Testing Symmetry and Homogeneity in the AIDS with Cointegrated Data Using Fully-modified Estimation and the Bootstrap.

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Abstract

Convential SUR estimation of the AIDS is shown to lead to small sample bias and distortions in the size of a Wald test for symmetry and homogeneity when the data are cointegrated. A fully-modified estimator is developed in an attempt to remedy these problems. It is shown that this estimator reduces the small sample bias but fails to eliminate the size distortion. Bootstrapping is shown to be ineffective as a method of removing small sample bias in both the conventional and fully modified estimators. Bootstrapping is effective however as a method of removing the size distortion and performs equally well in this respect with both estimators.

Key words: AIDS, cointegration, fully modified estimation, bootstrapping.

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1 Introduction

The Almost Ideal Demand System *(*AIDS*)* developed by Deaton & Muellbauer (1980) is derived from an indirect expenditure function and can approximate the conditions which are implied by static economic theory, while being sufficiently flexible to frame some of the implied properties as restrictions on a more general model. It has been widely applied to the estimation of food demand functions. A substantial literature now exists reporting applications of this model in a wide variety of contexts. Our intention in this paper is to address some of the criticisms that have been levelled at the model since its inception within the context of the recognition that economic time series are often non-stationary.

Early critiques of the AIDS were concerned with the static nature of the model and a number of attempts were made to incorporate dynamic aspects of consumer choice. Important examples of this literature include the work of Blancifiorti & Green (1983), Anderson & Blundell (1983) and Anderson & Blundell (1984). Blancifiorti and Green focus on the issue of habit persistence in estimating an AIDS where current expenditure patterns are a function of past consumption levels, and in which autoregressive disturbances are accounted for in order to allow for other omitted dynamic influences. Anderson and Blundell adopt a slightly more general approach in developing a dynamic specification which is "in the spirit of the error correction models of Hendry & Von Ungern Sternberg (1981)".

The AIDS is commonly estimated under the assumption that the right-hand-side variables in the model are predetermined. This assumption has been criticised and it has been argued that the errors in the AIDS are likely to be correlated with the regressors for two reasons. First, Eales & Unnevehr (1993) argue that many applications of the AIDS have involved the use of aggregate data and that in such cases it is reasonable to assume that prices and quantities are jointly determined. Second, Buse (1994) argues that construction of Stones's price index that is commonly used to linearise the AIDS leads to a violation of the assumption of predetermined right-hand-side variables.[1]

One of the attractions of the AIDS is that it allows the theoretically implied hypotheses of symmetry and homogeneity are testable. In many applications these hypotheses are rejected. Papers by Laitinen (1978), Meisner (1979) and Bera *et al.* (1981) consider the small sample performance of asymptotic tests of these hypotheses in large demand systems. Whilst these papers do not directly consider the AIDS they indicate that there is likely to be a tendency for over-rejection of the hypotheses in this model. Buse (1998) considers the effect on the performance of various tests for homogeneity of linearising the AIDS using a number of alternative indices. Using Monte Carlo evidence, he argues that test size distortions are sensitive to the "correlation structure of prices, the time-series properties of the data, and the choice of price index".

It is now recognised that many economic time series are nonstationary and it is therefore appropriate to consider the issues identified in the light of this. The estimation of the AIDS using integrated data has been addressed using a number of methods. Ng (1995) specifically considers the issue of testing the homogeneity restriction and uses a method in which the empirical distribution of the relevant test statistics are simulated by parameterising the data generating process and using this as the basis for a Monte Carlo exercise. Attfield (1997) uses the triangular error correction model (TECM) of Phillips (1991), and in considering the theoretically implied restrictions also focuses only on the homogeneity restriction. The

[1] Pashardes (1993) also draws attention to the potential for mis-specification arising through linearisation of the model.

most commonly used method is the Johansen technique as exemplified by Pesaran & Shin (1999), an approach which might be considered the direct descendent of the error correction model of Anderson & Blundell (1983) and Anderson & Blundell (1984).

We extend this literature in a number of directions. First we present a fully modified seemingly unrelated (FM-SUR) estimator, which is in the spirit of the fully modified estimator proposed by Phillips & Hansen (1990). We argue that this estimator adjusts for endogeneity and the presence of nuisance parameters whilst accounting for the possible presence of omitted dynamics and thus, in principle, addresses the issues that have been identified above. We present Monte Carlo evidence in consideration of this contention. The estimator which we propose differs from those of Ng (1995) and Attfield (1997) because it is possible to impose and test cross-equation restrictions of the type implied by symmetry in addition to the within-equation homogeneity restrictions. We also investigate the use of a bootstrapping procedure to derive the small sample properties of both conventional and fully modified SUR estimation. Finally we illustrate our methods with an application to the Blanciforti *et al.* (1986) data and show that, as a by-product of the bootstrapping procedure we are able to impose concavity on the model.

Our results show that the fully modified estimator which we develop performs well when the long-run covariance matrix is known. However when, as in any real application, the matrix must be estimated, the performance of the estimator is comparable to that of conventional estimation with the bootstrap. For this reason we keep the technical details of the fully-modified estimator to a minimum with the aim of focussing attention on the implications of our work for the applied researcher. Full details are available in Balcombe & Tiffin (2002) and on request from the authors.

2 Method

Assuming $m+1$ commodities, in applying the AIDS m share equations of the form:

$$s_{it} = \alpha_i + \sum_{j=1}^{m+1} \gamma_{ij} \ln p_{jt} + \beta_i \ln h_t + u_{it} \tag{1}$$

are estimated, where s_{it} is the share of total expenditure on all m commodities accounted for by the i^{th}commodity, p_{jt} is the price of the j^{th}commodity and h_t is total expenditure deflated by Stones price index. We assume that the price and deflated expenditure series are generated by the following:

$$\ln p_{it} = \mu_i + \ln p_{it-1} + v_{it} \tag{2}$$

$$\ln h_t = \mu + \ln h_{t-1} + v_t \tag{3}$$

In specifying the properties of the error processes in equations 1 to 3 omitted dynamics and endogeneity are accounted for. Thus, a long-run covariance matrix is assumed to represent the correlation structure between u_{it}, v_{it} and v_t. Banerjee *et al.* (1993, p. 240) show how the long-run covariance matrix collapses to the conventional contemporaneous covariance matrix in the absence of serial correlation whilst correlation between the v's and u_{it} leads to endogeneity. Moon (1999) considers the unrestricted estimation of a model such as that in equations 1 to 3 by SUR and suggests replacing the conventional covariance matrix with its long-run counterpart. Paralleling the seminal results of Phillips & Hansen (1990) it is shown that the estimator is consistent but also that the asymptotic distribution is not centred on zero. Noting that that the non-centrality disappears when there is no long-run correlation between the v's and u_{it} Phillips & Hansen (1990, p 112) interpret the non-centrality as a form of conventional simultaneous equations bias arising from the endogeneity of the explanatory variables in equation 1. It is important to note that this bias does not affect the consistency of the estimates as it disappears asymptotically. For this reason it is commonly termed 'second-order bias'.

These results imply that, with integrated data conventional estimation is consistent even in the presence of the type of endogeneity identified with the AIDS. The second order bias which arises in the presence of long-run endogeneity disappears asymptotically. However, in the context of single equation models it has been shown, using Monte Carlo evidence that conventional estimators are biased in small samples when applied to integrated data with endogenous regressors (see Banerjee *et al.* (1986) and Phillips & Hansen (1990)). Furthermore Phillips & Hansen (1990, p. 112) argue that conventional methods of dealing with this bias such as the use of instrumental variables do not eliminate it. In addition, endogeneity leads to the presence nuisance parameters in the asymptotic distribution of the parameters and associated test statistics. Thus whilst conventional methods of estimation may be valid asymptotically in the presence of integrated data, the problems of endogeneity and omitted dynamics continue to bedevil estimation in finite samples.

One solution is to use fully-modified estimation as pioneered by Phillips & Hansen (1990). A fully modified SUR estimator in which it is possible to impose the restrictions involved in the AIDS is presented by Balcombe & Tiffin (2002). The model is reparameterised to impose the restrictions and the two adjustments of fully modified estimation are applied. In the first the data are transformed to remove the (long-run) correlation between the regressors and the disturbance terms in equation 1 and in the second an estimate of the second-order bias term is used to counter its effect. It can be shown that when the true long-run covariance (LRV) matrix is used, the fully modified estimator is asymptotically normal and unbiased.[2] In practice an estimate of the LRV matrix must be used and this is likely to impinge on the properties of the estimator in small samples. Thus we also investigate the use of the bootstrap as a method of simulating the small sample distribution.

Li & Maddala (1997) consider issues involved in the use of the bootstrapping of cointegrating regressions. In particular they consider ways in which dependencies between the observed variables in the model may be preserved in the bootstrap sample. It is clear that this aspect of the bootstrapping problem is important here where intertemporal correlation between observations due to the integrated nature of the data and omitted dynamics, together with contemporaneous correlation between the explanatory variables and shocks in the share equations as a result of endogeneity lead to second order bias and non-normality. In devising the method we use here we follow Li & Maddala (1997) first in using the stationary bootstrap as our method of resampling, and second, in using a sampling scheme in which the residuals generated under the null hypothesis are resampled and used to produce pseudo-data using the estimates obtained with the null hypothesis imposed.

Defining $\hat{\boldsymbol{\eta}}'= \left(\hat{\boldsymbol{\eta}}_1^{'}, \ldots \hat{\boldsymbol{\eta}}_T^{'}\right)$ where $\hat{\boldsymbol{\eta}}_t = (\hat{u}_{1t}, \ldots, \hat{u}_{mt}, \hat{v}_{1t}, \ldots \hat{v}_{m+1,t}, \hat{v}_t)$ is the estimated vector of innovations and residuals from the restricted model in equations 1 to 3, the stationary bootstrap produces a bootstrap sample $\{\boldsymbol{\eta}_t^* | t = 1 \ldots T\}$ by repeatedly drawing a random number of sequential rows from $\hat{\boldsymbol{\eta}}$. $\boldsymbol{\eta}^*$ is then used with

[2] A concise proof is given in Balcombe & Tiffin (2002), extended proofs are available on request.

equations 1 to 3 with parameters estimated under the homogeneity and symmetry restrictions to produce a data-set which is consistent with the null hypothesis in a test of these restrictions. The model is re-estimated repeatedly using such data to simulate the distributions of the parameters and the Wald-test under the null. Bias corrected estimates of the parameters are obtained as:

$$\hat{\mathbf{A}}^*_{BC} = \hat{\mathbf{A}}^* + \left(\hat{\mathbf{A}}^* - \bar{\mathbf{A}}\right) \tag{4}$$

where $\hat{\mathbf{A}}^*$are the parameter estimates form the original sample data, used in producing the bootstrap samples, and $\bar{\mathbf{A}}$ is the mean of all of the estimates obtained from the bootstrap samples.

3 Monte Carlo Study

In order to assess the performance of the estimators in small samples we carry out a Monte Carlo study. The aim is to show how conventional estimation can lead to misleading results in the presence of integrated data and to examine how fully modified estimation and the bootstrap can be used to improve matters. The data are generated using equation 1 to 3 with:

$$\mathbf{A} = \begin{pmatrix} \alpha_1 & \cdots & \alpha_m \\ \gamma_{11} & \cdots & \gamma_{1m} \\ \vdots & & \vdots \\ \gamma_{m+1,1} & \cdots & \gamma_{m+1,m+1} \\ \beta_1 & \cdots & \beta_m \end{pmatrix} = vec \begin{pmatrix} 0 & 0 & \cdots & 0 \\ (m+1)^{-\frac{1}{2}} & -\frac{(m+1)^{-\frac{1}{2}}}{m} & \cdots & -\frac{(m+1)^{-\frac{1}{2}}}{m} \\ -\frac{(m+1)^{-\frac{1}{2}}}{m} & (m+1)^{-\frac{1}{2}} & \cdots & -\frac{(m+1)^{-\frac{1}{2}}}{m} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{(m+1)^{-\frac{1}{2}}}{m} & -\frac{(m+1)^{-\frac{1}{2}}}{m} & \cdots & (m+1)^{-\frac{1}{2}} \\ (-1)^1 & (-1)^2 & \cdots & (-1)^m \end{pmatrix} \tag{5}$$

In this design above, the signal to noise ratio is approximately preserved. For a given error variance, when the regressors are independent and have the same variances in their innovations, prices and incomes in each equation contribute to the same proportion of variance in the dependent variable in both systems. This will not be the case when there is covariance in the regressors, and the finite sample performance will not be invariant to the experimental design. Further, the matrices above do not necessarily approximate the sort of parameters that we would expect to find in an AIDS model. However our aim is to be illustrative rather than comprehensive. The regressors and errors in the model are generated according to:

$$\mathbf{\Delta x}_t = \begin{pmatrix} \Delta \ln p_{1t} \\ \vdots \\ \Delta \ln p_{m+1,t} \\ \Delta \ln h_t \end{pmatrix} = \mathbf{v}_t + \mathbf{J}_{m+2,1} e_t \tag{6}$$

$$\mathbf{u}_t = \begin{pmatrix} u_{1t} \\ \vdots \\ u_{mt} \end{pmatrix} = \mathbf{w}_t + \mathbf{J}_{m,1} e_t \tag{7}$$

$$e_t = \rho e_{t-1} + \nu_t \tag{8}$$

where the vector of innovations

$$(\nu_t \colon \mathbf{w}_t' \colon \mathbf{u}_t') \tag{9}$$

is IID multivariate normal with the identity as its covariance matrix, $\mathbf{v}_t$ is $(m+2)\times 1$, $\mathbf{w}_t$ is $m \times 1$, $\mathbf{J}_{r,c}$ is a $(r \times c)$ matrix of ones and e_t and ν_t are scalars. This simple design has endogeneity and serial correlation, and has the advantage that the true

	$m = 2$		$m = 3$	
Nominal Size	Wald-FM	Wald-SUR	Wald-FM	Wald-SUR
0.1	0.1057	0.3277	0.1109	0.4171
0.05	0.0507	0.2335	0.0588	0.3119

Table 1: Comparison of theoretical and empirical size with theoretical long-run covariance matrix ($m = 2$)

LRV matrix can be obtained analytically.

4 Results

In real applications the long-run covariance matrix must be estimated. However, in order to validate the FM-SUR estimator that has been introduced above we carry out a limited Monte Carlo exercise using the theoretical covariance matrix. We focus on the performance of the Wald test of symmetry and homogeneity restrictions since this provides a useful summary of the joint distribution of the parameter estimates. Table 1 gives a comparison of empirical and nominal size of the Wald test for 2 and 3 equation systems ($m = 2$, $m = 3$) respectively. The results were obtained using the Monte Carlo designed outlined above with $T = 50$, $\rho = 0.5$ and 1000 trials. The results indicate that, for both systems, the Wald test for the fully modified estimator (Wald-FM) has an empirical size close to the nominal size. By comparison the equivalent tests applied to the conventional SUR estimator (Wald-SUR and FM-SUR) substantially over-reject the null hypothesis.

We now turn to a more comprehensive comparison of the conventional and fully modified estimators in which the covariance matrix is estimated. Assuming weak stationarity of the innovations and following Andrews (1991), a consistent estimate of the long run covariance matrix for use in the fully modified estimator can be obtained using a truncated spectral kernel, with automatic bandwidth

T		m 2	3	4	5
25	SUR	14.428	9.734	7.431	5.966
	FM	5.515	4.192	4.508	3.327
50	SUR	7.795	5.639	4.931	3.251
	FM	3.179	1.260	1.701	1.718
100	SUR	4.258	3.119	1.931	1.736
	FM	0.689	0.754	0.324	0.850

Table 2: Bias summary statistic ($\times 10^3$)

selection based on an AR1 approximation of the covariance structure. An initial estimate of the long-run covariance matrix is obtained using the residuals from SUR estimation. A subsequent iteration is employed in which the residuals from FM-SUR are used. The SUR estimator used for comparison is the conventional one in which the conventional variance covariance matrix is estimated iteratively in the usual way. We compare the estimators in terms of their bias and the empirical size of the Wald test of the symmetry and homogeneity restrictions. In all of the ensuing results $\rho = 0.5$ and we perform 1000 Monte Carlo trials to compare the performance of the conventional and fully modified SUR estimators for a variety of sample and system sizes.

In order to summarise the magnitude of the bias across all of the estimated parameters we compute the average of the absolute value of the bias across all parameters and refer to this as the 'bias summary statistic'. Table 2 reports the statistic decreases with sample and system size. The reduction in the bias of the SUR estimator with sample size illustrates the consistency of the estimator that was discussed above. It can also be seen that the use of the fully modified estimator reduces the second-order bias but does not fully remove it.

Tables 3, 4 and 5 report the empirical size of the Wald test. As expected the SUR estimator exhibits a substantial distortion in size. At the nominal 10%

M		nominal size 0.10	0.05	0.01
2	SUR	0.264	0.166	0.000
	FM	0.494	0.416	0.167
3	SUR	0.697	0.533	0.003
	FM	0.888	0.827	0.364
4	SUR	0.973	0.914	0.001
	FM	0.990	0.976	0.535
5	SUR	0.999	0.993	0.001
	FM	1.000	0.999	0.678

Table 3: Empirical size of Wald tests (T=25)

M		nominal size 0.10	0.05	0.01
2	SUR	0.279	0.187	0.010
	FM	0.332	0.239	0.044
3	SUR	0.679	0.526	0.006
	FM	0.764	0.668	0.113
4	SUR	0.942	0.876	0.008
	FM	0.971	0.949	0.258
5	SUR	0.995	0.986	0.005
	FM	1.000	0.998	0.451

Table 4: Empirical size of Wald tests (T=50)

M		nominal size 0.10	0.05	0.01
2	SUR	0.291	0.197	0.014
	FM	0.223	0.146	0.004
3	SUR	0.687	0.557	0.005
	FM	0.640	0.490	0.017
4	SUR	0.934	0.868	0.012
	FM	0.936	0.864	0.051
5	SUR	0.999	0.993	0.014
	FM	0.998	0.995	0.128

Table 5: Empirical size of Wald tests (T=100)

T		m 2	3	4	5
25	SUR	24.805	14.801	16.094	17.495
	FM	17.369	13.794	13.749	17.241
50	SUR	9.466	7.862	4.539	7.092
	FM	8.040	6.882	4.875	4.361
100	SUR	5.325	2.863	2.014	3.883
	FM	3.041	1.534	2.460	3.160

Table 6: Bias summary statistic ($\times 10^3$) with bootstrap

and 5% levels there is over-rejection whilst at the 1% level there is under-rejection. Increasing the dimensions of the system increases the size distortion at the nominal 10% and 5% levels. Increasing the sample size has a limited impact on the pattern of size distortion. The FM-SUR estimator also exhibits a size distortion. Given the results in Table 1 this suggests that problems in the estimation of the long-run covariance matrix result in the correction for non-normality in the FM estimator performing badly. The size distortion results in an over-rejection at all significance levels when using the FM estimator. The effects of increasing the dimension of the system are similar to those for the SUR estimator but there is some evidence that there is a slight reduction in the distortion as the sample size increases, particularly in the smaller systems.

In order to examine the performance of the bootstrap bias correction method we perform a Monte Carlo trial of the Bootstrap. Thus for each of the 1000 Monte Carlo samples we carry out 500 bootstrap trials and use equation 4 to correct the parameter estimates obtained with the Monte Carlo sample. Table 6 reports the average bias of the bias corrected estimates. Comparing the results in this Table with those in Table 2 it can be seen that the use of the bootstrap certainly does not reduce the bias and may increase it. This is contrary to the findings of Li & Maddala (1997) who find that the bootstrap performs well in reducing the

M		nominal size 0.10	0.05	0.01
2	SUR	0.100	0.050	0.024
	FM	0.102	0.052	0.010
3	SUR	0.116	0.050	0.006
	FM	0.108	0.038	0.010
4	SUR	0.102	0.066	0.028
	FM	0.110	0.058	0.004
5	SUR	0.120	0.076	0.030
	FM	0.112	0.070	0.016

Table 7: Empirical size of Wald tests with bootstrapping (T=25)

small sample bias of a Fully Modified estimator applied to a simple single equation model. The failure of the bootstrap to achieve any correction of the small sample bias may be attributed to the fact that the asymptotic distribution of the regression coefficients is dependent on the unknown population value of the parameter. When the asymptotic distribution of a statistic or parameter is independent of unknown population parameters it is described a asymptotically pivotal. Horowitz (1997) argues that bootstrapping only improves on the approximation to the small sample statistic that is provided by the asymptotic distribution, when the statistic that is being bootstrapped is pivotal. Hence the mean of the bootstrap distribution that is used in the bias correction (equation 4) is likely to be no more accurate in estimating the mean of the small sample distribution than the mean of the appropriate asymptotic distribution.

Tables 7, 8 and 9 report the empirical sizes of the Wald test for symmetry and homogeneity when bootstrapping is used to produce the critical values. It can be seen that in this case the use of the bootstrap is effective in improving performance as the empirical size is close to the nominal size in all cases. In particular, it is interesting to note that the performance of the SUR estimator is

M		nominal size 0.10	0.05	0.01
2	SUR	0.102	0.066	0.028
	FM	0.102	0.058	0.010
3	SUR	0.112	0.066	0.016
	FM	0.088	0.048	0.014
4	SUR	0.122	0.060	0.022
	FM	0.092	0.052	0.008
5	SUR	0.106	0.056	0.012
	FM	0.104	0.042	0.006

Table 8: Empirical size of Wald tests with bootstrapping (T=50)

M		nominal size 0.10	0.05	0.01
2	SUR	0.114	0.076	0.028
	FM	0.108	0.054	0.014
3	SUR	0.126	0.062	0.016
	FM	0.090	0.052	0.012
4	SUR	0.098	0.054	0.020
	FM	0.082	0.040	0.006
5	SUR	0.104	0.064	0.018
	FM	0.092	0.058	0.008

Table 9: Empirical size of Wald tests with bootstrapping (T=100)

as good as that of the fully modified estimator when bootstrapping is employed. The improvement that results from the use of the bootstrap reflects the fact that the asymptotic distribution of the statistic in both estimators is independent of the data generating process and the statistic is therefore pivotal in the sense defined by Horowitz (1997).

5 Empirical Example

To illustrate the application of the methods that have been discussed we re-examine the data used by Blanciforti *et al.* (1986) to estimate food demand functions for the US over the period 1947-1981. The model is estimated for four food commodity groups: meats; fruits and vegetables; cereals and bakery products and miscellaneous foods. We estimate the share equations for the first three aggregate commodities and derive the parameters of the miscellaneous foods equation using the symmetry, adding-up and homogeneity restrictions. We begin by testing for unit roots using the augmented Dickey-Fuller test (see Greene (2000, p. 783)). The lag length used in the equation estimated for this test is chosen by setting a maximum lag length of 5 and then sequentially removing lags that are found to be insignificantly different from zero. In all cases a trend is included in the test equation. The results are given in Table 10 and it can be seen that in all cases it is not possible to reject the null hypothesis that a unit root is present.

We then proceed to estimate the model with SUR, FM-SUR and also apply the bootstrap to both methods to obtain critical values for the Wald test of the symmetry and homogeneity restrictions and to obtain 'bias corrected' estimates. We compute uncompensated and compensated own price elasticities using the

Variable	Aggregate Commodity	Test Statistic	Lags
shares	meats	-2.149	0
	fruit and veg.	-1.804	1
	cereals and bakery products	-2.96	4
prices	meats	-0.414	0
	fruit and veg.	0.234	0
	cereals and bakery products	-1.411	3
	miscellaneous foods	0.659	0
expenditure	-	-0.210	4
critical value	-	-3.50	

Table 10: Results of Unit Root Tests

following equations:

$$\varepsilon_{ii} = -1 + \frac{\gamma_{ii}}{s_i} - \beta_i \tag{10}$$

$$\varepsilon_{ij} = \frac{\gamma_{ij}}{s_i} - \beta_i \frac{s_j}{s_i} \tag{11}$$

$$\phi_{ii} = -1 + \frac{\gamma_{ii}}{s_i} + s_i \tag{12}$$

$$\phi_{ij} = \frac{\gamma_{ij}}{s_i} + s_j \tag{13}$$

$$\psi_i = 1 + \frac{\beta_i}{s_i} \tag{14}$$

where ε_{ij} is the uncompensated elasticity of demand for commodity i with respect to price j, ϕ_{ij} is the compensated elasticity of demand for commodity i with respect to price j and ψ_i is the expenditure elasticity for commodity i.

The Wald statistic obtained with SUR estimation is 13.385 whilst that obtained with FM-SUR is 67.451. Compared with a critical value at the 5% level of 12.592 these results lead to the rejection of homogeneity and symmetry. However bootstrapping gives 5% critical values of 26.797 and 116.662 for SUR and FM-SUR respectively and the hypothesis is not rejected. The own-price elasticity estimates obtained with SUR and FM-SUR are reported in Table 11 together

		SUR	FM-SUR	Bias Corrected SUR	Bias Corrected FM-SUR
Meats	compensated	−0.356	−0.396	−0.344	−0.413
	uncompensated	−0.337	−0.386	−0.280	−0.371
Fruit and Veg.	compensated	0.024	0.114	0.212	0.285
	uncompensated	−0.126	−0.095	0.037	0.056
Cereals and Bakery	compensated	−0.736	−0.762	−0.794	−0.804
	uncompensated	−0.945	−0.957	−0.997	−0.997
Misc. Foods	compensated	−0.724	−0.806	−0.771	−0.875
	uncompensated	−1.384	−1.411	−1.458	−1.495

Table 11: Own price elasticities obtained with alternative methods of estimation

	SUR	FM-SUR	Bias Corrected SUR	Bias Corrected FM-SUR
meats	2.060	2.029	2.209	2.029
meats	1.252	0.959	1.127	0.959
cereal and bakery	0.442	0.544	0.485	0.544
misc. foods	0.142	0.296	0.0667	0.296

Table 12: Expenditure elasticities obtained with alternative methods of estimation

with the bias corrected estimates obtained using the bootstrap. The compensated elasticities of demand for fruit and vegetables obtained with all four methods are positive implying that the model violates concavity. Bias correction actually worsens matters to such an extent that the uncompensated elasticity also becomes positive when bias correction is applied. Note also that for meat, the compensated elasticity is larger in magnitude than the uncompensated elasticity with all four methods of estimation. In Table 13 and 14 we report the full set of compensated and uncompensated elasticities obtained with FM-SUR, imposing curvature with the bootstrap method in the spirit of Chalfant *et al.* (1991). Thus we compute the elasticities using only the draws for which the restriction is satisfied. Table 15 reports the own price elasticities estimated with the alternative estimators and Table 16 reports the expenditure elasticities for all of the estimators.

Quantity	Price			
	meats	fruit and veg.	cereal and bakery	misc. foods
meats	-0.440	-0.176	0.115	0.502
fruit and veg.	-0.273	-0.321	0.153	0.441
cereal and bakery	0.266	0.228	-0.751	0.256
misc. foods	0.438	0.249	0.097	-0.784

Table 13: Compensated elasticities of demand: FM-SUR estimation with bootstrap concavity restriction

Quantity	Price			
	meats	green veg.	cereal and bakery	misc. foods
meats	-0.512	-0.222	0.084	0.420
fruit and veg.	-0.437	-0.426	0.082	0.254
cereal and bakery	-0.184	-0.062	-0.945	-0.259
misc. foods	-0.111	-0.106	-0.141	-1.413

Table 14: Uncompensated elasticities of demand: FM-SUR estimation with bootstrap concavity restriction

			Bias Corrected	
		FM-SUR	SUR	FM-SUR
Meats	compensated	-0.440	-0.402	-0.473
	uncompensated	-0.512	-0.415	-0.526
Fruit and Veg.	compensated	-0.321	-0.180	-0.271
	uncompensated	-0.426	-0.267	-0.352
Cereals and Bakery	compensated	-0.751	-0.758	-0.778
	uncompensated	-0.945	-0.963	-0.975
Misc. Foods	compensated	-0.784	-0.771	-0.870
	uncompensated	-1.413	-1.465	-1.539

Table 15: Own price elasticities obtained with alternative methods of estimation incorporating the bootstrap concavity restriction

			Bias Corrected	
	SUR	FM-SUR	SUR	FM-SUR
meats	1.832	1.771	1.956	1.828
Fruit and veg.	1.496	1.474	1.567	1.596
cereal and bakery	0.434	0.550	0.470	0.533
misc. foods	0.207	0.230	0.045	0.117

Table 16: Expenditure elasticities obtained with alternative methods of estimation incorporating the concavity restriction

Comparing the own-price elasticities in Table 13, 14 and 15 it can be seen that in all cases the elasticities for fruit and vegetables are now negative and the compensated elasticities are smaller in absolute value than their uncompensated counterparts. It is also apparent that the alternative methods of estimation do not lead to substantial differences in the estimates except perhaps in the case of fruit and vegetables where concavity was violated in the earlier results. We find that the demand for meat and miscellaneous food is less elastic than reported by Blanciforti *et al.* (1986) whilst demand for cereal and bakery products is more elastic. Our uncompensated cross price elasticity estimates differ quite substantially from those of Blanciforti *et al.* (1986), and in the majority of cases are of the opposite sign to those reported there. The expenditure elasticities reported in Table 16 are broadly similar across the range of methods applied here and also to those reported by Blanciforti *et al.* (1986).

6 Conclusion

The literature on estimating and testing restrictions in the AIDS has been extended to the case where the data are cointegrated. Earlier work on cointegrated AIDS has concentrated on testing only the homogeneity restriction. Here we extend this literature by developing a fully modified estimator in which symmetry and homogeneity can be tested. We note that, unlike the case where the data are stationary, where the data are integrated conventional estimation is consistent even in the presence of endogeneity. However it is also recognised that substantial small sample bias may remain as a result of the presence of second-order bias in the asymptotic distribution. We show that fully modified estimation can be used to reduce, but not eliminate, bias in small samples. We also note that the non-normality of the conventional estimator when applied to integrated data leads to substantial size distortions in the Wald test of symmetry and homogeneity. We show that the correction applied in the fully modified estimator to make the asymptotic distribution normal has little effect on the size distortion with similar distortion remaining in the fully modified statistic. We believe that the poor performance of the fully modified estimator in this respect may be attributed to difficulties in estimating the long-run covariance matrix. In an effort to remedy the small sample problems of the estimators we explore the use of the bootstrap. We find that the bootstrap has no impact as a method of bias correction and attribute this to the non-pivotal nature of individual parameter estimates. It should be noted that the method of bias correction employed here is amongst the most simple. It may be that more sophisticated methods (see Efron & Tibshriani (1993)) lead to an improvement in performance. By contrast we show that the bootstrap can be used to give empirical sizes which are very close to the nominal counterparts for both SUR and FM-SUR estimation.

Overall, when data are cointegrated, we argue that where interest focuses on individual parameter estimates our fully modified estimator gives a reduction in the small sample bias of the SUR estimator sufficient to justify its use. In cases where interest is solely focused on the testing of restrictions, the use of the bootstrap is vital and its use with conventional SUR estimation is equally appropriate to its use with the fully modified estimator.

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