NC

96

FACULTY WORKING PAPERS

Effects of Exchange Rates
on Domestic Agricultural Prices

John C. Dutton, Jr.

Faculty Working Paper No. 96 January 1987

DEPARTMENT OF ECONOMICS AND BUSINESS
NORTH CAROLINA STATE UNIVERSITY
RALEIGH, NORTH CAROLINA

Effects of Exchange Rates on Domestic Agricultural Prices

John C. Dutton, Jr.

Faculty Working Paper No. 96 January 1987

Working papers in this series are preliminary material and should not be quoted or reproduced without written permission of the authors. Comments are welcome. I wish to acknowledge cooperative agreement support from the International Economics Division of E.R.S., U.S.D.A., for the completion of research for this paper.

Effects of Exchange Rates on Domestic Agricultural Prices

John Dutton
North Carolina State University*

Effects of Exchange Rates
on Domestic Agricultural Prices

1. Introduction.

One of the important effects of exchange rate changes is that they alter domestic prices. An unsettled question for the agricultural sector is just how much domestic agricultural prices can be expected to change for a given shift of the exchange rate. The importance of the issue is obvious, given the continuous and substantial shifting of the exchange rate since the advent of floating rates.

In a frequently cited paper, Chambers and Just (1979) (hereafter CJ) analyze the theoretical relationship between changes in the exchange rate and resulting changes in domestic commodity prices. Their work stimulated quite a bit of interest, as is evident from the number of responses to their paper. They include Grennes, Johnson, and Thursby; Reed; Bredahl et al.; and Orden. The literature also includes CJ (1980), a reply to the first two of these responses.

To understand the effects of exchange rates empirically, we must first divine as much as possible from theory. CJ set out to do this by analyzing the elasticity of a commodity price with respect to the exchange rate. Previous work on the subject (e.g., Kost; Bredahl and Gallagher) had generally considered that elasticity to be bounded between 0 and -1. I.e., the domestic price moves in the opposite direction from the value of the dollar, and it moves by a percentage no more than the percentage change of the dollar. CJ in their paper reject this boundedness of the elasticity. They assert that, when cross price effects are accounted for, the elasticity of price with respect to the exchange rate may be more than 0 or less than -1. Their model containing cross price effects seems to indicate greater ambiguity of exchange rate effects than previous analyses. To support their claim, they present the comparative statics of an exchange rate change. Unfortunately, they present their analysis with a model which contains changes in nominal prices but no changes in real prices and therefore no interesting price responses. Their model does not show what they assert.

In this paper, I correct and extend their analysis, arriving at some of the same conclusions they attempt to reach, but using a model of real rather than nominal domestic price changes and exchange rate changes. The main conclusions are: (1) cross price effects do matter, and can put the exchange rate elasticity of a domestic price outside the 0 to -1 bounds, although the empirical importance of that phenomenon has still to be explored, and (2) a change of the real exchange rate can take many forms, with the form of that change being at least as important as cross price effects in determining what happens to domestic prices. In addition to indicating the range of possible elasticities, the analysis which follows includes simulations to begin assessing the empirical importance of some of the possibilities. Hopefully future work will extend that assessment process, so as to more strictly limit what we can expect effects of exchange rates on domestic prices to be.

The paper contains the introduction and five additional sections. In the second section, I use the CJ model to demonstrate that the exchange rate elasticity of domestic price which they present is in fact not ambiguous, but can be determined exactly, given the assumptions of their model. I do this by solving their system for exchange rate effects on all endogenous prices, rather than solving only for one in terms of the others. I indicate that determinateness of the model is dependent on a critical but arbitrary assumption about nominal foreign income. The fact of this indeterminacy without an arbitrary assumption limits the usefulness of the CJ model for understanding exchange rate effects. More critical, however, is the casting of their model entirely in nominal terms. The model is not designed to analyze real exchange rate shocks, and in fact demonstrates no price responses to the exchange rate.[1]

The third section of the paper raises an important issue concerning analysis of real exchange rates. Results of a simple model with three goods indicate that real exchange rate changes can originate from a wide variety of relative price changes among those goods. I show that results of a real exchange rate shock vary substantially according to how that shock originated.

The fourth section is a correction and extension of the CJ model.[2] The analysis incorporates two major changes. First, the model is treated as a general equilibrium system and solved simultaneously for effects of the exchange rate on all prices. The solution gives each effect in terms only of exogenous changes. In contrast, the CJ solution for a single effect contains other endogenous effects.

Second, the analysis is performed in terms of shocks to the real exchange rate. These shocks to the real exchange rate by definition involve real relative price changes. The CJ analysis, by contrast, is cast in terms of shocks to the nominal rate only. Real relative prices do not change at all. Orden, in a recent paper, also presents an expanded analysis of the CJ model. He makes the point that exchange rate analysis should be conducted in terms of real rates. His model is couched in terms of real rates, and, like this one, incorporates nontraded goods. Orden's paper deals with relative effects on exchange rates and prices of changes in current account balances and implied changes in contemporaneous disposable income. His use of Cobb-Douglas utility functions limits the role of cross price effects. In contrast, this paper re-emphasizes the cross price effects featured by CJ, but does so in a model allowing changes in real exchange rates. The present model can demonstrate that, as CJ maintain, cross price effects do matter, and can cause the exchange rate elasticity of a price to fall outside the 0 and -1 bounds. And just as important, the form of the exchange rate change is critical. As I show in a fifth section containing simulations, a given change in the real exchange rate is consistent with a wide variety of relative price changes.

The fifth section uses reasonable elasticities and stylized facts to simulate the effects of real exchange rate shocks under a variety of conditions. Both variations in assumed elasticities and variations in sources of exchange rate change are considered. A final section concludes the paper.

2. The Chambers and Just model.

To obtain their results, CJ start with an equilibrium condition in the market for good i. In the work which follows, I denote the good of primary concern as 1 rather than i; this can be done without loss of generality. The condition is that excess supply at home of good 1 (S_1) equals excess demand abroad (D_1). Excess supply is a function of all n goods in the economy. Excess demand is a function of those n goods plus foreign income. There are no nontraded goods. Thus:

$S_1 = g(p)$ and $D_1 = f(\theta, M)$

where p is a vector of the n goods prices in home currency, θ is a vector of those same prices in foreign currency, and M is foreign income in foreign currency. The law of one price is assumed to apply to each commodity so $\theta =$

pe, where e is the nominal exchange rate, measured in foreign currency per unit of home currency.

To obtain their results, CJ differentiate this equilibrium condition. Thus:

$$\sum_{j=1}^{n} \epsilon_{1j}\hat{p}_j = \sum_{j=1}^{n} \eta_{1j}(\hat{p}_j + \hat{e}) + \eta_{1M}\hat{M} \tag{1}$$

where $\epsilon_{1j} = p_j(\partial S_1/\partial p_j)$, $\eta_{1j} = p_j(\partial D_1/\partial p_j)$, $\eta_{1M} = M(\partial D_1/\partial M)$, and a "^" over a variable indicates a proportional change. (For convenience, I have altered somewhat the symbols used in CJ. Also, because elasticities of excess supply or demand are undefined in cases where $S_1 = 0$ or $D_1 = 0$, I use semi-elasticities in equation (1) and much of the analysis following.) Rearranging yields:

$$\frac{\hat{p}_1}{\hat{e}} = \frac{\eta_{11}}{\epsilon_{11}-\eta_{11}} + \frac{1}{\epsilon_{11}-\eta_{11}}\left\{\sum_{j=2}^{n}\eta_{1j} + \sum_{j=2}^{n}(\eta_{1j}-\epsilon_{1j})(\hat{p}_j/\hat{e}) + \eta_{1M}\frac{\hat{M}}{\hat{e}}\right\} \tag{2}$$

As CJ point out, assuming $\eta_{1M} = 0$ and $\eta_{1j} = \epsilon_{1j} = 0$ for all $j \neq 1$ gives:

$$\frac{\hat{p}_1}{\hat{e}} = \frac{\eta_{11}}{\epsilon_{11}-\eta_{11}}$$

This clearly bounds the elasticity between 0 and -1. Of course, by assuming away the cross price and income terms we violate the zero degree homogeneity conditions for demand and supply. These conditions imply that:

$$\eta_{1M} + \sum_{j=1}^{n}\eta_{1j} = 0 \quad \text{and} \quad \sum_{j=1}^{n}\epsilon_{1j} = 0$$

CJ rightly point out also that cross price effects empirically are likely to be non-zero. They conclude from equation (2) that $\hat{p}_1/\hat{e}$ is not bounded between 0 and -1 and in fact is ambiguous. I show below that, given their assumptions, its value can actually be ascertained exactly.

The key is to solve simultaneously for all the endogenous $\hat{p}_j/\hat{e}$ terms. For each good, I derive an equation similar to (1). The first n-1 of these equations, stacked and rearranged slightly, can be expressed in matrix form as:

$$[\boldsymbol{\epsilon} - \boldsymbol{\eta}]\,\hat{\mathbf{p}} - \boldsymbol{\eta}_{\mathbf{M}}\hat{M} = \boldsymbol{\eta}\mathbf{s}\hat{e} \qquad (3)$$

where ϵ and $\boldsymbol{\eta}$ are (n-1 x n) matrices, with ϵ_{ij} and η_{ij} equal to $p_j \partial S_i/\partial p_j$ and $p_j \partial D_i/\partial p_j$ as before, $\hat{\mathbf{p}}$ is an (n x 1) vector of $\hat{p}_j$ terms, $\mathbf{s}$ is an (n x 1) vector of ones, and $\boldsymbol{\eta}_{\mathbf{M}}$ is an (n-1 x 1) vector of $M\partial D_i/\partial M$ terms. I drop the equation for the nth good because Walras' Law implies it is not independent of the others. With these n-1 equations and two more we can solve for $\hat{p}_i/\hat{e}$ while considering the full general equilibrium aspects of the solution. Note that two additional equations are needed because (3) contains n-1 equations and n+1 unknowns.

The nth equation is the identity for disposable income:

$$M = \sum_i e p_i q_i + \sum_i e p_i D_i = \sum_i e p_i c_i$$

where q_i and c_i are production and consumption of good i in the foreign country. Differentiating gives:

$$dM = \sum_i p_i c_i \, de + \sum_i e c_i \, dp_i + \sum_i e p_i \, dc_i$$

Assuming the foreign economy is allocating its current consumption expenditure optimally, then by the envelope theorem the third term on the right is zero. Converting to percentage change form gives:

$$\sum_i \alpha_i \hat{p}_i - \hat{M} = -\sum_i \alpha_i \hat{e}$$

where α_i is the share of the ith good in expenditure for current consumption. This is the nth equation. CJ make the nontrivial arbitrary assumption that nominal foreign income is fixed, so that $\hat{M} = 0$. In order eliminate one of the n+1 unknowns, I follow suit in this derivation. The system of n equations and n unknowns in partitioned matrix form is then:

$$\begin{bmatrix} \epsilon - \eta \\ \alpha' \end{bmatrix} \hat{p} = \begin{bmatrix} \eta \\ -\alpha' \end{bmatrix} s\,\hat{e}$$

where α' is a (1 x n) row vector, and the ith element of α' is α_i.

Zero degree homogeneity of supply implies that $\epsilon s = 0$, where 0 is an (n x 1) vector of zeros. Thus we can subtract $\epsilon s \hat{e}$ from the right side of the equation without altering it. From this, the previous matrix equation becomes:

$$\begin{bmatrix} \epsilon - \eta \\ \alpha' \end{bmatrix} \hat{p} = \begin{bmatrix} \eta - \epsilon \\ -\alpha' \end{bmatrix} s\hat{e}$$

Premultiplication by the inverse of the matrix yields:

$$\frac{\hat{p}}{\hat{e}} = -s$$

<u>Each and every price p_j has an elasticity with respect to e of -1</u>.[3]

This result seems surprising. However, if we consider the economics of the situation rather than the mathematics it becomes much less so. In fact, it would be more surprising if a change in only the nominal exchange rate should change relative real world prices, unless some price stickiness caused differences in adjustment speeds. This model has no such stickiness: in fact it posits no adjustment process. Given no particular differences in adjustment speed and given the law of one price, one possible response to a change in e is for all prices in one of the countries to make an exactly compensating proportional change. In the CJ model, nominal income in the foreign country is by assumption fixed. This could be achieved with fixed real income and fixed nominal prices in the foreign country. In such a case, all price adjustment would take place in the home country; prices and nominal income would rise or fall exactly in proportion with the exchange rate. This solution, fixed prices abroad and price changes at home exactly compensating the change in exchange rate, is certainly possible. Given the mathematics of the model, it turns out also to be the unique solution. Notice that no effects on trade or real relative prices arise from the change in the nominal exchange rate.

This analysis demonstrates that, contrary to the findings of CJ, $\hat{p}_1/\hat{e}$ is not ambiguous but can be determined to be -1. The assumption $\hat{M} = 0$ is critical for obtaining this particular value. Some other, no less arbitrary assumption could be made and a different value would result. E.g., assume $\hat{M}^* = 0$, where M^* is foreign income denominated in dollars. In that case, since $\hat{M} = \hat{M}^* + \hat{e}$, equation (4) would be replaced by:

$$\begin{bmatrix} \epsilon - \eta \\ \alpha' \end{bmatrix} \hat{p} = \begin{bmatrix} \eta s + \eta_{1M} \\ -\alpha' s + 1 \end{bmatrix} \hat{e}$$

The terms in the right hand matrix equal zero, the top term because of zero degree homogeneity and the bottom because the α_i's sum to 1; therefore, the $\hat{p}_j$ terms all equal zero as well. Home prices in this example would be fixed; prices in terms of foreign currency would have to compensate for the exchange rate change.

Other assumptions would lead to still different results. Without some arbitrary normalizing assumption, though, equation (3) is indeterminate. In economic terms, we know that when the exchange rate changes, home nominal prices will move relative to foreign nominal prices, but without more information, we do not know how the movement will be divided between the two. One country's prices could experience the entire movement, or the other country's could, or some combination of the two. It is even possible that both would move in the same direction, but by different amounts. In order to determine the price movements in the two countries, it is necessary to impose some normalization. CJ's use of fixed foreign nominal income amounts to such a normalization. Other, just as arbitrary normalizations are also reasonable. In the model developed below, the issue disappears because I make no attempt to determine nominal price changes, but analyze in terms only of real (i.e., relative) price changes.

A good reference on the importance of an arbitrary assumption in models with nominal exchange rate shocks is Dornbusch. In a somewhat similar model, Dornbusch explores the issue of effects of exchange rate changes on prices and concludes that little can be said without specification of some such assumption. Although the Dornbusch model is somewhat like that in CJ, it differs in the important respect that Dornbusch specifies his model so the exchange rate shocks lead to real relative price changes.

3. Causes of Changes in the Real Exchange Rate

To gain important insights on exchange rate effects, it is clear that we should focus on real exchange rates and real relative price changes at home. The sections which follow do so. In this section I explore an important difficulty in such an analysis. The difficulty arises from the need to know the form of a real exchange rate change before we can know its effects. A simple U.S., rest-of-world three-good model will illustrate.

The three goods are a traded good T with price p_T, a home nontraded good N with price p_N, and a foreign nontraded good F with price p_F. In contrast with the preceding section, here all prices are denoted in dollars. From these come U.S. and rest of world price levels, in dollars also:

$$\beta_T\hat{p}_T + \beta_N\hat{p}_N = \hat{p}_U \tag{4a}$$

$$\alpha_T\hat{p}_T + \alpha_F\hat{p}_F = \hat{p}_R \tag{4b}$$

As usual we can choose a numeraire. In order to be able for all prices to be in conventional "real" terms, we let the home price level be the numeraire. Thus $p_U = 1$ and (4a) becomes:

$$\beta_T\hat{p}_T + \beta_N\hat{p}_N = 0 \tag{4a}$$

The real exchange rate is defined to be $r = p_U/p_R$. From that we can revise (4b) above to:

$$\alpha_T\hat{p}_T + \alpha_F\hat{p}_F = -\hat{r} \tag{4b}$$

The three goods also imply three excess supply equations which are functions of the three prices. (For simplicity here I leave out income terms.) With those three equations, plus (4a) and (4b) above, it would appear that we have a set of five equations in three unknowns (p_N, p_T, and p_F) plus one exogenous variable (r). However, Walras' Law applies and turns out to eliminate all three excess supplies as independent equations.

Walras' Law rests on the presumption that for an individual, the value of his excess supply for one commodity must be compensated by the value of his excess supplies of other commodities. The individual's budget constrains him in this manner. If he is able to trade one good for another then his excess supplies for the individual goods need not each be zero. His overall value of excess supply for all goods must be zero, however. Thus

$$\sum_j \sum_i p_i S_{ij} = 0 \tag{5}$$

where j indexes individuals and i indexes goods. Also, the total of excess supplies summed over individuals for one good must be zero.

$$\sum_{j} p_i S_{ij} = 0 \qquad (6)$$

Equation (5) plus (6) applied to all but the last good, together imply that (6) will also hold for that last good. Thus (6) for the last good is not independent.

This is Walras' Law. If we divided the goods into two or more groups, with spillovers of excess supply from one group to the other not allowed, then there would be two or more systems of excess supplies like the one just described, with the last equation in each being implied by the others. Thus two or more equations, one in each system, would be redundant because of Walras' Law. This division is exactly what the tradeables-nontradeables dichotomy implies for our model. There are three groups, tradeables plus one nontradeable for each country. In each country, the nontradeables sector is insulated from the tradeables. Thus one nontradeable equation for each country, plus one tradeables equation for the two countries together, are all redundant. All three of the excess supply equations in the system are therefore redundant in this case.[4]

We are left with (4a) and (4b) as a set of two equations in three unknowns (with $\hat{r}$ exogenous). We need one more equation to determine a unique set of relative prices. There is insufficient information in the equations set forth so far. In particular, given those two, we need to know exactly what has changed to affect the real exchange rate.

One possibility is that prices in the foreign country remain constant relative to each other, but move together relative to the U.S. price level (actually, in nominal terms the U.S. price level could be the one moving). Such a situation could occur, for example, from a shock in the U.S. which had little effect on prices abroad relative to each other. This situation would imply the equation:

$$\hat{p}_T - \hat{p}_F = 0 \qquad (7a)$$

Solving the three equations, with $\hat{r} = 1$ (a 1% appreciation of the dollar) yields the solution:

$$\hat{p}_N = \beta_T/\beta_N , \quad \hat{p}_T = -1 , \quad \hat{p}_F = -1 \qquad (8a)$$

Foreign prices expressed in dollars, both tradeable and nontradeable, move relative to U.S. prices by the full amount of the exchange rate change. Relative prices at home shift. Given that relative tradeables prices drop and the overall price level is the numeraire, the home nontradeables relative price must rise, with the extent of the rise determined by the sizes of the two sectors.

A second possibility is that the real exchange rate change comes entirely from a shift in p_T, perhaps from a commodity price shock, with the foreign nontradeables price p_F remaining constant relative to U.S. prices:

$$\hat{p}_F = 0 \qquad (7b)$$

This case yields the solution:

$$\hat{p}_N = \beta_T/\beta_N\alpha_T , \quad \hat{p}_T = -1/\alpha_T , \quad \hat{p}_F = 0 \qquad (8b)$$

In this case the tradeables price must move more than proportionally with the real exchange rate. For the whole foreign price level to shift 1%, with the nontradeables price constant, the tradeables price must shift more than 1%. At home, since p_T declines, p_N must rise in order to keep the home price level fixed.

A third case would be for relative prices at home to stay constant (or equivalently, for the home nontradeables price to remain fixed), so that:

$$\hat{p}_N = \hat{p}_T \tag{7c}$$

The solution then is:

$$\hat{p}_N = 0 , \quad \hat{p}_T = 0 , \quad \hat{p}_F = -1/\alpha_F \tag{8c}$$

In this case the foreign price level changes only because the foreign nontradeables price changes. Foreign nontradeables prices change perhaps from money-induced inflation abroad, yet with tradeables prices abroad pegged to U.S. markets. Home prices are constrained to remain constant overall, as well as relative to each other. Note that the real dollar price of tradeables does not move at all in response to the exchange rate.

Ingenuity could doubtless produce additional cases. Various combinations of monetary and other events at home and abroad, for example, could produce a wide variety of possibilities. The point is that to know how a real exchange rate shift will affect prices in general, we must know what relative prices are changing to bring about the change in the real rate. Given the three prices, there are many combinations of changes in relative prices which can bring about a given change in the real exchange rate.

A fertile area for future research would be to go beyond the real exchange rate to its components to see how these components are moved and in turn how they affect domestic real agricultural and other prices. In the remainder of this paper, we usually assume the price level of foreign nontradeables to remain constant relative to an index of tradeables prices. Thus we adopt the first case above as our "usual" one.

4. Price Responses to Real Exchange Rates.

In this section we describe a model in the spirit of CJ, but using changes in the real exchange rate rather than the nominal. We retain their assumption of the law of one price for traded goods and include nontraded goods so as to allow for real exchange rate shocks.[5]

The model is a two-country one with three goods produced in the home country and one good produced abroad. The home goods include a nontraded good with price P_N, an agricultural export good with price P_A, and a second export good with price P_E. This second export good is included to allow for the possibility of cross price effects of substitutability or complementarity. In this simple model the nontraded good serves as numeraire good; also, for simplicity P_N will be treated as the domestic price level, an assumption which is not too strong if the nontraded sector comprises the largest part of the economy. The foreign country consumes a domestically produced traded good with price P_O, as well as importing the agricultural good and the other export good from the home country. Adding complexity would improve the realism of the model, but would not alter the nature of the qualitative results. See appendix A for a more general model.

Equilibrium conditions for the four goods are:

$$S_i(P_A, P_E, P_N, P_O, M_U) = D_i(P_A^*, P_E^*, P_O^*, M_R^*)$$

where S_i and D_i denote excess supply at home and excess demand abroad. M_U and M_R are home and foreign disposable income and starred quantities indicate values in foreign currency. Zero degree homogeneity of demand allows us to divide all foreign currency values by e, the nominal exchange rate in foreign currency per unit of home currency. We can also use zero degree homogeneity to divide all prices and income by P_N and thus convert them to real values, denoted by lower case letters:

$$S_i(p_A, p_E, p_O, m_U) = D_i(p_A, p_E, p_O, m_R)$$

This model contains two groups of goods, nontradeables at home and tradeables; we have left out nontradeables abroad. This is one solution to the problem posed by section 3. In terms of the model of that section, we have eliminated one variable (the foreign nontradeable good price) so that equations (4a) and (4b) alone would be sufficient to determine a solution. In this simpler model we can focus on the effects of cross price terms.

As in section 3, Walras' Law implies that one of the equations in each group is redundant. Dropping the equations for N and O, we are left with two equations and five unknowns (p_A,p_E,p_O,m_R, and m_U).

Two more equations come from defining income:

$$p_A c_A + p_E c_E + p_O c_O = m_R \tag{9a}$$

$$p_A g_A + p_E g_E + p_O g_O + g_N = m_U \tag{9b}$$

where c_i and g_i are consumption of good i abroad and at home.

Finally, we note that the real exchange rate is $r = eP_N/P_O$, the nominal rate adjusted for price level differences in the two countries. But $p_O = P_O/eP_N$. Thus $p_O = 1/r$; the foreign price level in real dollars is the reciprocal of the real exchange rate. We will treat the real exchange rate and therefore p_O as exogenous.

We are left with the four unknowns p_A, p_E, m_R, and m_U, and two excess demand and supply equations:

$$D_A(p_A, p_E, p_O, m_R) = S_A(p_A, p_E, p_O, m_U)$$

$$D_E(p_A, p_E, p_O, m_R) = S_E(p_A, p_E, p_O, m_U)$$

plus equations (9a) and (9b). Differentiating these and rearranging we obtain the system:

$$\begin{bmatrix} D_{AA}-S_{AA} & D_{AE}-S_{AE} & D_{Am} & -S_{Am} \\ D_{EA}-S_{EA} & D_{EE}-S_{EE} & D_{Em} & -S_{Em} \\ c_A & c_E & -1 & 0 \\ g_A & g_E & 0 & -1 \end{bmatrix} \begin{bmatrix} dp_A \\ dp_E \\ dm_R \\ dm_U \end{bmatrix} = \begin{bmatrix} -D_{A0}+S_{A0} \\ -D_{E0}+S_{E0} \\ -c_0 \\ -g_0 \end{bmatrix} dp_0 \qquad (10)$$

Note that in the differentiation of (9a) and (9b), $\sum_i p_i\ dc_i = \sum_i p_i dg_i = 0$, by the envelope theorem.

The matrix equation can be solved by inverting the left-hand side matrix B, which in general will be non-singular. The parameter of particular interest here is dp_A/dp_0, which can readily be converted to elasticity form. Even for this relatively simple model the resulting solution is a complex combination of own and cross price terms, marginal propensities to consume out of income, and consumption quantities. All sorts of results are possible. In the paragraphs which follow, I first check the implications of stability conditions, and then review some special cases of interest obtained by restricting parameter values.

The Routh-Hurwitz Theorem for Stability (see Quirk and Saposnik) requires that $k_i > 0$, where $k_i = (-1)^i$ times the sum of all ith order principal minors of the matrix B. Thus:

$$k_1 = -[(D_{AA} - S_{AA}) + (D_{EE} - S_{EE}) - 2] > 0 \qquad (11a)$$

$$\begin{aligned} k_2 = {} & [(D_{AA} - S_{AA})(D_{EE} - S_{EE}) - (D_{AE} - S_{AE})(D_{EA} - S_{EA})] \\ & - 2(D_{AA} - S_{AA}) - (D_{Am}c_A - S_{Am}g_A) \\ & - 2(D_{EE} - S_{EE}) - (D_{Em}c_E - S_{Em}g_E) + 1 > 0 \end{aligned} \qquad (11b)$$

$$k_3 = 2\,[(D_{AA} - S_{AA})(D_{EE} - S_{EE}) - (D_{AE} - S_{AE})(D_{EA} - S_{EA})]$$
$$- (D_{AE} - S_{AE})(D_{Em}c_A - S_{Em}g_A) - (D_{EA} - S_{EA})(D_{Am}c_E - S_{Am}g_E)$$
$$+ (D_{AA} - S_{AA})(D_{Em}c_E - S_{Em}g_E) + (D_{EE} - S_{EE})(D_{Am}c_A - S_{Am}g_A)$$
$$- (D_{Am}c_A - S_{Am}g_A) - (D_{Em}c_E - S_{Em}g_E)$$
$$- [(D_{AA} - S_{AA}) + (D_{EE} - S_{EE})] \quad > \quad 0 \tag{11c}$$

$$k_4 = [(D_{AA} - S_{AA})(D_{EE} - S_{EE}) - (D_{AE} - S_{AE})(D_{EA} - S_{EA})]$$
$$- D_{Am}[(D_{EA} - S_{EA})c_E - (D_{EE} - S_{EE})c_A]$$
$$- D_{Em}[(D_{AE} - S_{AE})c_A - (D_{AA} - S_{AA})c_E]$$
$$+ S_{Am}\,[(D_{EA} - S_{EA})g_E - (D_{EE} - S_{EE})g_A]$$
$$+ S_{Em}\,[(D_{AE} - S_{AE})g_A - (D_{AA} - S_{AA})g_E]$$
$$+ (S_{Am}D_{Em} - S_{Em}D_{Am})(c_Ag_E - c_Eg_A) \quad > \quad 0 \tag{11d}$$

Also, it is required that:

$$k_1 k_2 - k_3 > 0 \quad \text{and} \quad k_1 k_2 k_3 - k_3^2 - k_4 k_1^2 > 0. \tag{11e}$$

Some limited conclusions can be drawn from these conditions. First, (11a) will be met if own price effects have the usual signs, so that $D_{AA} - S_{AA} < 0$ and $D_{EE} - S_{EE} < 0$. These are sufficient but not necessary for $k_1 > 0$. Condition (11b) will more likely be met if cross price effects for A and E are less than own price effects in absolute value, in the sense that:

$$(D_{AA} - S_{AA})(D_{EE} - S_{EE}) - (D_{AE} - S_{AE})(D_{EA} - S_{EA}) \quad > \quad 0 \tag{12}$$

This condition is neither sufficient nor necessary in general. However, if income effects for A and E at home and abroad are zero, so that $D_{Am} = S_{Am} = D_{Em} = S_{Em} = 0$, then (12), together with the $D_{AA} - S_{AA} < 0$ and $D_{EE} - S_{EE} < 0$, becomes sufficient for $k_2 > 0$.

Conditions (11c) and (11d) are more forbidding. However, if the income effects are zero and the conditions given above for k_1 and k_2 are met, then k_3 and k_4 will exceed zero as well. Finally, meeting these conditions will also guarantee that (11e) is met. It should be stressed that the parameter restrictions mentioned are together sufficient, but not necessary for stability. It appears that here, as in many instances, smaller income effects enhance prospects for stability.

I next describe what happens to the exchange rate elasticity of P_A under some special cases. First, consider the implications of the set of sufficient conditions for stability described above. Converting to elasticities, and noting that $\hat{p}_D = -\hat{r}$, we get:

$$\frac{\hat{p}_A}{\hat{e}} = \frac{(\eta_{AD} - \epsilon_{AD})(\eta_{EE} - \epsilon_{EE}) - (\eta_{ED} - \epsilon_{ED})(\eta_{AE} - \epsilon_{AE})}{\Delta} \qquad (13a)$$

where $\Delta = (\eta_{AA} - \epsilon_{AA})(\eta_{EE} - \epsilon_{EE}) - (\eta_{AE} - \epsilon_{AE})(\eta_{EA} - \epsilon_{EA})$.

Zero degree homogeneity of excess demand and supply implies that:

$$\sum_{j=A,E,F} \eta_{ij} + \eta_{im} \quad \text{for } i=A,E$$

$$\sum_{j=A,E,F,N} \epsilon_{ij} + \epsilon_{im} \quad \text{for } i=A,E$$

Substituting for η_{AD}, ϵ_{AD}, η_{ED} and ϵ_{ED} and rearranging gives the expression:

$$\frac{\hat{p}_A}{\hat{r}} = -1 + \frac{(\eta_{EE} - \epsilon_{EE})\epsilon_{AN} - (\eta_{AE} - \epsilon_{AE})\epsilon_{EN}}{\Delta} \qquad (13b)$$

In general, we would expect cross price elasticities of supply to be negative and cross price elasticities of demand to be positive. If that is true here, and if own price elasticities exceed cross price, then the right-hand term in

(13b) will be positive and less than one. The $\hat{p}_A/\hat{r}$ elasticity will lie in the 0 to -1 interval.

Note, however, that if excess supplies at home of A and E are unaffected by the price of the nontraded good, then the right-hand term is zero and the elasticity is exactly -1. Further, if A and E are both complements in production with the nontraded good, so that $\epsilon_{AN}>0$ and $\epsilon_{EN}>0$, and A and E are substitutes in consumption and production with each other, then the right hand term is negative, and the price of A changes more than proportionally with the real exchange rate. Such conditions seem unlikely, though they are certainly possible.

Alternatively, from (13a) it is evident that if cross price effects of A and E with respect to P_O are zero, the whole elasticity becomes exactly 0. Or if good O is a complement of A and E (and A and E are substitutes), then $\hat{p}_A/\hat{r}$ will exceed zero, thus lying outside the 0 to -1 bounds in the opposite direction. The price of A will respond counter to intuition.

A particularly interesting case occurs from (13a) if cross price effects of A with respect to E are zero and $\epsilon_{AO} = 0$. In such a case $\eta_{AO} = -\eta_{AA}$. Equation (13a) collapses to the usual partial equilibrium formulation:

$$\frac{\hat{p}_A}{\hat{r}} = \frac{-\eta_{AA}}{\eta_{AA} - \epsilon_{AA}} \qquad (14)$$

The response elasticity of p_A with respect to r depends only on its own price excess demand and supply elasticities. Because formula (14) is appealing, it is important to remember how many parameter restrictions we imposed to get it. (Actually, for this particular result, we can relax the restrictions somewhat. It is not necessary for η_{EA} or ϵ_{EA} to be zero, nor for income effects for good E to be zero. The other mentioned restrictions still apply.)

It is difficult to generalize about the results we have presented. We have shown that in our model the most typical cross price elasticity values lead to changes in export goods prices less than in proportion to exchange rate changes. This is the "typical" expected result. However, we have also shown that an export good price can "overadjust," or it can adjust in the "wrong" direction, given somewhat less usual cross price elasticities. This supports the original CJ contention that cross price elasticities do matter and that they may lead to domestic price movements outside the 0 to -1 bounds assumed in some of the literature.

It is important to point out that the results of this section depend on the form we assumed for the real exchange rate change. The results of section 3 imply that how the rate changes may be quite important. That conclusion will be born out in the simulations of section 5 below.

Since so many results are possible, one must insert actual parameters to draw any further conclusions. The section which follows contains simulated response elasticities of agricultural prices to the exchange rate. I use elasticities from a well-known Longmire-Morey paper, and supply reasonable estimates for any additional required parameters.

5. Simulations.

This section contains simulations to illustrate the effects of exchange rate changes on real wheat, corn, and soybean prices. The model is that of appendix A. To complete the simulation model, three additional goods are specified -- another traded good (O), a home nontraded good (H), and a foreign nontraded good (F). Data are meant to represent approximately the situation of 1980. Consumption expenditures used for the model are given in table 1. Wheat, corn, and soybean expenditures are from Longmire and Morey. Total consumption numbers are based on IMF and World Bank data. The most speculative expenditure number to estimate is other tradeables. For the U.S. that number is set at approximately twice the level of imports. For the rest of the world (ROW), a similar proportion relative to total consumption is employed. The nontraded goods are calculated as residuals.

The most speculative aspect of the model is choosing the elasticities not specified in Longmire and Morey. We are let off the hook to some extent by Walras' Law, since it requires omission of one equation from each of the three groups (tradeables, U.S. nontradeables, and R.O.W. nontradeables). The left-out goods will be "other" tradeables and the two nontraded goods. Elasticities used for the base line case are presented in table 2. The own and cross price elasticities for wheat, corn, and soybeans with respect to wheat, corn, and soybean prices are from Longmire and Morey. The others are rough guesses chosen with the zero degree homogeneity constraint applied. Like Longmire and Morey, we in most cases assume the rest of the world elasticities to equal those for the U.S. (except, of course, that in each country elasticities for the nontraded good of the other are zero). Table 2 presents the U.S. elasticities.

In addition to the elasticities of table 2, the model employs expenditure shares generated from table 1, as well as price index weights. In these simulations, we use the expenditure shares also as price index weights.

The results of simulating a -1% change in the real exchange rate are presented in table 3. Case 1 indicates results using the base line elasticity assumptions. Cases 2 through 10 are derived in each instance by varying the assumptions of the base line case. Cases 1 through 6 employ the assumption that the price level of the tradeables sector relative to the nontradeables sector abroad is fixed. Cases 7 and 8 are based on as assumption of a fixed nontradeables price abroad. Cases 9 and 10 are computed keeping the price level of tradeables relative to nontradeables at home fixed. The ten cases thus illustrate the effects of changes along two major dimensions -- variations in elasticity combinations and variations in the particular set of relative price changes leading to alteration of the real exchange rate. Table 4 indicates the changes in elasticities, highlighted with underlining, made from the base line case.

In the base line case a one percent decrease in the value of the dollar results in "standard" price movements for wheat and soybeans and "nonstandard" price movements for corn and the "other" tradeable good. I.e., for wheat and soybeans the prices change in the appropriate direction by less than the full amount of the exchange rate change; whereas, corn and the "other" tradeable prices move more than in proportion to the exchange rate. It is important to note that traded goods prices must move on average by 1%.

Case 2 is the "partial equilibrium" case for wheat. By restricting certain elasticities to zero, we end up with an elasticity for wheat equal to the excess demand elasticity abroad divided by the sum of excess supply and demand elasticities. Interestingly, not only does wheat end up with that value, but given the restrictions required, corn and soybeans end up with exactly the same value. Again the "other" tradeables price move by more than 1%.

Case 3 is similar to case 1, but with lower cross price effects of other tradeables on wheat, corn, and soybean markets. The elasticities for the three agricultural crops all fall within the "expected" 0 to 1 range.

Case 4 is obtained by setting income elasticities for wheat, corn, and soybeans to zero. As is evident, the corn price again responds more than proportionally with the exchange rate.

Case 5 is derived by setting cross price elasticities with respect to nontradeables to zero. As is evident, the three price effects for the agricultural products again fall within the "standard" range.

Case 6 represents the situation if own price elasticities for wheat, corn, and soybeans are doubled. The resulting price effects are very close to those of case 1, although all three agricultural price effects move even more closely with the exchange rate.

Cases 7 and 8 represent the "standard" and "partial equilibrium" cases computed under an assumption of a fixed foreign nontradeables price. Case 7 illustrates the importance of determining what particular relative prices shift to cause the real exchange rate change. Given the assumptions here, the tradeables prices all move by several times the exchange rate change. The home nontradeable relative price of course compensates in the opposite direction by a substantial amount. Case 8 is similar to case 7, except that the particular elasticity assumptions create the "partial equilibrium" result for wheat, corn, and soybeans.

Cases 9 and 10 employ still another assumption about how the real exchange rate changes. In this case the home nontradeables price is kept constant. Given that assumption, the price level of tradeables at home must also remain constant. Case 9, like case 7, demonstrates how much differently prices respond under different assumptions. In this case, wheat, corn, and soybean prices respond much less strongly to the exchange rate than in most of the other cases. Case 10 is similar to case 9, but with the "partial equilibrium" results for the three crops.

What conclusions are we to draw overall? First, it is obvious that the form of a real exchange rate change can make a tremendous difference in how the exchange rate affects domestic prices. Second, the "partial equilibrium" case can arise, but would seem to appear only under quite restrictive conditions. Third, for cases 1 through 6, prices of the three agricultural goods seem to respond on average close to proportionately with the exchange rate, but with a fairly wide variety among the various possibilities. Fourth, without more specific knowledge of the system, it is difficult to make broad a priori judgments about how exchange rates can be expected to affect individual prices.

6. Conclusion.

This paper discusses the important question of how exchange rates affect domestic prices. A major aspect is an issue raised by CJ: whether in estimations of price and exchange rate effects on trade, the price effects can be expected a priori to be equal to or less than the exchange rate effects (with signs specified to take account of how the exchange rate is defined). I demonstrate that the formal model used by CJ is not rich enough to answer the question being asked. This is mainly because their model treats the effect of one nominal variable on a set of others, with nothing in the model to cause real relative price changes. In fact their model essentially precludes a most important real price change, that of the real exchange rate. This paper demonstrates that exchange rate effects on prices are determinate in the model, but only because of an arbitrary assumption concerning nominal income abroad.

Section 3 shifts the analysis to one of the real exchange rate and describe a critical issue for determining exchange rate effects. I show that there are many sets of relative price changes which can lead to any given change in the real exchange rate. In order to understand how the real exchange rate affects domestic prices, we must also understand just what underlying price changes are causing the real exchange rate to move.

Section 4 concentrates on the issue of how cross price and income elasticities can change the effects of exchange rates on domestic prices. The intuition of CJ on this point turns out to be generally valid. When their model is replaced by one incorporating real exchange rate shocks, we find that results of the shocks depend substantially on cross price and income elasticities. Also, it is clear that, as CJ suggests, a domestic price can move more than in proportion to the exchange rate, or it can move in the "wrong" direction.

Section 5 contains simulations which bear out the theoretical conclusions of sections 3 and 4. The simulations indicate that the form of a real exchange rate change is quite important for determining domestic price effects. Changes from altering elasticity assumptions do not appear to be so dramatic, though they are certainly significant. The cases of price responses outside the 0 to 1 range appear quite commonly in the simulations.

These results are important as guides to what constraints on price and exchange rate coefficients are reasonable in empirical work. Certainly they indicate some important questions which should be asked in evaluating theory and empirical results concerning exchange rate effects. They also suggest important areas for future research. Work should be continued on estimating individual own price, cross price, and income elasticities. Just as significant, however, would be more detailed investigation of the form of real exchange rate movements and how particular forms differ in their effects on domestic markets.

Table 1

Expenditure Levels for Simulations, Billions of Dollars

	Wheat	Corn	Soybeans	Other Traded	Nontraded	Total
U.S.	4.25	16.25	8.0	514.0	2087.5	2630.0
R.O.W.	71.4	36.25	13.0	1925.0	7580.35	9626.0

Table 2

U.S. Elasticities Used for Base Line (Case 1) Simulation

Good \ Price	W	C	S	O	N	M
Demand						
W	-.2	.05	.05	.1	0	0
C	.05	-.4	.1	.1	.075	.075
S	.05	.1	-.4	.1	.075	.075
Supply						
W	.4	-.15	-.05	-.1	-.1	0
C	-.15	.4	-.3	-.1	.15	0
S	-.05	-.3	.4	-.1	.05	0

Table 3

Simulated Results of 1% Decline in Exchange Value of Dollar

Case Price	1	2	3	4	5
W	.969	.869	.866	.960	.926
C	1.025	.869	.861	1.005	.845
S	.977	.869	.810	.957	.843
O	1.001	1.008	1.009	1.002	1.007
H	-.260	-.260	-.260	-.260	-.260
F	1.000	1.000	1.000	1.000	1.000

Case Price	6	7	8	9	10
W	.984	3.537	.869	.279	.869
C	1.017	4.080	.869	.203	.869
S	.988	3.736	.869	.235	.869
O	1.000	4.767	4.947	-.012	-.048
H	-.260	-1.227	-1.230	0	0
F	1.000	0	0	1.269	1.268

Table 4
Elasticity Values for Cases 2 Through 10

Case 2
"Partial Equilibrium Case"
Demand

Price / Good	W	C	S	O	N	M
W	-.2	0	0	0	0	.2
C	.4	-.4	0	0	0	0
S	.2	.2	-.4	0	0	0

Supply

Good	W	C	S	O	N	M
W	.4	0	0	0	0	-.4
C	-.4	.4	0	0	0	0
S	-.1	-.3	.4	0	0	0

Case 3
Lower Cross Price Elasticities w.r.t. Other Tradeable
Demand

Price / Good	W	C	S	O	N	M
W	-.2	.05	.05	.05	0	.05
C	.05	-.4	.1	.05	.075	.125
S	.05	.1	-.4	.05	.075	.125

Supply

	W	C	S	O	N	M
W	.4	-.15	-.05	-.05	-.15	0
C	-.15	.4	-.3	-.05	.1	0
S	-.05	-.3	.4	-.05	0	0

Case 4
Income Elasticities Zero
Demand

Good \ Price	W	C	S	O	N	M
W	-.2	.05	.05	.1	0	0
C	.05	-.4	.1	.1	.15	0
S	.05	.1	-.4	.1	.15	0

Case 5
Cross Price Elasticities w.r.t. Nontradeables Zero
Demand

Good \ Price	W	C	S	O	N	M
W	-.2	.05	.05	.1	0	0
C	.05	-.4	.1	.1	0	.15
S	.05	.1	-.4	.1	0	.15

Supply

Good	W	C	S	O	N	M
W	.4	-.15	-.05	-.2	0	0
C	-.15	.4	-.3	.05	0	0
S	-.05	-.3	.4	-.05	0	0

Case 6
Own Price Elasticities Doubled
Demand

Price / Good	W	C	S	O	N	M
W	-.4	.05	.05	.3	0	0
C	.05	-.8	.1	.5	.075	.075
S	.05	.1	-.8	.5	.075	.075

Supply

Good	W	C	S	O	N	M
W	.8	-.15	-.05	-.5	-.1	0
C	-.15	.8	-.3	-.5	.15	0
S	-.05	-.3	.8	-.5	.05	0

Case 7
Standard Case, Foreign Nontradeables Price Fixed

Same as Standard Case above

Case 8
"Partial Equilibrium" Case, Foreign Nontradeables Price Fixed

Same as Case 2 above

Case 9
Standard Case, Home Nontradeables Price Fixed

Same as Standard Case above

Case 10
"Partial Equilibrium" Case, Home Nontradeables Price Fixed

Same as Case 2 above

References

Bredahl, Maury E., and Paul Gallagher. "Comment on 'Effects of an Exchange Rate Change on Agricultural Trade." Agricultural Economics Research 29(2) (April 1977), pp. 45-48.

Bredahl, Maury, et al. "The Impact of Currency Value Changes Given Free Trade, Non-Tariff Barriers and Non-Traded Goods." Agricultural Economics Paper No. 1979-29, University of Missouri-Columbia, 1979.

Chambers, Robert G., and Richard E. Just. "A Critique of Exchange Rate Treatment in Agricultural Trade Models." American Journal of Agricultural Economics (May 1979): 249-257.

Chambers, Robert G., and Richard E. Just. "A Critique of Exchange Rate Treatment in Agricultural Trade Models: Reply." American Journal of Agricultural Economics (May 1980): 255-259.

Dornbusch, Rudiger. "Exchange Rates and Fiscal Policy in a Popular Model of International Trade." American Economic Review (December 1975): 859-871.

Grennes, Thomas, Paul R. Johnson, and Marie Thursby. "A Critique of Exchange Rate Treatment in Agricultural Trade Models: Comment." American Journal of Agricultural Economics (May 1980): 249-252.

Kost, William E. "Effects of an Exchange Rate Change on Agricultural Trade." Agricultural Economics Research 28(3) (July 1976), pp. 99-106.

Longmire, Jim, and Art Morey. Strong Dollar Dampens Demand for U.S. Farm Exports. Foreign Agricultural Economic Report Number 193, Economic Research Service, U.S. Department of Agriculture, December 1983.

Orden, David. "The Exchange Rate and Agricultural Prices: A Comment on Some Neglected Income Effects." American Journal of Agricultural Economics (forthcoming Nov. 1986).

Quirk, James, and Rubin Saposnik. Introduction to General Equilibrium Theory and Welfare Economics. New York: McGraw-Hill, 1968.

Reed, Michael R. "A Critique of Exchange Rate Treatment in Agricultural Trade Models: Comment." American Journal of Agricultural Economics (May 1980): 253-254.

Notes

1. The main text contains discussion of the CJ 1979 model. A revised model in CJ 1980 is not materially different; appendix B provides a complete discussion of that model as well.)

2. A more general version of this model is presented in appendix A.

3. Bredahl et al. make the point that the CJ analysis leads to exchange rate elasticities all equal to -1. However, their own model is incorrectly specified since they ignore Walras' Law. One of their equations is redundant so their elasticity matrix is singular.

4. In this model, and in the other models described in this paper, countries are allowed to run non-zero balances on their trade accounts. Implicitly they are allowed to borrow from each other. Also implicitly there is another market, a market in capital (or possibly in money, as a store of value). However, separability is maintained between consumption in the present period and consumption in any other period. Therefore we are able to analyze consumption allocations within the present without considering intertemporal choice. As a result, we can ignore the prices of goods in other periods. They do not show up in our excess demand and supply functions for the present period. Also, the excess demands and supplies for those goods can be left out of our system.

5. CJ 1980 includes nontraded goods, but not real exchange rate changes. See appendix B for a discussion of the CJ 1980 model.

Appendix A

In this appendix I present a more general version of the analysis of section 4. The number of goods of each type is allowed to vary. Also, home and foreign price indices p_U and p_R are defined covering all consumption goods for the two countries.

I begin with market clearing equations similar to equation (10) in CJ 1980:

$$S(p,\bar{p},m_U) = D(\delta,p^*,M_R) \qquad \text{(A1)}$$

where S and D are n+q+r vectors of excess supplies at home and excess demands abroad, p and δ are nx1 vectors of traded goods prices, $\bar{p}$ is a qx1 vector of home nontraded goods prices, and p^* is an rx1 vector of foreign nontraded goods prices. As indicated in section 3, three equations are redundant, one for traded and one for each group of nontraded goods. Therefore, I drop the last of each group. Converting the prices in the excess demand function to dollars (allowed by zero degree homogeneity of S and D) gives n+q+r-3 market clearing conditions in terms of dollar prices:

$$S(p,\bar{p},m_U) = D(p,p',m_R)$$

Differentiating totally and converting to semi-elasticities yields:

$$(\epsilon - \eta)\hat{\pi} + \epsilon_m\hat{m}_U - \eta_M\hat{m}_R = 0 \qquad \text{(A2)}$$

where ϵ and η are (n+q+r-3) x (n+q+r) matrices of semi-elasticities from the independent equations of (A1), ϵ_m and η_M are (n+q+r-3) x 1 vectors of semi-elasticities, and $\pi' = (p\ \bar{p}\ p')$, an (n+q+r)x1 vector of dollar prices. Note the absence of an exchange rate term. Equation (A2) has n+q+r-3 equations and n+q+r+2 unknowns. We can obtain two other equations by differentiating income identities.

$$\alpha'\hat{\pi} + \alpha'\hat{c} = \hat{m}_R \quad \text{and} \quad \beta'\hat{\pi} + \beta'\hat{\pi} = \hat{m}_U$$

where α and β are vectors of consumption shares, and c and g are vectors of consumptions levels. The envelope theorem implies that the second term in each equals 0, so the equations become:

$$\alpha'\hat{\pi} = \hat{m}_R \quad \text{and} \quad \beta'\hat{\pi} = \hat{m}_U \tag{A3}$$

In addition, two price indexes can be defined as:

$$\varnothing'\hat{\pi}^* = \hat{p}^*_R \quad \text{and} \quad \lambda'\hat{\pi} = \hat{p}_U \tag{A4}$$

where p^*_R and p_U are price levels in the foreign and home countries and $\varnothing$ and λ are vectors of weights, with each country's weights containing zeros in positions corresponding to nontraded goods of the other country. The foreign country equation can be converted to a dollar version:

$$\varnothing'\hat{\pi} = \hat{p}_R \tag{A5}$$

(Note that $\sum_{i=1}^{n+q+r} \varnothing_i = 1$ and $\sum_{i=1}^{n+q+r} \lambda_i = 1$.)

To convert to "real" terms, the home price level will be used as numeraire, so $\hat{p}_U = 0$. Finally, I define the real exchange rate as:

$$r = \frac{e p_U}{p^*_R} = \frac{p_U}{p_R}$$

with:

$$\hat{p}_U - \hat{p}_R = \hat{r} \tag{A6}$$

If we combine relevant portions of (A2) through (A6), we have a system of n+q+r+3 equations and n+q+r+4 unknowns (n+q+r prices, plus the two income levels, plus the two price levels). To solve the system we need one more equation. As pointed out in section 3, including three groups of goods with the implied restrictions on trade leaves us one extra degree of freedom. As indicated in that section, the restriction I will add is that the overall level of tradeables prices relative to nontradeables prices in the foreign country be fixed. Let $\boldsymbol{\emptyset}_T$ be an (n+q+r) x 1 vector. In the first n positions is that portion of the foreign price index weights applicable to the n traded goods, scaled up so that $\sum_{i=1}^{n} \emptyset_{Ti} = 1$. The other elements are zero. Likewise, let $\boldsymbol{\emptyset}_N$ be a similar vector for foreign nontradeables, also scaled up to sum to 1, with zeros in positions 1 to n and n+q+1 on. Then our last equation is:

$$\boldsymbol{\emptyset}_T'\hat{\pi} - \boldsymbol{\emptyset}_N'\hat{\pi} = 0 \tag{A7}$$

Adding this equation gives us n+q+r+4 independent equations altogether, a sufficient number to solve the system. This system can be written in partitioned matrix form as:

$$\begin{bmatrix} \epsilon - \eta & \epsilon_m & -\eta_M & 0 & 0 \\ \alpha' & 0 & -1 & 0 & 0 \\ \beta' & -1 & 0 & 0 & 0 \\ \lambda' & 0 & 0 & -1 & 0 \\ \boldsymbol{\emptyset}' & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ \boldsymbol{\emptyset}_T' - \boldsymbol{\emptyset}_N' & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{\pi} \\ \hat{m}_U \\ \hat{m}_R \\ \hat{p}_U \\ \hat{p}_R \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hat{r} \end{bmatrix} \tag{A8}$$

We can simplify (A8) somewhat by combining some of the equations:

$$\begin{bmatrix} \epsilon - \eta & \epsilon_{\blacksquare} & -\eta_M \\ \alpha' & 0 & -1 \\ \beta' & -1 & 0 \\ \lambda' & 0 & 0 \\ \emptyset_T' - \emptyset_N' & 0 & 0 \\ -\emptyset' & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\pi} \\ \hat{m}_U \\ \hat{m}_R \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \hat{r} \end{bmatrix} \qquad \text{(A9)}$$

The solution is then obtained by inverting the matrix on the left hand side. The solution for $\hat{\pi}_1/\hat{r}$ is obviously a complex function of own and cross price elasticities, as well as income and price index weights. It is not obvious, however, whether that elasticity will necessarily lie between 0 and -1. In the text I present simulated results to evaluate both the possibility and the likelihood that that parameter will lie outside of the 0 to -1 interval.

Appendix B

Following publication of the CJ piece, two comments, Grennes, Johnson, and Thursby (GJT); and Reed were published. Reed stressed the lack of nontraded goods in the original CJ model. In a reply, CJ 1980, the authors sketch out a model containing nontraded goods in each country. They assert that the addition of the extra detail does not alter the basic conclusion of their analysis. In this section, I support that assertion also with respect to the present critique of their model; the critique still carries through.

Their new model contains the equilibrium condition for excess demand and supply:

$$S_i(p,\bar{p},m) = D_i(\delta,p^*,M)$$

where $\bar{p}$ and p^* are vectors of nontraded goods prices in the two countries, m is income in the exporting country, and the other variables follow their definitions in the original paper.

Let δ and p be n × 1 vectors, $\bar{p}$ be q × 1, and p^* be r × 1. As before, the law of one price for traded goods implies $\delta = ep$. Stack the vectors p, $\bar{p}$, and p^* to form the (n+q+r) × 1 vector π. (Note: π here contains n+q prices denominated in home currency and r prices denominated in foreign currency.) Also, form the (n+q+r-1) × 1 vectors S and D of excess demands and supplies, where $S_i = 0$ for i = n+1 to i = n+q and $D_i = 0$ for i = n+q+1 to i = n+q+r-1. I drop the last traded good equation and the two last nontraded goods equations because Walras' Law implies redundancy. Taking the derivatives $\partial D/\partial\pi$ and $\partial S/\partial\pi$ and forming semi-elasticities of the elements gives (n+q+r-3) × (n+q+r) matrices η and ϵ of own and cross price effects. Note that some elements of each matrix are identically zero. Thus, $\eta_{ij} = 0$ where i corresponds to a nontraded good in the supplying country and $\epsilon_{ij} = 0$ where i corresponds to a nontraded good in the buying country. Differentiating the equilibrium condition $S(p,\bar{p},m) = D(ep,p^*,M)$ and converting to elasticities yields:

$$\epsilon\hat{\pi} + \epsilon_m\hat{m} = \eta\hat{\pi} + \eta s\hat{e} + \eta_M \hat{M}$$

where ϵ_m and η_M are (n+q+r-2) x 1 vectors of income elasticities, and s is an (n+q+r) x 1 vector, with 1's in the first n+q positions and zeros in the last r. Rearranging gives:

$$(\epsilon - \eta)\hat{\pi} + \epsilon_m\hat{m} - \eta_M\hat{M} = \eta s\hat{e} \tag{B1}$$

a system of n+q+r-3 equations and n+q+r+2 unknowns. To this system we can add two more equations. Let c and g be (n+q+r) x 1 vectors of consumption levels in the foreign and home country respectively, with zeros in spots corresponding to nontraded goods of the other country. Then:

$$c' \begin{bmatrix} ep \\ \bar{p} \\ p^* \end{bmatrix} = M \text{ and } g'\pi = m \quad .$$

Note that c and g consist of zeros in spots corresponding to nontraded goods outside the relevant country. Let α be a vector of expenditure shares in the foreign country and β the corresponding vector at home. Differentiating logarithmically results in:

$$\alpha' \begin{bmatrix} \hat{p} \\ \hat{\bar{p}} \\ \hat{p}^* \end{bmatrix} + \alpha' s\hat{e} + \alpha'\hat{c} = \hat{M}$$

$$\beta'\hat{\pi} + \beta'\hat{g} = \hat{m}$$

To simplify I use the envelope theorem result for an optimizing country that $\alpha'\hat{c} = \beta'\hat{g} = 0$. The resulting simplified equations are:

$$\alpha' \begin{bmatrix} \hat{\rho} \\ \hat{\mathbf{p}} \\ \hat{\rho}^* \end{bmatrix} + \alpha' s\hat{e} = \hat{M} \quad \text{and} \quad \beta'\hat{\pi} = \hat{m} \tag{B2}$$

Adding these two to (B1) yields a system of n+q+r-1 equations and n+q+r+2 unknowns. The system is not soluble. We are short three equations.

CJ (1980) would solve this problem by assuming arbitrarily that $\hat{M} = 0$ and $\hat{m} = 0$. A neater solution results from using the former but substituting $\hat{m}^* = 0$, where m^* is home income measured in foreign currency. Thus, we gain two equations. However, the system is still indeterminate.

To gain the last equation, we again assume the price level of tradeables relative to nontradeables remains constant abroad. Thus:

$$[\phi_T' - \phi_N'] \begin{bmatrix} \hat{\rho} \\ \hat{\mathbf{p}} \\ \hat{\rho}^* \end{bmatrix} + [\phi_T' - \phi_N']s\hat{e} = 0 \tag{B3}$$

(See appendix A for a definition of ϕ_T' and ϕ_N'.)

Using these assumptions, we can reformulate the system in terms of partitioned matrices:

$$\begin{bmatrix} \epsilon - \eta & \epsilon_m & -\eta_M \\ \beta' & -1 & 0 \\ \alpha' & 0 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ \phi_T' - \phi_N' & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\pi} \\ \\ \hat{m} \\ \\ \hat{M} \\ \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & 0 \\ -\alpha' & 0 \\ 0 & 0 \\ 0 & 1 \\ \phi_T' - \phi_N' & 0 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix} \hat{e} \tag{B4}$$

Since $\epsilon s + \epsilon_m = 0$ and $1 - \beta' s = 0$, we can modify the right hand side. Also, using the constancy of M, we can alter the left hand side. The resulting equation is:

$$\begin{bmatrix} \epsilon - \eta & \epsilon_m \\ \beta' & -1 \\ \alpha' & 0 \\ 0 & -1 \\ \phi'_T - \phi'_N & 0 \end{bmatrix} \begin{bmatrix} \hat{\pi} \\ \hat{m} \end{bmatrix} = - \begin{bmatrix} \epsilon - \eta & \epsilon_m \\ \beta' & -1 \\ \alpha' & 0 \\ 0 & -1 \\ \phi'_T - \phi'_N & 0 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix} \hat{e}$$

Inverting the matrix on the left hand side results in the solution for $\hat{\pi}$ and $\hat{m}$:

$$\begin{bmatrix} \hat{\pi} \\ \hat{m} \end{bmatrix} = - \begin{bmatrix} s \\ 1 \end{bmatrix} \hat{e}$$

This is essentially the solution arrived at above for the CJ simpler model. All prices, traded and nontraded, in the home country change exactly enough to offset the change in nominal exchange rate. Nominal income also changes by that amount. Prices in foreign currency do not change. As in the main text, this solution follows from the arbitrary assumptions of constant nominal income levels measured in foreign currency.

Other results would follow under other assumptions. For example, if we made nominal incomes constant in terms of home currency, then home prices would remain constant and foreign prices would change to offset the exchange rate change.

It is clear from this exercise that it is not the lack of nontraded goods in the CJ model which is its chief limitation. Rather, it is the lack of any mechanism to indicate the presence of real relative price shocks.