Optimal Forest Fire Management with Applications to Florida

Ibtisam Al Abri\textsuperscript{a}, Kelly Grogan\textsuperscript{a}, Adam Daigneault\textsuperscript{b}

\textsuperscript{a} University of Florida, \textsuperscript{b}University of Maine

Selected Paper prepared for presentation at the Agricultural & Applied Economics Association’s 2017 AAEA Annual Meeting, Chicago, Illinois, July 30\textsuperscript{th} – August 1\textsuperscript{st}, 2017

Copyright 2017 by [Al Abri, Grogan, Daigneault]. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided that this copyright notice appears on all such copies.
ABSTRACT. This study develops a stochastic dynamic model to optimize net present value from timber for a landowner who integrates wildfire risk and different processes of fuel accumulation into his fuel prevention decisions. The derived model is capable of determining optimal thinning frequencies, timing, and level simultaneously and as a function of fire risk and fuel biomass dynamics. Numerical analysis reveals that the landowner’s prevention decision depends closely on the type of fuel biomass growth in his site and the association between fire arrival rate and fuel accumulation. The complexity of the landowner’s problem in the presence of endogenous fire arrival and fuel accumulation risks implies that government programs could be utilized to improve landowner’s awareness and responses to these risks.

I. INTRODUCTION

Florida’s landscape is one that has evolved with a natural fire cycle. However, wildfires often threaten valuable timberland and human property. Wildfires burned a total of 158,579 acres just during the first quarter of 2017, compared to 71,620 acres in 2016 (FDACS, 2017). Lightning is a common cause of wildfires, and Florida ranks second in the U.S. for annual lighting strikes and first for lightning density (NOAA, 2016). Regular fire incidents in Florida increases forest lands vulnerability. Florida’s timberland covered almost 16 million acres of total forested land. The industry contributed $16.09 billion to the state economy and created more than almost 77,000 jobs (FFS, 2016). Florida timber production predominantly utilizes pine species, including the longleaf pine, sand pine, slash pine, and loblolly pine. Combining the high flammability of dead pine needles with frequent lightning strikes and an environment that evolved around regular wildfires creates a high-risk situation for timber producers.

However, forest managers can better protect their land by understanding the behavior of fire in their regions. Two main factors play a significant role in increase the power and size of wildfires in Florida: weather and fuels (Monroe and Marynowski, 1999). Harsh weather conditions like high temperatures, gusty winds, and low humidity stimulate the spread of fire and result in a damaging wildfire. Additionally, accumulated surface fuels are a major factor that increases the spread rate of fire. The common fuels in Florida’s pine forests are dead materials (e.g. dried grasses, weeds, and pine needles) and living vegetation (e.g. palmettos, shrubs,
grasses, plants and small trees). If surface-level fuels are left unmanaged, accumulation of flammable dead and dying trees boost the rapid spread of fire and destroy whole series of valuable pine forests. If a fire starts on land with high fuel loads, this fire is more likely to also spread to neighboring stands (Monroe and Marynowski, 1999). The devastating wildfires of 1998 were hard to manage due to the heavy accumulation of coarse woody debris and “ladder” fuels that allowed the fire to reach the tree canopy. Therefore, the type, size, and density of surface-level fuels determine the resulting damages from forest fires (Monroe and Long, 2001).

Silvicultural management activities can be used to mitigate damages from forest. Thinning is one common management activity, where landowners remove some of the trees partway through the stand’s rotation period. These trees can often be sold to sawtimber markets, and thinning can increase yields of the remaining trees by reducing competition for resources (Brose and Wade, 2002; Miller, Clendenen, and Bruce 1987; Haynes 2003). Prescribed burning is a second common option which reduces the amount of fuel present, by burning the underbrush (Brose and Wade, 2002). While preventative measures can be worthwhile from an individual landowner’s perspective, these measures can also be worthwhile from a social perspective. The U.S. government regularly spends more than $3 billion on fire suppression and prevention each year (Gorte, 2013).

This study utilizes a stochastic dynamic model an individual landowner’s timber stand. The model maximizes the landowner’s net present value by choosing the optimal thinning frequencies, and timing and degree of thinning as a function of fire risk and fuel biomass accumulation. We also derive the optimal rotation length under these circumstances. Unlike previous work in this area, this study also integrates different processes of fuel accumulation (e.g. exponential or logistic accumulation) in the modeling. In reality, private landowners may have inaccurate perceptions of risk or may lack the tools to determine the optimal fuel removal frequency and/or effort level across an entire rotation. This study seeks to improve landowner knowledge about optimal practices. The model reveals the landowner’s intra-rotation management decisions and demonstrates that the kind of fuel accumulation process (e.g. exponential or logistic accumulation) critically influences the landowner’s optimal decisions. Previous work that does not consider the accumulation process cannot provide management
options under a wide range of fuel accumulation processes that occur across under different timber and ecological systems.

Previous work has studied optimal rotation length in southeastern U.S. forests and researchers have extended the work to cover the optimal rotation lengths for forest in the presence of fire risk (e.g. Reed 1984 and 1987; Englin, Boxall, and Hauer 2000; Alvarez and Koskela, 2006; Clark and Reed 1989; Sims 2011). Few studies have considered optimizing within rotation forest management activities (e.g. Amacher, Malik, and Haight, 2005; Daigneault, Miranda, and Sohngen, 2010; Stainback and Alavalapati, 2004). Stainback and Alavalapati (2004) examine the impact of wildfire on slash pine rotation lengths for landowners who receive carbon payments; however, these payments need to be repaid in the event of a forest fire. In terms of the amount of destruction in the event of a fire, they consider two severity options: 70% and 100% destruction of the stand, which are independent of landowner actions or stand characteristics. Amacher, Malik, and Haight (2005) studied fire prevention practices in the management of loblolly pine species. The model allows the landowners to decide on the level of fire prevention and the age at which the fire prevention occurs. However, the model does not allow the landowners to undertake fire prevention more than once during the rotation. Unlike Amacher, Malik, and Haight (2005), the model utilized by Daigneault, Miranda, and Sohngen (2010) does not restrict the number and timing of fire prevention practices. However, their model does not allow the landowners to decide on the level and efforts of management activities. Busby, Albers, and Montgomery (2012) studied the risk associated with forest fire in a landscape with fragmented ownership and spatial interactions. Fuel removal is a binary variable and the landowner makes the decision to undertake removal or not in each time period. They represent the impact of fire using a damage function with increasing, decreasing, or constant returns to scale with respect to fuel removal, and model fuel accumulation as a linear function.

This study contributes to the existing literature by combining the models of Amacher, Malik, and Haight (2005), Daigneault, Miranda, and Sohngen (2010), and Busby, Albers, and Montgomery (2012) in order to allow the landowners to choose the level of fire prevention practices as well as the number and timing of these practices during the rotation. In addition, this study is the first to integrate various patterns of fuel accumulation, which previous models largely ignore or simplify. The model also contributes to the literature through its use of the fuel
stock in both the salvage value and the probability of fire arrival. A stand with a higher amount of flammable fuel is more likely to ignite from a lightning strike, and increased fuel stock is likely to increase the size of the fire and its resulting damages (Van Wagtendonk 1996; Brose and Wade 2002; Outcalt and Wade 2004, Crowley et al. 2009).

II. MODEL FORMULATION FOR A TIMBER-STAND OWNER PROBLEM

A landowner who manages his stand in a region that is highly prone to wildfire like in Florida can mitigate the fire risk by undertaking fuel removal practices such as thinning stands and by using prescribed burning to reduce the amount of fuel present for the wildfire. Therefore, a landowner has to make three nested decisions each period: whether to clearcut the stand or wait, whether to undertake fire prevention if waiting to harvest the timber, and the level of fuel removal if undertaking fire prevention. The landowner decision process is modeled as an infinite horizon, discrete time, stochastic dynamic optimization model. Then, the model is numerically solved to grant the landowner more flexibility in making simultaneous decisions regarding level, number, and time of fuel treatment practices based on his site.

Timber Revenue Maximization and Endogenous Fire Arrival Rate

This study includes two continuous state variables and a binary state variable. The first continuous state variable is stand biomass $s_t \in [s_{min}, s_{max}]$ measured in cubic meters per hectare at the beginning of each period. The index of fuel biomass in the site $f_t \in [f_{min}, f_{max}]$ is the second continuous state variable, defined at the beginning of each period. The random binary state variable represents wildfire occurrence $\theta_t \in [0,1]$ during each period.

For the landowner to protect his investment in valuable timber against fire damage, the model assumes that the timber grower decides whether to harvest or wait another period; $\alpha_t$ is a binary action variable with $\alpha_t=1$ indicating harvest and $\alpha_t=0$ indicating at least one period to harvest. In the case of harvesting, $\alpha_t=1$, a forest manager sells the timber and replants immediately. Simultaneously, the timber manager is assumed to undertake some fuel prevention activities if waiting to harvest the timber. A continuous action variable $x_t \in [0, f_t]$ indicates that
a landowner chooses the level of treatment range from doing nothing \((x_t = 0)\) to completely cleaning the site by removing the maximum fuel load that exists in that period \((x_t = f_t)\). These practices would enhance merchantable timber growing and increase the proportion of salvaged timber in case of fire. If the stand ignites \((\theta_t = 1)\) in any period, the landowner harvests the salvageable timber. The salvageable percentage of timber is inversely related to the fuel stock at the time of the fire.

Consistent with the literature, fire arrival is characterized as a Poisson distribution; further, this study models the fire arrival rate as a function of fuel biomass with the average fire arrival rate that increasing in fuel accumulation. We assume the fuel stock linearly scales the normal Poisson arrival rate parameter, \(\lambda\), implying the following probabilities.

\[
\lambda_{f_t} = \begin{cases} 
1 - e^{-\lambda(f_t)} & \theta_t = 1 \quad \text{Fire Occurs} \\
 e^{-\lambda(f_t)} & \theta_t = 0 \quad \text{No Fire} 
\end{cases}
\]

[1]

**Fuel Accumulation**

Landowner management decisions \((a_t \text{ and } x_t)\) determine the stock and the change in the stock of fuel present on the site. The model assumes that every period the stand is harvested or salvaged, the forest manager cleans the site in preparation for replanting in which most of flammable surface-level fuels are removed and less-hazardous minimal fuels remains in the site\(^1\), \(f_0\). However, if the stand is thinned, the amount of fuel is reduced by the amount of fuel removal. The state of fuel on the stand evolves each period according to:

\[
f_{t+1} = \begin{cases} 
k(f_t) & a_t = 0, x_t = 0 \\
k(f_t) - x_t & a_t = 0, x_t \in (0, f_t] \\
f_0 & a_t = 1, Fire
\end{cases}
\]

[2]

Where \(f_0\) is the minimal fuels remaining after harvested or salvaged stand, and \(k(f_t)\) is the path of accumulated surface-level fuel in the site over time.

---

\(^1\) Consistent with Daigneault, Miranda, and Sohngen (2010) who assumes when a stand is thinned, harvested, or salvaged, the fuel stock is reset to an initial level such that the proportion of salvageable timber after fire is minimized.
Timber Growth and Management Decisions

Based on the biological timber growth function, the volume of merchantable timber, $s_t$, increases as the stand grows naturally over time according to $y(s_t)$ and with fuel removal actions as a function of fuel load according to $g(f_t)$, such that $\partial y(s)/\partial s > 0$ and $\partial g(f)/\partial f > 0$. Studies reveal that the volume of harvested from thinned stands is higher by about 15% compared to harvests from a non-thinned stand (Miller, Clendenen, and Bruce 1987; Haynes 2003). Given that the amount of fuel present in the site is influenced by either the management actions ($s_t$ and $x_t$) or wildfire ($\theta_t = 1$) each period, the timber biomass the following period is evolves according to:

$$
\begin{align*}
    s_{t+1} &= \begin{cases} 
    y(s_t)g(f_t) & a_t = 0, x_t = 0 \\
    y(s_t)g(f_t - x) & a_t = 0, x_t \in (0, f_t] \\
    s_0 & a_t = 1, Fire
    \end{cases} \\
\end{align*}
$$

Where $s_0$ is the initial level of timber replanted after a fire or planned harvest, and $y(s_t)$ is the annual growth of timber. The change in timber quantity during the period is affected by the volume of fuel in the stand, $g(f_t)$.

Timber Revenue

The forest manager receives a net rent from selling timber and intermediate fuel removal in a given period, $l(s, f, \theta, \alpha, x)$. The net rent depends on the volume of timber ($s_t$), the level of fuel biomass ($f_t$), wildfire incident ($\theta_t = 0, 1$), and the landowner’s management decisions ($\alpha_t$ and $x_t$) during the period. If a stand ignites ($\theta_t = 1$), the landowner receives the discounted salvageable timber value after the wildfire and re-establishes a new forest immediately. If a fire does not occur in a given period ($\theta_t = 0$), the stand owner gets rent or incurs costs that depends on the action taken:
\begin{align*}
l(s, f, \theta, a, x) &= \begin{cases} 
  h(f_t)\zeta p_t s_t - c_{h=1} & \theta_t = 1 \\
  -c_{per} & a_t = 0, x_t = 0 \\
  -c(x) & a_t = 0, x_t \in (0, f_t ] \\
  p_t s_t - c_{new} & a_t = 1
\end{cases} \tag{4}
\end{align*}

Where \( p_t \) is the stumpage price, \( \zeta \) is the fraction of the timber price that the market assigns to the ignited log, \( h(f_t) \) is the proportion of salvageable timber in the case of fire, and \( c_{h=1} \) is the cost of replanting after fire incident. \( c_{per} \), and \( c_{new} \) are the costs of periodically maintaining and replanting the stand after harvest, respectively. \( c(x) \) is the cost of intermediate treatment which depends on the amount of fuel removed each period. Studies have found that the yield benefits of increasing timber yield by thinning practices often do not outweigh the cost of thinning (Rummer et al. 2003; Barbour et al. 2004). The cost function used here increases as the level of intermediate action increases; higher treatment efforts costs more and nonindustrial landowners are solely responsible for these costs. This model assumes that thinning does not produce any commercial timber, but it could be modified to consider this case.

\textit{Stochastic Dynamic Model and Numerical Optimization}

For our landowner’s problem, the optimal management path can be solved with a single discrete-time, stochastic dynamic Bellman equation:

\begin{align*}
V\{s, f, \theta\} &= \max_{a, x} \\
& \begin{cases} 
  h(f_t)\zeta p_t s_t - c_{h=1} + \delta E_\theta V(s_0, f_0) & \theta_t = 1 \\
  -c_{per} + \delta E_\theta V(s_{t+1}, f_{t+1}) & a_t = 0, x_t = 0 \\
  -c(x) + \delta E_\theta V(s_{t+1}, f_{t+1}) & a_t = 0, x_t \in (0, f_t ] \\
  p_t s_t - c_{new} + \delta E_\theta V(s_0, f_0) & a_t = 1
\end{cases} \tag{5}
\end{align*}

where \( V\{s, f, \theta\} \) is the expression for the expected present bare-land value of a stand for an infinite sequence of future periods derived from applying the optimal policy path or “contingency plan” \( a_t^* \) and \( x_t^* \). The optimal value of the problem is constituted from the optimal expected rent in the first period “immediate reward” plus optimal expected rent from all future periods “expected future rewards”. The Bellman equation characterizes how a rational, future-regarding forest manager tradeoffs immediate rewards and expected future rewards in order to...
get the dynamically optimal revenue (Miranda and Fackler 2002). The risk of random wildfire is integrated in equation [5] by modeling the probability of fire arrival rate as an increasing function on fuel accumulation.

Equation [5] does not have a close-form solution due to the specification of functional equations and endogenous variables. Therefore, this study follows Miranda and Fackler (2002) and Judd (1998) and uses a collocation method as an approximation to solve the Bellman equation. The method of collocation requires finding an approximant to the value function by a linear combination of n known basis functions defined on the state space whose coefficients are fixed in order for the value function to satisfy the Bellman equation, not at all possible states, but rather at n judiciously chosen collocation nodes (Miranda and Fackler 2002). Checking the residuals from the selected collocation method is essential to confirm that errors are minimized across the entire domain of the value function (Miranda and Fackler 2002).

This study solves the Bellman equation by implementing cubic spline basis functions in MATLAB_R2017a using the dpsolve routine suggested by Miranda and Fackler (2002) COMPECON library. The result of the dynamic programming shows the maximum residuals were on the order of $10^{-11}$ times the value of the firm, specifying that numerical solution to the Bellman equation is accurate.

III. DATA SOURCES AND APPLICATION

The numerical analysis is implemented for loblolly pine (*pinus taeda*) in the state of Florida. Loblolly pine species is one of the most economically valuable pines in the world, and it is the single most significant forest species in the United States (Schultz, 1997; Worthington, 1954). The popularity of loblolly forests in the southern United States is due to its fast growth, relatively easy management, and its value for both commodity and non-commodity uses (Schultz, 1997). A large literature discusses the economic benefits of managing loblolly pine forests and related policy implications; with some focus on managing loblolly stand in the presence of fire risk.

Table 1 lists the functional forms and parameter values used in the numerical analysis for Loblolly Pine in Florida. The existing literature provides substantial information regarding
timber volume growth functions and associated parameters. The annual timber growth function used in this study was extracted from Chang (1984) and Amacher, Brazee, and Thompson (1991). Volume in boardfeet is based on age 25 site index of 80 feet. Another group of studies estimates the effect of thinning and intermediate practices of the growth rate and yield of loblolly pine stand (e.g. Sharma et al., 2006 and Baldwin et al., 2000). Sharma et al. (2006) have shown that the level of thinning applied by the landowner matters and influences the increase in annual growth and yield.

The price of stumpage is assumed constant and taken from Timber Mart-South as $80 per 1000 boardfeet (TMS, 2010). The periodic maintenance ($c_{per}$) of the stand is assumed to be $10, consistent with Daigneault, Miranda, and Sohngen (2010) and Bair and Alig (2006); and the replanting cost after planned harvest ($c_{new} =$ $171.36$) and after fire incident ($c_{fire} =$ $122.4$) are calculated based on Amacher, Malik, and Haight (2005) using the derived optimal planting density and Dubois et al. (2001). The cost of establishing new stands after wildfire is lower because soil on burned land requires less preparation (Waldrop 1997; Dubois et al. 2001). Intermediate treatment cost varies by the level of fuel removal. Consistent with Amacher, Malik, and Haight (2005), the cost function of intermediate actions $c(x)$ is assumed to be a linear function of fuel removal efforts.

The fraction of the salvageable timber ($h(f_i)$) as presented in Table 1 is a function of the surface fuel accumulation. The severe forest fire of 1998 in Florida reveals that the mortality rate of non-treated stands was more than twice that of treated stands on average (Outcalt and Wade 2004). In addition, silvicultural practices like prescribed burning have been proven to mitigate tree mortality rate in fire prone regions (Moore, Smith, and Little 1955; Cumming 1964; Outcalt and Wade 2004). The specification of the function is similar to that derived by Daigneault, Miranda, and Sohngen (2010). Furthermore, previous studies in this contest have largely ignored fuel accumulation dynamics. This study assumes three types of functional forms to demonstrate possible fuel accumulation patterns in Florida: exponential, logistic, and concave functions. Table 1 presents the specification of each function. The selected exponential function is based on Daigneault, Miranda, and Sohngen (2010), Brown, Reinhardt, and Kramer (2003), Omi and Martinson (2002), and Smith, Heath, and Jenkins (2003).
### TABLE 1
Optimal Management Model Specification for Loblolly Pine in Florida

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Function/symbol</th>
<th>Assumed form/value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\delta$</td>
<td>0.95</td>
</tr>
<tr>
<td>Annual timber growth</td>
<td>$y(s_t)g(f_t)$</td>
<td>( \frac{2}{1+f_t}(0.532s_t-1\times10^{-3}s_t^2+1\times10^{-6}s_t^3-6\times10^{-11}s_t^4) )</td>
</tr>
<tr>
<td>Stumpage price</td>
<td>$p_t$</td>
<td>$80</td>
</tr>
<tr>
<td>Periodic Maintenance cost</td>
<td>$c_{per}$</td>
<td>$10</td>
</tr>
<tr>
<td>Replanting cost after harvest</td>
<td>$c_{new}$</td>
<td>$171.36</td>
</tr>
<tr>
<td>Replanting cost after fire</td>
<td>$c_{\theta=1}$</td>
<td>$122.4</td>
</tr>
<tr>
<td>Fuel removal cost</td>
<td>$c(x)$</td>
<td>$c_0+c_3x(x=c_0=c_1=100,c_3=50)$</td>
</tr>
<tr>
<td>Salvageable timber function</td>
<td>$h(f_t)$</td>
<td>$1-f_t^{0.93}$</td>
</tr>
<tr>
<td>Fraction of stumpage price for salvage sales</td>
<td>$\zeta$</td>
<td>0.75</td>
</tr>
<tr>
<td>Exponential fuel accumulation function</td>
<td>$\lambda(f_t)_{exp}$</td>
<td>$15^{(f_t^{0.05})}$</td>
</tr>
<tr>
<td>Logistic fuel accumulation function</td>
<td>$\lambda(f_t)_{log}$</td>
<td>$15(1-0.93e^{-0.93f_t})$</td>
</tr>
<tr>
<td>Concave fuel accumulation function</td>
<td>$\lambda(f_t)_{con}$</td>
<td>$0.93(1-\frac{f_t}{f_{max}})$</td>
</tr>
</tbody>
</table>

### IV. RESULTS

Results suggest that the pattern of fuel accumulation substantially affects the optimal pattern of effort. Figure 1 shows stand value and stand age at harvest are $1,302.25/ha$ and 59 years, respectively. Table 2 and Figure 2 indicate that when fuel accumulation follows an exponential pattern, the landowner should control fuel accumulation starting earlier on in the rotation. In addition, the optimal pattern of fuel removal increases in level and frequency as the stand ages and the value of the trees increases.

The optimal policy path leads to a non-monotonic level of fuel removal effort as the stand ages and the value of the trees increases. Although the highest level of removal occurs at the middle of the rotation, the frequency increases at the end of the rotation indicating landowner’s awareness of fire risk against his valuable almost mature stands. Additional economic and risk...
parameters will be considered to determine if there are cases where effort should remain constant and to further explore the patterns of the timing of fuel removal effort over the length of the rotation.

**FIGURE 1**
Stand Biomass and Harvest Age

**FIGURE 2**
Fuel Accumulation Index and Fire Prevention Decisions
TABLE 2
Optimal Timing, Frequency, and Level of Fuel Removal Actions during One Rotation

<table>
<thead>
<tr>
<th>Removal</th>
<th>Early (t&lt;20)</th>
<th>Middle (21&lt;t&lt;40)</th>
<th>Late (t&gt;41)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timing</td>
<td>5,6,9,15</td>
<td>25,38,39</td>
<td>47,49,55,57,58</td>
</tr>
<tr>
<td>Frequency</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Level (relatively)</td>
<td>low</td>
<td>high</td>
<td>Low to medium</td>
</tr>
</tbody>
</table>

V. POLICY IMPLICATION AND CONCLUSION

This study formulates a stochastic dynamic model utilizing a Bellman equation to integrate fuel accumulation and endogenous arrival rate of fire into the process of determining the landowner’s optimal fuel management path to maximize net revenue. Unlike previous studies, the derived model is able to report the timing, frequency, and level of intermediate management actions simultaneously. Further, the timber landowner is expected to manage his stand based on his site quality and characteristics, such as surface fuel growth pattern. Therefore, this study incorporates various fuel accumulation dynamics which describe possible fuel biomass growth in Florida. Numerical simulation was conducted for loblolly pine using dpsolve routine suggested by Miranda and Fackler (2002). Numerical analysis reveals that landowner’s prevention decision depends closely on the type of fuel biomass growth in his site and the association between fire arrival rate and fuel accumulation. This study reveals that understanding landowners’ behaviors and deriving the optimal policy in the presence of endogenous fire occurrence and fuel accumulation risks is more complicated than reported by previous studies. With an increasing cost of fire suppression in the United States, government programs intended to encourage stand owners to invest in cleaning their site to mitigate fire damages should more focus on forestland where fire risk and fuel growth dynamics are more intense.
References


Reed, W. J. 1984. ‘‘The Effects of the Risk of Fire on the Optimal Rotation of a Forest.’’


Worthington, N.P., 1954. The loblolly pine of the south versus the douglas fir of pacific north west (Vol. 842).