Strategy and Policy in the Food System: Emerging Issues

EDITED BY
Julie A. Caswell and Ronald W. Cotterill

Proceedings of NE-165 Conference
June 20-21, 1996
Washington, D.C.

PART TWO: Measurement of Market Power

7. Simulating the Effects of Differentiated Products Mergers: A Practitioners' Guide

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Keywords: Mergers, market power, antitrust analysis, product differentiation

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Simulating the Effects of Differentiated Products Mergers: A Practitioners' Guide

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Public policy toward horizontal mergers—as embodied in the case law and in the Horizontal Merger Guidelines issued in 1992 by the U.S. Department of Justice and Federal Trade Commission—is primarily structural. Relevant markets are first delineated; market shares are assigned; and particular market shares and levels of market concentration give rise to presumptions of illegality. As the Supreme Court recently noted, "market definition generally determines the result of the case."[2]

This essay is a status report on the development of a new approach to the analysis of mergers. This approach replaces market-share-based presumptions with simulations of the effects of mergers within the context of tractable oligopoly models calibrated to the particular characteristics of the industry in question.[3] The simulation approach is both most needed for, and best suited to, significantly differentiated consumer products, and it certainly can be applied to products for which detailed, high-frequency data is available for use in estimating demand.[4] Simulations have been used with encouraging results by the author and others in actual antitrust investigations of differentiated products mergers.[5]

This essay proceeds by explaining the basic rationale for the simulation approach, describing its conceptual steps, presenting different demand systems that can be used, and discussing important issues in the implementation of the approach.

The Basic Rationale of the Simulation Approach

Limitations of Traditional Structural Merger Policy[6]

The basic principles followed by the courts today were laid out by the Supreme Court more than three decades ago. In *Brown Shoe* the Supreme Court held that market delineation is a prerequisite in horizontal merger cases, and it emphasized market shares in its analysis of the effects of the merger.[7] In *Philadelphia National Bank* the Court held that sufficiently high market shares establish the presumptive illegality of horizontal mergers.[8] The Supreme Court has not revisited these decisions, nor have the numerous lower court decisions greatly elaborated on these themes.[9]

The most influential elaboration is that in the Horizontal Merger Guidelines.[10] The Guidelines articulate general enforcement standards for horizontal mergers based on the "increase in the HHI," defined as twice the product of the pre-merger shares of the merging firms, and the "post-merger HHI," defined as the pre-merger Herfindahl-Hirschman Index (HHI) for the relevant market plus the "increase in the HHI." The Guidelines separately discuss the "unilateral effects" (i.e., effects not involving collusion) of mergers in differentiated products industries, stating that a merger will be presumed to harm consumers significantly if the combined share of the merging firms is at least 35 percent.

Neither the case law nor the Horizontal Merger Guidelines explicitly link market-share-based presumptions to effects of mergers on prices or welfare. Neither states any association between particular market shares and particular quanta of price and welfare effects, and neither articulates specific price-increase or welfare-decrease thresholds for illegality. This gives rise to three problems: There is no opportunity to examine critically the thresholds for illegality in terms of price and welfare effects. Efficiency effects of mergers cannot be explicitly traded-off against anticompetitive effects because anticompetitive effects are not quantified. Finally, expert witnesses and judges associated with particular cases are likely to have widely differing notions of the likely magnitudes of the price and welfare effects associated with particular merging firms' market shares, so a merger trial may turn out to be a massive failure to communicate.

The application of the traditional structural approach to differentiated products mergers presents additional problems.[11] Consumers typically have differing and complex subjective preferences with respect to price and other product attributes, and different products generally appear over a broad and fairly continuous range of prices and attributes. These conditions provide the opportunity for the defense to argue that no meaningful boundaries can be drawn within a price and quality continuum. The upshot of this argument is that the relevant market must be delineated very broadly, resulting in small market shares. Courts have often accepted this argument, failing to recognize that market shares are meaningless if markets are delineated very broadly. What matters are things like how often the product of either merging firm is viewed by consumers as the next-best substitute of the other, and how close other substitutes are in such cases.

Of course, plaintiffs also play the market delineation game, arguing for a very narrow market when the merging firms are particularly close in product space. While shares in these narrow markets are likely to be far more meaningful than those in the broad markets proposed by defendants, they still tell only part of the story. There will always be some competition at the margin, and it must be accounted for in a proper analysis of the likely competitive effects of a merger. More importantly, the key to a proper competitive analysis of a differentiated products merger is a careful consideration of the competition between the merging firms, and market shares do not necessarily indicate much about that.

Limitations of Estimation Without Simulation

Antitrust analysis increasingly relies on estimation to quantify critical issues such as market delineation. This is a positive development because it adds clarity and concreteness to the analysis. On the other hand, reliance on estimation does not alter the fundamental nature of merger analysis or merger litigation. The analysis remains primarily structural, and litigation still may involve a failure to communicate. Three cases illustrate.

Mrs. Smith's Pie was the first antitrust case of which I am aware in which a litigant explicitly relied on a demand elasticity estimate to delineate a relevant market. The government introduced an estimate of the own elasticity of demand for frozen dessert pies as evidence that they constituted a relevant market. Although highly relevant to the question at hand, the court declined to base its findings on that evidence because the court had "no basis for evaluating what a particular elasticity coefficient means."[12] Lacking a way to translate demand elasticities into market delineation conclusions, the court declined to trust expert judgment.[13]

The recent *Kodak* consent decree proceedings produced much the opposite result. The defense expert estimated the Kodak's own elasticity of demand for color print film to be approximately two. He testified that this implied that Kodak *did not* possess market power but made no attempt to explain why in his direct testimony. The district court was willing to trust the expert's judgment, and to do so notwithstanding contrary testimony from the government's expert on the implication of the estimated elasticity.[14]

The *Kraft* case is a recent high-profile merger case in which a central issue was whether the merger of Post Grape-Nuts and Nabisco Shredded Wheat likely would lead to significant price increases. Both sides addressed this issue using empirical analyses of demand for ready-to-eat breakfast cereals. As I read Judge Kimba Wood's lengthy and much praised opinion, she determined that the merger did not violate the antitrust laws largely on the basis of the defense expert's estimates of, and especially his interpretations of, cross elasticities of demand.[15] Nothing in the opinion suggests any way in which cross elasticities could be translated into price increase predictions or otherwise systematically assessed.

While economists can glean something about likely price and welfare effects of mergers from raw elasticities,[16] the process has appeared to the courts as a black box. District judges have been asked to choose between the expert judgments of opposing witness as to the significance of the estimated elasticities. Judges probably have not fully appreciated the subjectivity and imprecision of the black box, and to the extent they have appreciated it, they were forced to rely on more familiar evidence or fall back on prior beliefs. Incorporating the elasticity estimates into a simulation of the post-merger equilibrium opens the black box to inspection by the court and makes the analysis both far more precise and objective. Simulations depend on assumptions, and different experts are apt to make different assumptions, but the assumptions would have to be stated explicitly. They can be debated on their merits, and in many cases they can be tested statistically.

The Fundamentals of Differentiated Products Merger Simulations

The Basic Steps

The simulation of a differentiated product merger consists of a "front-end" analysis in which the elasticity parameters of a specified demand system are estimated,[17] and a "back-end" analysis in which those parameters are combined with pre-merger data on market shares and prices to predict the price effects of a merger.[18] The back-end analysis consists of the simulations themselves and two preliminary steps—calibration of the demand system and recovery of marginal costs.

If the estimated demand system does not fit the pre-merger equilibrium, there is no sensible way in which to compare the predicted post-merger equilibrium with that observed pre-merger. Thus, the first task in the back-end analysis is the calibration of the demand system to fit the pre-merger equilibrium. Essentially what is involved is calculating values of the shift parameters (e.g., intercepts) of the demand system that make it precisely predict the observed pre-merger equilibrium, given the estimated elasticity parameters.

The calibration of the demand system entails the choice of a particular set of shares and prices to represent the pre-merger equilibrium. The theoretically correct shares and prices are those that would prevail in the near future but for the merger. In the vast majority of cases, the best indication of those shares and prices will be the shares and prices for some recent time period. That time period should be of sufficient length so that seasonal and other transitory phenomena are averaged out—perhaps a year. If there are pronounced trends in the shares, or some solid basis for predicting changes in either shares or prices, the "pre-merger" shares and prices should be forecast or just guessed at.[19]

Recovery of the marginal costs is the final preliminary step before the actual simulations, and it requires an assumption about the particular oligopoly interaction that generated the pre-merger shares and prices. I believe that noncooperative price-setting, i.e., Bertrand competition, is the only reasonable assumption in most instances, and most economists seem to agree.[20] With Bertrand competition, it is relatively straightforward to infer the pre-merger marginal costs from the pre-merger first-order conditions for profit maximization.

The simulations themselves require three additional assumptions. First, a functional form for marginal cost must be assumed and, as a practical matter, constant marginal cost is likely to be the only tractable assumption unless there is information on the shape of marginal cost curves other than inferences from the pre-merger first-order conditions. Constant marginal cost is also likely to be a reasonable assumption, since marginal costs generally are likely to be approximately constant over the relevant output range.[21]

The simulations also require an assumption about the post-merger oligopoly interaction, and the Bertrand assumption is just as reasonable post-merger as it is pre-merger. In any event, simulations cannot predict any effect a merger might have on the oligopoly interaction itself (as opposed to effects on the equilibrium, holding the interaction constant).

Finally, it is necessary to assume a particular demand system.[22] There is a lot of choice as to the demand system, and the predictions certainly may depend on the assumption made, so the choice of a demand system can be a bone of contention in a litigated case. Local properties of functional forms generally animate functional form discussions in the estimation context, but global properties are critical in simulations, because at least some of the quantity changes associated with a merger are likely to be substantial.[23] The next section considers many possible demand systems and discusses some of their desirable and undesirable global properties.

The First-Order Conditions for Profit Maximization

To clarify how marginal costs are recovered and how the simulations are performed, it is useful to consider the general first-order conditions for a Bertrand equilibrium. Define:

p_i = the price of product i

q_i = the quantity of product i

c_i = the constant marginal cost of product i

m_i = the margin of product i, $\dfrac{p_i - c_i}{p_i}$

ϵ_{ij} = the elasticity of demand for product i with respect to the price of product j

θ_{ij} = the diversion ratio to product i from product j, $-\dfrac{\epsilon_{ij}\, q_i}{\epsilon_{jj}\, q_j}$.

The diversion ratio is the share of the sales lost by one merging product that is recaptured by another when the price of the former increases.[24]

Each firm may sell multiple products pre-merger. Let:

m = the column vector consisting of elements m_i
e = the column vector consisting of elements $-\epsilon_{ii}^{-1}$
Ω = matrix consisting of elements ω_{ij},

$$\omega_{ij} = \begin{cases} 1, & i = j \\ -\theta_{ij} p_i / p_j, & i \neq j,\ i \text{ and } j \text{ are both sold by the same firm} \\ 0, & i \neq j,\ i \text{ and } j \text{ are not both sold by the same firm.} \end{cases}$$

The pre- and post-merger first-order conditions *both* can be written:

$$\mathbf{m} = \Omega^{-1}\mathbf{e}. \tag{1}$$

The recovery of the marginal costs uses the pre-merger first-order conditions, with the ω_{ij} defined on the basis of the pre-merger position of the firms. In the simplest case, with each firm selling just one product pre-merger, the first-order conditions simplify to the familiar form: $m_i = -\epsilon_{ii}^{-1}$. It follows that $c_i = p_i(1 + \epsilon_{ii}^{-1})$. If firms sell multiple products pre-merger, the algebra is more complex, but the principle is the same.

Simulations use the post-merger first-order conditions, with the ω_{ij} defined on the basis of the post-merger position of the firms. The pre- and post-merger first-order conditions differ in that some pairs of products i and j are sold by the same firm—the merged firm—only after the merger. Consequently, the ω_{ij} for those pairs equal zero pre-merger but not post-merger.

Demand Systems for Merger Simulations[25]

Log-Linear and Linear Demand Systems[26]

Denoting demand shift variables by z_k, the log-linear demand for product i is:

$$\log q_i = \alpha_i + \sum_j \beta_{ij} \log p_j + \sum_k \gamma_{ik} z_k. \tag{2}$$

This probably is the demand regression model most familiar to many economists, and it has the convenient property that the own and cross elasticities of demand are both constant and are the regression coefficients themselves.[27]

There is, however, a serious problem with the assumption of constant elasticities when simulating the effects of significantly anticompetitive mergers. A merger that significantly affects prices and relative shares could be expected to affect significantly demand elasticities, and even small changes in the elasticities tend to have substantial effects on the price increase calculations. This is why the global properties of demand systems are so important and why the choice of a demand system is likely to be contentious. Erroneously assuming constant elasticity has been shown to yield price increase predictions many times the true value when the actual demand system is linear (see Shapiro 1996: 26–27) or logit (see Werden and Froeb 1994: 421).

The linear demand for product i is:

$$q_i = a_i + \sum_j b_{ij} p_j + \sum_k \gamma_{ik} z_k. \tag{3}$$

Linear demand is even more convenient than log-linear demand in that it permits a simple analytical solution for the back-end. Collapsing the demand shifters into the constant terms, the full set of demand equations is given by:

$$\mathbf{q} = \mathbf{a} + \mathbf{B}\mathbf{p}, \tag{4}$$

where $\mathbf{q}$, $\mathbf{p}$, and $\mathbf{a}$ are vectors consisting of elements q_i, p_i, and a_i, and where $\mathbf{B}$ is a matrix consisting of elements b_{ij}. Let $\mathbf{c}$ be the vector consisting of elements c_i, and define the matrix $\mathbf{D} = \{d_{ij}\}$ such that $d_{ij} = b_{ji}$ (note the transposition), if products i and j are sold by the same firm, and $d_{ij} = 0$, otherwise.

The first-order conditions for profit maximization both pre- and post-merger are

$$\mathbf{a} - \mathbf{D}\mathbf{c} + (\mathbf{B} + \mathbf{D})\,\mathbf{p} = \mathbf{0}. \tag{5}$$

Using $\mathbf{D}_0$, which pertains pre-merger, marginal costs are recovered:

$$\mathbf{c} = \mathbf{D}_0^{-1}\mathbf{a} + (\mathbf{D}_0^{-1}\mathbf{B} + \mathbf{I})\,\mathbf{p}_0. \tag{6}$$

Using $\mathbf{D}_1$, which pertains post-merger, post-merger prices are computed:

$$\mathbf{p}_1 = (\mathbf{D}_1 + \mathbf{B})^{-1}(\mathbf{D}_1\mathbf{c} - \mathbf{a}). \tag{7}$$

The last three equations continue to hold if **a**, **B**, and **D** are all multiplied by a positive scalar. Thus, in calibrating the demand system, i.e., in solving for **a**, shares (in physical quantity units) can be used instead of quantities.[28] The foregoing makes the linear demand system so simple to use in simulations that the back-end analysis can be tacked on to the regression routines used for the front-end, provided that the regression software has linear algebra programming capabilities.

The linear demand system is not without serious problems, however. Obviously, demands are not likely to be linear in fact, nor is linear demand generally thought to be a good approximation. In practice, the assumption of linear demand commonly results in the prediction of negative quantities unless nonnegativity constraints are imposed. Thus, the global properties of the linear demand system do not appear to be entirely satisfactory.

Logit Demand

In several coauthored papers, I have advocated the use of a version of the logit model as a demand system for differentiated merger simulations, at least as a starting point.[29] A logit demand system generally is motivated by a random utility model in which consumers make a discrete choice from a set of products, C, consisting of n alternatives. Consumers select the alternative yielding the greatest utility. The simplest model specifies the indirect utility of consumer k associated with the choice of product i as:

$$U_{ik} = \alpha_i - \beta p_i + \nu_{ik}. \tag{8}$$

The price coefficient, β, is the same for all consumers and products, and all generally perceived quality differences among products are summarized by the α_i, which can be linear functions of product characteristics. The disturbance term, ν_{ik}, represents an individual-specific component of utility that is uncorrelated with price, p_i. If the ν_{ij} are independently and identically distributed according to the extreme value distribution, the choice probabilities have been shown to take the familiar logistic form:

$$\pi_i = \frac{\exp(\alpha_i - \beta p_i)}{\sum_{j \in C} \exp(\alpha_j - \beta p_j)}. \tag{9}$$

This model is sometimes referred to as the "flat" logit model to contrast it with the more complex nested logit model discussed below.

For use in merger simulations, it is convenient to express the model in traditional antitrust terms. Product n is defined to reflect the choice of "none of the above," i.e., to be the outside good, and it is assumed that $p_n \equiv 0$ to make the utility of the outside good a constant. The share-weighted average pre-merger price for the inside goods is denoted $\bar{p}$. The choice probabilities for the "inside goods" (i.e., those other than the outside good), conditional on the choice being an inside good, are termed "shares" and denoted s_i. They correspond to conventional market shares, except that they do not presume a proper market delineation.[30] This reparameterized model has been dubbed the Antitrust Logit Model or ALM.

The aggregate elasticity of demand for the inside goods can be defined as:

$$\epsilon \equiv \frac{\partial \log \sum_{i=1}^{n-1} \pi_i(\lambda p_i)}{\partial \lambda} \bar{p} \Bigg]_{\lambda=1} = -\beta \bar{p} \pi_n. \tag{10}$$

The elasticities of demand for inside goods are easily shown to be:

$$\epsilon_{ii} = -\beta p_i (1 - \pi_i) = -\left[\beta \bar{p}(1 - s_i) + \epsilon s_i\right] p_i / \bar{p} \tag{11}$$

$$\epsilon_{ij} = \beta p_j \pi_j = s_j (\beta \bar{p} - \epsilon) p_j / \bar{p}. \tag{12}$$

Although π_n is not a choice probability at all in this parameterization, it is still necessary that $\pi_n \in (0, 1)$, which requires $\beta\bar{p} > \epsilon$. Otherwise, the cross elasticities are all negative, and no inside good is a substitute for any other inside good.

There are just two demand parameters in this model—ϵ, which controls substitutability between the inside goods and the outside good,[31] and β, which controls substitutabilities among the inside goods, given ϵ. This parameter parsimony facilitates both the front-end and back-end analyses, but it also makes the model highly restrictive. The best way in which to see the nature of the restriction is to observe that ϵ_{ij} does not depend on the identity of product *i*. This is the consequence of the Independence of Irrelevant Alternatives (IIA) property of the logit model. Formally, the IIA property is that for all products *i* and *j* and all possible subsets, *S*, of *C*:

$$\frac{\text{Prob}(i|S)}{\text{Prob}(j|S)} = \frac{\text{Prob}(i|C)}{\text{Prob}(j|C)}, \tag{13}$$

provided that all the probabilities are positive. Verbally, the IIA property is that the odds ratio of any two choices is independent of the other possible choices.[32]

The IIA property can be terribly useful, but it is also terribly restrictive. In some industries, some pairs of products surely are closer substitutes than others, and that is inconsistent with the IIA property. Thus, imposing the IIA property is inappropriate in at least some cases. It is possible to statistically test the restrictions IIA implies, and those restrictions can be relaxed.

The ALM is calibrated by solving the logit probability functions for the α_i after setting α_n to an arbitrary value (the choice of which has no effect):

$$\alpha_i = \alpha_n + \beta p_i + \log s_i + \log(\beta\bar{p}/\epsilon - 1). \tag{14}$$

The marginal costs are recovered from the first-order conditions. For a firm selling the set of products for which the choice probabilities sum to π_f and the shares sum to s_f, the first-order conditions for profit maximization can be shown to be:

$$p_i - c_i = \frac{1}{\beta\bar{p}(1 - \pi_f)} = \frac{\bar{p}}{\beta\bar{p}(1 - s_f) + \epsilon s_f}. \tag{15}$$

These seemingly simple equations have no simple analytic solution and must be solved numerically because π_f and s_f are complex functions rather than constants. The mathematical properties of the logit functions assure both that the solution is unique and that it is not difficult to find. The post-merger first-order conditions solved in the actual simulations differ only with respect to the set of products sold by the merged firm.

The properties of the ALM were thoroughly explored by Werden and Froeb (1994). Werden and Froeb (1994) and Froeb et al. (1997) apply it to hypothetical mergers of long distance carriers in the

United States and Japan. This model has been applied by the author to several actual mergers with satisfactory results. Unfortunately, the details of those applications must be kept confidential.

The random utility model that motivates the flat logit also can be used, with a more complicated distribution assumption for the ν_{ij}, to generate the nested logit model, in which the structure of choice probabilities suggests a hierarchial decision process. For example, a consumer purchasing a car might first decide on a size class, then on a degree of luxury or performance, and so on. The nested logit model can have an arbitrary number of nests at each level of the decision hierarchy and can have an arbitrary number of levels of decisions; however, the version of the model presented here has only one level with an arbitrary number of nests at that level.

The unconditional choice probability for product i is again denoted π_i. The marginal choice probability for the products in nest k is π_{N_k}, and the conditional choice probability for product i given a choice in nest k is $\pi_{i|N_k}$:

$$\pi_i = \frac{\exp\left((\alpha_i - \beta p_i)\delta_k^{-1}\right)\left[\sum_{j \in N_k} \exp\left((\alpha_j - \beta p_j)\delta_k^{-1}\right)\right]^{\delta_k - 1}}{\exp(\alpha_n) + \sum_{s=1}^{n-1}\left[\sum_{t \in N_s} \exp\left((\alpha_t - \beta p_t)\delta_s^{-1}\right)\right]^{\delta_s}} \tag{16}$$

$$\pi_{N_k} = \frac{\left[\sum_{j \in N_k} \exp\left((\alpha_j - \beta p_j)\delta_k^{-1}\right)\right]^{\delta_k}}{\exp(\alpha_n) + \sum_{s=1}^{n-1}\left[\sum_{t \in N_s} \exp\left((\alpha_t - \beta p_t)\delta_s^{-1}\right)\right]^{\delta_s}} \tag{17}$$

$$\pi_{i|N_k} = \frac{\exp\left((\alpha_i - \beta p_i)\delta_k^{-1}\right)}{\sum_{j \in N_k} \exp\left((\alpha_j - \beta p_j)\delta_k^{-1}\right)}. \tag{18}$$

The elasticities of demand are not quite so simple to derive from this model, but they can be shown to be:

$$\epsilon_{ii} = -\beta p_i\left[\left(1 - \pi_{i|N_k}\right)\delta_k^{-1} + \pi_{i|N_k} - \pi_i\right] \tag{19}$$

$$\epsilon_{ij} = \beta p_j\left[\pi_j + \pi_{j|N_k}\delta_k^{-1} - \pi_{N_k}\right], \quad i, j \in N_k \tag{20}$$

$$\epsilon_{ij} = \beta p_j \pi_j, \quad i \in N_k,\ j \notin N_k. \tag{21}$$

The aggregate elasticity is as in the ALM, and each of these formulas can be written as a function of ϵ and the s_i.

The δ_j are parameters in the interval (0, 1) that control the strengths of the nests. If all the δ_j equal 1, the model collapses to the flat logit. The closer δ_j is to 0, the stronger the nest and the more distant substitutes the products in nest j are for products in other nests. Some intuition of the meaning of particular nest parameters can be gotten from a simple example. Suppose that there are two nests, each with two products, and that all prices and shares are the same. From the elasticity formulae, it is easy to compute diversion ratios for a pair of products in the same nest and for a pair of products in different nests. The ratio of these two—the "relative diversion ratio"—indicates how many times more the

TABLE 7.1 Relative Diversion Ratios Associated with Various Nest Parameters

Nest Parameter, δ_j	1	.9	.8	.7	.6	.5	.4	.3	.2	.1
Relative Diversion Ratio	1	1.2	1.5	1.9	2.3	3	4	5.7	9	19

diversion is to the other product in the same nest than to either product in the other nest. Table 7.1 displays this information for various values of the nest parameter.

Simulations using this model proceed very much as those using the flat logit model, although the α_i must be solved for numerically, and with nest parameters below .5, that procedure may fail to converge. This is not a serious problem for reasons discussed below. The nested logit model is much more flexible than the flat logit model, but it still restricts substitution possibilities.

Other Candidate Demand Systems

While the author has previously advocated the use of the logit model, that was never to the exclusion of other models that might fit the data better and also have good global properties. The universal logit model[33] retains the attractive mathematical properties of the logit functional form but eliminates the restrictions on substitution possibilities. The IIA property is not imposed, and for maximum flexibility, there are as many elasticity parameters as there are own and cross elasticities of demand. This model is not derived directly from a random utility model, but instead simply specifies choice probabilities as:

$$\pi_i = \frac{\exp\left(\alpha_i + \sum_{j=1}^{n-1} \beta_{ij} p_j\right)}{\exp(\alpha_n) + \sum_{k=1}^{n-1} \exp\left(\alpha_k + \sum_{j=1}^{n-1} \beta_{kj} p_j\right)} . \tag{22}$$

It is not difficult to show that:

$$\epsilon_{ij} = \left[\beta_{ij} - \sum_{k=1}^{n-1} \beta_{kj} \pi_k\right] p_j \tag{23}$$

$$\epsilon = \bar{p} \pi_n \sum_{i=1}^{n-1} \sum_{k=1}^{n-1} s_k \beta_{ki} . \tag{24}$$

For the front-end estimation, the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980a: 75–78, 1980b) has become popular.[34] AIDS is a flexible functional form that satisfies (with appropriate parameter restrictions) the dictates of utility theory. Defining Y as the total expenditure for a group of products, and p as a price index for those products, the demand equations are of the form:

$$\frac{p_i q_i}{Y} = \alpha_i + \sum_j \beta_{ij} \log p_j + \gamma_i \log (Y/p). \tag{25}$$

The theoretically correct price index is a complicated function that makes the demand equation nonlinear in parameters, so in practice it is replaced with a Stone price index. Using weights, w_j, that are the averages of the expenditure shares for the individual products over the data set, the Stone price index is given by:

$$\log p = \sum_j w_j \log p_j. \tag{26}$$

AIDS regressions are used as the "bottom level" of a two-stage (or multi-stage) budgeting system (see Gorman 1995). The "top level" regresses deflated group expenditure shares on a constant, the price index for the group, price indices for other groups, total deflated expenditures, and other demand shifters. Unconditional demand elasticities in this two-stage budgeting context involve effects at both levels.

Work has begun on using an AIDS back-end. The global properties of the AIDS demand system, however, have not been yet explored, so its suitability for use in merger simulations is not known at this time.

The latest innovation in demand estimation is the characteristics model of Berry et al. (1995) (see also Berry and Pakes 1993: 248–249). This model is motivated by a discrete choice random utility model much like that which motivates the logit model, and this model is predicated on some of the same distributional assumptions. On the other hand, a variety of very different assumptions also are made, introducing much greater complexity and much greater flexibility in substitution patterns. The estimation of these models is also quite different in several respects, and a supply side is estimated at the same time (see also Berry 1994). Some work has been done on using these models in a back-end analysis, but it has not been reported in the literature. The global properties of these models do not appear to have been much explored.

Issues in Simulating Differentiated Products Mergers

Fitting the Model

The results of the simulations cannot be considered reliable unless the model can be shown to fit the industry reasonably well. One good indication of goodness-of-fit is a comparison of the marginal costs inferred from the first-order conditions with other available indications of marginal costs.[35] If the inferred marginal costs do not jibe with the evidence on actual marginal cost, the problem most likely is that pre-merger pricing in the industry is not consistent with the Bertrand assumption. In the author's experience, the Bertrand assumption generally explains pre-merger pricing of most products quite well, but it sometimes cannot explain pricing of all products. Indeed, the first-order conditions sometimes imply significantly negative marginal costs for some products. Such products typically have small shares and are being priced aggressively to build share. If, across the board, inferred marginal costs appear implausible, the problem is more likely to be serious specification error or inappropriate parameter estimates. In either case, the simulations should not proceed without addressing these issues.

The selection of products to be included in a simulation resembles the process of market delineation superficially, but it is very different in practical effect. To exclude a product from a delineated relevant market, is basically to deem it irrelevant to the competitive analysis, since it will be assigned a market share of zero. To exclude a product from a merger simulation is to deem irrelevant only the accommodating price response of that product, and not its price constraining effect. The prices of products included in merger simulations are allowed to respond to those of the merging firms, while prices of the excluded products are treated as parametric. The prices of all substitutes increase in response to the price increase of the merging firms, and that reinforces the price increase by the merging firms. Thus, the exclusion of a substitute biases downward the price increases from a merger.[36] The bias is very slight unless the products individually have very large shares. Consequently, excluding products can be freely done for a variety of reasons.

In light of the foregoing, it is probably best to significantly limit the number of products included in back-end simulations. Very minor products can always be safely excluded, since they would have no significant response to the price increases of the merging firms. Excluding minor products alone can drastically reduce the number of products in the simulations. Other prime candidates for exclusion are products thought to be, or estimated to be, relatively poor substitutes. In the context of the flat logit model, their exclusion surely makes the model fit better. In the context of the nested logit model, this means that very strong nests need not be used, which eliminates the problems that arise in solving models with strong nests. It may also be the case that some products are obviously not priced in a manner consistent with the Bertrand assumption. Such products should be excluded because there is no reason to believe that the model is a good predictor of such products' post-merger prices.

Finally, to the extent that the assumed demand system constrains substitution patterns, some consideration should be given to whether those constraints are appropriate and how they affect the prediction of the simulations. For example, if a flat logit model is used, the IIA assumption is maintained, and individual cross elasticities should be examined when possible to determine whether maintaining the IIA assumption is likely to bias the predictions significantly. If it is clear that there is a bias in a particular direction, the predictions can still be used as long as the bias is recognized.

Translating Retail Data into Wholesale Predictions

Only retail price and quantity normally are available at the level of aggregation and the frequency desired for the front-end estimation. The use of retail data is clearly desirable for the front-end estimation, because it is consumer demand that is of interest; however, the use of retail data presents a small problem for the back-end simulations. The effect of a merger of manufacturers is of interest, and manufacturers (normally) sell at wholesale. To do the simulations, it must be assumed that suppliers of distribution services, including retailers, do not participate strategically in the oligopoly interaction. Two alternative assumptions can be made about their behavior.

One assumption is that retailers (and any middlemen) have constant absolute margins, i.e., the mark-ups are a fixed number of cents per unit. This assumption is maintained implicitly if the retail price and quantity data are used in both the front- and back-ends without any adjustment. The inferred marginal costs then include both the manufacturing marginal cost and the implicit distribution marginal cost. This complicates checking the fit of the model.

Another, typically better, assumption is that retailers have constant relative margins, i.e., the mark-ups are a fixed percentage of the wholesale price. When this assumption is maintained, the calibration of the model involves backing out the retail mark-ups in determining the pre-merger prices. A similar adjustment must be made to some parameter estimates from the front-end, although no adjustment is necessary in a demand elasticity itself, since constant percentage mark-ups make the retail and wholesale elasticities the same. Using the flat logit model, no adjustment is necessary in ϵ, but the estimated β must be multiplied by the ratio of the retail price to the wholesale price to back out the effects of retail margins.

Whatever assumption is maintained it is advisable to verify that retail prices are actually set in such a way. It is imperative to determine what the retail margins are.[37]

Presentation and Interpretation of Results

Since no two firms in an industry are apt to increase price by precisely the same amount following a merger, the question arises of how best to present the array of information from the simulations. My inclination at this point is to report the percentage price increase for each of the merging firms and the average price increase for a reasonable product group, typically broader than the products used in the simulations. The appropriate group would be the one delineated as the relevant market, if one is delineated, and a comparable group otherwise.[38]

Calculating an average price increase for a group of products raises traditional index number problems. There is a natural inclination to use contemporaneous quantity weights in averaging prices in this context. The resulting price index has the dubious property, however, that average price may fall even if all individual prices rise. Though uncommon, this can occur with elastic demands for products that experience large price increases. Even if nothing so dramatic occurs, the use of contemporaneous quantity weights tends to minimize greatly the impact of a large price increase by just one merging firm (which occurs with highly asymmetric shares). The same effect flows from the use of a Paasche index, which uses post-merger quantities as weights. Thus, I have used a Laspeyre index, with pre-merger quantity weights. This, however, exaggerates the effect of the merger on prices. There is no economically correct index.[39]

Since price increase and other predictions are functions of parameters estimated with sampling error, there is statistical uncertainty in those predictions. In principle that uncertainly can be expressed in the form of a confidence interval. The calculation of a confidence interval is feasible in principle, because there are standard methods for approximating the variance of a function of estimated parameters. Nevertheless, the calculation of confidence intervals presents a few problems that have not yet been fully addressed. One is that parameters in different estimated equations are functions of the same data and have a covariance. Thus, calculation of confidence intervals for simulation predictions requires the system estimation of several equations and the recovery of cross-equation covariances. Alternatively, resampling methods could be used to generate the confidence intervals.

Apart from the sampling error, which in principle can be accounted for in confidence intervals, there remains a potentially more important source of error—that from specification error. There are numerous potential sources of specification error in the front-end estimation,[40] and the simulation results can be no better than demand parameter estimates used to generate them. The assumption of Bertrand competition in the back-end is an additional source of possible specification error.[41] Thus, the predictions from the simulations should be taken as reasonable, if rough, approximations, rather than precise measurements. A rough approximation of price effects is still vastly more useful than an exact measurement of market share (and there is no such thing, since market delineation is uncertain and contentious).

Notes

[1]Gregory J. Werden is Director of Research, Economic Analysis Group, Antitrust Division, U.S. Department of Justice. The research reported here was conducted jointly with Luke Froeb (some was also joint with Tim Tardiff). Further insights resulted from applying the research in actual antitrust investigations and from interaction with Economic Analysis Group colleagues. Nevertheless, the views expressed herein are not purported to be those of the U.S. Department of Justice or any other person.

[2]Eastman Kodak Co. v. Image Technical Services, Inc., 504 U.S. 451, 469 n.15 (1992).

[3]This approach is comparable to that of many modern macroeconomic studies (see Kydland and Prescott 1996).

[4]Commercial data vendors such as IRI and Nielsen provide such data, derived from point-of-sale scanners, for products sold in supermarkets and certain other retail outlets.

[5]The author prepared expert testimony, based largely on simulations, on the effects of proposed mergers involving white bread and cosmetics. Significant divestitures were required to address the anticompetitive effects of the bread merger. United States v. Interstate Bakeries Corp., 1996-1 Trade Cas. (CCH) ¶ 71,271 (E.D. Ill.) (consent decree). The cosmetics merger was not challenged. Other economists in the Antitrust Division have used simulations in analyzing mergers involving tissue (United States v. Kimberly-Clark Corp., 1996-1 Trade Cas. (CCH) ¶ 71,405 (N.D. Tex.) (consent decree)) and frozen sea food. Feldman (1994) used simulations to analyze the effects of an actual merger of HMOs. Simulations have not, however, been tested in litigation nor relied on by a court.

[6]The material in this section and the next is developed at greater length by Werden (1997).

[7]Brown Shoe Co. v. United States, 370 U.S. 294, 335, 343 (1962).

[8]United States v. Philadelphia National Bank, 374 U.S. 321, 363 (1963).

[9]A useful treatment of the case law and associated policy issues is ABA Antitrust Section (1986), and a brief economic analysis of the state of the law is provided by Hay and Werden (1993).

[10]The 1982 Merger Guidelines made substantial contributions to market delineation (see Werden 1992a, 1993), and the 1992 Horizontal Merger Guidelines made substantial contributions to the analysis of the competitive effects of mergers.

[11]These points are developed at greater length by Werden and Rozanski (1994).

[12]United States v. Mrs. Smith's Pie Co., 440 F. Supp. 220, 227–228 (E.D. Pa. 1976).

[13]There are ways to make that translation using the methodology of the Merger Guidelines; however, the Guidelines' methodology came later, and the means for using elasticities to make inferences came later still. See Werden (1992b: 115–117, 119–121; 1993: 528–532, 537–538).

[14]United States v. Eastman Kodak Co., 853 F. Supp. 1454, 1473 (W.D.N.Y. 1994). The court granted Kodak's motion to terminate long-standing consent decrees. The court of appeals affirmed, but it held that an own elasticity of two *did not* prove that Kodak *had* market power. United States v. Eastman Kodak Co., 63 F.3d 95, 108–109 (2d Cir. 1995). The defense expert was Jerry Hausman.

[15]New York v. Kraft General Foods, Inc., 926 F. Supp. 321, 333, 356 (S.D.N.Y. 1995). The defense expert was Daniel Rubinfeld.

[16]Recent work by Shapiro (1996) has facilitated this process.

[17]The front-end analysis may entail complex econometrics posing challenging technical problems, but they are neglected here because the purpose of this essay is to discuss the back-end. It should be noted, however, that the front-end estimation requires the assumption of a particular functional form for the demand system, and since there are good reasons to use the same demand system in the back-end, the demand system used in the front end should be chosen with the back-end in mind.

[18]Welfare analysis of the effects of mergers may involve significant complications which are not discussed here.

[19]Elasticities of demand depend on the shares and prices, and time span of the data used in the front-end analysis typically will be longer than that used to determine pre-merger shares and prices. Calibration, thus, may involve more than the selection of shift parameters; it also may involve the calibration of the elasticities themselves.

[20]Exceptions presumably include Baker and Bresnahan (1985). They do not assume Bertrand competition but rather seek to estimate the nature of the oligopoly interaction in conjectural variations terms. Simulations can be done in such a context.

[21]The merging firms lose share following a merger, so their relevant output range is below pre-merger levels. Although constant marginal cost is a safe assumption for outputs slightly below pre-merger levels, the output of the smaller merging firm may be cut dramatically if shares are highly asymmetric (see Werden and Froeb 1994: 413–414, 418). Nonmerging competitors gain share following a merger, so the relevant output range for them is slightly above pre-merger levels. Constant marginal cost generally is safe in this range, but capacity constraints should be considered.

[22]As detailed in Werden (1996), there is one sort of welfare analysis that does not depend on the functional form of the demand system. It is possible to calculate the marginal cost reductions for the merging firms that would be necessary to restore precisely the pre-merger prices. This provides a sufficient condition for a merger to enhance consumer welfare.

[23]With highly asymmetric pre-merger shares, output of the smaller-share product can fall 90 percent!

[24]Shapiro (1996) offers the diversion ratio as a useful way in which to gauge the effects of differentiated products mergers on competition and consumers. Although estimating a diversion ratio requires estimating the underlying demand elasticities, it is easier to intuit likely diversion ratios from qualitative evidence than to intuit cross elasticities of demand.

[25]So far as I am aware, constant elasticity was always assumed in analyses like those described here until quite recently. That assumption was made by Baker and Bresnahan (1985) in making their calculations, and by Hausman et al. (1994). The first published work to assume otherwise appears to have been Feldman (1994) and Werden and Froeb (1994), and both used logit models. Feldman did not calibrate the model as suggested here.

[26]Neither of these demand systems is derived from utility theory, so both could be considered unsatisfactory for that reason alone.

[27]Log-linear demand simplifies the back-end analysis, but not dramatically. Hausman et al. (1994: 174) present a simple and elegant solution based on a slightly different representation of the first-order condition than that presented above; however, it is a direct solution only if it is assumed that relative revenue shares are unaffected by the merger. That assumption is problematic unless the merging firms are symmetric and the accommodating price responses of competitors are ignored.

[28]It also follows that the foregoing solution method can be applied to any subset of products without bothering to recompute shares for just that set of products.

[29]Froeb and Werden (1996), Froeb et al. (1997), Werden and Froeb (1994, 1996), and Werden et al. (1996). These papers greatly elaborate the presentation of the model. The use of the logit model of analysis of differentiated products mergers was first proposed by Willig (1991: 299–304), but Willig used it only to motivate reliance on market shares. Feldman (1994) has also used the logit model in simulations.

[30]In estimating this model in the front-end, this reparameterization is essentially a two-stage budgeting approach (see Gorman 1995). Estimation of the logit parameters themselves is done through standard maximum likelihood methods, which can be applied to individual or grouped data.

[31]It may be misleading to call ϵ a parameter because it is not a constant. As prices and choice probabilities change following a merger, so too does ϵ, since it is a function of them. The pre-merger value of ϵ is an input to the simulation, while the post-merger value of ϵ is an output from it.

[32]The IIA property is discussed at length by Werden et al. (1996: 86–87) and references therein. It serves as a useful benchmark by providing a definition for “equally close substitutes.”

[33]The term and the model are borrowed from the econometrics literature. The term generally is used in that literature to describe a model that does not have linear functions in the exponentials, as the model here does.

[34]This model has been used by Hausman (1997), by Hausman and Leonard (1997), by Hausman et al. (1994), and in several actual antitrust investigations.

[35]Marginal costs approximately constant over a broad range can be closely approximated by average variable cost, although there remain potentially difficult issues in the determination of which costs should be treated as variable, and in getting sufficiently disaggregated data.

[36]There is a more complex effect on the welfare analysis. The profits associated with the excluded products are ignored in a total welfare calculation.

[37]Using an approach like that of Berry et al. (1995), the retail margin can be estimated, but it is still necessary to make the same sort of assumption.

[38]If the results are presented in terms of changes in consumer or total welfare, some of the presentation issues disappear. The scope of the product group remains an issue, however.

[39]A fairly comprehensive discussion of price indices is provided by Diewert (1987).

[40]Examples include the use of average revenue as the price variable, the imposition of a particular functional form for demand, and the failure to model consumer inventory behavior.

[41]Bertrand competition is not a complete description of the competitive interaction when, as is often the case, substantial temporal variation in prices and quantities is observed because of sales and other promotions. This fact does not appear to warrant significant concern about the reliability of the simulations, however. And, of course, substantial temporal variation in price and quantities is highly desirable in the estimation.

References

ABA Antitrust Section. 1986. *Horizontal Mergers: Law and Policy*. Monograph No. 12.

Baker, Jonathan B. and Timothy F. Bresnahan. 1985. The Gains from Merger or Collusion in Product-Differentiated Industries. *Journal of Industrial Economics* 33:427–444.

Berry, Steven T. 1994. Estimating Discrete Choice Models of Product Differentiation. *Rand Journal of Economics* 25:242–262.

Berry, Steven T. and Ariel Pakes. 1993. Some Applications and Limitations of Recent Advances in Empirical Industrial Organization. *American Economic Review (Papers & Proceedings)* 83:247–252.

Berry, Steven T., James Levinsohn, and Ariel Pakes. 1995. Automobile Prices in Market Equilibrium. *Econometrica* 63:841–890.

Deaton, Angus and John Muellbauer. 1980a. *Economics and Consumer Behavior*. Cambridge: Cambridge University Press.

Deaton, Angus and John Muellbauer. 1980b. An Almost Ideal Demand System. *American Economic Review* 70:312–326.

Diewert, W. Erwin. 1987. Index Numbers. In *The New Palgrave: A Dictionary of Economics*, ed. J. Eatwell, M. Milgate, and P. Newman. London: Macmillan.

Feldman, Roger. 1994. The Welfare Economics of a Health Plan Merger. *Journal of Regulatory Economics* 6:67–86.

Froeb, Luke M. and Gregory J. Werden. 1996. Simulating the Effects of Mergers Among Noncooperative Oligopolists. In *Computational Economics and Finance: Modeling and Analysis with Mathematica*, ed. Hal R. Varian. New York: TELOS.

Froeb, Luke M., Timothy J. Tardiff, and Gregory J. Werden. 1997. The Demsetz Postulate and the Effects of Mergers in Differentiated Product Industries. In *The Role of Economists in Modern Antitrust*, ed. Fred S. McChesney. New York: Wiley.

Gorman, W. M. 1995. Two-Stage Budgeting. In *Separability and Aggregation: Collected Works of W. M. Gorman, Volume I*, ed. C. Blackorby and A. F. Shorrocks, Chapter 2. Oxford: Clarendon Press.

Hausman, Jerry A. 1997. Valuation of New Goods under Perfect and Imperfect Competition. In *The Economics of New Goods*, ed. Timothy F. Bresnahan and Robert J. Gordon. Chicago, IL: University of Chicago Press.

Hausman, Jerry, Gregory Leonard, and J. Douglas Zona. 1994. Competitive Analysis with Differentiated Products. *Annales d'Economie et de Statistique* 34:159–180.

Hausman, Jerry A. and Gregory K. Leonard. 1997. Economic Analysis of Differentiated Products Mergers Using Real World Data. *George Mason Law Review* 5:321-346.

Hay, George A. and Gregory J. Werden. 1993. Horizontal Mergers: Law, Policy, and Economics. *American Economic Review (Papers & Proceedings)* 83:173–177.

Kydland, Finn E. and Edward C. Prescott. 1996. The Computational Experiment: An Econometric Tool. *Journal of Economic Perspectives* 10:69–85.

Shapiro, Carl. 1996. Mergers with Differentiated Products. *Antitrust* 10(2):23–30.

U.S. Department of Justice. 1982. Merger Guidelines, reprinted in 4 Trade Reg. Rep. (CCH) ¶ 13,102.

U.S. Department of Justice and Federal Trade Commission. 1992. Horizontal Merger Guidelines, reprinted in 4 Trade Reg. Rep. (CCH) ¶ 13,104.

Werden, Gregory J. 1992a. The History of Antitrust Market Delineation. *Marquette Law Review* 76:123–215.

Werden, Gregory J. 1992b. Four Suggestions on Market Delineation. *Antitrust Bulletin* 37:107–121.

Werden, Gregory J. 1993. Market Delineation under the Merger Guidelines: A Tenth Anniversary Retrospective. *Antitrust Bulletin* 38:517–555.

Werden, Gregory J. 1996. A Robust Test for Consumer Welfare Enhancing Mergers Among Sellers of Differentiated Products. *Journal of Industrial Economics* 44:409-413.

Werden, Gregory J. 1997. Simulating the Effects of Differentiated Products Mergers: A Practical Alternative to Structural Merger Policy. *George Mason Law Review* 5:363-386.

Werden, Gregory J. and George A. Rozanski. 1994. The Application of Section 7 to Differentiated Products Industries: The Market Delineation Dilemma. *Antitrust* 8(3):40–43.

Werden, Gregory J. and Luke M. Froeb. 1994. The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy. *Journal of Law, Economics, & Organization* 10:407–426.

Werden, Gregory J. and Luke M. Froeb. 1996. Simulation as an Alternative to Structural Merger Policy in Differentiated Products Industries. In *The Economics of the Antitrust Process*, ed. Malcolm B. Coate and Andrew N. Kleit. Boston: Kluwer.

Werden, Gregory J., Luke M. Froeb, and Timothy J. Tardiff. 1996. The Use of the Logit Model in Applied Industrial Organization. *International Journal of the Economics of Business* 3:83–105.

Willig, Robert D. 1991. Merger Analysis, Industrial Organization Theory, and Merger Guidelines. *Brookings Papers on Economic Activity, Microeconomics*, 281–332.