligned}$$

Again using the intertemporal budget constraint as well as (A9) itself lagged one period, it becomes clear that, in the absence of uncertainty, the argument of the instantaneous utility function, $\bar{c}_i - (m_{it} - g_{it} \cdot p_{it}) / P_{it}$, grows at the rate

$$(P_{it} / P_{i,t-1}) / (P_t^* / P_{t-1}^*). \quad (A10)$$

We infer from this result that the proper deflator for consumption is

$$\bar{P}_{it} = P_{it}^2 / P_t^*.$$

After application of this deflator, (A9) yields

$$c_{it} = m_{it}/\bar{P}_{it} = \gamma_{it}(H_{it} + A_{it} + K_{it}) + g_{it} \cdot p_{it}/\bar{P}_{it} - \gamma_{it}G_{it}, \quad (A11)$$

where H_{it} , A_{it} , K_{it} , G_{it} are the deflated values of \tilde{H}_{it} , \tilde{A}_{it} , \tilde{K}_{it} , \tilde{G}_{it} , respectively. We interpret the first term on the right of this equation as real permanent income and the difference between the two last terms as real transitory consumption.

B. MODEL OF INCOME AND CONSUMPTION

We use two alternative assumptions about the timing of income announcements.

1. Income Announcements Made at the Beginning of Each Calendar Year

Let y_{it} denote the stochastic part of observed real disposable labor income (deflated by the CPI) for household i in calendar year t . We treat y_{it} as composed of a lifetime component, a transitory component, and a measurement error:

$$y_{it} = y_{it}^L + y_{it}^T + u_{it}. \quad (B1)$$

Lifetime income follows a random walk,

$$y_{it}^L = y_{i,t-1}^L + \epsilon_{it}, \quad E(\epsilon_{it}) = 0, \quad V(\epsilon_{it}) = \sigma_\epsilon^2,$$

and stochastic income, a first-order moving average:

$$y_{it}^T = \eta_{it} + \lambda \eta_{i,t-1}, \quad E(\eta_{it}) = 0, \quad V(\eta_{it}) = \sigma_\eta^2.$$

We assume the measurement errors are definitional rather than random, so that $u_{it} = u_{i,t-1} = u_i$, but u_i has a positive variance in the cross section.

The assumptions imply

$$\Delta y_{it} = \epsilon_{it} + \eta_{it} - (1-\lambda)\eta_{i,t-1} - \lambda\eta_{i,t-2}, \quad (B2)$$

$$\Delta y_{i,t-1} = \epsilon_{i,t-1} + \eta_{i,t-1} - (1-\lambda)\eta_{i,t-2} - \lambda\eta_{i,t-3}, \quad (B3)$$

$$y_{i,t-2} = \epsilon_{i,t-2} + \eta_{i,t-2} + \lambda\eta_{i,t-3} + y_{i,t-3}^L + u_i, \quad (B4)$$

$$V(\Delta y_{it}) = V(\Delta y_{i,t-1}) = \sigma_\epsilon^2 + 2(1+\lambda^2-\lambda)\sigma_\eta^2, \quad (B5)$$

$$V(y_{i,t-2}) = \sigma_\epsilon^2 + (1+\lambda^2)\sigma_\eta^2 + V(y_{i,t-3}^L) + V(u_i) \equiv \sigma^2, \quad (B6)$$

$$\text{cov}(\Delta y_{it}, \Delta y_{i,t-1}) = - (1-\lambda)^2\sigma_\eta^2 \quad (B7)$$

$$\text{cov}(\Delta y_{it}, y_{i,t-2}) = - \lambda\sigma_\eta^2, \quad (B8)$$

$$\text{cov}(\Delta y_{i,t-1}, y_{i,t-2}) = \text{cov}(\Delta y_{it}, \Delta y_{i,t-1}) + \text{cov}(\Delta y_{it}, y_{i,t-2}). \quad (B9)$$

Since $V(y_{i,t-3}^L) + V(u_i)$ is of no independent interest, σ^2 is estimated as a separate parameter.

The implied annual innovation in income is

$$\epsilon_{it} + \beta_{it}\eta_{it}, \quad (B10)$$

where

$$\beta_{it} = \gamma_{it}[1+\lambda/(1+r)].$$

(All) implies that the first difference in real annual consumption equals (B10) plus a transitory-consumption term. From (A2), a similar formula holds for the first difference for each composite good j , provided it is "purged" of relative-price effects in the form

$$c_{jit} = [(m_{jit} - \xi_{jit}p_{jit})/b_j + \xi_{it} \cdot p_{it}] / \bar{p}_{it}. \quad (B11)$$

Our consumption data are weighted averages of a "snapshot" observation from the diaries for the biweekly observation periods a ($a=1, \dots, 26$) and an unweighted average of spending during the 26 accounting periods ending with a . Thus, if household i is observed during period a and f_j ($0 \leq f_j \leq 1$) is the marginal budget share for those components of good j that are covered by the interview, we have

$$\begin{aligned} \Delta c_{jia} = & (1-f_j)(\epsilon_{it} + \beta_i \eta_{it}) + f_j [(a/26)(\epsilon_{it} + \beta_i \eta_{it}) \\ & + (1-a/26)(\epsilon_{i,t-1} + \beta_i \eta_{i,t-1})] + e_{jia}, \quad j = 1, \dots, m, \end{aligned} \quad (B12)$$

where e_{jia} is a good-specific stochastic term including transitory consumption as well as measurement errors and we have assumed, as a close approximation,

$$\beta_{ia} = \beta_{i,a-1} = \beta_i.$$

Given the remaining parameters, the variances of Δc_{jia} and their covariances with each other just identify the variance-covariance matrix of the e_{jia} terms. Since this matrix is of no interest by itself, the variances and covariances of Δc_{jia} are estimated as independent parameters. Assuming the distributions of ϵ , η , β , and a all are independent of each other, letting β denote the mean of the β_i , and noting that the mean of a is $(1 + 2 + \dots + 26)/26 = 13.5$, the cross-sectional covariances with the income variables are, for $j = 1, \dots, s$,

$$\text{cov}(\Delta c_{jt}, \Delta y_t) = [1 - (12.5/26)f_j] \sigma_\epsilon^2 \quad (\text{B13})$$

$$+ [1 - f_j(25 - 12.5\lambda)/26] \beta \sigma_\eta^2,$$

$$\text{cov}(\Delta c_{jt}, \Delta y_{t-1}) = (12.5/26)f_j(\sigma_\epsilon^2 + \beta \sigma_\eta^2), \quad (\text{B14})$$

$$\text{cov}(\Delta c_{jt}, y_{t-2}) = 0. \quad (\text{B15})$$

2. Income Announcements Made at the Beginning of Each Two-Week Accounting Period

Let y_{iat} denote the true disposable real income received by household i during accounting period a in year t . Assume

$$y_{iat} = y_{iat}^L + y_{iat}^T, \quad (\text{B16})$$

where

$$y_{iat}^L = y_{i,a-1,t}^L + \epsilon_{iat}, \quad (\text{B17})$$

$$y_{iat}^T = \eta_{iat} + \left(\frac{38+\lambda}{39}\right) \eta_{i,a-1,t} + \dots + \left(\frac{1+38\lambda}{39}\right) \eta_{i,a-12,t-1} \quad (\text{B18})$$

$$+ \lambda \eta_{i,a-13,t-1}.$$

Assume annual income is subject to the same observation error as in the preceding model. Then, (B16) - (B18) imply the following formula for annual income:

$$\begin{aligned}
y_{it} = & \sum_{a=1}^{26} (27-a) \epsilon_{iat} + (1/39) \left[\sum_{a=1}^{26} \eta_{iat} \left(\sum_{b=a+13}^{39} b + \lambda \sum_{b=1}^{26-a} b \right) \right. \\
& + \sum_{a=14}^{26} \eta_{ia,t-1} \left(\sum_{b=a-13}^{a+12} b + \lambda \sum_{b=27-a}^{52-a} b \right) \quad (B19) \\
& + \sum_{a=1}^{13} \eta_{ia,t-1} \left(\sum_{b=1}^{a+12} b + \lambda \sum_{b=27-a}^{39} b \right) + \sum_{a=14}^{26} \eta_{ia,t-2} \left(\sum_{b=0}^{a-14} b + \lambda \sum_{b=53-a}^{39} b \right) \\
& \left. + 27y_{i,26,t-1}^L + u_i \right]
\end{aligned}$$

The expressions for Δy_{it} , $\Delta y_{i,t-1}$, and $y_{i,t-2}$ follow directly from this formula.

The first differences for consumption are computed relative to the same accounting period in the preceding year. So,

$$\begin{aligned}
\Delta c_{jiat} = & 26(1-f_j)(c_{iat} - c_{ia,t-1}) + f_j \left[\sum_{b=1}^a (c_{ibt} - c_{ib,t-1}) \right. \\
& \left. + \sum_{b=a+1}^{26} (c_{ib,t-1} - c_{ib,t-2}) \right], \quad j = 1, \dots, s.
\end{aligned} \quad (B20)$$

(The last sum is omitted if $a = 26$.)

The life-cycle hypothesis implies

$$\begin{aligned}
 c_{iat} - c_{ia,t-1} &= \sum_{b=1}^a (\epsilon_{ibt} + \beta_i \eta_{ibt}) \\
 &\quad + \sum_{b=a+1}^{26} (\epsilon_{ib,t-1} + \beta_i \eta_{ib,t-1}), \\
 \sum_{b=1}^a (c_{ibt} - c_{ib,t-1}) &+ \sum_{b=a+1}^{26} (c_{ib,t-1} - c_{ib,t-2}) \\
 &= \sum_{b=1}^a (a+1-b)(\epsilon_{ibt} + \beta_i \eta_{ibt}) + \sum_{b=a+1}^{26} (a+27-b)(\epsilon_{ib,t-1} + \beta_i \eta_{ib,t-1}) \\
 &\quad + \sum_{b=1}^a (b+25-a)(\epsilon_{ib,t-1} + \beta_i \eta_{ib,t-1}) \\
 &\quad + \sum_{b=a+2}^{26} (b-a-1)(\epsilon_{ib,t-2} + \beta_i \eta_{ib,t-2}).
 \end{aligned}$$

(The sums from $a+2$ to 26 and from $a+1$ to 26 are omitted if $a \geq 25$ and $a = 26$, respectively.)

Using these formulae, the following cross-sectional moments emerge after a lot of tedious though straightforward algebra and arithmetic:

$$V(\Delta y_t) = 11,726V(\epsilon_{iat}) + 2[6,419,902(1+\lambda^2) + 4,552,836\lambda]V(\eta_{iat})/39^2 \quad (B21)$$

$$\text{cov}(\Delta y_t, \Delta y_{t-1}) = 2,925V(\epsilon_{iat}) - [3,365,806(1+\lambda^2) - 3,492,267\lambda]V(\eta_{iat})/39^2 \quad (B22)$$

$$\text{cov}(\Delta y_t, y_{t-2}) = - [3,204,675(1+\lambda^2) + 8,356,908\lambda]V(\eta_{iat})/39^2 \quad (B23)$$

$$\begin{aligned} \text{cov}(\Delta c_{jt}, \Delta y_t) &= (11,726 - 3,500f_j)V(\epsilon_{iat}) \\ &+ [235,590 + 210,470\lambda - f_j(6,137,078 + 190,292\lambda)/26]\beta V(\eta_{iat})/39 \end{aligned} \quad (B24)$$

$$j = 1, \dots, s,$$

$$\begin{aligned} \text{cov}(\Delta c_{jt}, \Delta y_{t-1}) &= (2,925 + 4,962f_j)V(\epsilon_{iat}) \\ &+ [96,705 + 17,550\lambda + f_j(3,022,902 + 1,979,848\lambda)/26]\beta V(\eta_{iat})/39 \end{aligned} \quad (B25)$$

$$j = 1, \dots, s,$$

$$\text{cov}(\Delta c_j, y_{t-2}) = f_j[675V(\epsilon_{iat}) + (605,250 + 80,730\lambda)\beta V(\eta_{iat})/(26 \cdot 39)] \quad (B26)$$

With the definitions of σ_ϵ^2 and σ_η^2 indicated in the main text, the formulae in the text now follow.

Table A1

Average Budget Shares (in percent) of the 28
Individual Spending Categories

1.	Flour and bread.	2.63
2.	Meat and eggs	6.70
3.	Fish	1.60
4.	Canned meat and fish	0.60
5.	Dairy products	3.73
6.	Butter and margarine	0.98
7.	Potatoes and vegetables.	5.13
8.	Other food	4.29
9.	Beverages.	2.35
10.	Tobacco.	1.74
11.	Clothing	7.58
12.	Footwear	1.72
13.	Housing.	11.04
14.	Fuel and power	4.78
15.	Furniture.	4.45
16.	Household appliances and equipment	2.93
17.	Miscellaneous household goods.	2.60
18.	Medical care	1.70
19.	Motorcars and bicycles	4.95
20.	Running costs of vehicles.	6.91
21.	Public transportation.	2.54
22.	Postage, telephone and telegraph charges	1.47
23.	Recreation	5.89
24.	Public entertainment	3.10
25.	Books and newspapers	2.26
26.	Personal care.	2.00
27.	Miscellaneous goods and services	1.49
28.	Restaurants, hotels, etc	2.82

Table A2

Disaggregation Schemes With and Without Durables

(a) Model with durables

I. Food, beverages, and tobacco

1. Flour and bread
2. Meat and eggs
3. Fish
4. Canned meat and fish
5. Dairy products
6. Butter and margarine
7. Potatoes and vegetables
8. Other foods
9. Beverages
10. Tobacco

II. Clothing and footwear

11. Clothing
12. Footwear

III. Housing, fuel, and furniture

13. Housing
14. Fuel and power
15. Furniture
16. Household appliances and equipment
17. Miscellaneous household goods

IV. Transportation and recreation

19. Motorcars and bicycles
20. Running costs of vehicles
21. Public transportation
22. Postage, telephone, and telegraph charges
23. Recreation
24. Public entertainment
25. Books and newspapers

V. Other goods and services

18. Medical care
26. Personal care
27. Miscellaneous goods and services
28. Restaurants, hotels, etc.

(b) Third variant, excluding durables

I. Food, beverages, and tobacco

1. Flour and bread
2. Meat and eggs
3. Fish
4. Canned meat and fish
5. Dairy products
6. Butter and margarine
7. Potatoes and vegetables
8. Other foods
9. Beverages
10. Tobacco

II. Clothing and footwear

11. Clothing
12. Footwear

III. Housing, fuel, and furniture

13. Housing
14. Fuel and power
15. Miscellaneous households goods

IV. Transportation and recreation

20. Running costs of vehicles
21. Public transportation
22. Postage, telephone, and telegraph charges
24. Public entertainment
25. Books and newspapers

V. Other goods and services

18. Medical care
26. Personal care
27. Miscellaneous goods and services
28. Restaurants, hotels, etc.

Table A3

Estimates of Stone-Geary Expenditure System with Translating Parameters a

Model	Model With Durables	Excluding Durables
	_____	_____
Marginal Budget Shares (b_i)		
I. Food, beverages, tobacco	.1897 (.0076)	.2667 (.0082)
II. Clothing and footwear	.0768 (.0041)	.1251 (.0059)
III. Housing, Fuel & Furniture	.2399 (.0088)	.2116 (.0080)
IV. Transportation & Recreation	.3627 (.0098)	.2582 (.0081)
c_1	162.92 (112.09)	271.71 (129.58)
c_2	-23.44 (6.81)	-13.27 (3.15)
c_3	-360.66 (359.70)	-607.56 (290.38)
D_1	1360.52 (95.63)	1347.09 (103.68)
D_2	297.64 (208.95)	236.09 (189.53)
D_3	327.08 (116.48)	173.48 (107.44)
D_4	7.65 (4.73)	10.73 (2.93)
Share Equation Pseudo R^2		
I.	.271	.187
II.	-.030	.020
III.	.139	.080
IV.	.040	.047

a. The basic model estimated is given as follows:

$$s_I = b_1 + (1-b_1)(D_1 \text{ Fsize} + D_2 \text{ Searn}) * (P_I/y)$$

$$-b_1 \cdot c_1 \cdot \text{Nchild} (P_{II}/y) - b_1 \cdot (D_3 \cdot \text{Fsize} + D_4 \text{ Age}) \cdot (P_{III}/y)$$

$$-b_1 \cdot c_2 \cdot \text{Age} \cdot (P_{IV}/y) - b_1 \cdot c_3 (P_V/y)$$

$$s_{II} = b_2 - b_2 (D_1 \text{ Fsize} + D_2 \text{ Searn}) \cdot (P_I/y)$$

$$+(1-b_2) \cdot c_1 \text{Nchild} (P_{II}/y) - b_2 \cdot (D_3 \text{ Fsize} + D_4 \text{ Age}) \cdot (P_{III}/y)$$

$$-b_2 \cdot c_2 \cdot \text{Age} (P_{IV}/y) - b_2 \cdot c_3 \cdot (P_V/y)$$

$$s_{III} = b_3 - b_3 (D_1 \cdot \text{Fsize} + D_2 \cdot \text{Searn}) \cdot (P_I/y)$$

$$-b_3 \cdot c_1 \text{Nchild} (P_{II}/y) + (1 - b_3) \cdot (D_3 \text{ Fsize} + D_4 \text{ Age}) \cdot (P_{III}/y)$$

$$-b_3 \cdot c_2 \cdot \text{Age} \cdot (P_{IV}/y) - b_3 \cdot c_3 (P_V/y)$$

$$s_{IV} = b_4 - b_4 (D_1 \text{ Fsize} + D_2 \text{ Searn}) \cdot (P_I/y)$$

$$-b_4 \cdot c_1 \text{Nchild} (P_{II}/y) + (1-b_4) \cdot (D_3 \text{ Fsize} + D_4 \text{ Age}) \cdot (P_{III}/y)$$

$$+(1-b_4) \cdot c_2 \cdot \text{Age} \cdot (P_{IV}/Y) - b_4 \cdot c_3 (P_V/y)$$

where

s_i = expenditure share for commodity i

y = total expenditures

Fsize = numbers of individuals in household

Searn = qualitative variable for single-earner households (=1)

Age = age of household head

Nchild = number of children under 18 years of age

