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TO HARVEST OR NOT TO HARVEST?
AN ANALYSIS OF CUTTING BEHAVIOR
ON
FEDERAL TIMBER SALES CONTRACTS

Randal R. Rucker
North Carolina State University
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Keith B. Leffler
University of Washington

No. 86 June 1986

DEPARTMENT OF ECONOMICS AND BUSINESS
NORTH CAROLINA STATE UNIVERSITY
RALEIGH, NORTH CAROLINA

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In recent decades, sales of publicly owned natural resources have expanded significantly.[1] These sales have generated substantial revenues for the public sector. For example, more than $5.7 billion was paid for federal timber and mineral leases in 1980 with states and counties receiving $699 million of these funds through revenue sharing arrangements.[2] In those states and counties with large shares of federally owned land, revenues from the sale of natural resources constitute an important part of total government funds. State and county governments in Oregon, for example, received roughly $180 million from federal timber sales revenues in 1980.

Economists have devoted substantial research effort to studying the environmental consequences and the competitiveness of alternative sales techniques.[3] There has, however, been little attention paid to the potential for extreme fluctuations in revenues as a result of the "option" nature of public resource sales. While economists have analyzed the revenue stability of income, excise, sales, and value added taxes, we are unaware of a single study of the potential instability of revenues from public resource sales and the effects of this instability on state and local economies.[4]

That this instability can have serious effects has been demonstrated by recent events in the Pacific Northwest. Lumber prices in that region fell by

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[1] See Clawson (1983, Chapter 3).

[2] USDA, Report of the Forest Service, Fiscal Year 1980, and USDOI, Public Land Statistics, 1980.

[3] See, for example, Hyde (1981), Johnson (1979b), Leland (1978), McDonald (1979), Mead (1967), Mead, Schniepp, and Watson (1983), Reece (1979), Riley and Samuelson (1981), and Smith (1982).

[4] Boskin et al. (1985) demonstrate that the value of federal mineral rights and land fluctuate greatly over time. They do not, however, address the question of how different provisions of public resource sale and lease arrangements affect the stability of revenue flows.

as much as 60 percent in the early 1980s.[5] Private operators who had purchased cutting rights to public timber in the late 1970s found themselves with obligations to pay prices for stumpage far in excess of its value. They responded by delaying harvesting operations and claiming that unless relief from their contractual obligations were to be granted, massive defaults and widespread bankruptcies would be inevitable. The impacts of these events were widespread. Half of the sawmills in the West either curtailed operations or shut down during most of 1982; the unemployment rate in the wood products industry was 22 percent in October of 1982; and the value of federal timber harvested in Washington and Oregon declined from $737 in 1979 to $277 million in 1982.[6]

In response to these problems, public agencies have instituted changes in the basic provisions of their timber sales contracts that are designed to prevent contract defaults and stabilize revenues. Unfortunately, the effects of these changes on the bidding and cutting behavior of timber purchasers are not known with any degree of certainty. In particular, the effects of different policies and provisions on cutting behavior have not been subjected to systematic analysis. We provide such an analysis in this paper.

In Section I, we develop a model of timber contracting in which the bid price and the probability of default are endogenous. We begin the section with an overview of United States Forest Service timber sales contracts, then

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[5] Ruderman (1982).

[6] See "Reagan Looks at NW Timber Troubles," Seattle Post-Intelligencer, March 6, 1983; "Timber Woes Crimping State Trust Funds," Seattle Post-Intelligencer, October 22, 1981; "Future of Timber Relief Legislation Seems Dim," Great Falls Tribune, September 6, 1982; USDA, Report of the Forest Service (Fiscal years 1979 and 1982); and USDOI, Public Land Statistics (1979 and 1982).

model the effects of important features of such contracts, and conclude with a series of empirical propositions developed from the model. In Section II, we present an empirical examination of cutting behavior on Forest Service timber sales contracts. We find general support for the analytical approach taken in the first section and develop estimates of the actual impact of various policy changes that have been discussed for controlling the default problems.

I. Analysis of the Cutting and Default Incentives in Timber Contracting

The value of the timber on a tract of land is subject to substantial uncertainty. In addition to the uncertainty about the price of the harvested logs, the quantity and quality of the lumber products that can be produced from standing timber is costly to determine. If the owner of timber sells the cutting right for a lump sum, the potential buyers have incentives to expend considerable resources evaluating the timber stand. This prepurchase measurement expense does not increase the resource's value, and under competition, will be expected to reduce the seller's sale revenue.[7]

Sellers of timber can capture the resource value that might be dissipated in excessive prepurchase measurement by offering a type of contingent sale, known as a scale sale. In these sales, the buyer agrees on a price per unit of harvested timber. The total payment is then contingent on the quality and volume of harvest which is "scaled" cheaply during transportation to the mill. Scale sales are the dominant sales technique of several

[7]See Barzel (1982) for an extended discussion of the potential waste induced by "excess" measurement incentives. See Johnson (1979a) and French and McCormick (1984) for discussions of the effects of presale measurement expenses on sellers' revenues.

federal and state agencies, and are also used by private timber sellers. Similar contingent sales contracts are used by public agencies and private landowners to sell petroleum and mineral rights. These sales provisions make the revenues from the sale dependent not only on the actual volume of the resource, but also on purchasers' decisions concerning if and when to "harvest." Below we analyze in detail the incentives of the purchasers of United States Forest Service timber to actually cut and pay for timber "purchased" in scale contracts.

Forest Service Timber Sales Procedures and Contracts[8]

The first step taken by the Forest Service in preparing a timber tract for sale is to conduct an inspection or "cruise" of the site. Information from this cruise on the volume and quality of timber and on various physical characteristics of the tract is used to obtain an "appraised price." This price is the Forest Service's estimate of current stumpage value[9] and is also the minimum bid they will accept for the timber on the tract. Following the completion of the cruise and appraisal, the sale is advertised and a public auction is held at which the tract is sold to the highest bidder.

Forest Service contracts typically provide buyers with two to four years to complete harvesting operations. Contract length varies directly with the

[8]The features discussed below are those that play central roles in affecting bidding and cutting behavior. Actual Forest Service contracts are considerably more detailed and contain numerous requirements and provisions not described in this section. Procedures, policies, and provisions for Forest Service sales are detailed in the U.S. Forest Service Manual and USFS Contract Form 2400-6.

[9]The term "stumpage value" refers to the net value of timber on the stump, i.e., the difference between the value of the lumber products produced from that timber and the costs of transforming standing timber into final lumber products.

volume of timber on the tract and is generally agreed to be considerably greater than the minimum time required to complete operations.[10]

With the exception of a relatively small cash deposit (currently 10 percent), purchasers pay for contracted timber as they cut. Under a flat rate payment scheme, the per unit price offered by the winning bidder is the price he actually pays when he cuts the timber. Some Forest Service contracts contain escalation provisions (stumpage rate adjustment clauses) specifying quarterly adjustments in the purchase price of stumpage in response to changes in an index of the value of final lumber products.

A purchaser who does not complete harvesting operations by the scheduled termination date has two options. The first is to request a contract extension. Extensions are granted only if a specified percentage of the advertised volume of timber on the tract has been harvested by the contracted termination date.[11] When an extension is requested, the Forest Service reappraises the tract. If the new estimate of stumpage value exceeds the original estimate, then the price paid under the extension is the new estimate plus the difference between the bid price and the original estimate. Otherwise, the price paid with an extension is simply the original bid price.

The second alternative to harvesting is to default on the contract. If a purchaser of a Forest Service contract defaults, the unharvested portion

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[10] See, for example, Mead, et al. (1983, p. 16) and Dowdle (1983, p. 26).

[11] The normal requirements for extensions were waived between 1980 and 1984 in response to purchasers' pleas for relief from adverse market conditions. Between 1978 and 1980, the minimum cutting requirement was 75 percent. The policy in effect between 1971 and 1978 required extension applicants to have cut at least 50 percent of the advertised volume of timber on their tract. Prior to 1971, there were no minimum cutting requirements for extensions.

of the tract is resold. The defaulting purchaser must pay damages equal to the administrative costs of the resale auction plus the difference between the original bid (multiplied by the fraction of the tract not harvested) and the bid when the tract is resold.[12]

A Model of Bidding and Cutting Incentives

Two intuitive propositions are important in understanding the incentives created in scale sales by delaying payments until the timber is harvested. First, bid prices will reflect any anticipated increases in stumpage values. Second, the purchaser in effect buys an option that includes the alternatives of cutting, extending, and defaulting.[13]

To facilitate the analytical modeling of these features of timber contracts, we make the following simplifying assumptions.

A1. Forest Service timber auctions are competitive.

A2. Prospective purchasers are risk neutral and homogeneous with respect to both production costs and expectations concerning future stumpage values.

A3. The initial stumpage value (S_0) is known with certainty by all buyers. We normalize by setting S_0 equal to one.

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[12] Until recently, the Forest Service required no interest payments by defaulting operators, even though the receipts from the resale of a defaulted contract are delayed until the winner of the resale auction harvests the timber. A policy change, in effect on sales since July 1982, requires defaulting purchasers to pay interest on the unpaid balance on their contracts for half the resale contract period and for the period between the initially specified termination date and the date that the resale contract is awarded. This modification increases the cost of defaulting, but still does not cover all the costs resulting from delayed receipt of the rebid.

[13] Timber sales contracts differ from standard options because the cost of not exercising the option varies with the realized value of the stumpage. In a standard call option, the cost of not exercising the option is zero (because the price paid for the option is a sunk cost), regardless of the end-of-period value of the stock or commodity. Discussions of option pricing models for such contracts can be found in Copeland and Weston and Cox et al.

A4. The rate of growth in stumpage values is a random variable (R) whose density function, $h(r)$, is symmetric around its mean (μ) and has a nonzero variance, (σ^2).[14] The transformation of variables relating R and the end of contract stumpage value (S) is $S = So(1+R) = (1 + R)$.

A5. The expectation of bidders at the time of the initial sale is that (independent of the realized stumpage values at the end of the first contract period) the distribution of random growth rates in stumpage values over subsequent contract periods is the same as during the initial contract period.

Contractual arrangements for Forest Service flat rate pay-as-cut scale sales are characterized in the following manner. The purchaser of a timber contract can elect to cut the timber, default, or request an extension. Assuming a positive expected rate of growth in timber prices, this decision will tend to be delayed until the end of the initial contract period. If a purchaser elects to default, the tract is resold and the defaulter pays the difference between his bid price and the rebid price, plus any "fixed cost" of default (D).[15] Defaulting purchasers do not keep the difference if the

[14] Most of the timber sold by the Forest Service in the Pacific Northwest is old growth whose rate of physical growth is zero. Insofar as lumber produced from old growth has unique attributes, it is appropriate to analyze this timber as a nonrenewable resource. Assuming (as seems reasonable) that there are no stock effects associated with harvesting old growth timber, then stumpage values will increase at the rate of interest in equilibrium. See Fisher (1981) for a review of exhaustible resource models.

[15] These fixed costs include the administrative costs of conducting the resale, potential costs imposed by future noncooperative Forest Service actions, and possible increased future contract costs from increases in the price of performance bonds. The Forest Service will not sell a tract to a purchaser who has defaulted and not paid the associated penalties. After a defaulting purchaser has paid the specified penalties, he is again eligible to bid for Forest Service tracts. We speculate that "noncooperative Forest Service actions" may take such subtle forms as closer monitoring of a purchaser's compliance with his future operating schedules.

rebid price is greater than the original bid price, implying that the minimum cost of default is D.[16]

A request for a contract extension is granted only if a specified portion of the timber is first harvested. If an extension is granted, the purchaser pays the original bid price plus a "fixed cost" of extension (E).[17] In addition, extensions are granted only if the realized price is below the bid price.[18]

The determination of equilibrium bid prices and harvest probabilities on flat rate contracts can now be described. Suppose a timber contract is purchased at a bid of B and at the end of the contract the realized stumpage value is s. If s is less than B, the loss from harvesting, LH, will be that difference. Thus,

(1a) $LH = B - s$

If default is chosen, the loss, LD, will be the maximum of (1) the fixed costs of defaulting (D), or (2) D plus the difference between the original bid price and the resale price (B^r). Thus,

(1b) $LD = \max(B - B^r + D, D)$

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[16]Because the rebid price will include expected increases in stumpage value during the new contract period, the rebid price may exceed the original bid even if the stumpage value at the end of the initial contract period is less than the original bid.

[17]The Forest Service does not appear to impose any additional costs on purchasers receiving extensions through "noncooperative actions," suggesting that E is close to zero.

[18]The Forest Service has no stated policy of refusing to grant extensions if the realized stumpage value is above the bid price. This seems, however, to be a reasonable approximation because under the actual Forest Service policy (of increasing prices to be paid for timber on extended contracts under certain circumstances), purchasers will not find it profitable to extend if realized prices exceed bid prices by a significant amount.

If extension is chosen, any fixed costs of extension (E) are incurred and the minimum percentage (M) of the timber must be cut. In addition, the contract must be completed by the end of the extension period.[19] The expected loss on timber remaining on the tract at that time will be the present value of $B - ES_2$ where ES_2 is the expected end-of-extension-period stumpage value. The "loss" from extending (LE) is therefore,

$$LE = M(B - s) + (1-M)[(B - ES_2)/(1+i)] + E \tag{1c}$$

where i is the rate of discount.

If the realized stumpage value is greater than the bid price ($s > B$), then harvesting is preferred to defaulting because the gains from harvesting are positive whereas purchasers always lose at least D if they default. By assumption, extensions are not granted if $s > B$. Harvesting is therefore the profit maximizing choice with a net gain given by

$$GH = s - B \tag{2}$$

The present value of expected profit at the time a timber contract is awarded is the discounted probability-weighted average of the gains or losses (corresponding to the optimal strategy) at each possible end-of-contract stumpage value. The discounted expected net revenues ($E\pi$) are thus given by

$$E\pi = (1+i)^{-1} \int_{S_{min}}^{S_{max}} \pi(B, s, \phi) f(s) ds \tag{3}$$

where f(s) is the pdf of S, the end-of-initial-contract-period stumpage value, $\pi(B, s, \phi)$ is the net revenue corresponding to the profit maximizing strategy, and ϕ represents a vector of policy parameters (the rules of the

[19] Again, this characterization of extension policies represents a simplification. In our data set, several contracts written in the 1960s (before minimum cutting requirements were in effect) received more than one extension and rare instances of purchasers defaulting after receiving an extension were found.

game for the contract) and factors exogenous to the contracting process (e.g., the rate of growth in stumpage values).

We can rewrite the expression for the expected net revenues from the contract as

$$(4) \qquad E\pi = (1+i)^{-1}\int_{B}^{S_{max}} GH(s,B,\phi)f(s)\,ds - (1+i)^{-1}\int_{S_{min}}^{B} \min\{LH(s,B,\phi),\ LE(s,B,\phi),\ LD(s,B,\phi)\}f(s)\,ds$$

This form emphasizes that purchasers always harvest for realized stumpage values greater than the bid price. For stumpage values less than the bid price, they choose the loss minimizing actions.

Under the assumptions of competition (A1) and risk neutrality (A2), the equilibrium bid price (B^*) will be that bid price for which expected profits are equal to zero. That is,

$$(5) \qquad \int_{B^*}^{S_{max}} GH(s,B^*,\phi)f(s)\,ds \equiv \int_{S_{min}}^{B^*} \min\{LH(s,B^*,\phi),\ LE(s,B^*,\phi),\ LD(s,B^*,\phi)\}f(s)\,ds$$

A difficulty in solving explicitly for B^* is that the equilibrium bid depends on the expected end-of-extension period stumpage value if a purchaser chooses to extend, and on the expected rebid price if a purchaser chooses to default. The solution to this difficulty is embodied in (A5) which implies that $ES_2 = (1+\mu)s$ and that the expected equilibrium rebid price will be $B^{r*} = (s/S_0)B^* = sB^*$. That is, if the realized end-of-contract stumpage value is 10 percent greater than the initial stumpage value, then the rebid will be 10 percent greater than the original bid price.[20]

[20] The relationship between B^{r*} and B^* is derived in Rucker (Appendix A).

Using expressions (1a) - (1c), and (2), and the relationships among s, B^{f*}, and B^*, the equilibrium condition, (5), can be rewritten as

$$(6) \qquad \int_{B^*}^{S_{max}} (s-B^*)f(s)ds \equiv \int_{S_{min}}^{B^*} \min\{B^*-s,\ M(B^*-s) + (1-M)(B^*-(1+u)s)/(1+i) + E,\ \max(D,\ B^* - sB^* + D)\}f(s)ds$$

An obvious implication is that if E and D are large enough, it will never be optimal to extend or default, and the equilibrium bid price will be the expected end-of-contract price. If D and E are small enough that defaulting or extending are optimal for some price ranges, then the equilibrium bid price will be greater than the expected future value of stumpage.[21] In this case, purchasers will harvest if realized stumpage values exceed the bid price and will either extend or default for all realizations less than the bid price.[22] Thus, if the realized stumpage value lies between the expected value of stumpage and the bid price, purchasers of timber sales contracts will not exercise their option to harvest even though realized stumpage values exceed their expected values at the time of sale.

A number of testable propositions concerning the properties of equilibrium bid prices and cutting probabilities can be derived from the preced-

[21]To see this, suppose initially that E and D are "prohibitively high." In this case purchasers always harvest and $B^* = ES = (1+\mu)$. If E and D fall, purchasers can reduce their losses by extending or defaulting for some end-of-contract stumpage values. This implies that expected profits will be positive for a bid price of B = ES and that the new equilibrium bid price must be greater than ES.

[22]That purchasers will choose not to harvest for all stumpage values less than B^* is demonstrated by the following. From (1a) and (1c), $LE - LH = (1-M)[(i-\mu)s-iB^*]/(1+i) - E$. For $E \approx 0$ and $\mu = i$, $LE - LH = (1-M)[-iB^*]/(1+i) < 0$.

ing framework. Several of these, each accompanied by a brief discussion designed to provide intuition for the result, are stated below.[23]

Propositions and Testable Implications

Proposition 1: A change in initial stumpage value will result in an equal proportional change in the equilibrium bid price and no change in the probability of harvest during the initial contract period.[24]

Discussion: For a given distribution of expected growth rates, an increase in initial stumpage value results in an equal proportional increase in the mean and standard deviation of the distribution of future stumpage values. Because this new distribution of end-of-contract stumpage values is simply the original distribution with newly defined units of measurement, the solution for the new equilibrium bid price is the "same" in terms of its standardized distances from both the initial stumpage value and the expected future price.

Proposition 2: An increase in the costs of an extension, e.g., via an increase in minimum cutting requirements (M), will lead to a decrease in the equilibrium bid price and an increase in the cutting probability.

Discussion: Suppose extending is initially optimal for some end-of-contract stumpage values. An increase in the costs of extending will result in negative expected returns at the original equilibrium bid price, implying that the equilibrium bid must fall and that the probability of harvest must increase.

Proposition 3: An increase in the expected rate of growth in stumpage values (i.e., a variance-preserving rightward shift in the distribution of end-of-contract stumpage values) will increase the bid price and reduce the initial period harvest probability.

Discussion: Suppose the expected end of contract stumpage value increases from μ_p to $\mu_p + k$. If the equilibrium bid price also increases by k to $B^* + k$, then there is no change in the probability of harvest. However, in the absence of appropriate interest charges for extensions and defaults, a bid price of $B^* + k$ yields positive expected net revenues. The equilibrium bid price must therefore increase by more than k and the

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[23]The implications presented below are limited to those that are tested empirically in Section II. Formal proofs can be found in an Appendix available upon request from the authors. Additional propositions can be found in Rucker.

[24]This assumes that the extension and default costs, E and D, are a "fixed" proportion of the stumpage value. If this assumption is not accurate empirically, then Proposition 1 must be modified. For example, if E and D are fixed in dollar value, such that they increase less than proportionately with stumpage values, then bid prices are predicted to increase more than proportionately with increases in stumpage values and the harvest probability will fall.

probability of cutting must fall as a result of an increase in the expected rate of growth in stumpage values.

__Proposition 4__: A proportional increase in the mean and standard deviation of the distribution of end-of-contract stumpage values will increase the equilibrium bid price and reduce the initial period harvest probability.
__Discussion__: Suppose the mean and standard deviation of the distribution of end-of-contract stumpage values increase from μ_p to $a\mu_p$ and from σ_p to $a\sigma_p$ (where $a > 1$). If the equilibrium bid increases to aB*, then there is no change in the probability of harvest. However, in the absence of appropriate interest charges for extensions and defaults, a bid price of aB* yields positive expected net revenues. The equilibrium bid price must therefore be greater than aB* and the probability of harvest must fall.

__Proposition 5__: An increase in the level of uncertainty concerning end-of-contract stumpage values (a mean preserving spread) has an ambiguous effect on both the bid price and the initial period harvest probability.
__Discussion:__ A mean preserving spread can be viewed as a proportional increase in the mean and standard deviation of the future stumpage value distribution followed by a variance-preserving leftward shift in that distribution. From propositions 3 and 4 these two changes will have opposing effects on bid prices and harvest probabilities.

__Proposition 6__: An increase in the duration of a timber contract may increase or decrease the probability of harvest during the initial contract.
__Corollary__: If mean preserving spreads decrease the probability of harvesting a particular set of contracts, then an increase in contract length necessarily reduces the probability of harvest.
__Discussion__: An increased contract length can be represented by a combination of a proportionate increase in the mean and standard deviation combined with a mean preserving spread.[23] Propositions 4 and 5 therefore generally imply an ambiguous effect on the initial period harvest probability. If mean preserving spreads reduce harvest probabilities in a particular set of contracts, then the effects described by Propositions 4 and 5 both act to decrease harvest probabilities.

__Proposition 7__: An increase in the discount rate (i) will reduce the initial period harvest probability.

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[23]To see this, let S_1 and S_2 be the random end-of-contract stumpage values for one period and two period contracts, respectively. By (A4), the mean and standard deviation of S_1 are $(1+\mu)$ and σ. By (A4) and (A5), the relationship between S_2 and R is $S_2 = (1+2R+R^2)$. Assuming that $R^2 \approx 0$, the mean and standard deviation of S_2 are $(1+2\mu)$ and 2σ. The difference between the ratios of the mean and standard deviation of S_1 and S_2 is

$$(1 + \mu)/\sigma - (1+2\mu)/2\sigma = 1/2\sigma > 0$$

implying the result stated in the text.

Discussion: As a nonrenewable resource with no stock effects, the expected rate of change in old growth stumpage values is equal to the rate of interest in equilibrium. A change in the rate of interest (i) therefore results in an equal change in the expected rate of growth in stumpage values (μ). The remaining discussion of this proposition is esentially the same as that of Proposition 3.

Proposition 8: Forest Service stumpage rate adjustment clauses have ambiguous effects on both bid prices and initial period harvest probabilities.

Discussion: For a given bid price, upward adjustment clauses reduce net revenues during periods of rising stumpage values, whereas downward adjustment clauses increase net revenues during periods of falling stumpage values. The former effect will increase equilibrium bid prices, while the latter causes them to fall. Similarly, (for a given bid price) downward adjustments may increase incentives to harvest during periods of falling stumpage values, while upward adjustments decrease the incentive to harvest during periods of rising stumpage values. The net effects of these opposing influences on both bid prices and harvest probabilities are indeterminate.

II. Empirical Effects of Changes in Policy Parameters on Cutting Probabilities

The harvest-don't harvest decisions by purchasers of public timber sales contracts fit into the framework of qualitative response models.[26] In this application, the harvest decision is determined by whether the realized value of the timber exceeds some "critical value" that is determined by tract characteristics, contractual provisions, agency policies, the level of competition, and various exogenous factors.

The specification of the logistic regression equation we use to explain the decision to cut or not to cut is

$$(7)\quad HARVEST_i = \alpha_0 + \alpha_1 SVZERO_i + \alpha_2 DISCRATE_i + \alpha_3 STDEV_i + \alpha_4 CLENGTH_i + \alpha_5 SRA_i + \alpha_6 EXTMIN_i + \epsilon_i$$

[26]Discussions of qualitative response models can be found in Judge, et al., and Amemiya. In our empirical analysis, the cutting decisions of purchasers are viewed as a dichotomous choice.

The definitions, empirical proxies, predicted effects, and sources for the variables in this regression are,

$HARVEST_i$ - the probability that tract i is harvested by the initially specified termination date. This variable is assigned a value of one if the tract is harvested, zero otherwise. Source: USFS Contract Form 2400-19a (Report of Timber Sale Rate Redetermination).

$SVZERO_i$ - the stumpage value (per mbf) of tract i at the time of sale. The proxy used for this variable is the Forest Service's appraised price. From proposition 1, we predict this variable to have no significant impact on cutting probability. Source: USFS Contract Form 2400-17 (Report of Timber Sale).

$DISCRATE_i$ - the annualized rate of discount over the initially specified contract period of tract i. This is measured using the nominal annualized interest rate on T-bills with sale and maturity dates corresponding to the sale and termination dates of individual timber sales. Proposition 7 suggests that an increase in this variable reduces the probability of harvest. Source: Databank (P. Rao, University of Washington).

$STDEV_i$ - the standard deviation of the distribution of end-of-contract stumpage values for a one-period contract. The proxy used for this variable was an eight quarter geometrically declining weighted moving average of the absolute values of changes in annualized rates of growth in lumber prices. As indicated in proposition 5, we cannot predict the direction of the effect of this variable a priori. Source for lumber price series: Western Wood Products Association lumber price index for Dry Douglas Fir-Larch.

$CLENGTH_i$ - contract length (in months). Proposition 6 suggests that the cutting probability is ambiguously affected by the contract length. Source: USFS Contract Form 2400-17.

SRA_i - a 0, 1 dummy variable that is assigned a value of one for contracts with stumpage rate adjustment clauses. From proposition 8, these clauses may increase or decrease the cutting probability. Source: USFS Contract Form 2400-17.

$EXTMIN_i$ - a 0, 1 dummy variable that is assigned a value of one for contracts written after June 30, 1971 when a 50 percent minimum cutting requirement for extensions was imposed. We expect this policy change to increase the cutting probability (Proposition 2).

To estimate equation (7), we use data from 679 timber sales contracts issued between 1965 and 1976 in two national forests in the state of

Washington.[27] Contracts from this period are chosen because (1) cutting behavior was not affected by the "after the fact" modifications of contractual provisions instituted in the early 1980s, and (2) two changes in contractual provisions and policies were made during this period: stumpage rate adjustment clauses were included in many contracts between 1965 and 1967, and the minimum cutting requirements for an extension were increased from 0 percent to 50 percent in 1971.

Empirical Results

Descriptive statistics for the data are presented in Table 1. The coefficient estimates from the logistic regression are displayed in Table 2.[28] The Goodness-of-fit chi square statistic provides a summary measure of the explanatory power of the model. The value of this statistic (37.24) leads to the rejection of the null hypothesis that the coefficients on the explanatory variables are jointly equal to zero.[29]

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[27] Because our model does not yield implications for non-competitive sales, we exclude sales with only one bidder from our data set. Our model is based on the equilibrium relation between the rate of interest and the expected rate of growth in stumpage values. In periods of adjustment to exogenous shocks to the timber market, this relationship may not hold. We identify periods of deviation from long run equilibrium by estimating a series of annualized expected rates of growth in nominal lumber prices (see Rucker, Appendix B for details). We then delete sales for which this proxy has negative values from our data set. To control for the effects of additional factors on bid prices, the sample of sales analyzed is restricted to scale sales with terms greater than six months and less than five years.

[28] We also estimated specifications that included a time series variable and a dummy variable to distinguish sales in different forests. The estimated coefficients on these variables were not statistically significant, and their inclusion did not affect the significance of the other explanatory variables.

[29] At a significance level of 5 percent, the critical value for this test statistic is 15.5. See Judge et al., or Amemiya for a discussion of this statistic.

As predicted by our analytical model, changes in initial stumpage values (SVZERO) do not significantly affect harvest probabilities. The negative (and marginally significant) estimated coefficient on DISCRATE is also consistent with our model's predictions. This suggests that an increase in the nominal rate of discount (with a concurrent increase in the expected rate of growth in stumpage values) reduces the cutting probability. The negative estimated coefficient on STDEV suggests that a reduction in the expected variability in timber prices increases the likelihood that a tract will be harvested by the scheduled termination date.

The estimated coefficients on CLENGTH, SRA, and EXTMIN provide information on the effects of policies that can be controlled directly by the Forest Service. Some public agencies recently have considered shortening the duration of their timber contracts as a way of reducing speculative incentives and increasing harvest probabilities.[30] The negative and marginally significant coefficient on CLENGTH in Table 2 indicates that reduced contracts do increase the probability of harvest. This negative coefficient provides support for the corollary to Proposition 6. That is, because the estimated coefficient on STDEV is negative, the model predicts that the coefficient on CLENGTH will also be negative.

The Forest Service recently reinstituted the use of stumpage rate adjustment clauses in timber contracts on the west side of the Cascades. The purpose of this policy change was to increase the likelihood of harvest. The statistically insignificant coefficient on SRA in Table 2 suggests that the inclusion of stumpage rate adjustment clauses has no significant influence on

[30]See for example, "Short-term Timber Pacts Under BLM Consideration," The Oregonian, January 14, 1984.

harvest probabilities.[31] Finally, the estimated coefficient on EXTMIN indicates that the imposition of minimum cutting requirements for extensions in 1971 increased the probability of harvest during the initial contract period.

Estimates of the impact of changes in the statistically significant explanatory variables on the probability of harvest are shown in the final column of Table 2. These indicate that a 10 percent increase in the nominal discount rate reduces the probability of harvest by 1.4 percentage points; that a 10 percent increase in expected stumpage value variability reduces the probability of harvest by .57 percentage points ; that a 10 percent increase in contract length decreases the probability of harvest by .50 percentage points; and that the imposition of a 50 percent minimum cutting requirement in 1971 increased the probability of harvest by 12.8 percentage points.

Conclusions

We have developed an analytical framework for investigating various features of timber contracts. Our framework models the endogenous determination of the equilibrium bid for the timber and the expected probability that the contract will be fulfilled by the specified termination date. Our empirical analysis focuses on the likelihood that the contracts are fulfilled, an issue that has been neglected by other researchers.

[31] The stumpage rate adjustment clauses included in Forest Service contracts in the 1960s stipulated symmetric upward and downward adjustments (50 percent up-50 percent down) in prices to be paid for stumpage in response to changes in indexes of lumber values. The stumpage rate adjustment clauses currently used by the Forest Service call for asymmetric adjustments in stumpage payments of 50 percent up and 100 percent down. These differences in past and present stumpage rate adjustment clauses suggest that caution should be employed in using the results of our empirical analysis to predict the effects of this recent policy change.

Recent events in the Pacific Northwest suggest that extensions and defaults on federal contracts can have significant impacts on government revenues and also on the fortunes of timber companies and communities whose economies rely heavily on income from publicly owned natural resources.

Our empirical results provide insights into the important factors influencing the decision to fulfill or not fulfill Forest Service timber contracts. Our estimates of the impacts of policy variables indicate that increased minimum cutting requirements for extensions, reduced contract lengths, and stabilization of timber prices increase the fulfill rate on these contracts. Our results do not provide support for the view that stumpage rate adjustment clauses increase the probability of harvest.

One message from our analysis is that the study of revenue impacts of different management policies for publicly owned natural resources requires detailed knowledge of the structure of the sales contracts for the different resources. Government oil and mineral leases, for example, have features of option contracts. The possibility that the value of these rights may fall suggests that "defaults" may also occur on these contracts. The potential for such and the impacts of alternative contractual arrangements can be determined only by analyzing the features of the particular contracts.

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Table 1

Descriptive Statistics

	Mean	Standard Deviation	Minimum	Maximum
HARVEST	.823	.382	0	1
SVZERO	40.0	25.8	2.15	146
DISCRATE	6.41	1.24	4.01	9.01
STDEV	31.4	17.7	7.92	67.7
CLENGTH	27.9	12.4	7	60
SRA	.138	.346	0	1
EXTMIN	.521	.500	0	1

Table 2

Determinants of Harvest Probabilities on Forest Service Timber Sales Contracts

Explanatory Variable	coefficient estimate	asymptotic t-ratio	P-value[1]	% change in probability of harvest due to 10% change in explanatory variable[2]
Constant	2.88	4.41	.00	---
SVZERO	.003	.58	.56	---
DISCRATE	-.165	-1.49	.07	-1.4
STDEV	-.014	-2.05	.04	-.57
CLENGTH	-.014	-1.56	.12	-.50
SRA	.259	.71	.48	---
EXTMIN	.912	3.51	.00	12.8[3]

Goodness of fit chi-square = 37.24
Number of observations = 679
Sample period: 1965(1) - 1976(2)

[1] P-value indicates the minimum level of test significance for which the null hypothesis that the coefficient is zero (one-tailed test for DISCRATE and EXTMIN, two-tailed test otherwise) is rejected.

[2] The predicted probability of harvest on tract i is $\hat{p}_i = [1 + \exp(-X_i\alpha)]^{-1}$ where X_i is the vector of explanatory variables for tract i. The marginal effect of a change in X_{ik} on the probability of harvest is $\delta\hat{p}_i/\delta X_{ik} = \hat{p}_i(1-\hat{p}_i)\hat{\alpha}_k$. With the exception of EXTMIN (see below), the figures in this column are calculated as $.1\overline{X}_k(\delta\hat{p}_i/\delta X_{ik})$, where $\overline{X}_k$ is the sample mean of X_k and $\hat{p}_i$ is evaluated at the sample means of the explanatory variables.

[3] Because EXTMIN is a zero-one dummy variable, this figure represents the effect on the probability of harvest of a change in minimum cutting requirements for extension from 0 to 50 percent. This effect is calculated as the difference between the predicted harvest probabilities when EXTMIN = 0 and when EXTMIN = 1.

Appendix

This appendix contains proofs of the propositions stated and discussed in section I. The assumptions made to facilitate the development of the model are stated in Section I and will be referred to occasionally. To reduce any potential confusion concerning notation, we define the following variables:

R = the rate of growth of stumpage values over the initial contract period. R is a random variable.

$h(r)$ = the probability density function of R. R has mean, μ, standard deviation, σ, and minimum and maximum values of R_{min} and R_{max}.

S = end-of-initial-contract-period stumpage value. S, a random variable, is related to R through the transformation $S = S_0(1+R)$, where S_0 is the initial stumpage value of the tract. S_0 is observable and assumed to be known to all buyers with certainty. We normalize by setting S_0 equal to one.

$f(s)$ = the probability density function of S. S has mean, $\mu_s = (1+\mu)$, standard deviation, $\sigma_s = \sigma$, and minimum and maximum values, S_{min} and S_{max}.

B_s^e = equilibrium bid price.

B_s^{er} = equilibrium rebid price (in case of default).

$$EG_s(B) = \int_B^{S_{max}} (s - B)f(s)ds$$

= weighted average (for all $s > B$) of gains from harvesting.

$$EL_s(B) = \int_{S_{min}}^{B} \min\{B - s,\ M(B - s) + (1-M)[B - (1+\mu)s]/(1+i),\ \max(D_s,\ B - sB + D_s)\}f(s)ds$$

= weighted average (for all $s < B$) of losses from harvesting, extending or defaulting.

Propositions and Proofs

Proposition 1: A change in the initial stumpage value of a tract will result in an equal proportional change in the equilibrium bid price and no change in the probability that the tract will be cut during the initial contract period.

Proof: Suppose the initial stumpage value changes from 1 to k. Let Y (a random variable) be the end-of-contract period stumpage value corresponding to initial stumpage value k, j(y) be the p.d.f. of Y, and B_y^e be the associated equilibrium bid price. The transformation of variables that relates S and Y is $Y = kS$. To determine the relationship between B_s^* and B_y^*, recall that B_y^* and B_y^{*r} are defined as the bid and rebid prices such that

$$(1.1)\quad \int_{B_y^*}^{Y_{max}} (y - B_y^*) j(y)\,dy \equiv \int_{Y_{min}}^{B_y^*} \min\{B_y^* - y,$$

$$M(B_y^* - y) + [(1-M)(B_y^* - (1+\mu)y)]/(1+i) + E_y,$$

$$\max(B_y^* - B_y^{*r} + D_y,\ D_y)\} j(y)\,dy$$

and that B_s^* and B_s^{*r} are the bid and rebid prices such that

$$(1.2)\quad \int_{B_s^*}^{S_{max}} (s - B_s^*) g(s)\,ds \equiv \int_{S_{min}}^{B_s^*} \min\{B_s^* - s,$$

$$M(B_s^* - s) + [(1-M)(B_s^* - (1+\mu)s)]/(1+i) + E_s,$$

$$\max(D_s,\ B_s^* - B_s^{*r} + D_s)\} f(s)\,ds$$

where E_y and D_y are the "fixed" costs of extending and defaulting associated with tracts whose initial stumpage value is k. The assumed relationships between these fixed extension and default costs and those on a tract with initial stumpage value of 1 are $E_y = kE_s$ and $D_y = kD_s$.

Given the transformation $Y = kS$, the following relationship holds

$$(1.3)\quad \int_a^b z(s) f(s)\,ds = \int_{ka}^{kb} z(y/k) j(y)\,dy$$

(1.2) can therefore be rewritten as,

$$(1.4)\quad \int_{kB_s^*}^{Y_{max}} (y/k - B_s^*) j(y)\,dy \equiv \int_{Y_{min}}^{kB_s^*} \min\{B_s^* - y/k,$$

$$M(B_s^* - y/k) + [(1-M)(B_s^* - (1+\mu)(y/k))]/(1+i) + E_y/k,$$

$$\max(D_y/k,\ B_s^* - B_s^{*r} + D_y/k)\} j(y)\,dy$$

Factoring 1/k out of each side of (1.4) gives

$$(1.5)\quad 1/k \int_{kB_s^*}^{Y_{max}} (y - kB_s^*) j(y)\,dy \equiv 1/k \int_{Y_{min}}^{kB_s^*} \min\{kB_s^* - y,$$

$$M(kB_s^* - y) + [(1-M)(kB_s^* - (1+\mu)y)]/(1+i) + E_y,$$

$$\max(kB_s^* - kB_s^{*r} + D_y,\ D_y)\} j(y)\,dy$$

Because B_s^* and B_s^{*r} solve this equation by definition, the problem in (1.1) is solved by $B_y^* = kB_s^*$ and $B_y^{*r} = kB_s^{*r}$. Thus, a change in the initial stumpage value results in a proportional change in the equilibrium bid price. That the probability of cut does not change, is demonstrated by the following

$$(1.6) \quad \int_{B_s^*}^{S_{max}} f(s)\,ds = \int_{kB_s^*}^{kS_{max}} j(y)\,dy = \int_{B_y^*}^{Y_{max}} j(y)\,dy$$

__Proposition 2__: An increase in M will decrease the equilibrium bid price and increase the initial period harvest probability.

__Proof__: This proposition can be proved with the following argument. Let B_s^* be the equilibrium bid for $M = M_0$ and E_s. Suppose M increases to M_0' and the equilibrium bid price remains B_s^*. At that bid price, the expected gains (for $s > B_s^*$) are unchanged, but if it was initially optimal to extend over some price ranges, then the expected losses (for $s < B_s^*$) have increased. This implies that the new equilibrium bid price must be less than B_s^* and that the probability of harvest will increase.

__Proposition 3:__ A variance-preserving rightward shift in the distribution of end-of-contract stumpage values will increase the bid price and reduce the initial period harvest probability.

__Proof:__ Let the random variable S be defined as above, and let $S' = S + a$ (where $a > 0$). Let $f'(s')$ be the p.d.f. of S' and note that $E(S') = \mu' = \mu_s + a$ and $Var(S') = Var(S) = \sigma^2$. Similarly, let $R' = R + a$, $h'(r')$ be the density function of R', and note that $E(R') = \mu' = \mu + a$. If we assume that $B' = B_s^* + a$ is the equilibrium bid price for S', then

$$(3.1) \quad EG'(B') = \int_{B'}^{S'_{max}} (s' - B')f'(s')\,ds'$$

and

$$(3.2) \quad EL'(B') = \int_{S'_{min}}^{B'} \min\{B' - s',\ M(B' - s') + (1-M)[B' - (1+\mu')s']/(1+i) + E_s,$$
$$\max(D_s,\ B' - s'B' + D_s)\}f'(s')\,ds'$$

The nature of the relationship between S' and S suggests that

$$(3.3) \quad EG(B_s^*) = \int_{B_s^*}^{S_{max}} (s - B_s^*)f(s)\,ds$$

$$= \int_{B_s^*+a}^{S_{max}+a} (s' - a - (B' - a)]f'(s')\,ds'$$

$$= \int_{B'}^{S'_{max}} (s' - B')f'(s')\,ds'$$

$$= EG'(B')$$

i.e., if the bid price increases by a, then the expected gains are not affected by this transformation. For realized stumpage values less than B_s^*,

$$(3.4) \quad EL(B_s^*) = \int_{S_{min}}^{B_s^*} \min\{B_s^* - s,\ M(B_s^* - s) + (1-M)[B_s^* - (1+\mu)s]/(1+i) + E_s,\ \max(D_s,\ B_s^* - sB_s^* + D_s)\}f(s)ds$$

$$= \int_{S_{min}+a}^{B_s^*+a} \min\{B' - s',\ M(B' - s') + (1-M)[B' - a - (1+\mu'-a)(s'-a)]/(1+i) + E_s,\ \max(D_s,\ B' - a - (s'-a)(B'-a) + D_s)\}f'(s')ds'$$

$$= \int_{S'_{min}}^{B'} \min\{LH',\ LE' + v,\ LD' + w\}f'(s')ds'$$

where v and w can both be shown to be positive,[1] implying that,

$$(3.5) \quad EL'(B') < EL(B_s^*)$$

and therefore, that

$$(3.6) \quad EL'(B') < EG'(B')$$

Thus, the new equilibrium bid price must be greater than B'. For a bid of B' = B_s^* + a, the probability of harvest is not affected by a variance preserving shift. This is demonstrated by the following,

$$(3.7) \quad \int_{B_s^*}^{S_{max}} f(s)ds = \int_{B_s^*+a}^{S_{max}+a} f'(s')ds' = \int_{B'}^{S'_{max}} f'(s')ds'$$

Because the new equilibrium bid price is greater than B', the probability of harvest must fall.

<u>Proposition 4</u>: A proportional increase in the mean and variance of the distribution of end-of-contract-period stumpage values will increase the bid price and reduce the initial period harvest probability.
<u>Proof</u>: Let S be defined as above, let S' = aS (where a > 1), and let f'(s') be the p.d.f. of S'. This change in the distribution of future stumpage values can be generated by a change in the underlying distribution of growth rates (R). Recall that the transformation relating R and S is S = 1 + R. Similarly, the relationship between S' and its underlying distribution (R') is S' = 1 + R'. It can be shown that the transformation between R and R' that results in a proportional change in the mean and standard deviation of

[1] This assumes that $S'_{min} > 0$.

the normalized distribution of end-of-contract stumpage value is $R' = a(1+R) - 1$. Note that $ER' = \mu' = a(1+\mu) - 1$.

Given the above, assume that the equilibrium bid price for S' is $B' = aB_s^*$. Then,

$$(4.1) \quad EG(B_s^*) = \int_{B_s^*}^{S_{max}} (s - B_s^*) f(s)\, ds$$

$$= \int_{aB_s^*}^{aS_{max}} (s'/a - B'/a)\ f'(s')\, ds'$$

$$= 1/a \int_{B'}^{S'_{max}} (s' - B') f'(s')\, ds' = (1/a) EG'(B')$$

and

$$(4.2) \quad EL(B_s^*) = \int_{S_{min}}^{B_s^*} \min\{B_s^* - s,\ M(B_s^* - s) + (1-M)[B_s^* - (1+\mu)s]/(1+i) + E_s,$$
$$\max(D_s,\ B_s^* - sB_s^* + D_s)\} f(s)\, ds$$

$$= \int_{aS_{min}}^{aB_s^*} \min\{B'/a - s'/a,\ M(B'/a - s'/a)$$
$$+ (1-M)[B'/a - (1+(\mu'+1)/a - 1)(s'/a)]/(1+i) + E_s,$$
$$\max(D_s,\ B'/a - (s'/a)(B'/a) + D_s)\} f'(s')\, ds'$$

$$= \int_{S'_{min}}^{B'} \min\{(1/a)LH',\ LE'',\ LD''\} f'(s')\, ds'$$

where $LE'' > (1/a)LE'$ and $LD'' > (1/a)LD'$.[2] Thus, if defaulting or extending are optimal for some price ranges, $EL(B_s^*) > (1/a)EL'(B')$ and $EG'(B') > EL'(B')$, implying that the equilibrium bid for S' exceeds aB_s^*.

As indicated by the following, for $B' = aB_s^*$, the probability of harvest is not altered by a proportional increase in the mean and variance

$$(4.3) \quad \int_{B_s^*}^{S_{max}} f(s)\, ds = \int_{aB_s^*}^{aS_{max}} f'(s')\, ds' = \int_{B'}^{S'_{max}} f'(s')\, ds'$$

The result that the new equilibrium bid price is greater than B' implies that the harvest probability must fall.

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[2]This assumes, again, that $S'_{min} > 0$.

__Proposition 5__: A mean-preserving spread (mps) in the distribution of future stumpage values has an ambiguous effect on both the bid price and the initial period harvest probability.
__Proof:__ Let S be defined as above, let $S' = (1+a)S - a\mu_s$ (where $a > 0$) and let $f'(s')$ be the p.d.f. of S'. Note that $ES' = \mu_s$ and that $Var(S') = (1+a)^2 Var(S) > Var(S)$. Next, assume the equilibrium bid price for S' is $B' = (1+a)B_s^* - a\mu_s > B_s^*$. Then,

$$(5.1)\qquad EG(B_s^*) = \int_{B_s^*}^{S_{max}} (s - B_s^*)f(s)ds$$

$$= \int_{(1+a)B_s^* - a\mu_s}^{(1+a)S_{max} - a\mu_s} [(s' + a\mu_s')/(1+a) - (B' + a\mu_s')/(1+a)]f'(s')ds'$$

$$= 1/(1+a)\int_{B'}^{S'_{max}} (s' - B')f'(s')ds' = EG'(B')/(1+a)$$

and

$$(5.2)\qquad EL(B_s^*) = \int_{S_{min}}^{B_s^*} \min\{B_s^* - s,\ M(B_s^* - s) + (1-M)[B_s^* - (1+\mu)s]/(1+i) + E_s,$$
$$\max(D_s,\ B_s^* - sB_s^* + D_s)\}f(s)ds$$

$$= \int_{(1+a)S_{min} - a\mu_s}^{(1+a)B_s^* - a\mu_s} \min\{(B'-s')/(1+a),\ M(B'-s')/(1+a)$$

$$+ (1-M)[B' + a\mu_s - (1+\mu)(s' + a\mu_s)]/(1+a)(1+i) + E_s,$$

$$\max[D_s,\ (B' + a\mu_s)/(1+a) - (s' + a\mu_s)(B' + a\mu_s)/(1+a)^2 + D_s]\}f'(s')ds'$$

$$= \int_{S'_{max}}^{B'} \min\{LH'/(1+a),\ LE'',\ LD''\}f'(s')ds'$$

where $LE'' = LE'/(1+a) + aE_s/(1+a) - (1-M)(\mu a\mu_s)/(1+a)(1+i) \gtrless LE'/(1+a)$, and $LD'' = \max[D_s,\ (B' - s'B' + D_s + a\mu_s(1-B' + a(1-\mu_s) - s') + aD_s)/(1+a)] \gtrless LD'/(1+a)$. The ambiguity of the terms for the losses from extending and defaulting suggests that EL may be greater than, less than, or equal to $EL'/(1+a)$. The possibility that $EL < EL'/(1+a)$ suggests that the new equilibrium bid price may be less than B_s^*, the original bid price.

As demonstrated by the following, for $B' = (1+a)B_s^* - a\mu_s$, the probability of harvest is not affected by a mean-preserving-spread,

$$(5.3)\qquad \int_{B_s^*}^{S_{max}} f(s)ds = \int_{(1+a)B_s^* - a\mu_s}^{(1+a)S_{max} - a\mu_s} f'(s')ds' = \int_{B'}^{S_{max}^*} f'(s')ds'$$

The result that the new equilibrium bid may be greater than or less than B' implies that the effect of an mps on probability of harvest is ambiguous.

Proposition 6: An increase in the duration of a timber contract may increase or decrease the probability of harvest during the initial contract.
Proof: Let S and S' be the end-of-contract stumpage values for a one period and a two period contract respectively. The relationship between S and R is $S = (1+R)$, while the relationship between S' and R is $S' = (1+2R+R^2)$. Assuming that $R^2 \approx 0$, the transformation of variables between S and S' is $S' = 2S - 1$. Let $f'(s')$ be the p.d.f. of S', $\mu_s' = 2\mu_s - 1$ be the mean of S', and $\sigma_s' = 2\sigma_s$ be the standard deviation of S'. Assume that the equilibrium bid price for S' is $B' = 2B_s^* - 1$. Then

$$(6.1)\qquad EG(B_s^*) = \int_{B_s^*}^{S_{max}} (s - B_s^*)f(s)ds$$

$$= \int_{2B_s^*-1}^{2S_{max}-1} (s'/2 + 1/2 - B'/2 - 1/2)f'(s')ds'$$

$$= \int_{B'}^{S'_{max}} (s'/2 - B'/2)f'(s')ds' = (1/2)EG'(B')$$

and[3]

$$(6.2)\qquad EL(B_s^*) = \int_{S_{min}}^{B_s^*} \min\{B_s^* - s,\ M(B_s^* - s) + (1-M)[B_s^* - (1+\mu)s]/(1+i) + E_s,$$
$$\max(D_s,\ B_s^* - sB_s^* + D_s)\}f(s)ds$$

$$= \int_{2S_{min}-1}^{2B_s^*-1} \min\{(B' - s')/2,\ M(B' - s')/2$$
$$+ (1-M)[(1/2)(B' + 1 - (1+\mu)(s'+1))/(1+i)] + E_s,$$
$$\max(D_s,\ (1/2)[(B' + 1 - (s'+1)(B'+1)/2] + D_s)\}f'(s')ds'$$

$$= \int_{S'_{min}}^{B'} \min\{(1/2)LH',\ LE'',\ LD''\}f'(s')ds'$$

where $LE'' = (1/2)LE' + [E_s - \mu/(1+i)]/2 \gtrless (1/2)LE'$ and $LD'' = \max(D_s,\ 1/2[B' - s'B' + D_s] + 1/4[(1-s')(1-B')] + (1/2)D_s \gtrless (1/2)LD'$. Thus $EL(B_s^*)$ may be greater than, less than, or equal to $(1/2)EL'(B')$.

As demonstrated by the following, for $B' = 2B_s^* - 1$, an increase in contract length does not affect the probability of harvest,

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[3]We assume for simplicity that one period extensions are granted for two period contracts.

$$(6.3)\quad \int_{B_0^*}^{S_{max}} f(s)\,ds = \int_{2B_0^*-1}^{2S_{max}-1} f'(s')\,ds' = \int_{B'}^{S'_{max}} f'(s')\,ds'$$

The result that the new equilibrium bid may be greater than, less than, or equal to B' implies that the effect of an increase in contract length has an ambiguous effect on the probability of harvest.

Proposition 7: An increase in the nominal discount rate (with a concomitant equal increase in the expected rate of growth in end-of-contract stumpage values) will reduce the initial period harvest probability.
Proof: Let $i' = i + a$ be the new discount rate and $\mu' = \mu + a$ be the new expected rate of growth in stumpage values. This effect consists of a variance preserving shift in the distribution of end-of-contract stumpage values, combined with a change in the rate at which future costs and receipts are discounted. Assume that the equilibrium bid price corresponding to i' and μ' is $B' = B_0^* + a$. Because i enters only the extension expression, the effects of this change on the gains and losses from harvesting and on the losses from defaulting will be identical to those of Proposition 3 (see 3.3 and 3.4 above).

To demonstrate that at corresponding stumpage values,[4] the original losses from extension (LE) exceed the new losses from extension (LE'), take the difference,

$$(7.1)\quad LE - LE' = M(B_0^* - s) + (1-M)[B_0^* - (1+\mu)s]/(1+i) + E$$
$$- M(B' - s') + (1-M)[B' - (1+\mu')s']/(1+i') + E$$

Using the assumed relationships between i and i', μ and μ', s and s', and B_0^* and B', (7.1) can be rewritten as,

$$(7.2)\quad LE - LE' = M(B' - s') + (1-M)[B' - a - (1+\mu'-a)(s'-a)]/(1+i'-a)$$
$$- M(B' - s') + (1-M)[B' - (1+\mu')s']/(1+i')$$
$$= [k]\{(1+i')[B' - a(1+\mu'-a)(s'-a)] - (1+i'-a)[B' - (1+\mu')s']\}$$
$$= [k]\{a[\mu(1+i') + s'(i-\mu) + B']\} > 0^{5}$$

where $k = (1-M)/(1+i')(1+i'-a)$. As shown in Proposition 3, this result implies that the new equilibrium bid price is greater than B', and that the probability of harvest must fall.

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[4]The stumpage value on the new distribution (s') that corresponds to the stumpage value on the original distribution (s) is $s' = s + a$. These "correspond" in the sense that the areas under the original and new p.d.f.'s over corresponding intervals are equal.

[5]This assumes that $S'_{min} > 0$.

__Proposition 8a__: Stumpage rate adjustment clauses (SRA's) with symmetric adjustment clauses will have ambiguous effects on bid prices and initial period harvest probabilities.

__Proposition 8b__: An increase in the downward adjustment factor in SRA's will increase the equilibrium bid price and decrease the initial period harvest probability.

__Proposition 8c__: SRA's similar to those used by the USFS (i.e., with asymmetric adjustment factors of 50%up - 100%down) will have ambiguous effects on bid prices and initial period harvest probabilities.

__Framework for Proofs__: In contracts with SRA's, the price purchasers pay for their timber differs from the bid price by specified percentages of the __absolute__ value of the change in an index of lumber values between the sale and harvest dates. We make three simplifying assumptions to analyze the effects of SRA's. First, we assume that changes in an index of lumber values perfectly reflect changes in stumpage values. Thus, the SRA's in our model call for changes in payments in response to changes in __stumpage values.__

Second we assume that the Forest Service does not allow purchasers to extend if end-of-contract stumpage values exceed the adjusted payment price as of the time of harvest (i.e., the current contract rate). This assumption is the most obvious extension of the earlier assumption that for "flat rate" contracts, purchasers are not allowed to extend if the realized end-of--contract stumpage value exceeds the bid price. Finally, we assume that in case of default, purchasers pay the difference between the current contract rate and the rebid price (plus any fixed costs of defaulting). This assumption corresponds to Forest Service policy (see USFS contract form 24400-6, provision B9.4).

Given these assumptions, the gains from harvesting a tract whose contract includes SRA's can be written as

$$(8.1) \quad GH^{SRA} = s - CCR$$

$$(8.1a) \qquad = s - (B_s^* + A_u(s - S_0)) \quad \text{if } s > S_h$$

where CCR is the current contract rate, A_u is the specified "up" adjustment factor, and S_h is the end-of-contract stumpage value above which the purchaser breaks even by harvesting. Note that with SRA's, if $B_s^* > S_0$ there is a range of stumpage values greater than B_s^* over which purchasers will lose money if they harvest. It can be seen from (8.1a) that

$$(8.2) \quad S_h = (B_s^* - A_u S_0)/(1-A_u)$$

If $s < S_h$, purchasers will choose the option that minimizes their losses. The losses from harvesting, extending and defaulting are, respectively,

$$(8.3a) \quad LH^{SRA} = B_s^* + A_u(s - S_0) - s \quad \text{if } S_0 < s < S_h$$

$$= B_s^* + A_d(s - S_0) - s \quad \text{if } s < S_0$$

$$(8.3b) \quad LE^{SRA} = M(CCR - s) + (1-M)[E(CCR_2 : s) - (1+\mu)s]/(1+i) + E_s$$

$$(8.3c) \quad LD^{SRA} = \max(D_s,\ CCR - B_s^{*r} + D_s)$$

where A_d is the specified "down" adjustment factor. $E(CCR_2: s)$, the expected payment at the end of the extension period (conditional on the realized end-of-contract stumpage value), can be written as,

$$(8.4a)\quad E(CCR_2: s) = \int_{Q_{min}}^{Q_{max}} (CCR_2)g(q)dq$$

$$= \int_{Q_{min}}^{S_0} \{B_s^* + A_d(q-S_0)\}g(q)dq + \int_{S_0}^{Q_{max}} \{B_s^* + A_u(q-S_0)\}g(q)dq$$

where q is the realization of the random variable Q, the value of stumpage at the end of the extension period, and g(q) is the p.d.f. of Q. The relationship among Q, R, and s = S_1 is,

$$Q = S_1(1+R)$$

For the case in which $A_d = A_u = A$, (8.4a) can be rewritten as,

$$(8.4b)\quad E(CCR_2: s) = B_s^* + A \int_{Q_{min}}^{Q_{max}} (q-S_0)g(q)dq = B_s^* + A[(1+\mu)s - S_0]$$

We now use the preceding framework to demonstrate the effects of current Forest Service SRA's (with asymmetric adjustment clauses) on bid prices and harvest probabilities. The approach to proving proposition 8c will be to derive the effects of (1) including SRA's with symmetric adjustment clauses and (2) increasing the downward adjustment factor. Because the effect of current Forest Service SRA's (with asymmetric adjustment clauses) can be thought of as the sum of the effects of SRA's with symmetric (50% up-50% down) adjustment clauses and an increase in the downward adjustment factor (from 50% to 100%), the proof of proposition 8c is complete once 8a and 8b have been proved.

<u>Proof</u> - <u>Proposition 8a</u>: For the situation in which $A_d = A_u = A$, suppose the adjustment factor is increased from A to A'. Suppose also, that we assume the equilibrium bid price remains B_s^*. This increase in the adjustment factor has the following effects on the break-even harvest price and the losses and gains from harvesting, extending and defaulting,

$$(1)\quad \partial S_h/\partial A = [-(1-A)S_0 + B_s^* - AS_0]/(1-A)^2$$

$$= (B_s^* - S_0)/(1-A)^2 > 0$$

i.e., an increase in A leads to an increase in the realized stumpage value above which the purchaser earns positive returns from harvesting.

$$(2)\quad \partial GH^{SRA}/\partial A = -(s - S_0) < 0 \quad \text{for all } s > S_0$$

From (8.1a), GH^{SRA} is only defined for $s > S_h > S_0$. Thus, the "expected gains" from harvesting must fall as a result of an increase in A.

(3) $\partial LH^{SRA}/\partial A = s - S_o > 0$ if $s > S_o$

< 0 if $s < S_o$

(4) $\partial LE^{SRA}/\partial A = M(s-S_o) + (1-M)[(1+u)s - S_o]/(1+i)$

$= 0$ if $s = [(iM + 1)S_o]/[1+\mu+M(i-\mu)]$

> 0 if $s >$ " "

< 0 if $s <$ " "

(5) $\partial LD^{SRA}/\partial A = 0$ if $s > S_o$

$= (s - S_o) < 0$ if $s < S_o$

From (2), an increase in A results in a decrease in the positive component of expected profits. It can be shown that for $E = 0$ and $\mu = i$, purchasers holding contracts with SRA's will minimize losses by extending or defaulting for all realized stumpage values less than the break-even price (S_h). The relevant changes in the negative portion of expected profits resulting from an increase in A are therefore the changes in the losses from extending and defaulting. The ambiguity of the sum of the changes in (4) and (5) suggest that it is not possible to determine the direction of the net effects on the equilibrium bid price of an increase in A.

In our framework, if SRA's are included, purchasers will choose to harvest only if $s > S_h$. An increase in A, given B_s^*, leads to an increase in S_h. However, because S_h is also a function of B_s^* and because the effect of an increase in A on B_s^* is indeterminate, the net change in S_h and in the probability of harvest are also ambiguous.

<u>Proof</u> - <u>Proposition 8b:</u> Suppose the downward adjustment factor increases from A_d to A_d' and that we assume the equilibrium bid remains B_s^*. The components of expected profits are affected as follows

(1) $\partial GH^{SRA}/\partial A_d = 0$ for all $s > S_h$

(2) $\partial LH^{SRA}/\partial A_d = 0$ if $S_o < s < S_h$

$= s - S_o < 0$ for all $s < S_o$

(3) $\partial LE^{SRA}/\partial A_d = [(1-M)/(1+i)][\partial E(CCR_2: s)/\partial A_d]$ if $S_o < s < S_h$

$= M(s - S_o) + [(1-M)/(1+i)][\partial(ECCR_2: s)/\partial A_d]$ if $s < S_o$

(4) $\partial LD^{SRA}/\partial A_d = 0$ for all $S_o < s < S_h$

$= s - S_o < 0$ for all $s < S_o$

Again, because purchasers always prefer extending or defaulting to harvesting for $s < S_h$, the effects of an increase in A_d depend on the algebraic signs of (3) and (4). The latter is unambiguously negative. The former depends on the algebraic sign of $\partial E(CCR_2:s)/\partial A_d$. To determine this sign, rewrite (8.4a) as

$$(8.5a)\quad E(CCR_2:s) = B_s^* + \int_{Q_{min}}^{S_0} A_d(q - S_0)g(q)dq + \int_{S_0}^{Q_{max}} A_u(q - S_0)g(q)ds$$

The derivative of (8.5a) with respect to A_d is,

$$\int_{Q_{min}}^{S_0} (q - S_0)g(q)dq$$

which is unambiguously negative. The algebraic sign of (3) is therefore also unambiguously negative and the effect of an increase in A_d is to decrease the losses from extending and defaulting. At the original equilibrium bid price, expected profits will therefore be positive (for A_d'), implying that the equilibrium bid price increases when A_d is increased. Because an increase in the equilibrium bid price results in an increase in S_h (the break-even price for harvesting) and because purchasers only harvest when $S > S_h$, the probability of harvest decreases.

<u>Proof</u> - <u>Proposition 8c</u>: The effect of the introduction of a SRA clause with asymmetric adjustment factors of 50% up and 100% down is the sum of the effects of the introduction of a symmetric (50% up - 50% down) stumpage rate adjustment provision and an increase in the downward adjustment provision. Because the effects of the former are ambiguous, the effects of the sum of these changes will also be ambiguous.