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# Formulating and Testing a New Conservation Auction Mechanism in an Experimental Setting 

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## 1 Abstract

This paper proposes a new auction mechanism, the provision point reverse auction (PPRA), as an alternative to the discriminative reverse auction. The PPRA requires that a certain number of contracts be affordable, given the buyer's budget, for any contracts to be made. The PPRA includes a public component where the probability of contract acceptance for one individual is affected by the offers of others. We first provide theoretical support for the new mechanism, comparing the new expected profit function for a PPRA to the expected profit function for the reverse discriminative auction. We follow with laboratory experiments where we compare average offers for the PPRA to other discriminative reverse auctions and find that the PPRA significantly reduces accepted offers by between either $21.55 \%$ to $58.17 \%$ or $12.57 \%$ to $21.59 \%$, depending on parameter values and alternate auction mechanisms. This decrease in average offers has the potential to increase societal welfare when the goods being purchased result in positive externalities, as we would expect in conservation or payment for ecosystem services auctions.

## 2 Introduction

Payment for environmental services programs have become an increasingly important component of conservation and environmental protection. Many of these programs use reverse (or procurement) discriminative auctions to allocate contracts to individuals who provide the environmental service (Latacz-Lohmann \& Schilizzi, 2005). Reverse discriminative auctions involve one buyer and many sellers, where the winners of the auctions receive their offer (or bid) as payment. In most reverse discriminative auctions, the buyer has a fixed budget and accepts offers in ascending order until the budget has been exhausted. In such an auction, sellers must balance potential gains in expected profit from a higher offer against corresponding decreases in the probability the offer will be given a contract by the buyer. The higher the offer, the less likely a contract will be won and the potential profit will be realized. If these auctions are conducted for multiple rounds, sellers gain information about
the costs of the other sellers each round and use that information to increase their profits at the expense of the buyer. More specifically, sellers slowly increase their offers until they discover the offer at which they would no longer receive a contract. We call submitting an offer above one's value "rent-seeking offers" or simply rent-seeking behavior. ${ }^{1}$ Overtime, as rent-seeking behavior becomes more pronounced, the buyer can afford fewer contracts and incurs a welfare loss. This is a particularly significant problem for payment for environmental services or conservation programs because each contract may provide an environmental positive externality. In such a case, a reduction in the number of contracts the buyer can purchase could decrease the environmental benefits from the program and harm society at large. Given the large number of PES or conservation programs which use reverse discriminative auctions, including the Conservation Reserve Program (CRP) in the United States, the Auction for Landscape Recovery (ALR) in Australia, Challenge Funding in Scotland, and others, rent-seeking behavior is likely decreasing social welfare by hundreds of millions of dollars.

With an eye toward reducing rent-seeking offers, and thus potentially increasing social welfare in conservation and PES contexts, we propose the "provision point reverse auction" (PPRA). The PPRA functions as a discriminative reverse auction in that there is one buyer with many sellers and each individual with an accepted offer receives their offer as payment. However, unlike other discriminative procurement auctions, in a PPRA the buyer declares a requirement that a certain pre-specified number of offers must be affordable for any offers to be accepted. That is, if the buyer cannot afford to purchase a certain number of the cheapest offers, given their budget constraint, then no contracts will be made with any individual.

Similar to a reverse discriminative auction, an individual participating in a provision point reverse auction must weigh increases in potential profit from a higher offer against corresponding decreases in the probability of realizing that profit. As an individual's offer becomes larger, it is also larger relative to the offers of their peers which decreases the probability the offer will be given a contract by the buyer. In a PPRA, however, a higher

[^0]offer not only increases the offer relative to its peers, it also reduces the chance that the buyer can afford the pre-specified number of units, further lowering the probability of contract acceptance. This incentivizes participants in a PPRA to submit offers closer to their costs, relative to a standard reverse discriminative auction.

The PPRA also includes a public component. When an individual increases their offer, they negatively affect the expected profit of all the other individuals in the auction by reducing the chance that any contracts are provided by the buyer. Thus, if individuals in a PPRA place positive utility on higher profits for their peers, they will be further incentivized to keep their offers close to their true costs. The authors believe this is particularly likely to be true in close-knit rural or developing communities where PES programs are often implemented.

This paper provides theoretical evidence which shows that, under various assumptions, optimal offers under a PPRA are less than the optimal offers under a reverse discriminative auction, given an opportunity cost. These theoretical predictions are supported by experimental evidence from the lab. Ten experimental sessions were conducted with 240 student participants in total. The experimental results suggest that the PPRA reduces accepted offers by between either $21.55 \%$ to $58.17 \%$ or $12.57 \%$ to $21.59 \%$, depending on parameter values and alternate auction mechanisms. The effect on offering behavior is particularly pronounced for the lowest offers, which are also the one's of greatest interest to the buyer.

Section 3 provides a literature review of conservation contracts and the relevant auctions. Section 4 introduces the formal model and provides some theoretical results. Section 5 describes the data and experimental methods. Section 6 provides the experimental results. Section 7 includes a discussion and Section 8 concludes.

## 3 Literature Review

Environmental goods or services are generally not exchanged on open markets, and thus do not have an easily observable price. Auctions are a convenient method for exchange when the values for a good are unknown, and thus present an attractive choice to policy makers
interested in purchasing environmental services. However, there are many types of auctions and it is not a priori obvious which auction format should be chosen, if an auction should be used at all. In the symmetric independent private values (SIPV) model, 1) there is a single indivisible unit available for sale, 2) each bidder knows their own private valuation, 3) all bidders are identical, 4) the valuations are independent and identically distributed, and 5) all bidders are risk neutral (Wolfstetter, 1996). Within this framework, there are four formats: the Dutch auction, the English auction, the first-price sealed bid auction, and the secondprice sealed bid auction. The famous Revenue Equivalence Theorem (RET) states that the auctioneer receives the same revenue, regardless of the chosen format (Myerson, 1981; Riley \& Samuelson, 1981, Vickrey, 1961). However, markets for environmental services do not satisfy many of the assumptions required for the RET to hold, and so we cannot apply this useful result to the questions of conservation and PES auctions. E.g, rather than one individual selling a single unit of a good to a pool of several individuals, one individual (or organization) is seeking to purchase multiple units of a good from several individuals.

One key difference between conservation and PES auctions and auctions in an SIPV model is that auctions for environmental services are generally multi-unit procurement auctions. That is, conservation or PES auctions generally involve one buyer purchasing multiple units of a good from multiple sellers. Unfortunately, the literature is less developed on the topic of multi-unit procurement auctions than on other mechanisms, particularly for auctions where the buyer is restricted by a budget (Nautz, 1995; Bower \& Bunn, 2001; Hailu, Schilizzi, \& Thoyer, 2005; Latacz-Lohmann \& Schilizzi, 2005; Schilizzi \& Latacz-Lohmann, 2007). Harris and Raviv (1981) and Cox et al. (1984) generalized Vickrey's original results and provided optimal offer functions for selling auctions with multiple units, symmetric, risk neutral sellers, and a fixed amount of the good the buyer was interested in purchasing. Hailu et al. (2005) extended this research and provided the optimal offer function for symmetric, risk neutral sellers in a multi-unit procurement auction where the buyer is only interested in purchasing a certain number of units, and is not constrained by a budget. To the best of our knowledge, no one has specified an optimal offer function for a multi-unit procurement
auction where the buyer is constrained by a budget. Without more robust theoretical guidance from the literature, researchers and policy makers are forced to rely on experience and experimental evidence when making their decisions about how to purchase environmental services.

When using auctions in payment for environmental services (PES) programs, buyers frequently opt for a uniform second price or discriminative auction (Latacz-Lohmann \& Schilizzi, 2005). In a uniform second price procurement auction, all individuals who submit winning offers are paid the first rejected offer. Because increasing one's offer cannot increase their own payoff, individuals have the incentive to offer their true cost to the seller. In a discriminative procurement auction, individuals who submit winning offers receive their offers as payment, analogous to a first price auction. Unlike the uniform second price procurement auction, in the discriminative procurement auction the optimal offering strategy is to submit an offer higher than one's true cost. Because only individual sellers have full information on their true costs, this offering behavior leads to information rents for the sellers.

There is disagreement in the literature about the relative cost effectiveness of the uniform second price and discriminative auctions from the perspective of the buyer. Each mechanism's efficiency and cost effectiveness is a function of the cost structure of the individual participants and the assumptions regarding information and communication. In their comprehensive review on the theoretical and empirical literature regarding conservation contracts, Latacz-Lohmann and Schilizzi (2005) provided several reasons that explain why funding agencies often choose discriminative procurement auctions over uniform price auctions, including the different risks of each auction mechanism for the buyer, which sellers profit the most from which auction, and the complexity of the different mechanisms.

To increase the efficiency of PES or conservation programs which use discriminative auction formats, we propose the provision point reverse auction (PPRA). The PPRA functions as a discriminative procurement auction with the added requirement that a certain number of units are purchased by the buyer, given a constant, exogenous budget. For example, if the provision point requirement is $80 \%$ participation, but the buyer can only afford contracts
for $75 \%$ of the sellers, then no contracts will be offered and the buyer will keep their money. Section 4 will provide more specifics.

The PPRA is connected to the research conducted on the provision point mechanism (PPM) for voluntary contributions to public goods (Davis \& Holt, 1993; Marks \& Croson, 1998; Rondeau, Schulze, \& Poe, 1999, Rondeau, Poe, \& Schulze, 2005). In a provision point mechanism, a public good is provided only if the total contributions exceed some predetermined threshold. If the total contributions do not exceed this "provision point," then all contributions are refunded to the participants and no amount of the public good is provided. The PPRA is essentially the reverse auction form of the provision point mechanism: instead of a total contribution requirement, the sum of the lowest cost offers must be less than the budget and the potential profits from the auction can be viewed as the public good offered through the mechanism.

The closest paper to the provision point reverse auction, as we formulate it, is Bush et al.'s use of a provision point in a contingent valuation study which attempted to reduce the upward bias in willingness to accept estimates. Their mechanism is called a provision point mechanism (PPM), after previous literature on contributions to public goods (Bush et al., 2013). We expand upon Bush et al.'s work by generalizing their mechanism to an auction with many possible provision point requirements and test the auction mechanism with real money in an experimental setting. We additionally provide theoretical support to substantiate the experimental evidence.

The voluntary agreement between local New York farmers and New York City over the Catskill-Delaware water system (or Cat-Del system) provides an example of a program with a functioning provision point. Albert Appleton, the New York City Commissioner of Environmental Protection and Director of the New York City Water and Sewer System during the program, provides an excellent overview (Appleton, 2016). New York City, which long had one of the cleanest sources of drinking water for any urban area, became concerned about the Cat-Del system in the 1980s. Instead of mandating regulation to preserve the clean water system, the government and farmers engaged in a voluntary program where the
city provided fixed payments to farmers in exchange for environmentally friendly practices. The city agreed that the program would be voluntary on the condition that if $15 \%$ of the farmers did not participate, costly regulation would take effect to achieve the desired water quality improvement. As such, this program was an example of both a provision point and a voluntary-threat, similar to the work done by Segerson and others (Segerson \& Miceli, 1999; Segerson \& Dawson, 2001; Poe et al., 2004; Taylor et al., 2004; Suter et al. 2010; Suter \& Vossler, 2013).

## 4 Theory and Model

The theory section is split into two parts. In the first subsection, we review the literature on target-constrained auctions, including a re-derivation of the symmetric Bayesian Nash Equilibrium optimal offer function. The second subsection introduces the provision point reverse auction and provides the expected profit function, along with several proofs that provide predictions for optimal offering behavior in a PPRA compared to a target-constrained auction.

### 4.1 Target-Constrained Optimal Offer Function

In the symmetric independent private values (SIPV) model, 1) there is a single indivisible unit available for sale, 2) each bidder knows their own private valuation, 3) all bidders are identical, 4) the valuations are independent and identically distributed, and 5) all bidders are risk neutral. In a seminal paper, Vickrey famously proved that the dominant strategy in a sealed-bid second-price auction was for an agent to reveal their value, regardless of the strategies of the other agents (Vickrey, 1961). This result only held for single-unit auctions, however, but Vickrey later expanded his model to include multi-unit auctions (Vickrey, 1962). Harris and Raviv expanded the model to general value distributions when all individuals have identical, concave utility functions (Harris \& Raviv, 1981). This generated a symmetric Bayesian Nash equilibrium optimal offer function for a first-price, multi-unit auction. Hailu, Schilizzi and Thoyer used a similar approach to generate a symmetric Bayesian

Nash equilibrium optimal offer function for a first-price, multi-unit procurement auction, also known as a "target-constrained auction" (Hailu et al., 2005).

Let $n$ denote the number of participants in an auction, $p$ denote the target if the auction is a target-constrained auction or provision point requirement if the auction is a PPRA, $B$ denote the budget if the auction is a budget-constrained auction or a PPRA, $v_{i}$ denote individual i's opportunity cost or value, $o_{i}$ denote their offer, $O_{j}\left(v_{j}\right)$ denote the assumed offering behavior of the other participants as a function of their values, and $O_{j}^{-1}$ denote its inverse. To simplify the theory and computations, we make the common assumption that all values are drawn from a standard uniform distribution. All of the auctions we consider with have the following properties:

1) They are multi-unit auctions, so that more than one unit is being exchanged in each round;
2) They are reverse auctions. That is, the auctions have one seller with multiple buyers. These auctions are also known as procurement auctions;
3) Values are independently drawn, so an individual's value provides no information about the values of the other participants;
4) Each bidder knows their own value but they do not know the value of any other participant. That is, all values are privately held;
5) All participants, as well as the units they are trying to sell, are symmetrical and indistinguishable;
6) All participants are risk-neutral.

The three auction formats we consider have expected profit functions given by:

$$
\begin{equation*}
E[\Pi]=\left(o_{i}-v_{i}\right) * \operatorname{Pr}\left(o_{i} \text { receives a contract }\right) \tag{1}
\end{equation*}
$$

The form of $\operatorname{Pr}\left(o_{i}\right.$ receives a contract $)$ depends on the auction used, as well as the parameter values chosen. As an individual increases their offer, potential profit, given by $\left(o_{i}-v_{i}\right)$,
increases, but $\operatorname{Pr}\left(o_{i}\right.$ receives a contract $)$, the probability of realizing the potential profit, decreases. Thus, picking the optimal offer for a given value requires balancing these two effects.

For the target-constrained auction, expected profit is given by:

$$
\begin{equation*}
E[\Pi]=\left(o_{i}-v_{i}\right) * \operatorname{Pr}\left(o_{i} \text { is one of the } p \text { lowest offers }\right) \tag{2}
\end{equation*}
$$

The closed form representation of $\operatorname{Pr}\left(o_{i}\right.$ is one of the $p$ lowest offers $)$, and much of the following theory, relies upon order statistics, so a brief set of definitions is in order. (See Wolfstetter (1996) for a brief and exceedingly useful overview of order statistics.) Out of a set of $n$ draws from a distribution, the random variable $V_{(r)}$, which represents the $r$ th lowest draw, is called the $r$ th order statistic. The probability distribution of $V_{(r)}$ is given by

$$
\begin{equation*}
f_{V_{(r)}}(x)=\frac{n!}{(r-1)!(n-1)!} F(x)^{r-1}(1-F(x))^{n-r} f(x) \tag{3}
\end{equation*}
$$

For a standard uniform distribution, $f(x)=1$ and $F(x)=x$, so that the above simplifies to

$$
\begin{equation*}
f_{V_{(r)}}(x)=\frac{n!}{(r-1)!(n-1)!} x^{r-1}(1-x)^{n-r} \tag{4}
\end{equation*}
$$

Notice that this is a beta distribution, $\mathrm{B}(r, n+1-r)$.
Thus, assuming that all values are drawn from a standard uniform distribution, the probability that an individual's offer is one of the $p$ smallest offers is given by the function:

$$
\begin{equation*}
g\left(n, p, O_{j}^{-1}\left(o_{i}\right)\right)=\frac{(n-1)!}{(p-1)!(n-p-1)!} \int_{O_{j}^{-1}\left(o_{i}\right)}^{1} u^{p-1}(1-u)^{n-p-1} d u \tag{5}
\end{equation*}
$$

Intuitively, the $g$ function takes in an individual's offer, $o_{i}$, and transforms it into an opportunity cost through $O_{j}^{-1}$. $O_{j}^{-1}\left(o_{i}\right)$ denotes the opportunity cost draw that would result in the offer $o_{i}$ from the other participants in the auction, assuming the common offering behavior $O_{j}$. This opportunity cost can then be used to calculate the probability the offer is one of the p smallest offers using the properties of order statistics and the given distribution for
opportunity costs. From this point on, $g\left(n, p, O_{j}^{-1}\left(o_{i}\right)\right)$ will be simplified as $g\left(O_{j}^{-1}\left(o_{i}\right)\right)$.

Given an expected profit function, we are interested in the offer which, for each possible value, maximizes expected profit. That is, we are interested in a function which takes in an individual's opportunity cost and returns their expected profit maximizing offer. Even more, we are interested in the offer function which is also a symmetric Bayesian Nash equilibrium. A symmetric Bayesian Nash equilibrium occurs when the best response to a given offer function is that offer function. More specifically, a symmetric Bayesian Nash equilibrium is an optimal offer function where, if an individual is participating in an auction where they assume the other individuals submit offers according to an offer function $O_{j}\left(v_{j}\right)$, the optimal response is to also submit offers according to $O_{j}\left(v_{j}\right)$.

Hailu, Schilizzi and Thoyer (2005) derive the symmetric Bayesian Nash equilibrium for a multi-unit procurement auction. We rederive, confirm, and expand upon their results here. In a multi-unit procurement auction (also known as a target-constrained auction), a participant in the auction is interested in the probability that their offer will be one of the $p$ lowest offers out of the $n$ offers submitted by the $n$ participants. This probability is represented by $g\left(O_{j}^{-1}\left(o_{i}\right)\right)$ in Equation 5. The expected profit for an individual in this auction is then represented by

$$
\begin{equation*}
E[\Pi]=\left(o_{i}-v_{i}\right) * g\left(O_{j}^{-1}\left(o_{i}\right)\right) \tag{6}
\end{equation*}
$$

which is, of course, a more specific representation of Equation 2. The first order conditions to maximize Equation 6 are

$$
\begin{equation*}
g\left(O_{j}^{-1}\left(o_{i}\right)\right)+\left(o_{i}-v_{i}\right) \frac{\partial g\left(O_{j}^{-1}\left(o_{i}\right)\right.}{\partial o_{i}} \frac{\partial O_{j}^{-1}\left(o_{i}\right)}{\partial o_{i}}=0 \tag{7}
\end{equation*}
$$

Recalling that

$$
\begin{equation*}
\frac{\partial f^{-1}(x)}{\partial x}=\frac{1}{f^{\prime}\left(f^{-1}(x)\right.} \tag{8}
\end{equation*}
$$

Equation 7 simplifies to

$$
\begin{equation*}
g\left(O_{j}^{-1}\left(o_{i}\right)\right)+\left(o_{i}-v_{i}\right) \frac{\frac{\partial g\left(O_{j}^{-1}\left(o_{i}\right)\right.}{\partial o_{i}}}{\frac{\partial O_{j}\left(O_{j}^{-1}\left(o_{i}\right)\right)}{\partial o_{i}}}=0 \tag{9}
\end{equation*}
$$

In equilibrium, $o_{i}=O_{j}\left(v_{i}\right)=O_{i, T C}^{*}\left(v_{i}\right)$. Equation 9 becomes

$$
\begin{equation*}
v_{i} \frac{\partial g\left(v_{i}\right)}{\partial o_{i}}=g\left(v_{i}\right) \frac{\partial O_{i, T C}^{*}\left(v_{i}\right)}{\partial o_{i}}+O_{i, T C}^{*}\left(v_{i}\right) \frac{\partial g\left(v_{i}\right)}{\partial o_{i}} \tag{10}
\end{equation*}
$$

Integrating both sides with respect to $v_{i}$ yields

$$
\begin{gather*}
-\int_{v_{i}}^{1} u \frac{\partial g(u)}{\partial o_{i}} d u=O_{i, T C}^{*}\left(v_{i}\right) g\left(v_{i}\right)  \tag{11}\\
O_{i, T C}^{*}\left(v_{i}\right)=\frac{-\int_{v_{i}}^{1} u \frac{\partial g(u)}{\partial o_{i}} d u}{-\int_{v_{i}}^{1} \frac{\partial g(u)}{\partial o_{i}} d u} \tag{12}
\end{gather*}
$$

Given that

$$
\begin{equation*}
\frac{\partial g(u)}{\partial o_{i}}=\frac{(n-1)!}{(p-1)!(n-p-1)!} u^{p-1}(1-u)^{n-p-1} \tag{13}
\end{equation*}
$$

the symmetric Bayesian Nash equilibrium for the target-constrained auction is given by

$$
\begin{equation*}
O_{i, T C}^{*}\left(v_{i}\right)=\frac{\int_{v_{i}}^{1} u^{p}(1-u)^{n-p-1} d u}{\int_{v_{i}}^{1} u^{p-1}(1-u)^{n-p-1} d u} \tag{14}
\end{equation*}
$$

The optimal offer function, $O_{i, T C}^{*}\left(v_{i}\right)$, takes in an individual's value and returns the optimal offer (i.e., the offer which maximizes expected profit) for that value. Figure 1 displays this optimal offer function, assuming $n=8$ and $p=5$ or $p=3$, where it can be clearly seen that, in target-constrained reverse discriminative auctions, low-value individuals can extract substantial rents (equivalent to many times their opportunity costs) from the buyer. Intuitively, for lower opportunity costs, an individual can increase their offer above their opportunity cost to increase potential profits while only slightly decreasing the probability that their offer will receive a contract. On the other hand, when a high opportunity cost individual submits an offer higher than their opportunity cost, they have a small chance
that their offer will be accepted. As a result, the optimal offer function converges to cost revealing offers as an individual's opportunity cost approaches 1 .

Figure 1:


Note that $O_{i, T C}^{*}\left(v_{i}\right)$ is not defined when $v_{i}=1$. Despite this, we can still make the following claim.

Proposition 1: As $v_{i}$ approaches $1, O_{i, T C}^{*}\left(v_{i}\right)$ converges to 1 .

Proof. For all $v_{i}$ between 0 and 1, the numerator of of $O_{i, T C}^{*}\left(v_{i}\right)$ is less than the denominator, so $O_{i, T C}^{*}\left(v_{i}\right)$ is bounded above by 1 for $v_{i}$ between 0 and 1 . Further, given that a non-negative expected profit requires $O_{i, T C}^{*}\left(v_{i}\right) \geq v_{i}, O_{i, T C}^{*}\left(v_{i}\right)$ is bounded below by $v_{i}$. Both $y=v_{i}$ and $y=1$ converge to 1 as $v_{i}$ approaches 1 , so $O_{i, T C}^{*}\left(v_{i}\right)$ converges to 1 as $v_{i}$ approaches 1 by the sandwich theorem.

It is also informative (and will be useful in future proofs) to show that $O_{i, T C}^{*}\left(v_{i}\right)$ is a strictly increasing function in $v_{i}$. But first, the following proposition and proof are made simpler by rewriting $O_{i, T C}^{*}\left(v_{i}\right)$ with the regularized beta function, given by:

$$
I_{x}(a, b)=\frac{\int_{0}^{x} t^{a-1}(1-t)^{b-1} d t}{B(a, b)}
$$

To rewrite $O_{i, T C}^{*}\left(v_{i}\right)$ in terms of the regularized beta function, we multiply the numerator and denominator of $O_{i, T C}^{*}\left(v_{i}\right)$ by $\frac{B(p+1, n-p)}{B(p+1, n-p)}$, where $B(p+1, n-p)$ is represents the beta function with parameters $p+1$ and $n-p$.

$$
\begin{equation*}
O_{i, T C}^{*}\left(v_{i}\right)=\frac{\int_{v_{i}}^{1} u^{p}(1-u)^{n-p-1} d u * \frac{B(p+1, n-p)}{B(p+1, n-p)}}{\int_{v_{i}}^{1} u^{p-1}(1-u)^{n-p-1} d u * \frac{B(p, n-p)}{B(p, n-p)}}=\frac{1-I_{v_{i}}(p+1, n-p)}{1-I_{v_{i}}(p, n-p)} * \frac{B(p+1, n-p)}{B(p, n-p)} \tag{15}
\end{equation*}
$$

Given that $B(x, y)=\frac{\Gamma(x) \Gamma(y)}{\Gamma(x+y)}$, Equation 15 simplifies to

$$
\begin{equation*}
O_{i, T C}^{*}\left(v_{i}\right)=\frac{1-I_{v_{i}}(p+1, n-p)}{1-I_{v_{i}}(p, n-p)} * \frac{p}{n} \tag{16}
\end{equation*}
$$

One convenient property of the regularized beta is that

$$
\begin{equation*}
I_{v_{i}}(p+1, n-p)=I_{v_{i}}(p, n-p)-\frac{v_{i}^{p}\left(1-v_{i}\right)^{n-p-1}}{p B(p, n-p)} \tag{17}
\end{equation*}
$$

and thus Equation 16 can be rewritten as:

$$
\begin{equation*}
O_{i, T C}^{*}\left(v_{i}\right)=\frac{1-I_{v_{i}}(p, n-p)+\frac{v_{i}^{p}\left(1-v_{i}\right)^{n-p-1}}{p B(p, n-p)}}{1-I_{v_{i}}(p, n-p)} * \frac{p}{n}=\frac{p}{n}+\frac{v_{i}^{p}\left(1-v_{i}\right)^{n-p-1}}{n B(p, n-p)\left(1-I_{v_{i}}(p, n-p)\right)} \tag{18}
\end{equation*}
$$

Proposition 2: $O_{i, T C}^{*}\left(v_{i}\right)$ is a strictly increasing function of $v_{i}$ for $v_{i}$ between 0 and 1 .

Proof. Applying the quotient rule, the derivative of $O_{i, T C}^{*}\left(v_{i}\right)$ with respect to $v_{i}$ is given by

$$
\begin{array}{r}
\frac{\partial O_{i, T C}^{*}\left(v_{i}\right)}{\partial v_{i}}=\left[\left(\left(p v_{i}^{p-1}\left(1-v_{i}\right)^{n-p-1}+(n-p-1)\left(1-v_{i}\right)^{n-p-2} v_{i}^{p}\right) *\right.\right. \\
\left.\left.\left(n B(p, n-p)\left(1-I_{v_{i}}(p, n-p)\right)\right)+n v_{i}^{p}\left(1-v_{i}\right)^{n-p-1} v_{i}^{p-1}\left(1-v_{i}\right)^{n-p-1}\right)\right] /  \tag{19}\\
n^{2} B(p, n-p)^{2}\left(1-I_{v_{i}}(p, n-p)\right)^{2}
\end{array}
$$

Factoring out $v_{i}^{p-1}$ and $\left(1-v_{i}\right)^{n-2 p-2}$, and dividing the numerator and denominator by $n$ yields

$$
\begin{equation*}
\frac{v_{i}^{p-1}\left(1-v_{i}\right)^{n-2 p-2}\left(\left(1-v_{i}\right)^{p}\left(p-n v_{i}+v_{i}\right) B(p, n-p)\left(1-I_{v_{i}}(p, n-p)\right)+\left(1-v_{i}\right)^{n} v_{i}^{p}\right)}{n B(p, n-p)^{2}\left(1-I_{v_{i}}(p, n-p)\right)^{2}} \tag{20}
\end{equation*}
$$

We want to show that

$$
\begin{equation*}
0<\frac{v_{i}^{p-1}\left(1-v_{i}\right)^{n-2 p-2}\left(\left(1-v_{i}\right)^{p}\left(p-n v_{i}+v_{i}\right) B(p, n-p)\left(1-I_{v_{i}}(p, n-p)\right)+\left(1-v_{i}\right)^{n} v_{i}^{p}\right)}{n B(p, n-p)^{2}\left(1-I_{v_{i}}(p, n-p)\right)^{2}} \tag{21}
\end{equation*}
$$

for all $v_{i}$ between 0 and 1 . Note that $v_{i}^{p-1}\left(1-v_{i}\right)^{n-2 p-2}$ and $p B(p, n-p)^{2}\left(1-I_{v_{i}}(p, n-p)\right)^{2}$ are both positive, and thus both can be cancelled out without affecting the direction of the inequality. Equation 21 thus holds when

$$
\begin{equation*}
-\left(1-v_{i}\right)^{p}\left(p-n v_{i}+v_{i}\right) B(p, n-p)\left(1-I_{v_{i}}(p, n-p)\right)<\left(1-v_{i}\right)^{n} v_{i}^{p} \tag{22}
\end{equation*}
$$

Dividing both sides by $n\left(1-v_{i}\right)^{p} B(p, n-p)\left(1-I_{v_{i}}(p, n-p)\right)$ yields

$$
\begin{equation*}
-\left(\frac{p}{n}-v_{i}+\frac{v_{i}}{n}\right)<\frac{\left(1-v_{i}\right)^{n-p} v_{i}^{p}}{n B(p, n-p)\left(1-I_{v_{i}}(p, n-p)\right)} \tag{23}
\end{equation*}
$$

A slight rearrangement of terms yields

$$
\begin{equation*}
v_{i}\left(1-\frac{1}{n}\right)<\frac{p}{n}+\frac{\left(1-v_{i}\right)^{n-p} v_{i}^{p}}{n B(p, n-p)\left(1-I_{v_{i}}(p, n-p)\right)} \tag{24}
\end{equation*}
$$

Notice that the righthand side of Equation 24 is the optimal offer function for $O_{i, T C}^{*}\left(v_{i}\right)$ from

Equation 18. Also note that $\left(1-\frac{1}{n}\right)<1$. Equation 24 thus implies Equation 25 below.

$$
\begin{equation*}
v_{i}\left(1-\frac{1}{n}\right)<v_{i}<O_{i, T C}^{*}\left(v_{i}\right) \tag{25}
\end{equation*}
$$

Given that profit maximization requires $O_{i, T C}^{*}\left(v_{i}\right)>v_{i}$ for all $v_{i}$ in $[0,1)$, the optimal offer function is increasing for all $v_{i}$ in $[0,1)$.

### 4.2 Set-up for Provision Point Reverse Auction

The provision point reverse auction is a discriminative reverse auction with the added requirement that $p$ of the $n$ total offers must be affordable for any contracts to be made, given the exogenous budget $B$. We call this additional requirement the "provision point requirement." In a PPRA, an individual has to consider several factors when choosing their offer. Like most discriminative auctions, the individual must weigh the increase in potential profit from a higher offer against the decreased probability that a given offer will be accepted. In a PPRA, a higher offer decreases the probability of contract acceptance through two avenues. First, a higher offer makes it less likely that the offer will be one of the $p$ lowest offers, and thus less likely that the offer will receive one of the $p$ possible contracts. Second, a higher offer decreases the probability that the provision point requirement will be met, and thus reduces the probability that any contracts will be provided.

The provision point requirement can be viewed as an "average" reservation price. In reverse auctions, a reservation price is the highest acceptable offer a seller can make to the buyer. That is, the buyer will not purchase any units for a price higher than the reservation price. By setting the budget and the provision point, the buyer implies that they will not spend more than $B / p$, on average, for the $p$ units. The average reservation price allows individuals with opportunity costs higher than the average reservation price to receive a contract by incentivizing lower opportunity cost individuals to submit lower offers. For example, in a PPRA, an individual can submit a bid higher than $B / p$ and still receive a contract if at least one of the other $p$ lowest offers is less than $B / p$, while in an auction with a reservation price of $B / p$ this is not possible.

Looking back to equation 1 , in a PPRA, the probability that an offer, $o_{i}$, receives a contract is the probability that $o_{i}$ is one of the $p$ lowest offers times the probability that the provision point requirement is met given that $o_{i}$ is one of the p-lowest offers. If either the provision point requirement is not met or $o_{i}$ is not one of the $p$ lowest offers, then $o_{i}$ will not receive a contract. Thus, the expected profit function for an individual participating in a PPRA is given by:

$$
\begin{gather*}
E[\Pi]=\left(o_{i}-v_{i}\right) * \operatorname{Pr}\left(o_{i} \text { is one of the } p \text { lowest offers }\right) *  \tag{26}\\
\operatorname{Pr}\left(\mathrm{PPR} \text { is met given } o_{i} \text { is one of the } \mathrm{p} \text { lowest offers }\right)
\end{gather*}
$$

When considering the probability the provision point requirement will be met, an individual is interested in the expected value of the excess budget, given the sum of the expected offers of the other low cost individuals. That is, the individual is interested in the difference between the budget and what they expect the sum of the other $p-1$ lowest bids to be. If their offer is one of the $p$ lowest and is greater than the excess budget, the provision point requirement will not be met because the sum of the $p$ lowest offers will exceed the budget. On the other hand, if their offer is one of the $p$ lowest and is less than the excess budget, the provision point requirement will be met as the sum of the $p$ lowest offers will be less than the budget. First, if we assume that the other individuals submit offers according to a common offer function, $O_{j}$, and we assume the budget, $B$, is given exogenously, then the expected value of the excess budget is

$$
\begin{equation*}
E[\text { Excess Budget }]=B-\sum_{j=1}^{p-1} E\left[O_{j}\left(v_{(j)}\right) \mid o_{i}<o_{(p)}\right] \tag{27}
\end{equation*}
$$

where $v_{(j)}$ is the jth lowest opportunity cost. Intuitively, the expected value of the excess budget tells an individual the expected offer which, on average, would just meet the provision point requirement. The variance in the distribution of the excess budget suggests the degree to which the probability the provision point requirement will be met changes with small changes in an individual's offer. Gupta and Sobel (1958) show that the sum of standard
uniform order statistics is asymptotically normal. Thus, if the assumed offering behavior, $O_{j}\left(v_{j}\right)$, is cost-revealing, then this distribution would be asymptotically normal. However, because individuals will not submit cost-revealing offers, we cannot use this approximation. In fact, given that the offer function for other individuals will generally not have a closed form, we believe it is unlikely that a closed form representation of Equation 27 exists.

To summarize, an individual's probability of submitting one of the $p$ lowest offers, given their offer and assumed offering behavior of other individuals, is described by Equation 5 Given the individual submits one of the $p$ lowest offers, the probability that the provision point requirement is met is given by the probability that $o_{i}$ is less than the excess budget, with the expected value of the excess budget given in Equation 27.

Before we proceed further, we require the following axiom which follows directly from Proposition 1.

Axiom 1: If the probability that an individual receives a contract is 0 in any auction, then their optimal offering behavior is to submit an offer at their opportunity cost.

This axiom is important because it defines optimal offering behavior for values for which the optimal offer function might not be defined. For example, the optimal offer function for the target-constrained auction (see Equation 14) is not defined when $v_{i}=1$. A natural conclusion from this fact is that the optimal offer for individuals with $v_{i}=1$ is 1 in both the target-constrained auction and the provision point reverse auction. With this background, we provide the following proposition.

Proposition 3: Suppose $O_{i, T C}^{*}\left(v_{i} \mid n, p\right)$ is the symmetric Bayesian Nash equilibrium optimal offer function for the target-constrained auction with a target of $p<n$ and $O_{i, P P}^{*}\left(v_{i} \mid n, p, B\right)$ is the symmetric Bayesian Nash equilibrium optimal offer function for the provision point reverse auction with a provision point requirement of $p<n$ and a budget of $B$. (From this point on, $O_{i, T C}^{*}\left(v_{i} \mid n, p\right)$ and $O_{i, P P}^{*}\left(v_{i} \mid n, p, B\right)$ will be simplified as $O_{i, T C}^{*}\left(v_{i}\right)$ and $O_{i, P P}^{*}\left(v_{i}\right)$, respectively.) Additionally, assume Axiom 1 holds. Then $O_{i, T C}^{*}\left(v_{i}\right)=O_{i, P P}^{*}\left(v_{i}\right)$ if and only
if either i) any single participant in the auction cannot affect the probability the provision point requirement is met by increasing or decreasing their offer or ii) $v_{i}=1$.

Proof. The expected profit function for the target-constrained auction is given by Equation 2. Let $g\left(n, p, o_{i}\right)$ represent the probability that an offer is one of the $p$ lowest and let $o_{i, T C}^{*}$ represent the optimal offer, given $v_{i}$, in a target-constrained auction. Note that $g\left(n, p, o_{i}\right)$ is a decreasing function in $o_{i}$; the larger $o_{i}$, the less likely it is one of the $p$ lowest offers. Expected profit for the target-constrained auction is maximized where the first order conditions are met:

$$
\begin{equation*}
\left(o_{i, T C}^{*}-v_{i}\right)=\frac{-g\left(n, p, o_{i, T C}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, T C}^{*}\right)}{\partial o_{i, T C}^{*}}\right)} \tag{28}
\end{equation*}
$$

Similarly, expected profit for the PPRA is maximized where the first order conditions are met. Let $w\left(n, p, B, o_{i}\right)$ represent the probability that the provision point requirement will be met and let $o_{i, P P}^{*}$ represent the optimal offer, given $v_{i}$, in a provision point reverse auction. Note that $w\left(n, p, B, o_{i}\right)$ is a non-increasing function of $o_{i}$; as a given offer gets larger, the likelihood that the provision point requirement is met does not increase. Then the first order condition for the PPRA is:

$$
\begin{equation*}
\left(o_{i, P P}^{*}-v_{i}\right)=\frac{-g\left(n, p, o_{i, P P}^{*}\right) * w\left(n, p, B, o_{i, P P}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * w\left(n, p, B, o_{i, P P}^{*}\right)+\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * g\left(n, p, o_{i, P P}^{*}\right)\right)} \tag{29}
\end{equation*}
$$

Suppose $O_{i, T C}^{*}\left(v_{i}\right)=O_{i, P P}^{*}\left(v_{i}\right)$. Then $o_{i, T C}^{*}=o_{i, P P}^{*}$ for all $v_{i}$. Multiplying the top and bottom of the right-hand side of Equation 29 by the reciprocal of its numerator yields

$$
\begin{equation*}
\left(o_{i, P P}^{*}-v_{i}\right)=\frac{-1}{\left(\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{g\left(n, p, o_{i, P P}^{*}\right)}+\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{w\left(n, p, B, o_{i, P P^{*}}\right)}\right)} \tag{30}
\end{equation*}
$$

From Equation 28, and given that $o_{i, T C}^{*}=o_{i, P P}^{*}$, we have

$$
\begin{equation*}
\left(o_{i, P P}^{*}-v_{i}\right)=\frac{-1}{\left(\frac{-1}{\left(o_{i, P P}^{*}-v_{i}\right)}+\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{w\left(n, p, B, o_{i, P P^{*}}\right)}\right)} \tag{31}
\end{equation*}
$$

Multiplying both sides by the denominator of the right-hand side yields

$$
\begin{equation*}
\left(o_{i, P P}^{*}-v_{i}\right) *\left(\frac{-1}{\left(o_{i, P P}^{*}-v_{i}\right)}+\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{w\left(n, p, B, o_{i, P P^{*}}\right)}\right)=-1 \tag{32}
\end{equation*}
$$

Which simplifies to

$$
\begin{equation*}
0=\left(o_{i, P P}^{*}-v_{i}\right) *\left(\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{w\left(n, p, B, o_{i, P P^{*}}\right)}\right) \tag{33}
\end{equation*}
$$

Equation 33 implies that either $o_{i, P P}^{*}=v_{i}$ or $\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}}=0$. Given that $o_{i, T C}^{*}=v_{i}$ only when $v_{i}=1$, the two optimal offer functions can be the same only when each participant cannot affect the probability the provision point requirement is met by changing their offer or $v_{i}=1$.

To prove the other direction, suppose that no individual can affect the probability the provision point requirement is met by changing their offer. Then, by definition, $\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}}=$ 0 and, using Equation 29

$$
\begin{equation*}
\left(o_{i, P P}^{*}-v_{i}\right)=\frac{-g\left(n, p, o_{i, P P}^{*}\right) * w\left(n, p, B, o_{i, P P}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * w\left(n, p, B, o_{i, P P}^{*}\right)+0 * g\left(n, p, o_{i, P P}^{*}\right)\right)} \tag{34}
\end{equation*}
$$

Simplifying Equation 34 provides

$$
\begin{equation*}
\left(o_{i, P P}^{*}-v_{i}\right)=\frac{-g\left(n, p, o_{i, P P}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}}\right)} \tag{35}
\end{equation*}
$$

which is the first order condition for the target-constrained auction. If instead of assuming that no individual can affect the probability the provision point requirement is met we assume
that $v_{i}=1$, the result follows immediately from Axiom 1 .

Proposition 3 provides our first theoretical prediction: when the parameters of a PPRA are such that no single participant can affect the probability that the provision point requirement is met, the optimal offer function for all participants in the auction is the optimal offer function for a target-constrained auction. Proposition 4 expands upon Proposition 3.

Proposition 4: Suppose $O_{i, T C}^{*}\left(v_{i}\right)$ is the symmetric Bayesian Nash equilibrium optimal offer function for the target-constrained auction with a target of $p<n$ and $O_{i, P P}^{*}\left(v_{i}\right)$ is the symmetric Bayesian Nash equilibrium optimal offer function for the provision point reverse auction with a provision point requirement of $p<n$ and a budget of $B$. Further suppose that $O_{i, T C}^{*}\left(v_{i}\right)$ is convex in $v_{i}$. If a participant in the auction can impact the probability that the provision point requirement is met, then $O_{i, T C}^{*}\left(v_{i}\right) \geq O_{i, P P}^{*}\left(v_{i}\right)$ for all $v_{i}$.

Proof. Equations 28 and 29 provide the first order conditions for the optimal offer for an individual competing in a target-constrained auction and a provision point reverse auction, respectively. Note that $g\left(n, p, o_{i}^{*}\right)$ and $w\left(n, p, B, o_{i}^{*}\right)$ are decreasing and non-increasing in $o_{i}^{*}$, respectively, so that both $\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}}$ and $\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}}$ are less than or equal to zero. We prove by contradiction. Suppose that $o_{i, T C}^{*} \leq o_{i, P P}^{*}$. Then, combining Equations 30 and 28, we have:

$$
\begin{array}{r}
\left(\frac{-1}{\left(\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{g\left(n, p, o_{i, P P}^{*}\right)}+\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{w\left(n, p, B, o_{i, P P^{*}}\right)}\right)}>\right. \\
\frac{-g\left(n, p, o_{i, T C}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, T C}^{*}\right)}{\partial o_{i, T C}^{*}}\right)} \tag{36}
\end{array}
$$

Note that both sides of the equation are positive. Thus, multiplying both sides of the equation by their reciprocal does not reverse the inequality. The resulting rearrangement is
given below.

$$
\begin{array}{r}
-\left(\frac{\partial g\left(n, p, o_{i, T C}^{*}\right)}{\partial o_{i, T C}^{*}}\right) * \frac{1}{g\left(n, p, o_{i, T C}^{*}\right)}>  \tag{37}\\
\left(-\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{g\left(n, p, o_{i, P P}^{*}\right)}-\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{w\left(n, p, B, o_{i, P P^{*}}\right)}\right)
\end{array}
$$

Given our assumptions about $w\left(n, p, B, o_{i, P P}\right)$ and $g\left(n, p, o_{i, P P}\right)$, we know that

$$
\begin{gather*}
-\frac{\partial w\left(n, p, B, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{w\left(n, p, B, o_{i, P P^{*}}\right)} \geq 0  \tag{38}\\
-\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{g\left(n, p, o_{i, P P}^{*}\right)} \geq 0 \tag{39}
\end{gather*}
$$

Returning to Equation 37, if the sum of Equations 38 and 39 is less than the left-hand side of Equation 37, then we know that Equation 39 is also less than the left-hand side of Equation 37

$$
\begin{equation*}
-\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}} * \frac{1}{g\left(n, p, o_{i, P P}^{*}\right)}<-\left(\frac{\partial g\left(n, p, o_{i, T C}^{*}\right)}{\partial o_{i, T C}^{*}}\right) * \frac{1}{g\left(n, p, o_{i, T C}^{*}\right)} \tag{40}
\end{equation*}
$$

which further implies that

$$
\begin{equation*}
-\frac{g\left(n, p, o_{i, P P}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}}\right)}>-\frac{g\left(n, p, o_{i, T C}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, T C}^{*}\right)}{\partial o_{i, T C}^{*}}\right)} \tag{41}
\end{equation*}
$$

The completion of this proof requires a lemma.

Lemma 1: Suppose $O_{i, T C}^{*}\left(v_{i}\right)$ is the symmetric Bayesian Nash equilibrium optimal offer function for the target-constrained auction with a target of $p<n$. Additionally, suppose that $O_{i, T C}^{*}\left(v_{i}\right)$ is a convex function. Then the difference between a given optimal offer, $o_{i, T C}^{*}$, and its corresponding value, $v_{i}$, is a decreasing function in $v_{i}$.

Proof. Equation 28 provides the first order condition for the optimal offer, given a value $v_{i}$, in a target-constrained auction. The left-hand side of Equation 28 provides the difference
between an optimal offer and its corresponding value. Taking a derivative with respect to $v_{i}$ on both sides yields

$$
\begin{equation*}
\frac{\partial O_{i, T C}^{*}\left(v_{i}\right)}{\partial v_{i}}-1=\partial\left(\frac{-g\left(n, p, o_{i, T C}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, T C}^{*}\right)}{\partial o_{i, T C}^{*}}\right)}\right) / \partial v_{i} \tag{42}
\end{equation*}
$$

Proposition 2 states that $O_{i, T C}^{*}\left(v_{i}\right)$ is an increasing function, and the convexity assumption implies that the second derivative of $O_{i, T C}^{*}\left(v_{i}\right)$ is positive over the range $[0,1)$ as well. If $\frac{\partial O_{i, T C}^{*}\left(v_{i}\right)}{\partial v_{i}}$ was greater than 1 for any $v_{i}$ in this range, then $\frac{\partial O_{i, T C}^{*}\left(v_{j}\right)}{\partial v_{i}}$ would also have to be greater than 1 , for any $v_{j}>v_{i}$, by convexity. Recall that $O_{i, T C}^{*}\left(v_{i}\right)$ is bounded below by the 45 degree line and that $O_{i, T C}^{*}\left(v_{i}\right)$ converges to 1 as $v_{i}$ converges to 1 , by Proposition 1 . If the derivative of $O_{i, T C}^{*}\left(v_{i}\right)$ was ever greater than 1 , then $O_{i, T C}^{*}\left(v_{i}\right)$ would not converge to 1 as $v_{i}$ converged to 1 . Thus, $\frac{\partial O_{i, T C}^{*}\left(v_{i}\right)}{\partial v_{i}}$ can never be greater than 1 . This fact, along with Equation 42, immediately provides the desired result.

Returning to the proof for Proposition 4, Lemma 1 states that

$$
\begin{equation*}
\partial\left(\frac{-g\left(n, p, o_{i, T C}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, T C}^{*}\right)}{\partial o_{i, T C}^{*}}\right)}\right) / \partial v_{i}<0 \tag{43}
\end{equation*}
$$

The only avenue through which $v_{i}$ affects $-\frac{g\left(n, p, o_{i, P P}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{i, P P}^{*}\right)}{\partial o_{i, P P}^{*}}\right)}$ is $o_{i}$. Further, because $o_{i}$ is an increasing function of $v_{i}$, we have

$$
\begin{equation*}
\partial\left(\frac{-g\left(n, p, o_{i, T C}^{*}\right)}{\left(\frac{\partial g\left(n, p, o_{, T C}^{*}\right)}{\partial o_{i, T C}^{*}}\right)}\right) / \partial o_{i}<0 \tag{44}
\end{equation*}
$$

Equation 44, along with the assumption that $o_{i, P P}^{*}>o_{i, T C}^{*}$, implies that Equation 41 is a contradiction.

Propositions 3 and 4 tell us that we expect the optimal offer curve for the PPRA to be weakly below the optimal offer curve for a target-constrained auction with the same parameter values. The degree to which the optimal offer curve for the PPRA lies below the
optimal offer curve for the TC auction depends on the parameter values chosen.

## 5 Experimental Design and Protocol

To test our predictions, we conducted ten experimental sessions totalling 240 undergraduate students in LEEDR (Lab for Experimental Economics and Decision Research) at Cornell University. The ten sessions were divided into five treatments of two sessions each. The five treatments consisted of one budget-constrained treatment with a budget of 4.42 , two targetconstrained treatments with targets of five and three, and two provision point treatments with a budget of 4.42 , one with a provision point requirement of five and the other with a provision point requirement of three. Each session lasted at most 40 minutes. Average earnings were approximately $\$ 24$ for each participant, with a range from $\$ 12$ to $\$ 35$. In each session, the 24 students were split evenly into three groups. Before the start of each session, the participants were given written instructions, which are included in the appendix. These written instructions include the following information:

1. The number of participants in a group (8).
2. The target or provision point requirement (5 or 3), if relevant.
3. The budget (\$4.42), if relevant.
4. The common distribution from which all opportunity costs were drawn, $\mathrm{U}(0,2)$.

We chose $n=8$ because we wanted relatively small group sizes to both increase our sample and to increase the impact of the provision point requirement. The first PPRA treatment chose a provision point requirement of $p=5$ because we wanted a relatively large number of participants to contribute to the provision point requirement, but we believed a target of 6 or 7 individuals would have led to larger offers in the target-constrained auction. In addition, we wanted the initial parameters to be such that the participants in the auction could not divide the budget equally among themselves. That is, we wanted the fifth highest opportunity cost in each group to be larger than the budget divided by 5 . If at least one
of the five lowest opportunity costs is greater than the budget divided by the provision point requirement, we say that the auction is "psychologically binding." To test for the robustness of the mechanism, we followed these sessions with an additional PPRA treatment but with a provision point requirement of three instead. This second PPRA treatment was not psychologically binding, as the budget divided by three was larger than the third highest opportunity cost in all groups. ${ }^{2}$

For the purpose of common knowledge, one author read from a series of PowerPoint slides which included an overview of the experimental instructions. After the PowerPoint presentation, all subjects participated in 5 practice rounds where parameter values varied. In each round, the participants selected an offer between $\$ 0$ and $\$ 7$, where $\$ 7$ was set as the maximum allowable offer. After each round, the participants were informed whether their offer was accepted and how much they were paid. If they were in the provision point reverse auction treatment, they were also informed if the provision point requirement was met. After the five practice rounds, opportunity costs were re-randomized and a series of 8 rounds began where the budget, target, provision point requirement and opportunity costs for each individual were fixed. Before the 9th round, the groups and opportunity costs were randomized once more and another 8 rounds were conducted to end the experiment. The results of the experiments are described in Section 6 .

## 6 Results

### 6.1 Difference in Means

The first comparison between auction formats is a simple difference in means test between treatments and within rounds. Our experimental format provided two sets of 8 rounds which we pooled to increase our statistical power. That is, we consider the offers from

[^1]Rounds 1 and 9 jointly, the offers from Rounds 2 and 10 jointly, and so on. Given our varying parameter values, we compare the differences between formats with comparable restrictions. For example, we compare the target-constrained auction with a target of 5 , the budget-constrained auction with a budget $=4.42$, and the PPRA with a provision point requirement of 5 and a budget of 4.42 , and then compare the target-constrained auction with a target of 3 , the budget-constrained auction with a budget $=4.42$, and the PPRA with a provision point requirement of 3 and a budget of 4.42 , etc. The results are given in Table 1 and 2 below.

In each table, columns (1), (2) and (3) provide the mean offers for each treatment in each set of rounds, while columns (4) and (5) provide the difference in means between the TC and BC treatments and the PPRA. There are several important results in Table 1. First, the target-constrained treatment has higher average offers than either of the other two treatments. Indeed, the difference in means between the the provision point reverse auction and the target-constrained treatment is above $\$ 1$ in most rounds. The theory predicted that average offers would be higher in the target-constrained auction than the provision point reverse auction, but the magnitude of the differences is somewhat surprising. Second, the budget-constrained treatment has higher average offers than the PPRA as well, albeit to a lesser extent. In most rounds, the budget-constrained treatment has offers more than $\$ 0.20$ higher than its provision point counterpart. Third, notice that while the average offers are relatively stable across rounds for the PPRA and budget-constrained auction, the target-constrained auction saw its average offers decrease over time. This runs contrary to previously established theoretical results, which suggest offers increase over time in a target-constrained auction. (See Section 3). Instead, it seems individuals submitted high offers in the first round, and their offers decreased over time as the participants competed over contracts. This may be a result of the relatively small group size, as conversation (and therefore collusion) between individuals was not permitted during the experiment. The statistically significant differences in means support our claim that the PPRA can reduce offers when compared to the target- or budget-constrained treatments.

Table 1: Mean Offers - Target $=5$, Budget $=4.42, \mathrm{PPR}=5$
Mean Offers Difference: PPRA \&

| Rounds | (PPRA) | (TC) | (BC) | TC | BC |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (5) |
| $1 \& 9$ | 1.124 | 2.716 | 1.383 | $1.593 * * *$ | 0.259*** |
|  | (0.450) | (1.105) | (0.687) | (0.123) | (0.086) |
| $2 \& 10$ | 1.115 | 2.440 | 1.401 | $1.324^{* * *}$ | 0.285*** |
|  | (0.487) | (0.740) | (0.670) | (0.090) | (0.085) |
| $3 \& 11$ | 1.143 | 2.389 | 1.372 | $1.246^{* * *}$ | 0.228*** |
|  | (0.498) | (0.644) | (0.620) | (0.083) | (0.081) |
| 4 \& 12 | 1.145 | 2.257 | 1.420 | $1.111^{* * *}$ | $0.274^{* * *}$ |
|  | (0.490) | (0.501) | (0.859) | (0.071) | (0.101) |
| $5 \& 13$ | 1.137 | 2.223 | 1.347 | $1.086^{* * *}$ | 0.209*** |
|  | (0.464) | (0.430) | (0.529) | (0.065) | (0.072) |
| 6 \& 14 | 1.200 | 2.161 | 1.335 | 0.961*** | 0.135* |
|  | (0.577) | (0.371) | (0.508) | (0.070) | (0.078) |
| 7 \& 15 | 1.167 | 2.090 | 1.360 | $0.923 * * *$ | 0.193** |
|  | (0.475) | (0.328) | (0.550) | (0.059) | (0.074) |
| $8 \& 16$ | 1.186 | 2.101 | 1.364 | 0.915*** | 0.178** |
|  | (0.576) | (0.604) | (0.587) | (0.085) | (0.084) |
| All | 1.152 | 2.297 | 1.373 | $1.115^{* * *}$ | 0.220*** |
|  | (0.507) | (0.662) | (0.633) | (0.030) | (0.029) |
|  |  | 0.01, * | $\mathrm{p}<0.05$, | $\mathrm{p}<0.1$ |  |

Note: The above table contains the mean for each of the three auction treatments and difference in means between the TC and BC auction treatments and the PPRA, with the standard errors below for the means or differences in means. PPRA denotes the provision point reverse auction, TC denotes the target-constrained auction and BC denotes the budget-constrained auction. The results above are for target-constrained auctions with a target of 5 , a budgetconstrained auction with a budget of 4.42 and a provision point auction with a provision point requirement of 5 and a budget of 4.42. The offers were pooled by rounds, so that the offers from rounds 1 and 9 were considered jointly, the offers from rounds 2 and 10 were considered jointly, etc.

Table 2 provides the results from additional experiments with different parameter values, where both the target-constraint and the provision point requirement were set to three. First, note that average offers are always less in the PPRA than either the target- or budgetconstrained auction, but that the differences are not statistically significant in most rounds. This agrees with our intuitive expectations, where a smaller target with a constant budget is less restrictive than a larger target with the same budget. Indeed, these results are

Table 2: Mean Offers - Target $=3$, Budget $=4.42, \mathrm{PPR}=3$
Mean Offers Difference: PPRA \&

|  | (PPRA) | (TC) | (BC) | TC | BC |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Rounds | (1) | (2) | (3) | (4) | (5) |
| $1 \& 9$ | 1.249 | 1.631 | 1.383 | 0.382*** | 0.134 |
|  | (0.628) | (0.556) | (0.687) | (0.086) | (0.095) |
| $2 \& 10$ | 1.197 | 1.454 | 1.401 | $0.257^{* * *}$ | 0.203** |
|  | (0.561) | (0.407) | (0.670) | (0.071) | (0.089) |
| 3 \& 11 | 1.252 | 1.409 | 1.372 | 0.157** | 0.120 |
|  | (0.639) | (0.381) | (0.620) | (0.076) | (0.091) |
| 4 \& 12 | 1.269 | 1.400 | 1.420 | 0.131 | 0.151 |
|  | (0.672) | (0.457) | (0.859) | (0.083) | (0.111) |
| $5 \& 13$ | 1.239 | 1.362 | 1.347 | 0.123 | 0.108 |
|  | (0.636) | (0.481) | (0.529) | (0.081) | (0.084) |
| 6 \& 14 | 1.225 | 1.361 | 1.335 | 0.137 | 0.111 |
|  | (0.601) | (0.695) | (0.508) | (0.094) | (0.080) |
| 7 \& 15 | 1.319 | 1.339 | 1.360 | 0.021 | 0.041 |
|  | (0.766) | (0.584) | (0.550) | (0.098) | (0.096) |
| $8 \& 16$ | 1.323 | 1.412 | 1.364 | 0.089 | 0.041 |
|  | (0.922) | (0.785) | (0.587) | (0.124) | (0.112) |
| All | 1.259 | 1.421 | 1.373 | 0.162*** | $0.114^{* * *}$ |
|  | (0.685) | (0.563) | (0.633) | (0.032) | (0.034) |

Note: The above table contains the mean for each of the three auction treatments and difference in means between the TC and BC auction treatments and the PPRA, with the standard errors below for the means or difference in means. PPRA denotes the provision point reverse auction, TC denotes the target-constrained auction and BC denotes the budget-constrained auction. The results above are for target-constrained auctions with a target of 3 , a budgetconstrained auction with a budget of 4.42 and a provision point auction with a provision point requirement of 3 and a budget of 4.42. The offers were pooled by rounds, so that the offers from rounds 1 and 9 were considered jointly, the offers from rounds 2 and 10 were considered jointly, etc.
generally consistent with the contention that, even when the provision point requirement is not more restrictive than the target or budget constraint, the provision point auction provides lower average offers. Also note that, with these parameter values, the target- and budget-constrained auctions provide more comparable average offers than seen in Table 1 , where the target-constrained auction resulted in substantially higher offers.

Tables 1 and 2 provide differences in means across all offers. The buyer, however, is
primarily interested in the lower offers because those offers actually receive contracts and result in payments from the buyer. Thus, a comparison of means of lower offers between auction formats would provide more information about improvements in the buyer's welfare than a comparison of all offers. The difference in means for the lowest five offers between the target-constrained treatment with a target of five, the budget-constrained treatment with a budget of 4.42 , and the PPRA with a budget of 4.42 and provision point requirement of five are given in Table 3. Table 3 shows comparable differences to Table 1 and provides additional support that the PPRA may be an attractive alternative to the target- and budget-constrained auctions from the perspective of the buyer. Indeed, the mean of the five lowest offers in a PPRA was between $19.4 \%$ and $25.6 \%$ smaller in the tested provision point reverse auctions than the comparable budget-constrained auction, depending on the round. One advantage of comparing the lower offers is that we remove large outliers from our comparison. Indeed, as seen in Table 3, we have statistically significant differences which were not observed in Table 1 because of an outlier in the provision point treatment.

The difference in means for the lowest three offers between the target-constrained treatment with a target of three, the budget-constrained treatment with a budget of 4.42, and the PPRA with a budget of 4.42 and provision point requirement of three are given in Table 4 Table 4 shows statistically significant differences in means between the three auction formats in most rounds, and thus suggests that the PPRA can yield improvements in the buyer's welfare for an additional set of parameter values. More specifically, the mean of the three lowest offers in tested provision point auctions was between $8.9 \%$ and $15.7 \%$ smaller than the comparable mean in the budget-constrained auctions, depending on the rounds. Indeed, Table 4 provides more compelling evidence than Table 2 that the PPRA can lower offers, even when the provision point requirement isn't "psychologically binding." (See Section 5)

### 6.2 Offer Functions

Figures 2 and 3 below display the fitted offer functions and individual offers (grouped by similar parameter values) observed from experiments across all rounds, assuming an exponential

Table 3: Mean Lowest 5 Offers - Pooled Rounds

|  | Mean Offers |  |  | Differenc |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Rounds | PPRA <br> (1) | $\mathrm{TC}$ <br> (2) | $\mathrm{BC}$ <br> (3) | $\mathrm{TC}$ <br> (4) | $\mathrm{BC}$ <br> (5) |
| $1 \& 9$ | $\begin{gathered} 0.822 \\ (0.316) \end{gathered}$ | $\begin{gathered} 2.160 \\ (0.765) \end{gathered}$ | $\begin{gathered} 1.042 \\ (0.394) \end{gathered}$ | $\begin{gathered} 1.339^{* * *} \\ (0.107) \end{gathered}$ | $\begin{gathered} 0.220^{* * *} \\ (0.065) \end{gathered}$ |
| $2 \& 10$ | $\begin{gathered} 0.814 \\ (0.292) \end{gathered}$ | $\begin{gathered} 2.069 \\ (0.608) \end{gathered}$ | $\begin{gathered} 1.093 \\ (0.320) \end{gathered}$ | $\begin{gathered} 1.255^{* * *} \\ (0.087) \end{gathered}$ | $\begin{gathered} 0.280^{* * *} \\ (0.056) \end{gathered}$ |
| $3 \& 12$ | $\begin{gathered} 0.838 \\ (0.323) \end{gathered}$ | $\begin{gathered} 2.104 \\ (0.495) \end{gathered}$ | $\begin{gathered} 1.095 \\ (0.307) \end{gathered}$ | $\begin{gathered} 1.266^{* * *} \\ (0.076) \end{gathered}$ | $\begin{gathered} 0.256^{* * *} \\ (0.058) \end{gathered}$ |
| $4 \& 12$ | $\begin{gathered} 0.848 \\ (0.323) \end{gathered}$ | $\begin{gathered} 2.027 \\ (0.397) \end{gathered}$ | $\begin{gathered} 1.070 \\ (0.306) \end{gathered}$ | $\begin{gathered} 1.179^{* * *} \\ (0.066) \end{gathered}$ | $\begin{gathered} 0.222^{* * *} \\ (0.057) \end{gathered}$ |
| $5 \& 13$ | $\begin{gathered} 0.854 \\ (0.299) \end{gathered}$ | $\begin{gathered} 2.025 \\ (0.352) \end{gathered}$ | $\begin{gathered} 1.089 \\ (0.246) \end{gathered}$ | $\begin{gathered} 1.171^{* * *} \\ (0.060) \end{gathered}$ | $\begin{gathered} 0.235^{* * *} \\ (0.050) \end{gathered}$ |
| 6 \& 14 | $\begin{gathered} 0.885 \\ (0.307) \end{gathered}$ | $\begin{gathered} 2.016 \\ (0.333) \end{gathered}$ | $\begin{gathered} 1.103 \\ (0.223) \end{gathered}$ | $\begin{gathered} 1.131^{* * *} \\ (0.058) \end{gathered}$ | $\begin{gathered} 0.218^{* * *} \\ (0.049) \end{gathered}$ |
| 7 \& 15 | $\begin{gathered} 0.886 \\ (0.325) \end{gathered}$ | $\begin{gathered} 1.962 \\ (0.281) \end{gathered}$ | $\begin{gathered} 1.099 \\ (0.223) \end{gathered}$ | $\begin{gathered} 1.076^{* * *} \\ (0.055) \end{gathered}$ | $\begin{gathered} 0.213^{* * *} \\ (0.051) \end{gathered}$ |
| $8 \& 16$ | $\begin{gathered} 0.870 \\ (0.310) \end{gathered}$ | $\begin{gathered} 1.934 \\ (0.298) \end{gathered}$ | $\begin{gathered} 1.094 \\ (0.224) \end{gathered}$ | $\begin{gathered} 1.064^{* * *} \\ (0.056) \end{gathered}$ | $\begin{gathered} 0.225^{* * *} \\ (0.049) \end{gathered}$ |
| All | $\begin{gathered} 0.852 \\ (0.311) \end{gathered}$ | $\begin{gathered} 2.037 \\ (0.471) \end{gathered}$ | $\begin{gathered} 1.086 \\ (0.285) \end{gathered}$ | $\begin{gathered} 1.185^{* * *} \\ (0.026) \end{gathered}$ | $\begin{gathered} 0.234^{* * *} \\ (0.019) \end{gathered}$ |

Note: The above table contains the mean of the lowest five offers for each of the three auction treatments and difference in means between the five lowest offers for the TC and BC auction treatments and the PPRA, with the standard errors for the means or difference in means below in parentheses. PPRA denotes the provision point reverse auction, TC denotes the target-constrained auction and BC denotes the budget-constrained auction. The results above are for target-constrained auctions with a target of 5 , a budget-constrained auction with a budget of 4.42 and a provision point auction with a provision point requirement of 5 and a budget of 4.42 . The offers were pooled by rounds, so that the offers from rounds 1 and 9 were considered jointly, the offers from rounds 2 and 10 were considered jointly, etc.
specification for the offer functions. We chose the exponential specification both because of it's similarity to the optimal offer curve for the target-constrained auction (see Figure 1) and because it fits the data well, particularly compared to either a linear or quadratic specification.

These figures show the degree to which individuals submitted offers above their opportunity costs across the different treatments and for the different parameter values. In some

Table 4: Mean Lowest 3 Offers - Pooled Rounds

| Table 4: Mean Lowest 3 Offers |  |  |  |  | Pooled Rounds |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mean Offers |  |  |  |  | Difference: PPRA \& |  |
|  | PPRA | TC | BC | TC | BC |  |
| Rounds | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |  |
| $1 \& 9$ | 0.750 | 1.142 | 0.823 | $0.392^{* * *}$ | 0.073 |  |
|  | $(0.279)$ | $(0.346)$ | $(0.284)$ | $(0.074)$ | $(0.066)$ |  |
| $2 \& 10$ | 0.777 | 1.094 | 0.922 | $0.317^{* * *}$ | $0.145^{* *}$ |  |
|  | $(0.268)$ | $(0.257)$ | $(0.253)$ | $(0.062)$ | $(0.061)$ |  |
| $3 \& 12$ | 0.845 | 1.100 | 0.937 | $0.255^{* * *}$ | 0.092 |  |
|  | $(0.237)$ | $(0.248)$ | $(0.266)$ | $(0.057)$ | $(0.059)$ |  |
| $4 \& 12$ | 0.827 | 1.081 | 0.918 | $0.254^{* * *}$ | 0.091 |  |
|  | $(0.241)$ | $(0.224)$ | $(0.269)$ | $(0.055)$ | $(0.060)$ |  |
| $5 \& 13$ | 0.836 | 1.033 | 0.979 | $0.197^{* * *}$ | $0.143^{* *}$ |  |
|  | $(0.238)$ | $(0.204)$ | $(0.231)$ | $(0.052)$ | $(0.055)$ |  |
| $6 \& 14$ | 0.864 | 1.010 | 0.995 | $0.176^{* * *}$ | $0.131^{* * *}$ |  |
|  | $(0.211)$ | $(0.219)$ | $(0.190)$ | $(0.051)$ | $(0.047)$ |  |
| $7 \& 15$ | 0.872 | 0.993 | 1.002 | $0.121^{* *}$ | $0.130^{* * *}$ |  |
|  | $(0.193)$ | $(0.223)$ | $(0.200)$ | $(0.049)$ | $(0.046)$ |  |
| $8 \& 16$ | 0.849 | 0.992 | 0.998 | $0.143^{* * *}$ | $0.149^{* * *}$ |  |
|  | $(0.206)$ | $(0.200)$ | $(0.207)$ | $(0.048)$ | $(0.049)$ |  |
| All | 0.828 | 1.056 | 0.947 | $0.228^{* * *}$ | $0.119^{* * *}$ |  |
|  | $(0.236)$ | $(0.247)$ | $(0.244)$ | $(0.020)$ | $(0.020)$ |  |
|  | $* * *$ | $\mathrm{p}<0.01$, | $* *$ | $\mathrm{p}<0.05$, | $*$ |  |
| $\mathrm{p}<0.1$ |  |  |  |  |  |  |

Note: The above table contains the mean of the lowest three offers for each of the three auction treatments and difference in means between the three lowest offers for the TC and BC auction treatments and the PPRA, with the standard errors below for means or differences in means. PPRA denotes the provision point reverse auction, TC denotes the target-constrained auction and BC denotes the budget-constrained auction. The results above are for target-constrained auctions with a target of 3, a budgetconstrained auction with a budget of 4.42 and a provision point auction with a provision point requirement of 3 and a budget of 4.42. The offers were pooled by rounds, so that the offers from rounds 1 and 9 were considered jointly, the offers from rounds 2 and 10 were considered jointly, etc.
instances, individuals actually submitted offers below their opportunity costs, represented by the 45 degree line. In a provision point reverse auction, it is possible that this behavior is altruistic: some individuals are decreasing their offers below their opportunity costs in the hope of satisfying the provision point requirement, and thus allowing some of their peers to receive contracts. Why some individuals in the budget-constrained auction chose to submit offers below their opportunity cost is less clear, although the behavior was largely limited to

Figure 2:

only a few participants. Each offer function is surrounded by a shaded region, representing a $95 \%$ confidence interval. Given the large variance in offers within treatments, we suggest greater consideration of the difference in average offers than the difference in coefficients on fitted functions. The variance in offers is consistent with individuals struggling to determine optimal offering behavior. It is hardly surprising given the computational difficulty of determining an optimal offer for any of the three auction formats.

Figures 4 and 5 provide the offers and fitted offer curves for the first and ninth rounds and the eighth and sixteenth rounds, respectively, for the treatments with a target or provision point requirement of five and a budget of 4.42 , while Figures 6 and 7 provide similar representations of the data for treatments with a target or provision point requirement of 3 and a budget of 4.42 .

Figure 3:


### 6.3 Efficiency Analysis

We are interested not only in comparing the three auction treatments with each other, but also against the theoretical predictions for the uniform reverse auction. In a uniform reverse auction, the buyer sets a target and the winning individuals receive the first rejected offer as payment, similar to a Vickrey second price auction. Theoretically, we expect individuals in a uniform procurement auction will submit their opportunity costs as their offers. To compare the auction formats we use three criteria to measure their efficacy. The first measure is social efficiency, which we define as follows:

$$
\begin{equation*}
\text { Social Efficiency }=\frac{\sum_{i}^{p} v_{(i)}}{\sum_{i}^{p} v_{i}} \times 100 \tag{45}
\end{equation*}
$$

Figure 4:

where $v_{(i)}$ is the ith smallest opportunity cost in the auction. In other words, social efficiency is the minimum opportunity cost required to supply five contracts divided by the opportunity cost of the individuals who received contracts. From society's perspective, welfare is maximized when the lowest opportunity cost individuals receive the available contracts. This result does not necessarily hold in instances with positive externalities like we might expect from PES programs, but the measure is informative nonetheless.

The second measure is simply the total cost to the buyer of purchasing five contracts. This allows us to compare cost savings for the buyer across the different auction mechanisms, and thus the amount of money the buyer must spend, on average, for the five units of environmental service.

Finally, we use a "cost effectiveness" measure to further compare how costly the auctions

Figure 5:

are for the buyer. We define this measure as follows:

$$
\begin{equation*}
\text { Cost Effectiveness }=\frac{\text { Uniform Auction Cost }- \text { Other Auction Cost }}{\text { Uniform Auction Cost }- \text { Total Opportunity Cost }} \tag{46}
\end{equation*}
$$

By definition, if the participants submitted offers equal to their opportunity costs, the cost efficiency measure would be $100 \%$, while the cost efficiency measure is $0 \%$ for the uniform auction.

Tables 5 and 6 below provide the efficiency and cost effectiveness measures for the various auctions by their parameter values. The OC column provides the measures for a hypothetical discriminative auction where individuals submit their opportunity costs as offers. In such an auction, all of the welfare gains would be given to the buyer and the auction would be $100 \%$ socially efficient. As such, it serves as the ideal auction from the perspective of the buyer.

Figure 6:


There are two important problems to discuss before continuing to the efficiency measures. First, we cannot compare the budget-constrained auction to the other formats directly with these measures because the buyer was almost never able to afford 5 contracts in the budgetconstrained treatment. Thus, questions like "how much did it cost the buyer to purchase five contracts" aren't reasonable. Second, the provision point auctions didn't always result in contracts in the treatment with $\mathrm{PPR}=5$, as the provision point requirement wasn't met in approximately $33 \%$ of the rounds. (The PPR was met in every round for the treatment with $\mathrm{PPR}=3$.) As a result, it isn't always sensible to compare the PPRA to the targetconstrained and uniform price auctions. Instead, we present only the efficiency measure for the PPRA when the provision point requirement was met. This alters the efficiency estimates slightly when the $\mathrm{PPR}=5$, but does not alter the analysis when the $\mathrm{PPR}=3$.

Table 5: Efficiency Measures, Target $/ \mathrm{PPR}=5$

|  | OC | Uniform | TC | PPRA |
| :--- | :---: | :---: | :---: | :---: |
| Social Efficiency | $100 \%$ | $100 \%$ | $71.46 \%$ <br> $(14.89 \%)$ | $95.98 \%$ <br> $(9.40 \%)$ |
| Avg. Total Cost of <br> Providing 5 Units | $\$ 3.02$ | $\$ 6.64$ | $\$ 10.19$ | $\$ 4.07$ |
| Cost Effectiveness | $100 \%$ | $0 \%$ | $-97.96 \%$ <br> $(29.65 \%)$ | $71.12 \%$ <br> $(12.03 \%)$ |

Note: The above table contains efficiency measures for several different auction formats. Standard errors are enclosed in parentheses below their given estimates. The OC column contains the results from a theoretical discriminative auction where all individuals submit their opportunity costs as offers. The Uniform column contains the predicted results from a uniform price auction. The TC column contains the experimental results for the target-constrained auction and the PPRA column contains the experimental results for the provision point reverse auction in rounds where the provision point requirement was met.

Table 6: Efficiency Measures, Target/PPR $=3$

|  | OC | Uniform | TC | PPRA |
| :--- | :---: | :---: | :---: | :---: |
| Social Efficiency | $100 \%$ | $100 \%$ | $64.61 \%$ <br> $(24.65 \%)$ | $76.8 \%$ <br> $(22.85 \%)$ |
| Avg. Total Cost of <br> Providing 3 Units | $\$ 1.06$ | $\$ 2.64$ | $\$ 3.17$ | $\$ 2.48$ |
| Cost Effectiveness | $100 \%$ | $0 \%$ | $-33.82 \%$ <br> $(34.21 \%)$ | $9.66 \%$ <br> $(35.92 \%)$ |

Note: The above table contains efficiency measures for several different auction formats. Standard errors are enclosed in parentheses below their given estimates. The OC column contains the results from a theoretical discriminative auction where all individuals submit their opportunity costs as offers. The Uniform column contains the predicted results from a uniform price auction. The TC column contains the experimental results for the target-constrained auction and the PPRA column contains the experimental results for the provision point reverse auction in rounds where the provision point requirement was met.

Figure 7:


Unsurprisingly, given the theoretical predictions, the target-constrained auction performs the worst by all three measures, regardless of the parameter values. Indeed, the targetconstrained auction costs over twice as much, on average, as the provision point reverse auction and costs nearly $80 \%$ more than the predictions for the uniform auction as well, when the target $=\mathrm{PPR}=5$. On the other hand, the provision point reverse auction was only slightly less socially efficient than the predictions for the uniform auction when the PPR $=5$, although the PPR achieved lower social efficiency than the predictions for the uniform price auction when the $\mathrm{PPR}=3$. In summary, the PPRA performs better than the uniform price auction from the perspective of the buyer, while it performs slightly worse than the uniform price auction by social efficiency. However, the difference in the social efficiency measure is not statistically significant for the session where the provision point requirement
equalled 5.

## 7 Discussion

Given the structure of the PPRA, we believe it will be particularly effective when three criteria hold true. First, the provision point requirement is most appealing when there is some threshold of service before which the buyer accrues less or no benefits. For instance, consider a situation where a government or NGO wishes to pay local farmers to adopt environmentally friendly practices to improve the water quality in a local lake. The government wishes to reintroduce fish to this lake, but in order for the fish to survive, pollution must be reduced by some quantity. If this threshold is not met, then the government is not interested in purchasing any contracts. In such a situation, the PPRA can ensure the government either some level of environmental service or the welfare they obtain from retaining their budget.

Second, we believe that the PPRA will be particularly effective for auctions with small numbers of participants who all operate in a given region. As the number of participants in a PPRA becomes smaller, the ability of any individual to affect the probability the provision point requirement is met increases, which increases the impact of the provision point requirement on offering behavior. Further, we believe that individuals who know each other will be more likely to take the welfare of the other participants into account. As such, a PPRA which takes place in a particular region may increase the salience of the provision point requirement even further.

Finally, the PPRA will be most effective at reducing offers when the cost of running an auction is low and when the buyer can move the program to a new location when the provision point requirement isn't met. The buyer may forgo substantial welfare opportunities if they cannot eventually provide contracts to some individuals, and thus the ability to move the auction to a new location at relatively little cost will decrease the chance the buyer is not able to purchase some environmental service.

As an example of a setting which satisfies these three criteria, consider the BirdReturns ${ }^{\ominus}$ program in California. In the BirdReturns ${ }^{\ominus}$ program, rice farmers in the Central Valley of

California are paid by conservationists and aviphiles to flood their paddies to create small habitats for migratory birds. The number of rice farmers in a given area is relatively small, and if a certain number of these "pop-up habitats" are not created, then the birds will not be able to use the regions as stepping stones along their journey. There are several potential areas in the Central Valley that could serve as pop-up habitats, so the conservationists and aviphiles could move to a new location if they cannot afford a certain number of contracts.

Finally, while the provision point reverse auction has the potential to function well in some settings, it certainly would not be appropriate for all procurement auctions. For example, electricity markets use reverse auctions to allocate contracts to energy producers. A PPRA in this context would mean that no electricity would be produced when the provision point requirement is not met, which would be an unacceptable outcome given that demand for electricity is inelastic.

## 8 Conclusion

The Provision Point Reverse Auction has the potential to increase the efficiency and cost effectiveness of conservation and PES programs, while simultaneously decreasing uncertainty for the purchasers of the environmental goods. Our experimental and theoretical results support this claim, showing that the PPRA can save the procurer between either $21.55 \%$ to $58.17 \%$ or $12.57 \%$ to $21.59 \%$ of their costs, on average, depending on parameter values and whether the alternative is a budget- or target-constrained auction, respectively. Further, the PPRA also improves social efficiency over the target-constrained auction, reducing the total cost of the environmental service to society.

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## 9 Appendix

This appendix contains the experiment instructions for the target-constrained auction, the budget-constrained auction and the provision point reverse auction in that order.

## InSTRUCTIONS

This experiment is a study of individual decision-making in a group setting. If you follow these instructions carefully and make informed decisions, you will earn money. The money you earn will be paid to you, in cash, after the experiment has concluded. We ask that you do not use any electronic devices during this experiment, including cell phones, tablets, etc. We further ask that you do not communicate with your peers in any capacity. If you have any questions, please raise your hand and the researchers will come to assist you.

In this experiment, you are a member of a group consisting of eight individuals. You and the other seven individuals each own one unit of a good that can be rented out each round. Each unit of the good is indistinguishable from any of the other units owned by your fellow participants. Additionally, each participant in your group will be given an individual valuation for their own unit of the good, which we will call their opportunity cost (more details will follow). Individuals who do not rent out their unit will receive this opportunity cost as payment at the end of each round. A single buyer is interested in renting five units of the good each round. (The buyer is not interested in renting more than five units) The buyer will pay individuals for their units using an auction. The auction will be conducted as follows:

- Each round, you will submit an offer representing the amount of money for which you would be willing to rent out your unit during that round. (Your offers will be capped at $\$ 7$ each round. If you try to submit an offer higher than $\$ 7$, you will be asked to enter a different offer.)
- Your peers will also submit their own offers for their units.
- The buyer will then rank these offers in ascending order and provide contracts to the lowest offer, then the second lowest offer, and so on until the fifth lowest offer.
- If you do receive a contract from the buyer, the buyer will take your unit (for that round) and you will receive your offer as payment (for that round).
- If you do not receive a contract from the buyer, you will keep your unit and receive your opportunity cost as payment for that round.

An individual's opportunity cost (in experimental dollars) will be drawn from a uniform distribution from 0 to 2, in increments of 0.01 . In other words, you will randomly receive a number between 0 and 2, where each number is equally likely to be drawn. For example, the odds that you receive opportunity cost $=1.15$ are the same as the odds that you receive opportunity cost $=0.82$ are the same as the odds that you receive opportunity cost $=0.23$, etc. As such, each of the eight individuals in your group will be randomly assigned an opportunity cost and will formulate offers given this information. All individuals in your group know only their own opportunity costs and that the other individuals in their group have opportunity costs drawn from the same distribution. You will not know the opportunity costs of any of your peers.

After eight rounds, you will be randomly assigned to a new group of eight individuals. (The random assignments were made before this session by drawing numbers from a hat. The assignment will not be based on the offers made in previous rounds.) In addition, you will receive a new opportunity cost, drawn from the same distribution as before. Each of the other 23 participants in this room will also be randomly assigned to a new group of eight individuals, and will also draw new opportunity costs. If there are any questions about this process, please raise your hand and ask one of the researchers.

The experiment will be complete after 16 rounds. All experimental dollars will be converted to real dollars using a one-to-one ratio. Before the experiment begins, the researcher will briefly discuss the experiment with you using a PowerPoint presentation. There will also be five practice rounds where all 24 participants will participate in rounds of the auction. After these rounds, new opportunity costs and groups will be assigned, and the experiment will begin.

## Summary

- At the beginning of the experiment, you will receive a randomly drawn opportunity cost between 0 and 2 , with each value in that range being equally likely.
- Based on that opportunity cost, each round you will submit an offer to the buyer for your unit.
- For each round, if out of your group of eight, your offer is one of the five lowest offers, you will receive your offer as payment.
- For additional clarification: in order to receive your offer as payment instead of your opportunity cost, your offer must be accepted by the buyer. For your offer to be accepted, your offer must be one of the five lowest. If your offer is not one of the five lowest offers, you will receive your opportunity cost as payment.
- After each round, you will once again have possession of your unit, and will be able to participate in the auction during the next round.
- After eight rounds, you and the other 23 participants in the experiment will be randomly assigned to new groups of eight with new, randomly assigned opportunity costs. You will maintain these new groups and opportunity costs for the remaining 8 rounds.


## InSTRUCTIONS

This experiment is a study of individual decision-making in a group setting. If you follow these instructions carefully and make informed decisions, you will earn money. The money you earn will be paid to you, in cash, after the experiment has concluded. We ask that you do not use any electronic devices during this experiment, including cell phones, tablets, etc. We further ask that you do not communicate with your peers in any capacity. If you have any questions, please raise your hand and the researchers will come to assist you.

In this experiment, you are a member of a group consisting of eight individuals. You and the other seven individuals each own one unit of a good that can be rented out each round. Each unit of the good is indistinguishable from any of the other units owned by your fellow participants. Additionally, each participant in your group will be given an individual valuation for their own unit of the good, which we will call their opportunity cost (more details will follow). Individuals who do not rent out their unit will receive this opportunity cost as payment at the end of each round. A single buyer is interested in renting units of the good each round. However, the buyer has a limited budget and thus will pay individuals for their units using an auction. The auction will be conducted as follows:

- Each round, you will submit an offer representing the amount of money for which you would be willing to rent out your unit during that round. (Your offers will be capped at $\$ 7$ each round. If you try to submit an offer higher than $\$ 7$, you will be asked to enter a different offer.)
- Your peers will also submit their own offers for their units.
- The buyer will then rank these offers in ascending order and provide contracts to the lowest offer, then the second lowest offer, and so on until the buyer's budget is exhausted.
- If you do receive a contract from the buyer, the buyer will take your unit (for that round) and you will receive your offer as payment (for that round).
- If you do not receive a contract from the buyer, you will keep your unit and receive your opportunity cost as payment for that round.

An individual's opportunity cost (in experimental dollars) will be drawn from a uniform distribution from 0 to 2, in increments of 0.01 . In other words, you will randomly receive a number between 0 and 2, where each number is equally likely to be drawn. For example, the odds that you receive opportunity cost $=1.15$ are the same as the odds that you receive opportunity cost $=0.82$ are the same as the odds that you receive opportunity cost $=0.23$, etc. As such, each of the eight individuals in your group will be randomly assigned an opportunity cost and will formulate offers given this information. All individuals in your group know only their own opportunity costs and that the other individuals in their group have opportunity costs drawn from the same distribution. You will not know the opportunity costs of any of your peers.

After eight rounds, you will be randomly assigned to a new group of eight individuals. (The random assignments were made before this session by drawing numbers from a hat. The assignment will not be based on the offers made in previous rounds.) In addition, you will receive a new opportunity cost, drawn from the same distribution as before. Each of the other 23 participants in this room will also be randomly assigned to a new group of eight individuals, and will also draw new opportunity costs. This process will occur every eight rounds. If there are any questions about this process, please raise your hand and ask one of the researchers.

The experiment will be complete after 16 rounds. All experimental dollars will be converted to real dollars using a one-to-one ratio. Before the experiment begins, the researcher will briefly discuss the experiment with you using a PowerPoint presentation. There will also be five practice rounds where all 24 participants will participate in rounds of the auction. After these rounds, new opportunity costs and groups will be assigned, and the experiment will begin.

## Summary

- At the beginning of the experiment, you will receive a randomly drawn opportunity cost between 0 and 2 , with each value in that range being equally likely.
- Based on that opportunity cost, each round you will submit an offer to the buyer for your unit.
- The buyer has a budget, $\$ 4.42$, with which to award contracts. The buyer accepts offers in ascending order, from smallest to largest until their budget is exhausted.
- In a given round, if you receive a contract, you will rent out your unit and receive your offer as payment for that round. (You will earn your offer instead of your opportunity cost as payment.)
- If you do not receive a contract, you will keep your unit and earn your opportunity cost as payment for that round.
- After each round, you will once again have possession of your unit, and will be able to participate in the auction during the next round.
- After eight rounds, you and the other 23 participants in the experiment will be randomly assigned to new groups of eight with new, randomly assigned opportunity costs. The budget will remain constant across all 16 rounds.


## INSTRUCTIONS

This experiment is a study of individual decision-making in a group setting. If you follow these instructions carefully and make informed decisions, you will earn money. The money you earn will be paid to you, in cash, after the experiment has concluded. We ask that you do not use any electronic devices during this experiment, including cell phones, tablets, etc. We further ask that you do not communicate with your peers in any capacity. If you have any questions, please raise your hand and the researchers will come to assist you.

In this experiment, you are a member of a group consisting of eight individuals. You and the other seven individuals each own one unit of a good that can be rented out each round. Each unit of the good is indistinguishable from any of the other units owned by your fellow participants. Additionally, each participant in your group will be given an individual valuation for their own unit of the good, which we will call their opportunity cost (more details will follow). Individuals who do not rent out their unit will receive this opportunity cost as payment at the end of each round. A single buyer is interested in renting five units of the good each round. (The buyer is not interested in renting more than five units) However, the buyer has a limited budget and thus will pay individuals for their units using an auction. The auction will be conducted as follows:

- Each round, you will submit an offer representing the amount of money for which you would be willing to rent out your unit during that round. (Your offers will be capped at $\$ 7$ each round. If you try to submit an offer higher than $\$ 7$, you will be asked to enter a different offer.)
- Your peers will also submit their own offers for their units.
- The buyer will then rank these offers in ascending order and provide contracts to the lowest offer, then the second lowest offer, and so on until the fifth lowest offer.
- If you do receive a contract from the buyer, the buyer will take your unit (for that round) and you will receive your offer as payment (for that round).
- If you do not receive a contract from the buyer, you will keep your unit and receive your opportunity cost as payment for that round.

However, the buyer is only interested in offering contracts to individuals in your group if they can afford at least five of the offers. From this point on, this number (five) will be referred to as the funding threshold. If, given the offers that your group submits, the buyer cannot afford the five lowest offers, then no individual will receive a contract, regardless of the magnitude of their offer. If the buyer can afford at least five offers then the buyer will offer contracts to the participants that submitted the five lowest offers, as described above.

An individual's opportunity cost (in experimental dollars) will be drawn from a uniform distribution from 0 to 2, in increments of 0.01 . In other words, you will randomly receive a number between 0 and 2 , where each number is equally likely to be drawn. For example, the odds that you receive opportunity cost $=1.15$ are the same as the odds that you receive opportunity cost $=0.82$ are the same as the odds that you receive opportunity
cost $=0.23$, etc. As such, each of the eight individuals in your group will be randomly assigned an opportunity cost and will formulate offers given this information. All individuals in your group know only their own opportunity costs and that the other individuals in their group have opportunity costs drawn from the same distribution. You will not know the opportunity costs of any of your peers.

After eight rounds, you will be randomly assigned to a new group of eight individuals. (The random assignments were made before this session by drawing numbers from a hat. The assignment will not be based on the offers made in previous rounds.) In addition, you will receive a new opportunity cost, drawn from the same distribution as before. Each of the other 23 participants in this room will also be randomly assigned to a new group of eight individuals, and will also draw new opportunity costs. This process will occur every eight rounds. If there are any questions about this process, please raise your hand and ask one of the researchers.

The experiment will be complete after 16 rounds. All experimental dollars will be converted to real dollars using a one-to-one ratio. Before the experiment begins, the researcher will briefly discuss the experiment with you using a PowerPoint presentation. There will also be five practice rounds where all 24 participants will participate in rounds of the auction. After these rounds, new opportunity costs and groups will be assigned, and the experiment will begin.

## Summary

- At the beginning of the experiment, you will receive a randomly drawn opportunity cost between 0 and 2 , with each value in that range being equally likely.
- Based on that opportunity cost, each round you will submit an offer to the buyer for your unit.
- The buyer has a budget, $\$ 4.42$, with which to award contracts. The buyer accepts offers in ascending order, from smallest to largest.
- For each round, if out of your group of eight, the buyer cannot afford the lowest five offers, then no contracts will be awarded. If the buyer can afford the five lowest offers, then exactly five contracts will be made with the participants who submitted the five lowest offers.
- In a given round, if you receive a contract, you will rent out your unit and receive your offer as payment for that round. (You will earn your offer instead of your opportunity cost as payment.)
- If you do not receive a contract, you will keep your unit and earn your opportunity cost as payment for that round.
- For additional clarification: in order to receive your offer as payment instead of your opportunity cost, your offer must be accepted by the buyer. For your offer to be accepted, your offer must be one of the five lowest, and the sum of the five
lowest offers must be less than the buyer's budget. If those two conditions are not met, you will receive your opportunity cost as payment.
- After each round, you will once again have possession of your unit, and will be able to participate in the auction during the next round.
- After eight rounds, you and the other 23 participants in the experiment will be randomly assigned to new groups of eight with new, randomly assigned opportunity costs. The budget and funding threshold will remain constant across all 16 rounds.


[^0]:    ${ }^{1}$ Much of the literature uses the term "bid-shading" instead of rent-seeking offers. This term is not appropriate for reverse auctions, however, as "bid-shading" literally means to make a slight reduction in bids, while in reverse auctions, individuals seek to increase profits by increasing their offers.

[^1]:    ${ }^{2}$ Note that individuals drew offers from a $\mathrm{U}(0,2)$ distribution rather than a $\mathrm{U}(0,1)$, as we assumed in the theory section. We made this decision after conducting a pilot experiment where individuals drew costs up to $\$ 1$. We found that, with such low opportunity costs, the individual rounds were no longer salient to the participants. Indeed, the participants became increasingly impatient as the session continued. As a result, we reduced the number of rounds to 16 and increased the maximum opportunity cost to $\$ 2$.

