Optimal economic length of leys:
a dynamic programming approach

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Abstract

A model is presented to investigate the optimal economic life cycle of grass leys with winter damage problems in northern Norway and to determine the threshold of winter damage before it is profitable to re-seed. A two-level hierarchic Markov process has been constructed using the MLHMP software. The model takes uncertainty concerning yield potential, damage estimation and weather dependent random fluctuations into account. A Kalman filter technique is used for updating of knowledge on yield potential and damage level.

The application of the model is demonstrated using data from two commercial Norwegian farms. Since parameter estimates vary considerably among farms, it is concluded that decision support concerning optimal economic life cycle of grass leys should be done at farm level. The results also show the importance of using a flexible dynamic replacement strategy. Use of the model for a specific farm situation is illustrated.

Keywords: Dynamic programming; Bayesian updating; Leys replacement; Norway

1. Introduction

Grass production is the main agricultural land use in many parts of Norway. In northern Norway, i.e. the three counties of Nordland, Troms, and Finnmark, as much as 94 per cent of agricultural land in use was under grass in 1999 (Statistics Norway, 2000). In the years 1975, 1978, 1985, 1995, and 1998 grass leys were severely damaged on many farms in northern Norway. Clearly, winter damage to grassland is a significant hazard in this area, and winter damage is the main reason why leys have to be resown.

Once established, young grass leys normally have higher yields than older leys (Nesheim, 1986). Therefore, even under "normal" as well as under winter damage conditions, it might be beneficial to plough the fields and re-seed grasses.

Deciding whether to re-establish grass leys is a typical replacement problem. Such problems are effectively handled by dynamic programming (DP). Agricultural applications of DP have been reviewed by Kennedy (1986) and Kristensen (1994). To our knowledge, there are no published studies of the use of DP to determine the optimal economic length of leys. However, DP has been used extensively to solve animal replacement problem (Kristensen, 1994) which corresponds closely to our replacement problem. The approach in this paper is to apply modern methods of DP modelling for the animal replacement research to the problem of replacement of leys.

A farmer who cultivates grassland is always faced with uncertainty, especially in the area of our study. For instance, the farmer will not know for sure what yields he will obtain in future. This uncertainty concerning future yields has several different sources:

- Even though the farmer knows the yield history of the field, there is still some uncertainty concerning the true yield potential.
- A ley may suffer damage in winter.

- In spring, the farmer may inspect the field and visually estimate the extent of a possible winter damage. There is a considerable uncertainty concerning the visual estimate.
- Weather conditions may influence the year-to-year yields considerably.

The aim of this study is to construct a model taking all the uncertainties mentioned into consideration. A two-level hierarchic Markov process (Kristensen and Jørgensen, 2000) is build using the MLHMP software (Kristensen, 2003). A Kalman filter model (West and Harrison, 1997) is used to represent and update knowledge on the yield potential of the field. An extended version of this paper will be published in Lien *et al*. (forthcoming).

2. The grassland yield model

2.1. Description of the problem

We assume a relatively flat and uniform area is sown with grasses on soil and under climatic conditions that lead to an unstable yield. In cold periods of the year when there is no snow cover, the soil freezes. In coastal areas the temperature often varies and, after a cold period there might be a milder period with rain. Frozen soil has low water permeability, so the water will not soak into the soil. If the mild, rainy period is followed by cold weather, the surface water freezes. An ice cover might damage the ley, and the yields can suddenly drop considerably between years because of winter damage.

In the first year after a winter damage, a partial recovery is often seen, but a moderately damaged ley that is not reseeded will normally not recover fully (Haraldsen *et al*., 1995). Decision strategies to keep or replace the ley after a year with winter damage should account for this potential partial recovery.

The sharp yield drops caused by winter damage are in addition to year-to-year yield uncertainty caused by for instance weather conditions in the growing season.

The optimal replacement decision involves selection of an optimal rotation strategy for grass leys that maximises expected present value of income over time. The decisions to replace or keep the ley are assumed to continue sufficiently far into the future to justify an assumption of an infinite time horizon. Stochastic DP problems with infinite time horizons can often be solved more easily than those with finite horizons, and are likely to give reliable results for real problems with time horizons of several years (Kennedy, 1986).

Each year the farmer must decide whether to replace or keep the existing grass ley. The decision depends on yield level, grass quality and roughage price, re-seeding cost, loss of production during the establishment phase, fieldwork and other variable costs, etc.

All of the above-mentioned factors affecting ley replacement decisions are uncertain, and so ideally should be modelled as stochastic variables. For simplicity we express only yield, assumed to be the main uncertainty in the evaluation, in stochastic terms. In addition to this uncertainty, we also assume that in years with winter damage the yields suddenly drop significantly, but then partially recover next year. Such drops with following increase in yield are accounted for with five possible discrete downward jump processes.

2.2. General model

Grassland yields are in general assumed to follow a quadratic function

$$y(n) = \begin{cases} y_0 - \dfrac{2(y_0 - \bar{y})n}{\bar{n}} + \dfrac{(y_0 - \bar{y})n^2}{\bar{n}^2}, & n \le \bar{n} \\ \bar{y}, & n > \bar{n} \end{cases} \tag{1}$$

where $y(n)$ is the expected yield for year $n+1$ since re-seeding (i.e. $y(0)$ is the expected yield for the first year after re-seeding). The parameters $y_0, \bar{y}$ and $\bar{n}$ are the initial yields, the minimum yield and year of minimum yield, respectively. Having reached the minimum yield, it is assumed to remain at this minimum value. When we apply the model (1) to a particular field i, we shall index the symbols by i (e.g. $y_i(n)$).

The actual yield of a field will vary at random from year to year, and from ley to ley. We may include those aspects by the following extension

$$Y_{ij}(n) = y_i(n)L_j^{'} + \varepsilon_n \tag{2}$$

where $Y_{ij}(n)$ is the observed actual yield at year n from field i, ley j, $L_j^{'} \sim \mathrm{N}(1, \sigma_L^2)$ is the random multiplicative effect of ley, and $\varepsilon_n \sim \mathrm{N}(0, \sigma^2)$ is a random year-to-year fluctuation. In other words, if $L_j^{'} = 1.05$, it means that the true expected yield of the ley is 5% higher than assumed. The effect of $L_j^{'}$ is partially a result of the climatic and other conditions around the year of re-seeding and partially we may interpret it as uncertainty regarding our choice of field characteristics expressed by the yield estimates $(y_0, \bar{y})$ and year of minimum yield $(\bar{n})$.

In order to model the effect of winter damage we further assume that the ley effect L'_j may be reduced by one or more winter damages. This means that the value is no longer constant over years, but depends on the winter damage history of the ley. In order to reflect this change the damage dependent ley effect is denoted as $L_j(n)$ for year n of ley j; that is, $L_j(n)$ represents the combined effects of the undamaged ley potential L'_j and the damage history of the ley. Extending the model with the effect of damages leads to the following model

$$Y_{ij}(n) = y_i(n)L_j(n) + \varepsilon_n. \tag{3}$$

In the modelling of the effect of winter damages, the following assumptions are made:

1. At the beginning of the season in spring, the farmer knows with certainty whether or not there has been a damage during the winter;
2. Through a visual inspection of the field, the farmer is able to give an unbiased estimate of the damage expressed as a ratio α_n defining the expected relative reduction in yield caused by the damage. The precision of the farmer's estimate is assumed known as well;
3. The effect of damage is reduced the following year as a result of partial recovering. After the partial recovery, the remaining effect is permanent;
4. Damages in subsequent years are independent of previous damages. In other words:

$$\mathrm{P}(\alpha_n | \alpha_1, \ldots, \alpha_{n-1}) = \mathrm{P}(\alpha_\mathrm{n}) \tag{4}$$

2.3. Field and ley level

In the model represented by Eqs. (3) and (4) only the resulting yield $Y_{ij}(n)$ is observable, but the estimated relative size α_n is known. In the following we shall only refer to one particular field and ley, and we may therefore skip the indexes for field and ley

$$Y(n) = y(n)L(n) + \varepsilon_n \tag{5}$$

If we apply the Kalman filter technique as described by West and Harrison (1997) we may interpret the yield model (5) as the observation equation. The corresponding system equation of the Kalman filter is

$$L(n) = F_n L(n-1) + e_n \tag{6}$$

where

$$F_n = \begin{cases} 1, & \text{no damage neither previous nor present year} \\ \dfrac{1-\beta\alpha_n}{1-\alpha_n}, & \text{damage last year, no new damage} \\ 1-\alpha_n, & \text{damage this year} \end{cases} \tag{7}$$

and $e_n \sim \mathrm{N}\left(0, \sigma_{Sn}^2\right)$ is a residual reflecting the system variance defined as

$$\sigma_{Sn}^2 = \begin{cases} 0, & \text{no damage this year} \\ \sigma_D^2, & \text{damage this year} \end{cases} \tag{8}$$

The interpretation of Eqs. (6) - (8) is that the yield potential $L(n)$ is assumed constant over time as long as no damage has occurred or if the latest damage is at least two years old. If a damage occurred last year, the effect of the damage is assumed to have decreased to a level defined by β due to partial recovery of the field. In years where a new winter damage is observed, α_n must express the size of the relative damage, and the loss is expressed as the expected value plus a random term of which the variance depends on the uncertainty $\left(\sigma_D^2\right)$ of the farmer's estimate.

It is obvious that the true value of $L(n)$ is important for the re-seeding decision since it represents a permanent trait of the ley and the size of a winter damage. Since we are not able to observe the true value, we have to estimate it using Bayesian updating (Kristensen, 1993).

In general, we shall denote the current estimate of $L(n)$ after the observation of n years of yields $\hat{L}_n$ and accordingly for other parameters. Now, assume that we know the estimates $\hat{L}_n$, and furthermore observe the yield $Y(n+1)$ of year $n+1$. We may then update our belief in the true value of $L(n+1)$ using the following relation taken from West and Harrison (1997):

$$\begin{aligned}
\hat{L}_{n+1} &= F_{n+1}\hat{L}_n + A_{n+1}\left(Y(n+1) - y(n+1)F_{n+1}\hat{L}_n\right) \\
A_{n+1} &= R_{n+1}\, y(n+1) V_{n+1}^{-1} \\
R_{n+1} &= F_{n+1}^2 C_n + \sigma_{Sn}^2 \\
V_{n+1} &= \left(y(n+1)\right)^2 R_{n+1} + \sigma^2 \\
C_{n+1} &= R_{n+1} - A_{n+1}^2 V_{n+1} \\
\hat{L}_0 &= 1 \\
C_0 &= \sigma_L^2
\end{aligned} \tag{9}$$

The value C_n is the variance of $\hat{L}_n$, and $\hat{L}_0$ and C_0 are the initial values of $\hat{L}_n$ and C_n for $n = 0$, i.e. before the first yield result is observed. The variance components σ_L^2 and σ^2 from Eq. (9) must be estimated from yield data.

In the prediction of the next yield we want to be as precise as possible using all previous information (i.e. all yield observations of the ley). The benefit of the Kalman filter technique combined with the proposed yield model (3) is that the expected value of $Y(n+1)$ given all previous information only depends on $\hat{L}_n$. The conditionally expected value of $Y(n+1)$ is

$$\mathrm{E}\left(Y(n+1) \middle| Y(n),\ldots,Y(0),\hat{L}_0,C_0\right) = \mathrm{E}\left(Y(n+1) \middle| \hat{L}_n\right) = y(n+1)F_{n+1}\hat{L}_n \tag{10}$$

and the conditional variance is V_{n+1} as defined in Eq. (9). It should be noticed that the conditional variance V_{n+1} varies with n, but is independent of the observations of $Y(n)$. From Eqs. (7) to (9), however, we conclude that we need to know the precise damage history $\alpha_1,\ldots,\alpha_{n+1}$ in order to know the precise values of V_{n+1} and C_n. Given this damage history, it is possible to calculate the sequence $V_1,\ldots,V_n$ for any n without observing $Y(1),\ldots,Y(n)$. That means that in order to predict the next yield result we only need to keep the most recent value of $\hat{L}_n$.

3. Optimisation model

3.1. Model structure

The tool to be used in this re-seeding model is a multi-level hierarchic Markov process as described by Kristensen and Jørgensen (2000). A multi-level hierarchic Markov process has an ordinary infinite time Markov decision process running at the *founder level*. For each combination of state and action, a stage of the founder may be represented by a *child (and grant-child) process*, which in turn is an ordinary finite time Markov decision process. In this case we use a model with only two levels as follows:

Founder process - Infinite time horizon.

> **Stage** - Stage length is equal to the lifetime of one particular ley.
>
> **State space** - Only one dummy state is defined.
>
> **Action space** - Only one dummy action is defined.

Child level 1 - Finite time.

> **Stage** - Stage length is equal to a year.
>
> **State space** - The state is defined by the values of the following *state variables* at stage n:
>
> > **Estimated ley potential** - The value of $\hat{L}_{n-1}$ - the estimated potential after $n-1$ observations (21 classes).

Variance of estimate - The value of C_{n-1} - variance of the estimated potential after $n-1$ observations (21 classes).

Winter damage - The value of α_n. Six levels are considered: α_n = 0.00, 0.20, 0.30, 0.40, 0.50 or 0.75.

Age of damage - Two levels: "This year", "Previous year".

If the winter damage is 75%, all other state variables are ignored, and the decision "Re-seed" is chosen without further consideration. The total number of states equals 3970 per stage.

Action space - Two options: "Keep" or "Re-seed". If the winter damage is 75% only the decision "Re-seed" is available.

The total number of state combinations is around 75 000. The model was constructed as a plug-in to the MLHMP software (Kristensen, 2003).

3.2. Parameters

The output, $m_i^d(n)$, of a stage under the action "Keep" is simply the expected yield calculated according to Eq. (10), and under the action "Re-seed" it is 0. As concern stage length, it is 1 year if the ley is kept and 0 if re-seeded. When we refer to state i at stage n, we mean the state $i=\left(\hat{L}_{n-1},\alpha_n,C_{n-1},a_n\right)$ where a_n is the age of the damage (if any).

The rewards, $r_i^d(n)$, of the model are defined as the expected net revenues in a particular state under a given action. The expected net revenue is calculated as the value of the coarse fodder produced minus the variable costs which are assumed to depend on expected yield according to a quadratic function. For the decision "Re-seed" $(d=1)$, the reward is simply zero since all costs related to re-seeding are included at stage 0 of the new ley.

The transition probabilities, $p_{ij}^d(n)$, define the probability of a transition from state $i=\left(\hat{L}_{n-1},\alpha_n,C_{n-1},a_n\right)$ at stage n to state $j=\left(\hat{L}_n,\alpha_{n+1},C_n,a_{n+1}\right)$ at stage $n+1$. The uncertainty concerning the transition is governed by two random events: The (later) observed yield $Y(n)$ of this year, which is normally distributed, and the observed level of winter damage next year, which has a simple discrete distribution. All other effects are deterministic. Based on the two known random distributions and all the deterministic transitions, the over-all transition probabilities are calculated analytically.

4. Example

4.1. Case farms and parameters

To illustrate the necessary input and the output produced using the model we applied it to two commercial farms in northern Norway. Farm A consists of 20 observations, which imply observations of several fields in some years. The youngest ley in the survey for farm A was one year old and the oldest one was fourteen years. For farm B we had 11 observations, the youngest leys being one year and the oldest seven. There was no (substantial) winter damage on the fields for these farms during the period we had data, 1985 to 1991.

For each farm, based on the historical field-specific data, we estimated $y_0,\bar{y},\bar{n},\sigma$ and σ_L. The standard deviation of estimated winter damage, σ_D, was subjectively estimated, and a sensitivity analysis was carried out to examine the effects of changing the value.

The probability for winter damage was assumed to follow a discrete distribution. Based on expert advice, we specified six discrete events, no winter damage (WD), 20%, 30%, 40%, 50% and 75% winter damage, with probability 0.85, 0.06, 0.05, 0.02, 0.01 and 0.01, where the percentages are the decline in yield of "no WD" yield on the field. We assumed, again based on expert advice, that the step down in yield following damage is reduced by 2/3 in the year after the year with winter damage due to a partial recovery of the damaged leys. In other words, β is set to 0.666. Maximum ley age is assumed to be 20 years, i.e., the model specifies that a ley that reaches 20 years is re-seeded. The input parameters for case farm A and B are shown in Table 1.

Table 1 Input parameters for farm A and farm B

Parameter	Farm A	Farm B
Yield model parameters		
First year yield: y_0	7 652	8 524
Minimum yield: $\bar{y}$	5 222	5 294
Age at minimum yield: $\bar{n}$	11	4
Year-to year standard deviation: σ	2 109	1 219
Standard deviation of ley effect: σ_L	0.023	0.013
Standard deviation of damage estimate: σ_D	0.030	0.030
Damage decrease first year after damage: β	0.666	0.666
Prices and costs		
Coarse fodder price, NOK kg^{-1} DM	1.65	1.65
Variable costs, constant term, NOK ha^{-1}	1 480.14	1 480.14
Variable costs, linear coefficient, NOK ha^{-1}	0.9792	0.9792
Variable costs, quadratic coefficient, NOK ha^{-1}	-0.0000235	-0.0000235
Net costs of re-seeding, NOK ha^{-1}	2 310	2 310

4.2. Results

Optimal policies maximising average net returns over time for the two farms were calculated.[1] Since farm-specific parameters are used, the results are specific for each outcome on that particular farm. Therefore, we report a small selection of the results for illustration. Table 2 shows the consequences of the optimal policies in terms of dry matter produced, net returns, and years between re-seeding. The figures of the table are calculated by means of the Markov chain simulation facility of the MLHMP software.

Table 2 Average net returns, average dry matter produced and average years between re-seeding for the two farms under optimal policies

Key figure	Farm A	Farm B
Average dry matter produced, kg DM per ha per year	6 018	5 752
Average net returns, NOK per ha per year	2 746	2 615
Average years between re-seeding of the leys	6.7	5.6

[1] The alternatives for optimality, maximisation of total expected discounted net returns and maximisation of average net returns over time, give about the same optimal policy (Kristensen, 1994).

That Farm B got a lower "average dry matter produced" than farm A is at first glance surprising, since farm B has a higher yield potential. But since farm B has a low minimum yield age (Table 1), the yield will be rather low in fields that are only three to four years old. For the same reason, after a winter damage of a ley that is not re-seeded, the future yield potential on farm B will be lower than on farm A. Farm B got a lower average net returns than farm A.

In Figure 1 we illustrate average future loss in net returns per ha per year by following a "always re-seed in year *X* strategy", where *X* varies from 1 to 20. Generally it is expensive to have an inflexible and short rotation strategy. Farm A showed considerable losses if, e.g., re-seeding every second or third year. Farm A has smallest losses with an "always re-seeding strategy" when the leys are six years. A strategy of always re-seeding older leys give increasing losses compared with the average optimal replacement age. Farm B shows much smaller losses, compared with farm A, with a fixed rotation strategy of three years. Farm B has smallest losses with a fixed rotation strategy when the leys are four years.

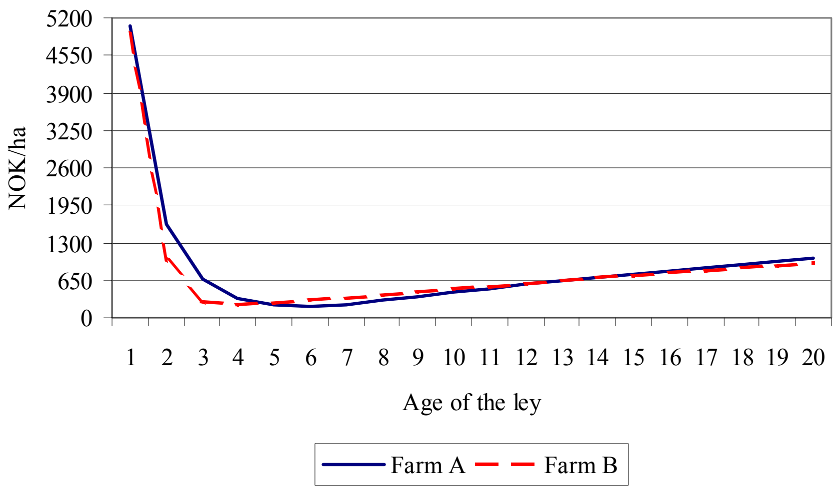

Figure 1 Farm A and farm B, average future loss in net returns in NOK per ha per year by following an "always re-seed in year X strategy".

So far we have only looked at what is optimal in average. To see what the farmers should do in each specific situation it is necessary to refer to the full set of solutions for that farm. In Figure 2 we have plotted the "future profitability" (also called retention pay-off, which show the profit by following the optimal strategy or losses by not follow the optimal strategy once (a negative value means that the leys should be re-seeded)) for different yields (represented as the relative yield potential, $\hat{L}_n$) and winter damage levels at a fixed ley age and a fixed variance of $\hat{L}_n$, C_n.[2]

Figure 2 shows that it is only optimal to keep the leys (positive values) in states with no or modest level of winter damage and relative high yield potential. Note also that farm B should keep the leys in many more of the states than farm A when the leys are five years old.

[2] We also did similar plots where "future profitability" was a function of C_n and $\hat{L}_n$ with fixed winter damage levels (WD) and ley age, and plots where "future profitability" was a function of C_n and WD with fixed $\hat{L}_n$ and ley age. These plots showed that C_n had insignificant influence on "future profitability". To save space these plots are not reported in this article.

Note also that Figure 2 only say something about what the farmer should do in the specific states. The probabilities for the different states are different, so the figures do not say anything about the average ratio of the cases that should be kept or re-seeded.

An analysis of the sensitivity of the choice of the model parameter standard deviation for estimated winter damage, σ_D, showed negligible influence on the results, which are therefore not reported here. The same is the case with the coarse fodder price, a 10 per cent change up or down had not at all any effect on the optimal replacement strategy.

As an illustration, Table 3 shows how the model can be used on specific fields with known field history (observed damage and yield in each year). The optimal policy for the field on farm A was calculated. Then the "future profitability" for the actual (constructed) field was identified using the field history for farm A. In this simple example we assume that the farmer considers to replace the leys that are five years old. In case 1 the farmer will lose NOK 1165 per ha if he re-seed the five years old leys, and in case 2 the gain is NOK 1972 per ha with a re-seeding choice.

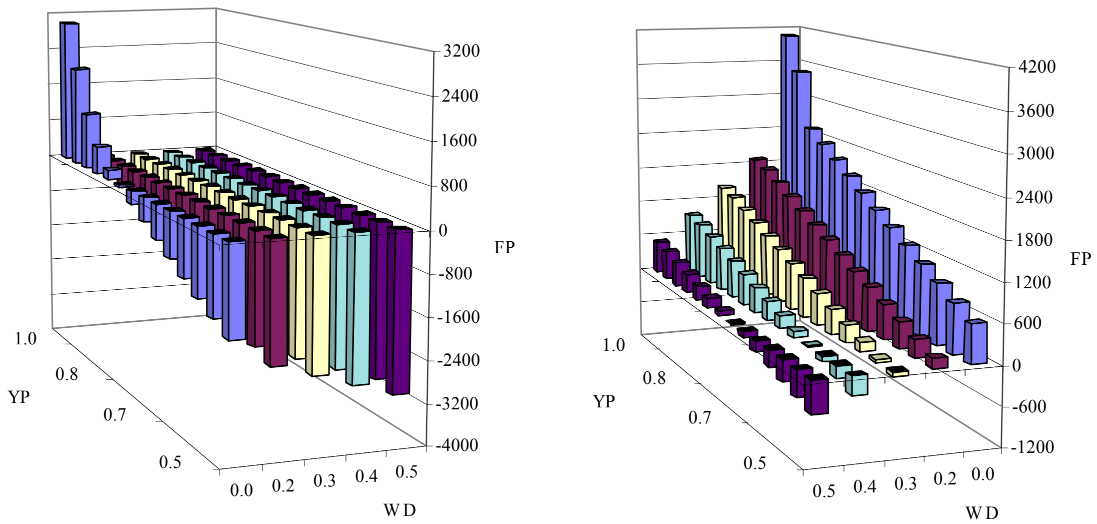

Figure 2 Farm A to the left, farm B to the right. "Future profitability" (as FP in the figure) as a function of the relative yield potential $\hat{L}_n$ (YP) and winter damage (WD), ley age is fixed to 5 years, and C_n is fixed to 0.0007. Note: to make the figure to the right (farm B) easy to read the WD states are reported in the opposite direction of that in the figure to the left (farm A).

Table 3 Some constructed actual field history data and "future profitability" on farm A, when the leys is replaced five years after re-seeding

	Years after reseeding	1	2	3	4	"Future profitability"
Case 1	Observed damage	0	0	0	0	
	Observed yield (kg DM/ha)	7850	7670	7620	7560	1165
Case 2	Observed damage	0	0	0.5	0	
	Observed yield (kg DM/ha)	7850	7670	3810	4300	-1972

5. Concluding comments

The application to the two case farms leads to two conclusions. First, the large variation in model parameters among fields/farms (as a consequence of differences in biological factors and management levels between fields/farms) implies that a ley replacement model should be at farm level rather than at an aggregated level intended to give general advice. Second, the farmers should have a flexible dynamic ley replacement strategy, and at each stage or year make decisions depending on yield level, winter damage, grass quality and coarse fodder price, re-seeding cost, age of the ley, loss of production during the establishment phase, fieldwork and other variable costs, etc.

The dynamic programming model for individual farmers used in this analysis is applicable to all replacement problems of leys in areas with winter damage problems. With some modification, our model may be used for other similar replacement problems with jumps in the transformation functions, for example optimal rotation of a forest with natural disaster problems.

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References

Haraldsen, T.K., Sveistrup, T.E., Lindberg, K., Johansen, T.J., 1995. Soil compaction and drainage systems of peat soils in northern Norway. Effects on yields and botanical composition in leys. Norsk landbruksforskning, 9, 11-28.

Kennedy, J.O.S., 1986. Dynamic Programming. Applications to Agricultural and Natural Resources. Elsevier. London, 341 pp.

Kristensen, A.R., 1993. Bayesian updating in hierarchic Markov processes applied to the animal replacement problem. Eur. R. Agric. Econ. 20, 223-239.

Kristensen, A.R., 1994. A survey of Markov decision programming techniques applied to the animal replacement problem. Eur. R. Agric. Econ. 21, 73-93.

Kristensen, A.R., 2003. A general software system for Markov decision processes in herd management applications. Computers and Electronics in Agriculture 38, 199-215.

Kristensen, A.R., Jørgensen, E., 2000. Multi-level hierarchic Markov processes as a framework for herd management support. Annals of Operations Research 94, 69-89.

Lien, G., Kristensen, A.R., Hegrenes, A., Hardaker, J.B. (forthcoming). Optimal length of leys in an area with winter damage problems. Grass and Forage Science.

Nesheim, L., 1986. A grassland survey in Nordland, North Norway III. Feed quality parameters and yield. Scientific reports of the Agricultural University of Norway. 65(20).

Statistics Norway, 2000. Jordbruksareal i drift, etter bruken av arealet, fylke og bruksstørrelse. 1969, 1979, 1989 og 1999. http://www.ssb.no/emner/10/04/10/jt1999/ arkiv/tab-2000-02-21-02.html.

West, M., Harrison, J., 1997. Bayesian Forecasting and Dynamic Models, 2nd edition. Springer Series in Statistics, Springer. New York, 403 pp.