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TESTING FOR WEAK SEPARABILITY IN FOOD DEMAND

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Abstract

Recognizing that parametric tests of separability are not invariant to functional form, the study employed two different demand systems--the AIDS and Rotterdam model--to conduct a test of weak separability for Canadian food demand. In general, the two models generated different results concerning separability. Test results conditional on the AIDS model suggest that the meats group and also the fats and oils group were not separable from the other food groups examined, while fruits and vegetables were considered separable from beverages, cheese, eggs, cereals, and sugar and syrups. Beverages, in turn, were considered separable only from fruits and vegetables, and cereals. Regarding group separability, test results derived from the Rotterdam model coincided with those of the AIDS model only in the case of fruits and vegetables being separable from beverages, cheese, eggs, cereals, and sugar and syrups. These contrasting results from the two models point out the value of considering alternative specifications when conducting parametric tests of weak separability. Also, these results suggest the need to employ non-parametric tests alongside parametric tests to guide applied work in demand analysis.

Testing for Weak Separability in Food Demand

By allowing individual preferences to be represented by utility functions, the neoclassical theory of consumer behaviour provides a convenient framework for analyzing consumer behaviour. From the maximization of the utility function, subject to income constraints, a complete set of demand equations can be obtained. These demand equations are a realization of the choices of the consumer, and are functions of income and all commodity prices. However, even after the restrictions implied by consumer theory (adding-up, homogeneity and symmetry) are imposed, data limitations often preclude the estimation of the complete set of demand equations that correspond to the choice set of the consumer. Aggregating individual goods into broad commodity categories reduces the estimation problem to manageable proportions, but, since many policy issues dictate economic analysis at a disaggregated level, this approach is often infeasible.

The notion of separability, conceived independently by Leontief (1947) and Sono (1961), has provided researchers with a systematic approach for circumventing the above estimation problem. Separability of preferences allows partitioning of the commodity space into groups such that the consumer's preference ordering of those commodities included in a group are independent or separable from quantities of other commodities. Separability of preferences, however, places restrictions on the preference structure of the consumer. If these restrictions are inconsistent with the true preference ordering of the consumer, resulting parameter estimates of demand equations are invalid.

Both parametric and non-parametric tests have been employed to test for separability of preferences. Non-parametric tests, as developed by Afriat (1967) and Varian (1983), have the desirable property of not being conditional on the functional form of the utility function. But these tests are nonstochastic, and, given time series data, implicitly require that preferences are strongly separable overtime (Swofford and Whitney, 1987). Parametric tests, on the other hand, have the disadvantage of being conditional on the functional form of the utility function. Most analysts who have employed parametric procedures to test for separability of preferences (e.g., Eales and Unnevehr, 1988; Pudney 1981; Jorgenson and Lau, 1975), however, have failed to address or account for the fact that test results are not invariant to the functional form chosen.

As far as the authors are aware, previous studies that have estimated food sub-demand systems for Canada have not conducted statistical tests to guide the grouping of food commodities into demand systems. Accordingly, this study intends to conduct parametric tests of weak separability in Canadian food demand. Moreover, recognizing that parametric tests of separability of preferences are conditional on functional form, two different demand specifications--the AIDS and Rotterdam model--will be employed to conduct tests of weak separability. A comparison of results derived from these two specifications may provide an indication of the robustness of test results to functional form.

Methodology

Weak separability applies when the marginal rate of substitution between any two goods in the same group is independent of quantities consumed outside the group. Weak separability is an important concept in empirical work because it is a necessary and sufficient condition for two stage budgeting (Deaton and Muellbauer 1980). Two stage budgeting assumes that, in response to the complexity involved in allocating total expenditures among many individual commodities, the consumer allocates total expenditures in two stages. First, total expenditures are allocated between broad commodity groupings, and next, each group expenditure is allocated among individual commodities within the group. This allocation scheme implies that the demand for each good appearing in a given group can be expressed as a function of total group expenditure and within group prices alone. This result, in addition to allowing the researcher to narrow the focus of study, reduces data requirements and conserves degrees of freedom.

The restrictions weak separability places on consumer behaviour can be characterized by the Slutsky substitution terms. Goldman and Uzawa (1964) show that a utility function is weakly separable if, and only if, the slutsky substitution terms π_{ik} can be expressed as

$$\pi_{ik} = \theta_{gh} \frac{\partial q_i}{\partial x} \frac{\partial q_k}{\partial x} \quad \text{for all } i \in G,\ k \in H,\ G \neq H, \qquad (1)$$

where θ_{gh} is a factor of proportionality, and G and H are separable commodity groupings. Equation 1 suggests that while weak separability places no restrictions on substitution between goods in the same group, substitution between goods in different groups occurs only through group expenditures and a factor of proportionality which characterizes the intergroup relationship.

From equation 1 it follows that

$$\pi_{ik} \frac{\partial q_j}{\partial x} \frac{\partial q_k}{\partial x} = \theta_{gh} = \pi_{jk} \frac{\partial q_i}{\partial x} \frac{\partial q_k}{\partial x} \quad \text{for all } i,j \in G,\ k \in H,\ G \neq H \qquad (2)$$

Thus a test of the hypothesis that the commodity group G is weakly separable from the H grouping can be constructed as

$$\pi_{ik}\, e_j - \pi_{jk}\, e_i = 0\ , \text{ where } e_i = \frac{\partial q_i}{\partial x} \text{ and } e_j = \frac{\partial q_j}{\partial x} \qquad (3)$$

The AIDS and Rotterdam models are considered for conducting weak separability tests in Canadian food demand. The AIDS model of Deaton and Muellbauer (1980) approximates an arbitrary expenditure function, while the

Rotterdam model developed by Barten (1964) and Theil (1965) represents a first order approximation to the demand function.

AIDS Model. The Aids model in linear form is specified as

$$w_i = \alpha_i + \Sigma_j \delta_{ij} \ln p_j + \beta_i \ln(x/P) \tag{4}$$

where w_i is the budget share of the ith good, p_j is the nominal price of the jth good, x is defined previously, and $\ln P = \Sigma_i w_i \ln p_i$ (Stones price index). The theoretical restrictions of adding-up, homogeneity, and symmetry, implied by demand theory are satisfied by the following parametric restrictions (on the AIDS model).

Adding-up: $\Sigma_i \alpha_i = 1$, $\Sigma_i \beta_i = \Sigma_i \delta_{ij} = 0$

Homogeneity: $\Sigma_i \delta_{ij} = 0$

Symmetry: $\delta_{ij} = \delta_{ji}$

For the AIDS model (4) $\pi_{ik} = \delta_{ik} + w_i w_k$ $(i \neq j)$, and $e_i = (\beta_i + w_i)/p_i$; Thus the weak separability restrictions specified in equation (3) take the following specific form

$$(\delta_{ik} + w_i w_k)(\beta_j + w_j)/p_j - (\delta_{jk} + w_j w_k)(\beta_i + w_i)/p_i = 0 \tag{5}$$

Rotterdam Model. Theil's estimable version of the Rotterdam model is given as

$$\bar{w}_i D(q_i) = \theta_i \Sigma_i \bar{w}_i D(q_i) + \Sigma_j \pi_{ij} D(p_j) + \mu_i \tag{6}$$

where D is the log change operator i.e., $D(z_t)=\ln(z_t/z_{t-1})$, $w_i = (w_{it} + w_{it-1})/2$, μ_i is a disturbance term, θ_i is the marginal budget share of the ith good, and π_{ij} are the Slutsky substitution terms. Both θ_i and π_{ij} are assumed to be constant over the sample. The adding-up condition implies

$$\Sigma_i \theta_i = 1, \ \Sigma_i \pi_{ij} = 0$$

Homogeneity is established by imposing the restrictions

$$\Sigma_j \pi_{ij} = 0$$

and Symmetry by the restrictions

$$\pi_{ij} = \pi_{ji}$$

In accordance with equation (3) a test of weak separability for the Rotterdam model can be constructed as

$$\pi_{ik} \; \theta_j / p_j \; - \; \pi_{jk} \; \theta_i / p_i = 0 \qquad (7)$$

where $\theta_i / p_i = e_i$

Test Procedure. The test procedure employed to perform the separability tests is illustrated in Figure 1. The utility tree depicted in panel A of figure 1, assumes that food demand is separable into a meats group, a fats and oils group and an aggregate group comprising of all other foods. This configuration facilitates the testing of the hypothesis that the meats group is weakly separable from the fats and oils group. Similar separability assumptions between meat and other food groupings are made in Panels B through D. Thus based on each panel a test of weak separability between meats and the respective food groups can be constructed. Although Figure 1 refers to separability between meats and the other food groups, a similar scheme can be employed to conduct weak separability tests between any given food group and the remaining food groups.

Data

The study employed annual data from 1960 to 1987. Quantity, price and price index data were obtained from various issues of the Handbook of Food expenditures, Prices and Consumption, published by Agriculture Canada.

Obtaining unit price data that span the time period under study proved illusive, therefore, recognizing that the price index data obtained had 1981 as base year, 1981 price data were employed to convert the price index data series into price data. The conversion for a given series was obtained by dividing the price index series by the 1981 price index datum of that series and then multiplying the result by the 1981 price. For example, the beef price index series was divided by, 100, the 1981 beef price index datum, and then the result was multiplied by, $6.28, the 1981 price per kilogram of beef. The 1981 beef and pork prices were weighted average prices of the various cuts of meat that comprise each carcass. Similarly, weighted average prices and price indices were used to represent group prices and price indices.

Price indexes and food expenditures associated with the "other foods" groups (see figure 1, for example) were derived, respectively, from the food CPI and total food expenditures. For example, the CPI for the "other foods" (foods other than meats, and fats and oils) category in panel A of Figure 1 was obtained by subtracting from the food CPI the CPI associated with the meats and fats and oils groups weighted by their respective food expenditure weights (0.3298, 0.0286) and then multiplying the result by the inverse of the food expenditure weight (0.6496) associated with the "other foods" group. Expenditures on the "other foods" group were obtained by multiplying total food expenditures by the "other foods" group food expenditure weight. The various food expenditure weights employed were weights averaged over the 1969-1986 period, and were obtained from the Handbook of Food expenditures, Prices and Consumption. Since the Rotterdam specification requires quantity values, these "other foods" expenditures were deflated by the general CPI to obtain quantity indexes.

Estimation and Test Results

Maximum likelihood estimates of the Aids and Rotterdam models, specified, respectively, in equations (4) and (6) with the homogeneity and symmetry restrictions imposed, were obtained using the LSQ option in TSP. A total of ten different demand systems were estimated, each associated with an utility tree such as those depicted in Figure 1. Since the adding-up condition renders the covariance matrix singular, to facilitate estimation, the other foods equation in each demand system was dropped prior to estimation. Estimation results are not presented, however, they will be made available to the reader upon request.

Chi-square tests statistic for weak separability tests, as constructed in equations (5) and (7), along with associated degrees of freedom and critical values at the 0.05 probability level are given in Table 1. Based on the AIDS model the results suggest that the meats group is not weakly separable from any of the food groups, but among the fats and oils group meat is separable from salad oils, and among the fruits and vegetable and beverage groups meat is separable from vegetables and from coffee and tea. In sharp contrast, results from the Rotterdam model suggest that meat is separable from all of the food groups. However, on an individual item basis, butter, shortening, fluid milk, fruit juices and soft drinks are considered not separable from meat.

Although test results derived from the AIDS model indicate that the fats and oils group is not separable from the fruits and vegetables group, results suggest that the fats and oils group is separable from fruits and from vegetables. Just as in the case of meat, results based on the Rotterdam model contradict those from the AIDS model in that the fats and oils group is considered separable from all of the food groups.

With regard to the separability of fruits and vegetables from beverages, cheese, eggs, cereal and sugar and syrups, results from both the AIDS and Rotterdam model indicate that the fruit and vegetable group is separable from these foods. However, while results from the AIDS model suggest that beverages are separable from cereals but not from cheese, eggs and sugar and syrups, the Rotterdam model indicates that beverages are separable from all of those food items.

Summary

The study intended to examine the pattern of separability in Canadian food demand, and the robustness of weak separability test results to functional form. To those ends, the AIDS and Rotterdam model were employed to conduct weak separability test in Canadian food demand. In general, the two models generated different results concerning separability. Test results conditional on the AIDS model suggest that the meats group (beef, pork, poultry and fish) and also the fats and oils group (butter, margarine, shortening and salad oils) were not separable from the other food groups examined, while fruits and vegetables were considered separable from the beverage group (milk, fruit juice, coffee & tea and soft drinks), cheese, eggs, cereals, and sugar and syrups. Beverages, in turn, were considered separable only from fruits and vegetables, and cereals. Regarding group separability, test results derived from the Rotterdam model coincided with those of the AIDS model only in the case of fruits and vegetables being separable from beverages, cheese, eggs, cereals, and sugar and syrups. However, on an individual food item basis, there were 20 cases where test results from the two models coincided.

The contrasting results from the two models points out the value of considering alternative specifications when conducting parametric test of weak separability. This study is but a first step in establishing the nature of separability in Canadian food demand. In light of the conflicting results provided by the two demand specifications, further research, probably along the lines of non-parametric methods, is needed to guide applied work in demand analysis.

References

Afriat, S. 1967. "The Construction of Separable Utility Functions from Expenditure Data." Mimeo, Purdue.

Barten, A.P. 1964. "Consumer Demand Functions Under Conditions of Almost Additive Preferences." Econometrica 32: 1-38.

Christensen, L.R., Jorgenson, D.W., Lau, L.J. 1975. "Transcendental Logarithmic Utility Functions." American Economic Review 65:367-82.

Deaton, A.S., and Muellbauer, J. 1980a. "An Almost Ideal Demand System." American Economic Review 70: 312-26.

__________________. 1980b. "Economics and Consumer Behaviour." New York: Cambridge University Press.

Eales, J.S., and Unnevehr, L. 1988. "Demand for Beef and Chicken: Separability and Structural Change." Amer. J. Agri. Econ. 70: 521-32.

Goldman, S., and Uzawa, H. 1964. "A Note on Separability in Demand Analysis." Econometrica 32: 387-98.

Jorgenson, D.W., and Lau, L. 1975. "The Structure of Consumer Preferences." Ann. Econ. and Soc. Measurement 4: 49-101.

Leontief, W.W. 1947. "Introduction to the Theory of the Internal Structure of Functional Relationships." Econometrica 15: 361-373.

Pudney, S. "An Empirical Method of Approximating the Separable Structure of Consumer Preferences." Review of Economic Studies 48: 561-77.

Sono, M. "The Effect of Price Changes on the Demand and Supply of Separable Goods." International Economic Review 2: 239-271.

Swofford, J.L., and Whitney, G.A. 1987. "Nonparametric Tests of Utility Maximization and Weak Separability for Consumption, Leisure and Money." The Review of Economics and Statistics 69: 458-64.

Theil, H. 1965. "The Information Approach to Demand Analysis." Econometrica 33: 67-87.

Varian, H.R. 1983. "Non-parametric Tests of Consumer Behaviour." Review of Economic Studies 50 :99-110.

Table 1. Testing for Weak Separability in Canadian Food Demand

Hypothesis	Degrees of Freedom	Critical Values	Test Statistic AIDS	Test Statistic Rotterdam
		(0.05)		
Meats Separable from:				
Fats & Oils	12	21.03	101.51	17.02
Butter	3	7.89	68.34	8.46
Margarine	3	7.89	26.89	2.21
Shortening	3	7.89	19.70	11.66
Salad Oils	3	7.89	5.51	3.12
Fruits and Vegetables	6	12.59	16.20	3.82
Fruits	3	7.89	13.87	0.38
Vegetables	3	7.89	2.27	3.60
Beverages	12	21.03	162.27	14.80
Milk	3	7.89	29.71	8.02
Fruit Juice	3	7.89	46.77	7.91
Coffee & Tea	3	7.89	2.52	2.27
Soft Drinks	3	7.89	88.27	13.01
Cheese	3	7.89	31.53	4.01
Eggs	3	7.89	22.30	0.74
Cereals	3	7.89	27.89	2.81
Sugar and Syrups	3	7.89	20.97	6.77
Fats and Oils Separable from:				
Fruits and Vegetables	6	12.59	17.62	3.70
Fruits	3	7.89	6.34	2.97
Vegetables	3	7.89	4.19	0.58
Beverages	12	21.03	49.04	13.81
Milk	3	7.89	15.30	4.66
Fruit Juice	3	7.89	21.58	10.54
Coffee & Tea	3	7.89	14.92	4.69
Soft Drinks	3	7.89	7.58	0.68
Cheese	3	7.89	8.30	2.07
Eggs	3	7.89	26.46	4.87
Cereals	3	7.89	9.44	3.31
Sugar and Syrups	3	7.89	11.66	4.28
Fruits and Vegetables Separable from:				
Beverages	4	9.49	9.27	1.58
Milk	1	3.84	0.29	0.03
Fruit Juice	1	3.84	0.37	0.37
Coffee & Tea	1	3.84	1.78	0.003
Soft Drinks	1	3.84	4.92	0.99
Cheese	1	3.84	0.39	1.90
Eggs	1	3.84	3.15	1.12
Cereals	1	3.84	0.25	2.00
Sugar and Syrups	1	3.84	1.44	0.49
Beverages Separable from:				
Cheese	3	7.89	9.08	6.95
Eggs	3	7.89	14.99	2.86
Cereals	3	7.89	2.30	1.66
Sugar and Syrups	3	7.89	8.03	3.70