Yardstick Based Procurement Design In Natural Resource Management

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Paper Selected for Presentation at the
25th International Conference of Agricultural Economists
Durban, South Africa August 16-22, 2003

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Abstract

This paper discuss the design of multidimensional yardstick based procurement auction. The suggested design combines Data Envelopment Analysis (DEA) based yardstick schemes with the multidimensional score auction. The principal select a single winner to perform a project, characterized by a multidimensional vector. The design is especially useful when there are uncertainty about the underlying common cost structure as well as the principal's valuation function. Potential applications in natural resource management is provided.

1 Introduction

Auctions has been successfully used to distribute and acquire a variety of goods. Traditionally, most auctions have been one-dimensional price auctions that allocate a single unit to the bidder with the highest price bid. In the past decades, however, more advanced auction mechanisms have been applied in practice and described in the literature.

In particular, the treasury bill auctions and the spectrum auctions have generated new auction forms and a considerable literature on multi-unit and combinatorial auctions. Combinatorial auctions price units individually or in packages.

Another mechanism that has received recent interest is the *multi-dimensional auction* form. Opposed to a traditional auction a multidimensional auction allows the bidders to bid over multiple attributes instead of just the price. Multidimensional auctions have been particularly important in public procurement. In this paper we present a new multidimensional auction.

Consider a *public procurement* of an item with qualities described by a vector z in a market with n potential sellers. By fixing the qualities at given levels (setting minimum constraints on z), the government can (in most cases) design an auction that will minimize the procurement cost for these quality levels. The outcome, however, may not be Pareto efficient - nor even individually rational for the buyer. Pareto efficiency requires that there is an optimal trade-off between the seller's costs of alternative quality levels and buyer's benefits here from. To make these trade-offs, we need more advanced multidimensional forms. We cannot fix the qualities a priori. The shortcoming of the traditional auction has been widely discussed in the case of spectrum auctions. Instead of revenue maximizing auctions some countries have applied negotiations - the so-called comparative hearings or beauty contests. A negotiation is clearly a multidimensional approach as opposed to a traditional auction with minimum quality levels. A negotiation, on the other hand, is less transparent than a traditional auction. It depends in part on negotiation power and lobbyism. For more on this, see e.g. Genty (1999).

Dealing with *externalities in natural resource management* is another case where a multidimensional mechanism may be relevant. Farmers and forest owners produce - and use - environmental goods like recreation, clean ground water and habitats for indigenous plants and animals. However, as no conventional market exists for these goods, the unregulated production levels are presumably below their social optima. A private farmer or forest owner is likely to prioritize the production of marketable goods, e.g. corn or timber. Environmental regulation plays an important role in trying to correct this market failure. One of the means is subsidies. Subsidies are usually granted on a flat-rate basis, say per hectare (ha), or they are determined from assessments of opportunity costs. Due to asymmetric information, the better-informed farmers and forest owners will usually extract information rents. The distribution of subsidies via a multidimensional auction might be relevant in limiting these rents with respect to all relevant parameters.

There are several *practical instances of multidimensional auctions*, e.g. the conservation reserve program in the USA (Babcock et al., 1997; Vukina et al., 2000), the Department of Defence procurement auctions for weapon systems in the USA (Che, 1993) and the television franchising in the UK (Galapo, 1999).

The theoretical literature on multi-dimensional auctions is sparse, however. The most well known multidimensional auction is the score auction. The *score auction* uses a score function to map multidimensional bids (quality vectors and prices (or compensations)), into one-dimensional scores. The score represents the principal's utility or the welfare function and is used to allocate the good and determine the price. The score auction takes the principal's utility or the welfare function as given. Cripps and Ireland (1994) tries to relax the assumption of a known utility function by setting quality thresholds that are unknown to the bidders. Other papers deal with the issue of learning the welfare function and the agents' cost functions (Beil, 2001).

This paper suggest a multidimensional auction that combine the score auction and the so-called DEA based yardstick schemes.

Data Envelopment Analysis (DEA) is a non-parametric approach to measure relative performance. It was first proposed by Charnes et al. (1978, 1979) and has subsequently become tremendously popular.

DEA can be used to model costs and can hereby assist in the design of an ex-ante regulation, i.e. a system where subsidies are based on past data. DEA can also be used in an ex-post regulation, where the additional information acquired during the regulation period, is used to set reasonable costs. The trick here is that the ex-ante commitment to ex-post regulation effectively creates a pseudo-market among the agents. Each agent therefore try to do at least as well as the others.

This paper shows how DEA can support procurement design. While both ex-ante and ex-post regulation seek to reduce the costs of producing given outputs, the focus in procurement is on the choice of agents (to operate in a market), as well as their multiple dimensional output vectors.

The outline of the paper is as follows. Section 2 provides a brief literature review. Section 3 gives a brief introduction to DEA and DEA based yardstick schemes. Section 4 presents and discuss the DEA based yardstick auction in different settings. In Section 5 concrete applications are provided. Final remarks are provided in Section 6.

2 Literature

This paper joins two bodies of literature, the literature on multi-dimensional score auctions on the one hand and the one on yardstick competition on the other.

The literature on auctions in general is impressive. In a standard auction or procurement context, where a single quality product is supplied, the *revenue equivalence* between first price and second price auctions is the most central result. It was suggested by Vickrey (1961), but remained a puzzle until 1981 where Riley and Samuelson (1981) and Myerson (1981) simultaneously solved the problem. They show that a class of independent private value auctions give the same expected revenue (or costs) to the principal.

Thiel (1988) uses tools from consumer theory to show that the optimal *multi-dimensional auctions* will be equivalent to the design of one-dimensional auctions. He consider the situation where the procurer decides on a budget, which becomes known to the agents. Also, the procurer does not not value any savings. The paper sketches the idea of the multi-dimensional score auction, but it offers no proof of optimality for such an auction. Also, it is unclear whether the assumptions of the preset budget and, especially, valueless savings are appropriate in most procurement situations (Branco, 1997).

Che (1993) is a central paper on *multi-dimensional score auctions*. He shows how the existing theory on auctions can be generalized to multi-dimensional auctions. He considers the allocation of a procurement contract that compensate the winner with a transfer t. The principal's score function value a one-dimensional quality parameter z and the transfer t. Che shows that the equivalence theorem can be applied to the multi-dimensional score auction. The score function provides a one-dimensional valuation that transforms the problem to the traditional private value single dimensional case. Using the result from Riley and Samuelson (1981), he proves that the first score and the second score auction leave the principal with the same utility. In a *first score auction*, the bidder with the highest score wins and has to meet the highest score. In a *second score auction*, the bidder with the highest score wins and has to meet the second highest score. Che arrives at his very strong result by restricting the bidders's cost type to be *monotonic*; the cost functions are assumed to be imbedded into each other in the sense that a lower cost type has lower costs for all possible qualities than a higher cost type has, i.e. the cost advantages and disadvantages are universal such that types can effectively be given a one-dimensional ordering.

Branco (1997) shows that Che's model fails with *correlated costs* among the agents. In such cases, the optimal quality will depend on all bidders' cost of producing the quality. To solve this problem he introduces a *2 stage model*. The first stage selects a winner and the second stage determines an optimal quality based on all bids submitted. He argues that this is typically what happens. The US defense auctions, for example, typically have a 2 stage system where the quality is negotiated after the winner have been found. The truth revealing model presented in this paper takes a somewhat similar 2 stage approach.

Even though multi-dimensional auctions have mainly been suggested and used in public procurement, there is an increasing focus on the use of multi-dimensional auctions in the private sector. In Bushnell and Oren (1995), a multi-dimensional auction is used to select supplier within a firm. Here the objective is to maximize the joint profit of the firm. An increasing number of papers on electronic markets also focus on redesigning ordinary markets along the line of multi-dimensional auctions e.g. Koppius and Heck (2002); Milgrom (2000). One of these, Milgrom (2000), show in general that the second score multi-dimensional auction achieves an efficient outcome if the auctioneer announces the utility function as the scoring rule. That is, there are no other deal that both the buyer and the seller would prefer.

Until very recently, the theoretical papers on multi-dimensional auctions have worked with one-dimensional types of players only. This makes it rather simple to ensure that a lower type has a higher chance of winning. However, there have been some attempts to model *multiple dimensional types*, e.g. Zheng (2000) and McAdams (2002). Zheng (2000) builds on result from the non-linear pricing, especially Armstrong (1996) who proved that monotonicity can be obtained essentially by excluding some low value costumers. McAdam (2002) is an example of an auction with multiple types modelled as a supermodular game. A supermodular game only requires that the players' strategies can be partially ordered and that the best-response functions of the players are upwards sloping, see Milgrom and Roberts (1990).

This paper takes a somewhat more simple and practical approach. Starting out with a real life regulatory problem, the existing multi-dimensional measurement technics from DEA is combined with the idea of the multi-dimensional score auction.

Like the general literature on auctions, there is also a large economic literature on *relative performance evaluations*. The seminal contribution is Holmstrom (1982). Relative performance evaluation as yardstick competition was introduced by Shleifer (1985). The extension to multiple dimensional performances and the combination with frontier models like Data Envelopment Analysis (DEA) was initiated in Bogetoft (1997, 2000) and expanded upon in Agrell et al. (2002). The uses of relative performance evaluations and yardstick schemes are numerous. Performance based payment schemes, where a manager's bonus depends on his performance relative to the sector or the market in general, is a prime example. Yardstick competition has, for example, been used to regulate transportation and electricity distribution, c.f. Dalen and Gomez-Lobo (1997) and Resende (2001). Similarly, elements of relative performance evaluation and yardstick competition are found in several production contracts in agriculture, see e.g. Olesen (2002).

3 DEA based yardstick competition

This section provides beief introductions to Data Envelopment Analysis (DEA) and DEA based yardstick competition. For more comperenhensive introductions, seeCharnes et al (1994) and Bogetoft(1997,2000).

3.1 Brief introduction to Data Envelopment Analysis

Consider n Decision Making Units (DMUs), that each transforms k inputs into r outputs. Let $x^i = (x^i_1, \ldots, x^i_k) \in \mathbb{R}^k$ be the inputs consumed and let $y^i = (y^i_1, \ldots, y^i_r) \in \mathbb{R}^r$ be the outputs produced by DMU^i, $i \in I = \{1, 2, \ldots, n\}$. The production possibility set is given by:

$$T = \{(x, y) \in \mathbb{R}^{k+r} | x \text{ can produce } y\} \tag{3.1}$$

Inefficiency is the ability to reduce inputs without affecting output or the ability to expand output without requiring more inputs. In the multiple inputs, multiple outputs case a popular measure has become the so-called Farrell index. It measures the possibility to make proportional input reductions E or output expansions F:

$$E^i = \min\{E \in \mathbb{R}_0 | (Ex^i, y^i) \in T\} \tag{3.2}$$

$$F^i = \max\{F \in \mathbb{R}_0 | (x^i, Fy^i) \in T\} \tag{3.3}$$

In DEA, the technology T is estimated from real observations using the so-called *minimal extrapolation principle*. The estimated technology is the smallest set containing the data and satisfying certain production economic regularities (like convexity and free disposability.)

When DEA is used in motivation games like DEA based yardstick competition, it is useful to exclude the evaluated unit from the estimation of the technology against which it is evaluated. In this way, Ratchet effects can be avoided. We denote the technology against which agent i is evaluated by T^{-i}. In the DEA literature, the resulting Farrell efficiency measures are known as super efficiencies. A problem with super efficiency is that it does not always exist. Some units may be *hyper-efficient* in the sense that $(Ex^i, y^i) \notin T^{-i}$ for arbitrary large value of E and $(x^i, Fy^i) \notin T^{-i}$ for arbitrary small values of F. This problem remains in the regulatory mechanisms presented later. The implications and the possible solutions of this problem is left for future research.

3.2 DEA based yardstick competition

The first game theoretical approach to the use of DEA is due to Banker (1980) and Banker et al. (1989). They provided game theoretical interpretations of the scoring problem in the standard DEA models given realized inputs and outputs. The study of the ex-ante motivation game of choosing inputs, outputs, efforts, skills etc. using formal agency models was initiated by Bogetoft (1990).

Throughout the paper we consider a stylized environmental regulation problem. Let the bidders be the agents and the government the principal. Assume - in the simplest possible version - that the only input is the realized costs, i.e. the input is one-dimensional $c^i \in \mathbb{R}, i = 1, \ldots, n$. The problem is to determine how much, B, to reimburse an agent using costs c to produce $y \in \mathbb{R}^r$ environmental goods. The goods are produced in a given context as defined by non-controllable variables $z \in \mathbb{R}^q$, z is common knowledge. z is (arbitrarily) modelled as outputs such that the DEA model consist of 1 input and $r + q$ outputs. Furthermore y and z can be perfectly verified and hence costlessly contracted upon. For concrete examples, see section 5.

In terms of technology and information, we assume that there is considerable uncertainty and asymmetric information about the underlying cost structure. The individual agents are supposed to have superior technological information. In an extreme case, they know with certainty the underlying true cost $C(y, z)$, i.e. the costs of producing y under environmental conditions z. Of course, we do not have to assume that the agents know these costs for all possible output profiles y and environmental conditions z. In fact, it will ease the design of good schemes if they only have local information, say the costs for a limited set of possible output vectors and given its specific local conditions.

The regulator, on the other hand, only knows the general nature of the cost function a priori, say that $C(.,.)$ belongs to a class $\mathcal{C}$ of possible cost functions[1]. In addition, the regulator knows that the realized production plans are possible, i.e. that

$$x^i \geq C(y^i, z^i), i = 1, \ldots, n \tag{3.4}$$

where x^i is the observed cost, which include slack. Slack (or inefficiency) is excess cost e.g. to much time used on a task.

The agent's resulting utility is assumed to be the utility from reimbursement and dis-utility from effort:

$$U^i(B, x^i, y^i, z^i) = (B - x^i) + \rho(x^i - C(y^i, z^i)) \tag{3.5}$$

The first term, $B - x^i$, is the profit. The second term is the excess costs or slack, $x^i - C(y^i, z^i)$, multiplied by ρ, the value of slack compared to

[1] This class is large, e.g. the set of all increasing convex functions. For more details, see Bogetoft (1994) and Bogetoft (1997)

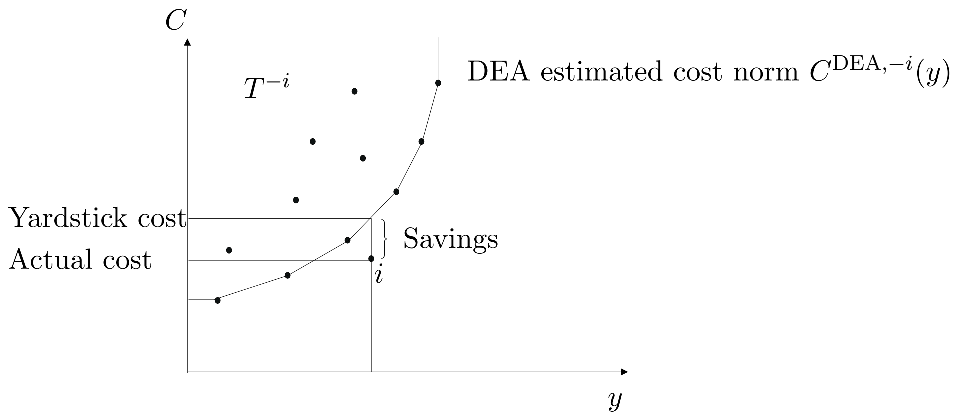

Figure 1: *The DEA yardstick model in the production-cost space*

profit. It is assumed that the agents prefer profit to slack, $\rho \leq 1$, and that the agents are risk neutral.

In this procurement setting we only consider the case where the actual costs cannot be contracted upon, i.e. $B(x, y, z) = B(y, z), \forall x$. In this case, the optimal solution, cf. Bogetoft(1997), is to use the following revenue cap with non-verifiable cost information:

$$B(y^i, z^i) = Q^i + C^{\mathrm{DEA}-i}(y^i, z^i) \tag{3.6}$$

i.e. the optimal reimbursement equals a lump sum payment to cover the reservation utility, Q^i, plus the DEA-estimated cost norm for the given output y and environmental variables z. The DEA-estimated cost norm is given by:

$$C^{DEA-i}(y^i, z^i) = \min_x \{x | (x, z^i, y^i) \in T^{-i}\} \tag{3.7}$$

The possibility set T^{-i} is a convex hull of the others actual performance. Figure 1 illustrates this reimbursement scheme.

In the DEA based yardstick literature, the principal's objective has been to minimize the cost of inducing the agents to produce given outputs in given contexts. Different combinations of y and z is not valued per se. The aim of this paper is to introduce preferences and output trade-offs into the yardstick regulation by applying the idea from the score auctions. Or vice versa, to introduce yardstick competition into multi-dimensional score auctions.

4 DEA Based Yardstick Auctions

In the DEA-incentive literature, the focus is on the use of historical or future production data to monitor the agents and to motivate them to take proper actions by committing ex-ante to a payment principle ex-post. The DEA based yardstick auction introduce the potential use of DEA to evaluate non-realized multi-dimensional bids (as opposed to realized production plans) in a procurement setting (as opposed to a control setting).

We consider a principal who wants to select a single agent to provide a good characterized by (y, z). To determine which agent to call upon and the compensation to award him, the regulator organize a multiple dimensional procurement auction. Initially, the agents submit bids, and based hereon the regulator determines which offers to use and how to compensate the corresponding agent. Next, the agent picks the actual production plan and payment is realized when the promised outputs are delivered.

A bid from agent i is therefore a r-dimensional environmental output vector $y^i \in \mathbb{R}^r$ and a corresponding cost $x^i \in \mathbb{R}$. In addition, a vector of non-controllable environmental state variables, $z^i \in \mathbb{R}^q$, characterize the individual bidder. z^i is common knowledge to both the agent and the regulator.

As in section 3.2, we assume that the agents are risk neutral and that they maximize profit and slack with a relative lower value of slack than profit. That is, when agent i gets compensated B^i for producing y^i and when he actually uses x^i in the context z^i, he is left with a utility of $B^i - x^i + \rho(x^i - c^i(y^i, z^i))$. The compensation B^i is determined by the DEA estimated yardstick cost, $C^{\mathrm{DEA}-i}(y^i, z^i)$.

Furthermore, we assume that the regulator knows and maximizes the true environmental gains $V(y^i, z^i)$ minus the costs of inducing the agents to undertake the production B^i. The procurement cost payed to the agents are inflated with $(1 + k) > 1$, to reflect the economy wide mis-allocations resulting from the generation of the necessary funding via tax payments. The principals welfare function is therefore given by:

$$V(y^i, z^i) - (1 + k)C^{\mathrm{DEA}-i}(y^i, z^i) \tag{4.1}$$

if agent i is selected. $V(.)$ is assumed to be strictly concave.

Below, different situations are considered. In model 1, we assume that all bidders can produce on the same underlying cost structure as in the DEA based yardstick scheme. In model 2, the individual bidders' cost functions are allowed to be more independent. We close the section by looking at the welfare function. First, we consider the two models in case of a linear welfare function. Second, the assumption that the welfare function is known with certainty is relaxed.

4.1 Model 1

Here we apply the same set of assumptions as in the literature on DEA based yardstick schemes. In terms of information, we have that the agents' actual costs, $C(y^i, z^i)$, are private information. The regulator simply knows that the costs originate from a common cost function $C(.,.)$ from a class $\mathbb{C}$ (also, see section 3.2).

Before formalizing the regulator's problem, we note that the multi-dimensional cost (types) of the different agents are correlated. We know that the actual costs of the agents all originate from the same underlying cost function. The set of possible cost functions, however, is very large. This means that types are not perfectly correlated from the point of view of the principal. On the other hand, they are also not independent. We argue that the assumed correlation is very natural by its relationship to production theory.

Combined with the DEA based yardstick schemes the regulator's problem can be set up as a general principal agent problem. To do so in a general format, let us assume that the agents make possibly false reports of their cost functions, say $x^i(.) : \mathbb{R}^r \to \mathbb{R}$ with the interpretation that the cost of producing y^i is claimed to be $x^i(y^i), i = 1,..,n$. Now, the principal must decide which agent to select, what he shall produce, and what to compensate him. A chosen agent is assigned an indicator value of one, say $d^i(x) = 1$, and a non-selected agent is assigned an indicator variable of 0, i.e. $d^i(x) = 0$. Note that the choice of agents - as all other choices by the principal - can depend on the full cost report of the agents. In a similar way, the production levels decided by the principal are indicated by $y^i(x), i = 1,..,n$, and the compensation levels are given by $B^i(x)$. Using the revelation principle, we can without loss of generality impose truth-telling constraints, i.e. assume that the costs reported by agent i is consistent with the underlying true cost function, $x^i(C, Y^i) = c^i(y^i, z^i)$, where $c^i(y^i, z^i) = C(y^i, z^i) \cdot 1_{Y^i}(y^i)$ is the true cost function for agent i. Note that we allow for the possibility that a given agent can only produce some of the possible outputs $Y^i \subseteq \mathbb{R}^r$. Let $Y = \Pi_{i=1}^n Y^i$.

The *regulator's problem* (with a variable budget) can therefore be formulated as:

$$
\begin{aligned}
&\max_{d(.),B(.),x(.),y(.)} \\
&E_c\left[U(d^1(c)(y^1(c),z^1),\ldots,d^n(c)(y^n(c),z^n) - (1+k)\textstyle\sum_i d^i(c)B^i(c)\right] \\
&\text{s.t.} \\
&E_{c^{-i}|c^i}\left[d^i(c)(B^i(c)+\rho(c^i(y^i(c),z^i) - C(y^i,z^i)) - Q^i)\right] \geq 0, \forall C \in \mathbb{C}(s), Y, i = 1,\ldots,n \quad (IR) \\
&E_{c^{-i}|c^i}\left[d^i(c)(B^i(c)+\rho(c^i(y^i(c),z^i) - C(y^i,z^i)) - Q^i)\right] \geq \\
&\quad E_{c^{-i}|c^i}\left[d^i(x^i,c^{-i},y,z)((B^i(x^i,c^{-i}) - x^i) + \rho(x^i - C(y^i(x^i,c^{-i}),z^i)) - Q^i)\right] \\
&\quad \forall C \in \mathbb{C}(s), Y, x^i, i = 1,\ldots,n \quad (IC) \\
&d^i(x) \in \{0,1\} \forall x, i = 1,\ldots,n \\
&c^i(y^i,z^i) = C(y^i,z^i)\cdot 1_{Y^i}(y^i), i = 1,\ldots,n
\end{aligned}
\tag{4.2}
$$

The objective function is the expected environmental value minus social costs. Expectation is taken with respect to the underlying, unknown costs, c, of the agents. The regulator's choice variables concern which agent to accept in the program, d, what they shall produce, y, and what to pay them, B. The first set of constraints is the individual rationality constraints. They ensure that all agents, given their private information about their costs, expect to get at least their reservation utility of Q^i if they are selected. Q^i is determined outside the model, e.g. by the state of nature z. For simplicity, Q^i is set equal to zero in the remaining of the paper. The second set of constraints is the usual incentive compatibility constraints. They say that no agent would ever like to deviate from truth-telling about costs, $x^i = c^i$.

The regulator's problem emphasize the complexity of the problem and the importance of a proper auction design. Note that in the chosen formulation, we assume that the agents only know their own costs, not the costs of the other agents. Since the compensation depends on all bidders' bids, each and everyone must form beliefs about the other bidders' $r + q$ dimensional cost functions. Mechanisms involving such belief structures are analytically extremely complex and we shall proceed below by analyzing simpler versions of the context. Also, we shall develop mechanisms, where the strategy of a given agent does not depend on the types of others. Hereby, the agents are effectively relieved of the burden of forming these complex beliefs.

Below we treat the case of a single and multiple production plan(s) separately. The case of multiple production plans is more complicated because we have to consider not only the other bidders' feasible production plans but also the individual bidder's own production plans.

Single production plan

For now, assume that agent i can only choose one production plan y^i. The report by the agent therefore effectively reduces to (x^i, y^i, z^i), where x^i is the reported cost, y^i is the reported production plan, and z^i is the (commonly known) context of agent i.

As in a score auction the submitted multi-dimensional bids are ranked with the following score function:

$$V(y^i, z^i) - (1 + k)x^i \tag{4.3}$$

The score function reflects the true welfare function. Now consider the following set of auction rules:

Step 1: The agents submit their bids $(x^i, y^i, z^i), i \in I$, and thereby commit to carry out the production of y^i at the cost x^i under the commonly known state of nature, $z^i, i \in I$.

Step 2: Each bid is assigned a score reflecting the welfare it would generate $V(y^i, z^i) - (1 + k)x^i$.

Step 3: The highest score wins.

Step 4: The compensation is determined by the yardstick cost $C^{\mathrm{DEA}-i}(y^i, z^i)$.

The idea of this procedure is simple. The regulator select the bidder that can provide the potential highest welfare, by selecting the highest score. The winner is asked to carry out the stated plan at the DEA estimated yardstick cost. The DEA based yardstick cost norms are estimated using all submitted bids except the winner's, like in the yardstick competition model presented in section 3.2.

Proposition 4.1. *The optimal bidding strategy for bidder i is to bid the true cost, $c^i = C^i(y^i, z^i)$, for the given feasible production plan, (y^i, z^i).*

Proof. The minimal extrapolation principle ensure that $C(y^i, z^i) \leq C^{\mathrm{DEA}-i}(y^i, z^i)$. Therefore we have that, since the compensation determined by the DEA based yardstick cost, is independent of i's bid and that i have only one feasible production plan:

Bidding below true cost, $c^i = C^i(y^i, z^i)$, will increase the chance of winning but the extra trade generated will generate negative profit. Therefore it never pays to bid below the true cost.

Bidding above true cost will lower the chance of winning without affecting the profit if winning. Furthermore, since slack is worth less than profit, including slack is unprofitable. Therefore it does not pay to bid above true cost.

Consequently, we have that the optimal bidding strategy is to bid the true cost. □

In this setting the DEA based yardstick auction is clearly individual rational and incentive compatible. Also, the auction ensures *allocative efficiency*, i.e. that the contract is allocated to the bidder that produces the highest welfare (Refereed to as *efficient auction* in the literature). To see this, note that the optimal strategy is to bid the true cost. Now, since the score reflects the welfare and the highest score win the contract, allocative efficiency is ensured.

So far we have have stressed allocative efficiency. This is relevant in a public procurement setting. In traditional private procurement setting, however, the principal would rather select the auction that maximizes the revenue or minimizes the procurement costs. It is therefore relevant also to consider the benefits generated to the principal (Refereed to as *optimal auction* in the literature). In the following we compare the DEA based yardstick auction and the second score auction with respect to the principal's welfare.

Let $S^{(2)}$ be the second highest score. Now, if the actual compensation $C^{\mathrm{DEA}-i}(y^i, z^i)$ produce a higher score than $S^{(2)}$ $(V(y^i, z^i) - C^{\mathrm{DEA}-i}(y^i, z^i) \geq S^{(2)})$, the DEA based yardstick auction is preferred to the principal. In general we have that none of the two auctions dominate each other. Figure

2 show two different situations, represented by score functions S^A and S^B. In situation A we have that the DEA based yardstick auction is preferred to the second score auction and vise versa in situation B.

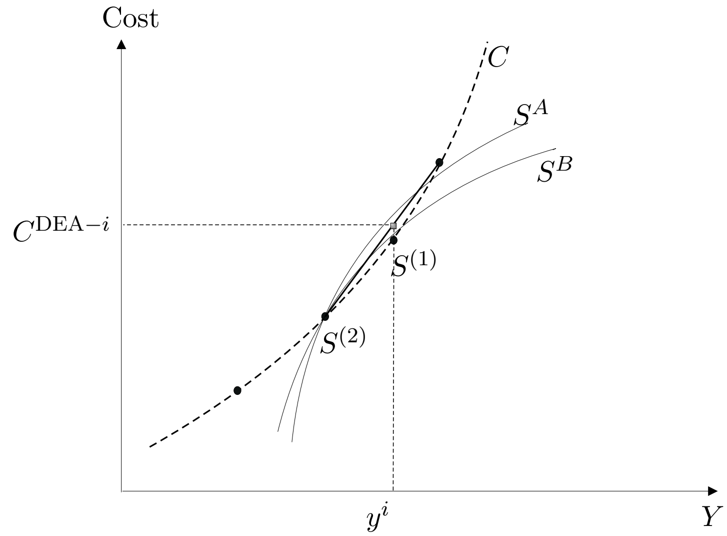

Figure 2: *Comparing the DEA based yardstick auction with the second score auction*

Figure 2 indicates that the highest scoring bidder might not be the one that generate the highest welfare, as oppose to the score auction. It might be that the compensation corresponding to $S^{(2)}$ (This yardstick compensation is not illustrated in Figure 2) would generate a higher welfare than the yardstick compensation corresponding to $S^{(1)}$, $C^{\text{DEA}-i}$ in Figure 2. Alternatively, the auction could select the winner that maximize the welfare generated to the principal. This can be done simply by replacing the actual costs bid with the corresponding yardstick cost (potential compensation). Hereby the selection mechanism directly tradeoff environmental benefits and the cost of acquiring them. Of course, such a mechanism does not ensure allocative efficiency, as oppose to the chosen design.

Also in traditional auctions allocative efficiency (*efficient auction*) and maximizing output (*optimal auction*) are conflicting. For more on this see Ausubel and Cramton (1999).

Multiple production plans

We now turn to the more general case where the agents have more feasible production plans. To make the general formulations in the regulator's problem more specific, we let each agent report his sets of possible productions, Y^i and the associated costs, $x^i(y^i), y^i \in Y^i$. As before the compensation is determine by the yardstick cost function. Unlike the case of a single production plan, we allow the bidders to submit any number of bids. We show

how this simplifies the bidding strategy by facilitating a complete revelation of the bidders' cost function.

Now, let the set of auction rules be:

Step 1: The agents submit their bids $(x^i(y^i), y^i, z^i), y^i \in Y^i, i \in I$, and thereby commit to carry out the production of y^i at the cost $x^i(y^i)$ under the commonly known state of nature, z^i, $i \in I$.

Step 2: Each bid is assigned a score reflecting the welfare it would generate $V(y^i, z^i) - (1+k)x^i$.

Step 3: The highest score wins.

Step 4: The compensation is determined by the most profitable bid $f(x^i(\widetilde{y}^i), \widetilde{y}^i, z^i)$ that produce a higher score than the highest losing bidder's bid.

Step 4 has been changed to cope with the multiple production plans. Step 4 simplifies the bidding strategy by selecting the most profitable bid among the highest bidder's non-dominated bids. Let $\tilde{Y}^i$ be the set of i's bids that produce a higher score than the highest losing bidder's bid. Now the most profitable bid is given by:

$$\max_{y \in \tilde{Y}^i} \{C^{\mathrm{DEA}-i}(y, z^i) - C(y, z^i)\} \tag{4.4}$$

Otherwise, if we simply select the highest score it would be optimal to form beliefs about the other bidders' cost functions in order to extract information rent. To see this, note that the bid that produce the highest score might not be the most profitable bid in $\tilde{Y}$.

Proposition 4.2. *The optimal bidding strategy for bidder i is to submit multiple bids, revealing the true cost, $C(y^i, z^i)$, for any feasible production plan, $(y^i, z^i), y^i \in Y^i$.*

Proof. Bidding below true cost will increase the chance of winning, but the extra wins will be unprofitable.

Bidding above the true cost function will lower the chance of winning. Furthermore, since slack is worth less than profit including slack in the bids is unprofitable. Therefore it never pays to bid above true cost.

Leaving out bids do not pay as the payment is determined as the maximum of the non-dominated bids.

From the minimal extrapolation principle we have that a truth revealing cost bid would always provide positive profit, $C(y^i, z^i) \leq C^{\mathrm{DEA}-i}(y^i, z^i)$. Therefore, leaving out bids corresponding to a feasible production plan is unprofitable. □

From the optimal bidding strategy we clearly see that the DEA based yardstick auction is individual rational and incentive compatible. Also, as in the single bid case we have allocative efficiency, that the contract is allocated to the bidder with the highest value. To see this, note that the optimal strategy is to bid the true cost and that the minimal extrapolation principal ensure that all bids are profitable. Now, since the score reflects the welfare and the highest score win the contract, allocative efficiency is ensured.

As in the case of a single production plan, it might be relevant to compare the DEA based yardstick auction and the second score auction in terms of welfare generated to the principal. Clearly, as before there are situations where either of the two auctions would be preferred, see Figure 2.

On the other hand multiple bids will typically provide a closer representation of the underlying cost structure, which might be used in redesigning the auction. To ensure the DEA based yardstick auction to outperform the second score auction, step 4 may be changed to:

Step 4': The winning bid is the most profitable among those where the corresponding yardstick cost ($C^{\mathrm{DEA}-i}(y^i, z^i)$) produce a higher score than the highest losing bidder's bid.

Applying step 4' the DEA based yardstick auction provide the principal with a higher welfare per definition. Since such a bid may not exist the auction may fail in selecting a winner.

Also, we suggest that the DEA based yardstick auction will often be near-optimal for the principal. To see this, note that the DEA approximation of the cost structure will provide a close fit in many cases. Having observed $n-1$ observations on the $C(.,.)$ curve, the piecewise linear approximation will typically not deviate too much for points close to the observed ones. The incentives to reveal, in principle, the entire cost function will make the approximation even better.

A general "problem" with the DEA based yardstick auction is that it does not ensure efficient trade per se. By efficient trade we mean that there may be production plans that are Pareto improving ex-post. This is in contrast to the second score auction where the winner gets to select the most optimal production plan in a second round (Milgrom, 2000). In the DEA based yardstick auction the actual contract is determine by the most efficient bid relative to the other bidders' bids, and not efficiently with respect to the score function. Therefore, with multiple production plans, there might be points along the iso-score curve (determined by the actual compensation plan) that would be preferred for both the principal and the selected agent. We return to efficient trade in section 4.4 where we relax the assumption that the true welfare is given.

4.2 Model2

We now relax the assumption that the bidders' true cost for any feasible production plan, $c^i(y^i, z^i)$, equals the overall cost function, $C(y^i, z^i)$. We only require the bidders cost functions, $c^i(y, z)$, to have a single point in common with $C(y, z)$. Clearly this allow for more independent cost functions, while still requiring some interdependence between the individual cost functions. As before we argue that the assumed correlation is very natural by its relationship to production theory. In fact the underlying cost function may be interpreted as the long run cost function while the costs of the individual DMUs may be thought of as originating from local, short run cost curves. Figure 3 shows how the individual cost functions may differ from each other.

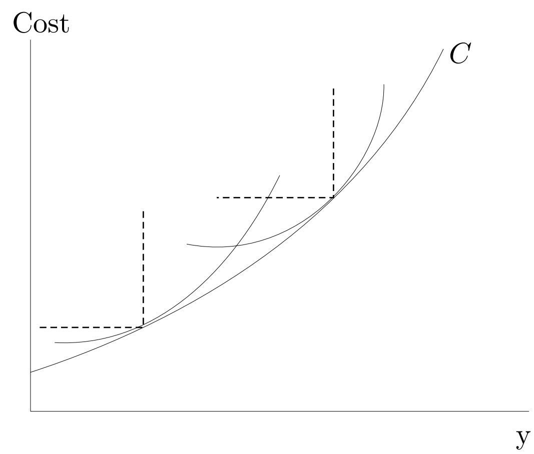

Figure 3: *The assumed cost structure*

Another interpretation is to say that proper investments would ensure any feasible production plan to be produced at the underlying minimum cost, $c^i(y^i, z^i) = C(y^i, z^i)$. Now if the auction provide proper incentives to invest we might actually return to cost structure described in model 1. This interpretation makes model 1 more realistic.

To begin with consider the special case, where the bidders are facing Leontief type of technologies (marked with dash lines in Figure 3). With this technology the single production plan where $c^i(y^i, z^i) = C(y^i, z^i)$ dominates i's other production plans. Now, since the only non-dominated production plan equals the common cost structure, this special case is similar to model 1 with a single production plan.

In the general case the cost functions are only required to be convex, represented by the solid lines in Figure 3. In this general case we might have bids where the yardstick cost $C^{\text{DEA}-i}(y^i, z^i)$ (the compensation) is smaller than the actual cost $c^i(y^i, z^i)$. This is a problem if the bidder with the highest score has no profitable bid with scores exceeding the highest losing bidder's bid. To avoid selecting one of these bids we simply exclude bids where $c^i(y^i, z^i) \geq C^{\text{DEA}-i}(y^i, z^i)$. The importance of this problem depends to a large extend on the actual cost functions. We will return to this problem below.

Now, consider the following auction rules:

Step 1: The agents submit their bids $(x^i(y^i), y^i, z^i), y^i \in Y^i, i \in I$, and thereby commit to carry out the production of y^i at the cost x^i under the commonly known state of nature, z.

Step 2: Each bid with positive profit $(C^{\text{DEA}-i}(y^i, z^i) \geq x^i(y^i))$ is assigned a score reflecting the welfare stated in the bid $V(y^i, z^i) - (1+k)x^i$.

Step 3: The highest score win.

Step 4: The compensation is determined by the most profitable bid $(x^i(\widetilde{y}^i), \widetilde{y}^i, z^i)$ that produce a higher score than the highest losing bidder's bid.

As in model 1 truth revealing is the optimal bidding strategy:

Proposition 4.3. *The optimal bidding strategy for bidder i is to submit multiple bids, that reveal the true cost, $c^i(y^i, z^i)$, for any given feasible production plan, (y^i, z^i).*

Proof. Bidding below true cost will increase the chance of winning, but the extra wins will be unprofitable.

Bidding above the true cost function will lower the chance of winning. Furthermore, since slack is worth less than profit including slack in the bids is unprofitable. Therefore it never pays to bid above true cost.

From the minimal extrapolation principle we have that a truth revealing cost bid would always provide positive profit, $x(y^i, z^i) = x^i(y^i) \leq C^{\text{DEA}-i}(y^i, z^i)$. Therefore, leaving out bids corresponding to a feasible production plan is unprofitable. □

Figure 4 give an example of these auction rules. The bold broken lines are the yardstick compensation that would provide the winner with a positive profit. The set of bids in the brackets produce a higher score than the highest losing bidder's bid and the auctioneer select the most profitable bid among these.

Among the eligible bids we have that allocative efficiency is ensured. That follows from the arguments given in model 1. If we consider all bids, however, we might have inefficient allocation. That would be the case if the highest score (among all bids) belong to a different bidder than the one with the highest eligible bid. Figure 5 illustrate the problem of allocative inefficiency in the extreme case where the underlying cost structure have constant return to scale (CRS).

Again, from the bidding strategy we have that the auction is individually rational and incentive compatible. Also, we might have situations where the

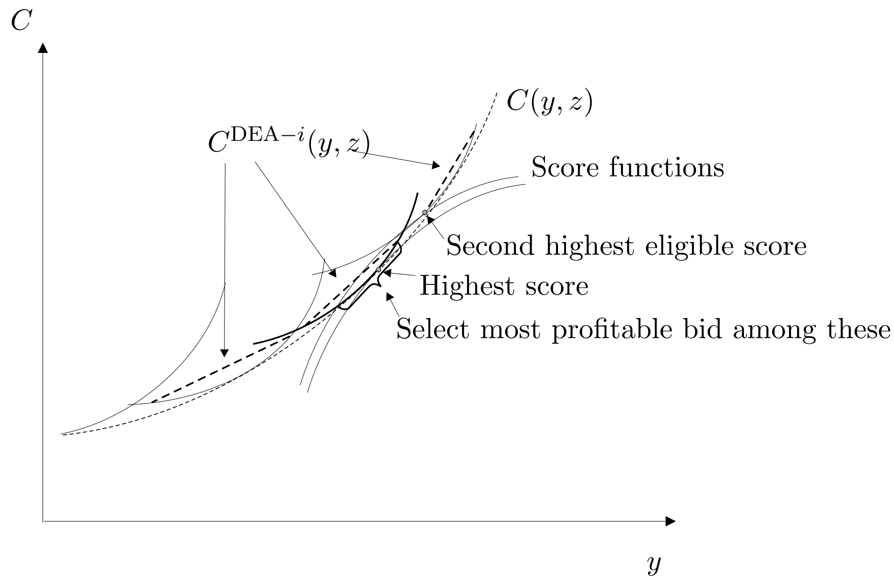

Figure 4: *DEA based yardstick auction in model 2*

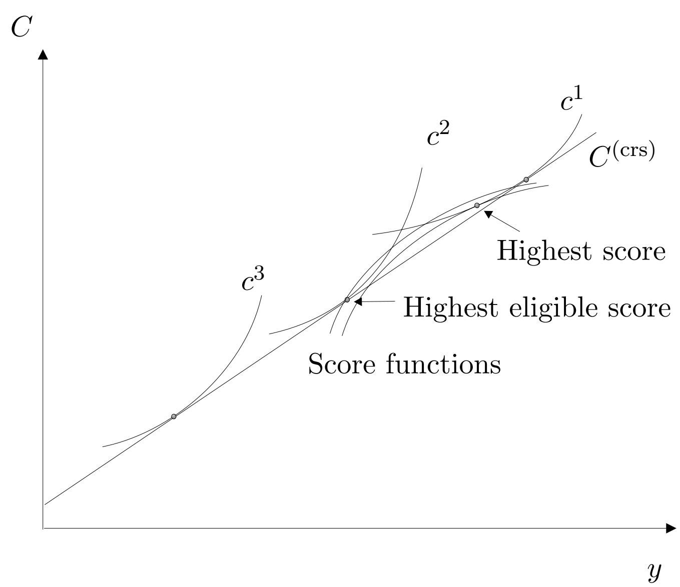

Figure 5: *Second score auction preferred to DEA based yardstick auction*

DEA based yardstick auction provides more welfare than the second score auction and vice versa as in model 1.

4.3 Linear score function

So fare we have assumed the welfare function to be a strictly concave function. Here we study the extreme case where the valuation function $V(.)$ is linear in all elements of y and z. As in model 1 and 2 above we will assume that the score function reflects the true welfare.

We show that in terms of welfare generated to the principal the second score auction weakly dominate the DEA based yardstick auction.

To see this, recall that the optimal bidding strategy in a second score auction is to bid the reservation score (the bid that generates the highest score) and that the actual compensation is determined by the most optimal point on the iso-score function determined by the second highest score. Also note that the compensation in a DEA based yardstick auction is determine by a convex envelopment of all the other bidders bids, which include the second highest score (or highest loosing bidder's score). Therefore, we have that the linear iso-score function that equal the second highest score will be tangent to the yardstick cost $C^{\text{DEA}-i}(.)$. It follows from this that the second score auction generate at least the same welfare as the DEA based yardstick auction. This is illustrated in Figure 6.

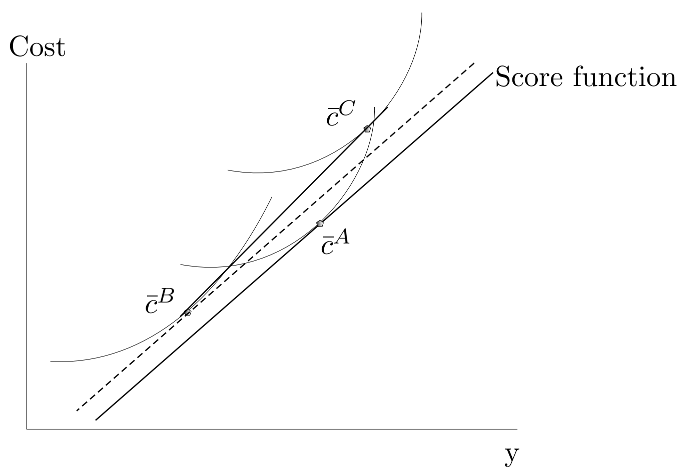

Figure 6: *Second score auction weakly dominate the DEA based yardstick auction if $V(.)$ is linear*

In Figure 6 $\bar{c}^A$ is the winner and $\bar{c}^B$ and $\bar{c}^C$ are two other bidders reservation values. It is easy to see from Figure 6 that the welfare generated from a second score auction will be higher than that of a DEA based yardstick auction, simply because the second highest iso-score function is tangent to any feasible compensation generated by the DEA based yardstick auction.

4.4 Uncertain welfare function

In all of the models above, the welfare function is assumed to be common knowledge and the score function is set equal to the welfare function. In this section we relax this assumption.

Again, if we look at the second score auction the winner is selected and the actual production plan and compensation are all determined by the score function. Therefore if the score function differ from the true welfare there will be a direct effect on efficient trade and on the actual welfare generated.

In the DEA based yardstick auction the winning bid is selected by the most profitable bid in the neighborhood of the second highest iso-score curve. This trade-off between maximizing welfare and efficiency (relative to the other bidders' performance) is especially relevant in case of uncertain welfare. In the DEA based yardstick auction the score function select the winner and select a set of bids. Therefore, a slightly wrong score function may not have any direct effects on the actual welfare generated.

In terms of allocative efficiency both types of auctions may fail in case of uncertain welfare. This follow simply from the fact that both auctions apply the same selection mechanism.

Changing the score function ex-post

Beil (2001) suggest an auction process where the bidders' true cost functions are revealed and the optimal score function determine afterwards. He suggest that the second score auction is applied successively with different score functions. After several rounds the most part of the true cost functions are revealed and the optimal scoring rule is determined. He assume myopic best response in each auction. This assumption is clearly unrealistic in repeated auctions. We suggest that a slightly modified version of the DEA based yardstick auction may allow the principal to select any score function ex-post, and still have complete revelation as the optimal bidding strategy.

Consider a DEA based yardstick auction where the principal selects the most profitable bid among the winning bidders efficient bids. In this case the optimal bidding strategy would be to submit the true cost function no matter which score function the principal may chose ex-post. To see this note that the set of efficient bids (bids where $C^{\mathrm{DEA}-i}(y^i, z^i) \geq c^i(y^i, z^i)$) is only determined by the other bidders bids. Also, since the principal commits to select the winner's most profitable bids deviating from truth telling is not optimal under any score function.

This type of auction is especially relevant in case of large uncertainty about the welfare and where the principal expects a large number of bidders and hereby a good representation of the underlying cost structure. This might very well be the case in natural resource management. On the other hand, if the principal expects only a few participants and thereby a relatively weak representation of the underlying cost structure, a better choice may be the DEA based yardstick auction suggested in model 1 and 2.

5 Applications in Natural Resource Management

Above, we have only refereed to a general example of compensating an agent for providing environmental goods. In natural resource management there are examples where only a single winner is needed, and others where a larger number of agents are required to provide environmental benefits. In this section we list examples where selecting a single winner makes sense and consider the problems involved in selecting more winners.

Clearly, regulating the farmers to lower the use of nitrogen or pesticides require the compensation (or taxes) to be distributed among a large number of agents. Though, there are a number of possible procurement situations that only require a single winner. Also, many situations that require more winners might be thought of as selecting a number of single winners in local areas. Below some applications for the *single winner* DEA based yardstick auction are given.

- Selecting location for a national park.
- Selecting location for protecting threatened species.
- Selecting drilling area for freshwater drilling (require extensive farming).
- etc.

Multiple winners

Now, consider the case of e.g. lowering the use of nitrogen by compensating the farmers. Here the problem is typically to maximize the environmental benefits subject to a certain budget. The DEA based yardstick auction might be altered to cover this situation of selecting any number of winners. In general this may be done in two ways, either by a discriminatory DEA based yardstick auction or a uniform DEA based yardstick auction.

In a discriminatory version of the DEA based yardstick auction, the principal simply repeat the case of a *single winner* until the budget is fulfilled. Clearly, this violates the truth revealing property. E.g. if the bidder with the potential highest score submit his true cost function he might risk getting a smaller compensation than a lower scoring bidder. This is easy to see, since the compensation might be determined by bidders who also win the auction. Therefore, better informed bidders might gain by lowering their bids. Properties of this auction is left for future research.

In a uniform version all winners will be compensated with respect to the same set of references - the same frontier. One solution could be to let the N winners chose a benchmark on the frontier expanded by the remaining bidders, $C^{DEA-N}(.)$. The properties of such an auction and whether such an auction would violate the truth revealing property is also left for future research.

6 Concluding remarks

The discrepancy between practical ad hoc procedures used in multiple dimensional procurements and the simplified, usually single dimensional theoretical models, is striking. The DEA yardstick auction treated in this paper is an attempt to solve real problems using sound theory.

Natural resource management problems often involve complex production structures, with joint production of multiple products simultaneously. Moreover, there are non-trivial inputs to the production process that cannot be controlled. There are also non-trivial elements of asymmetric information about the conditions and preferences of the different landowners. Earlier research has proved that DEA is useful in evaluating these rather complex situations that usually involve a large number of agents.

We showed how the suggested DEA based yardstick auction facilitate complete revelation of the agents cost functions. If the bidders cost function belong to the same underlying cost structure, as assumed in the literature on DEA based yardstick schemes, allocative efficiency is ensure in all cases. If we allow for more flexible cost functions, some inefficient bids are rationed away. This may cause allocative inefficiency in extreme cases. In general, however, the complete revelation provides a close fit to the true underlying cost structure and thereby reduces the information rent and provides more welfare.

The suggested mechanism select the most efficient bid in the neighborhood of the most preferred bid. Unlike the traditional multidimensional score auction the compensation is determine independently of the principal's preferences. Hereby we show that the DEA based yardstick auction may provide a higher welfare to the principal is some cases. This is also important in case of uncertain welfare, since efficiency is prioritized to uncertain welfare maximization.

Finally, we suggested a DEA based yardstick auction that allow the principal to change the score function ex-post and still provide complete revelation. In this auction the principal commits to select the winner's most profitable bid. This auction might be relevant in natural resource management with a potential large number of bidders and uncertain welfare measures.

There are several, relevant extensions of the research reported here. Extending the auction to select more winners is important for some applications in natural resource management. Another interesting extension would be to test the DEA based yardstick auction in a laboratory setting.

References

Agrell, P., Tind, J. and Bogetoft, P.: 2002, Incentive plans for productive efficiency, innovation and learning, *Int.Journal of Production Economics, 78, 1-11, 2002b.* **78**, 1–11.

Armstrong, M.: 1996, Multiproduct nonlinear pricing, *Econometrica* **64**(1), 51–75.

Ausubel, L. M. and Cramton, P.: 1999, The optimality of being efficient. University of Maryland.

Babcock, B., Lakshminarayan, P., Wu, J. and Zilberman, D.: 1997, Tergeting tools for the purchase of environmental amenities, *Land Economics* **73**(3), 325–339.

Banker, R.: 1980, A game theoretic approach to measuring efficiency, *European Journal of Operational Research* pp. 262–268.

Banker, R., A.Charnes, W.W.Cooper and R.Clarke: 1989, Constrained game formulations and interpretations for data envelopment analysis, *European Journal of Operational Research* pp. 299–308.

Beil, D. R.: 2001, An inverse-optimization-based auction mechanism, MIT working paper. Paper 143.

Bogetoft, P.: 1990, *Strategic Responses to DEA-Control - A Game Theoretical Analysis*, DASY, Copenhagen Buisness School, Copenhagen School of Economics and Buisness Administration, pp. 1–38.

Bogetoft, P.: 1994, Incentive efficient production frontiers: An agency perspective in dea, *Management Science* **40**, 959–968.

Bogetoft, P.: 1997, Dea-based yardstick competition: The optimality of best practice regulation, *Annals of Operations Research* **73**, 277–298.

Bogetoft, P.: 2000, Dea and activity planning under asymmetric infromation, *Management Science* **13**, 7–48.

Branco, F.: 1997, The design of multidimensional auctions, *RAND Journal of Economics* **28**(1), 63–81.

Bushnell, J. B. and Oren, S. S.: 1995, Internal auctions for the efficient sourcing of intermediate products, *Journal of Operations Management* **12**, 310–320.

Charnes, A., Cooper, W. W. and Rhodes, E.: 1978, Measuring the efficiency of decision making units, *European Journal of Operational Research* **2**, 429–444.

Charnes, A., Cooper, W. W. and Rhodes, E.: 1979, Short communication: Measuring the efficiency of decision making units, *European Journal of Operational Research* **3**, 339.

Che, Y.-K.: 1993, Design competition through multidimensional auctions, *RAND Journal of Economics* **24**(4).

Cripps, M. and Ireland, N.: 1994, The design of auctions nd tenders with quality thresholds: The symmetric case, *The Economic Journal* **423**.

Dalen, D. M. and Gomez-Lobo, A.: 1997, Estimating cost functions in regulated industries under asymmetric information, *European Economic Review* **31**, 935–942.

Galapo, S.: 1999, Report of the round-table conference on the auctioning of frequencies for broadcasting. Institute for Information Law, Amsterdam.

Genty, L.: 1999, Auctions and comparative hearings: Two ways to attribute spectrum licences, *Communications & Strategies* **35**(3), 11–43.

Holmstrom, B.: 1982, Moral hazard in teams, *Bell Journal of Economics* **13**, 324–340.

Koppius, O. R. and Heck, E. V.: 2002, Information architecture and electronic market performance in multidimensional auctions, Rotterdam School of Management, Erasmus University Rotterdam. Working paper.

McAdams, D.: 2002, Monotone equiilibrium in multi-unit auctions, *Working paper 4254-02*, MIT Sloan School of Management.

Milgrom, P.: 2000, An economist's vision of the b-to-b marketplace, an executive white paper, Perfect.com.

Milgrom, P. and Roberts, J.: 1990, Rationalizability, learning, and equilibrium in games with strategic complementarities, *Econometrica* **58**(6), 12555–1277.

Myerson, R. B.: 1981, Optimal auction design, *Mathematics of Operations Research* **6**(1), 58–73.

Olesen, H. B.: 2002, Contract production of peas, *In Press, Food Policy* .

Resende, M.: 2001, Relative efficiency measurement and prospects for yardstick competition in brazilian electricity distribution, *In Press, Energy Policy* .

Riley, J. G. and Samuelson, W. F.: 1981, Optimal auctions, *American Journal of Economics* **71**(3), 184–194.

Shleifer, A.: 1985, A theory of yardstick competition, *Rand Journal of Economics* **16**, 319–327.

Thiel, S. E.: 1988, Multidimensional auctions, *Economics Letters* **28**, 37–40.

Vickrey, W.: 1961, Counterspeculation, auctions, and competitive sealed tenders, *The Journal of Finance* **16**, 8–37.

Vukina, T., Levy, A. and Marra, M.: 2000, Auctions with environmental criteria: Evidence from the conservation reserve program. Working paper.

Zheng, C. Z.: 2000, Optimal auction in a multidimensional world, *Working paper*, Department of economics, Northwestern University.