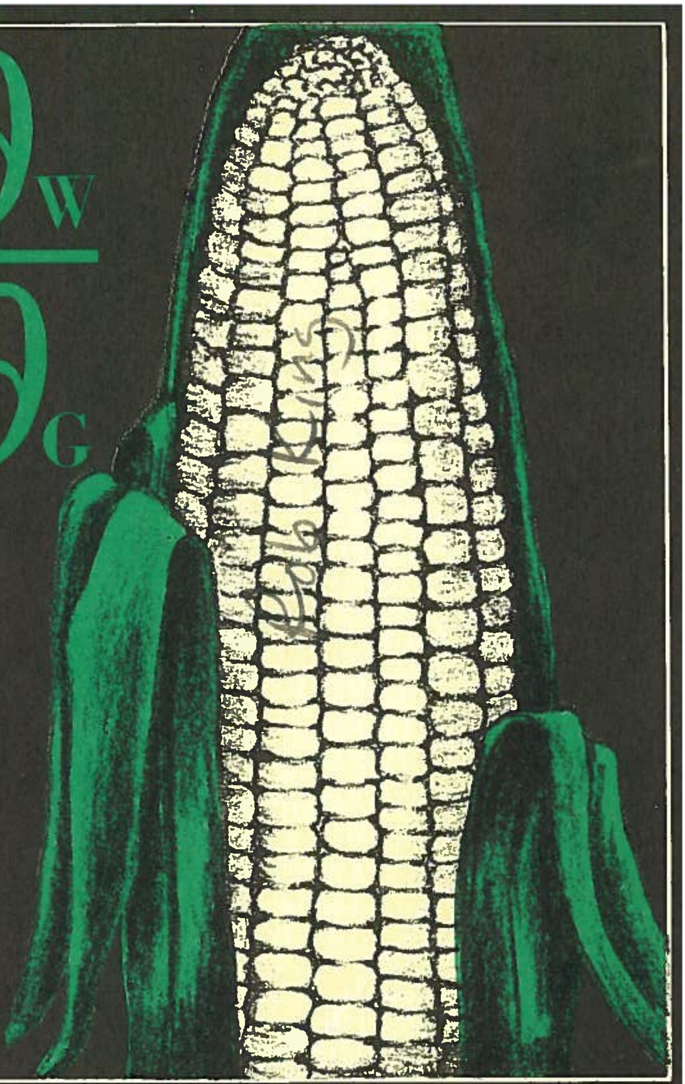
$$\frac{\partial G}{\partial N} \quad \frac{\partial W}{\partial G}$$



EVALUATION OF IRRIGATION WATER AND NITROGEN FERTILIZER IN CORN PRODUCTION

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Evaluation of Irrigation Water and Nitrogen Fertilizer in Corn Production

by

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Introduction

This study is part of a project contracted by Iowa State University and Colorado State University with the Bureau of Reclamation to provide information for agronomic and economic evaluation of irrigation development on individual farms. The main objective of this part of the study is to empirically establish corn production functions for varying applications of water and nitrogen fertilizer. Several economic results can be derived from the production functions. The results of the analysis provide information for predicting economic returns expected from irrigation development.

Production function analyses can provide information useful to the efficient allocation of resources. The estimates of the production surface and rates of substitution between resources can be used along with input and product price relationships to derive the most profitable pattern of allocating resources.

Experimental Design

The design of the field investigation, specific factors evaluated, and procedures used are essentially those recommended by the Center for Agricultural and Economic Development, Iowa State University. The requirements for the estimation of a production surface with a limited number of plots were best met by a 5 x 5 incomplete factorial design with 22 plots per block and three replications of each block. The specific design and treatments are given in Table 1.

TABLE 1. *Experimental design and treatments - figures represent number of replications in each of three blocks.*

Nitrogen Fertilizer Treatments (lbs./acre)	Irrigation Treatments				
	I	II	III	IV	V
	0.7*	1.0	3.0	6.0	9.0
200	2		2		2
150		1		1	
100	2		2		2
50		1		1	
0	2		2		2

*(Maximum soil water tension allowed at 12-inch soil depth in bars. Bars are a measure of soil suction. For a clay loam soil, 0.7 bar indicates that about 25 percent of available moisture is depleted; at 1.0 bar, 40 percent is depleted; at 3.0 bars, 70 percent is depleted; at 6.0 bars, 85 percent is depleted; and at 9.0 bars, 95 percent is depleted.)

This design (Table 1) allows the examination of block by treatment interactions. Most of the points are concentrated about the periphery of the factor space where they are most useful for estimating the coefficients in a second order polynominal. But the design does provide adequate points for a reasonably balanced test of goodness of fit.

Description of Field Study

The field study was established on Nunn clay loam (formerly Fort Collins clay loam) soil at the Agronomy Research Center, Colorado State University. The plot size consisted of eight rows of corn with 28-inch spacing and 35-foot lengths. Therefore, each plot represented 653 square feet or approximately 0.015 acres. The field was planted with a (105-day season) hybrid on May 9, 1968.

Prior to planting, the soil moisture approached that of field capacity. Irrigation treatments refer to water applied during the growing season. Soil moisture and amounts of nitrate and ammonium nitrogen were measured May 14, 1968. The soil moisture was measured to a depth of 6 feet and the amounts of available nitrogen were measured to 4 feet. Measurements were made at 12 locations in the 2-acre field and the averages of these measurements are given in Table 2.

TABLE 2. *Average soil moisture measured to a depth of 6 feet and at 12 locations and average amounts of nitrate and ammonium nitrogen measured to a depth of 4 feet at 12 locations on May 14, 1968.*

Soil Depth (feet)	Total Available Water (inches)	NH_4^+ and NO_3^- (lbs./acre)
0-1	0.55	59
1-2	0.99	79
2-3	0.79	71
3-4	1.21	29
4-5	0.86	—
5-6	1.16	—

Following emergence, the corn was side-dressed uniformly with phosphate and zinc and the nitrogen treatments were applied. The plots were ridged so that irrigation water could be measured into each basin and a near perfect distribution attained. Irrigation treatments were based upon tensiometer and electrical resistance block readings at 12-inch depth. The actual water treatments are given in Table 3. Additional soil moisture data were obtained throughout the season using electrical resistance blocks at 2- and 3-foot depths and neutron probe access tubes to 6-foot depth.

TABLE 3. *Irrigation schedule - figures represent inches of water applied.*

Date	Irrigation Treatment I	II	III	IV	V
July 5	2.57				
July 11		2.02			
July 15			2.94		
July 18	2.21				
July 23		2.02			
July 29	2.21				
August 2				2.76	
August 6			2.21		
August 7		1.84			
August 23	1.84				
Total	8.83	5.88	5.15	2.76	0.00

Measurements were made of the corn crop to obtain estimates of both grain and forage yields. Selected rows of each plot were harvested September 16-20 in an attempt to obtain the maximum forage (corn silage) yields. Other rows in each plot were harvested for grain when the crop matured. The yields of grain and forage are given in Tables 4 and 5, respectively. The yields are reported in pounds per acre adjusted to 15.5 percent moisture; conversion to bushels per acre can be accomplished by dividing each yield item by 56 pounds. Forage yields are given in oven-dry weights. Conversion to field weights as corn silage is commonly made by multiplying the weights given by 4.0.

TABLE 4. *Grain yields — pounds per acre at 15.5 percent water.*

Irr	N	BLOCK I Repl. 1	BLOCK I Repl. 2	BLOCK II Repl. 1	BLOCK II Repl. 2	BLOCK III Repl. 1	BLOCK III Repl. 2
I	0	4,918.6	7,087.5	7,791.3	6,931.9	7,474.8	8,209.1
I	100	2 7,895.4	15 7,144.9	24 8,540.1	37 7,585.2	46 8,147.4	59 7,847.2
I	200	7,714.9	9,005.8	8,277.5	9,306.0	6,392.3	7,563.0
II	50	8,350.3		8,447.6		8,446.9	
II	150	7,432.2		8,507.8		7,031.8	
III	0	6,117.6	6,761.0	7,548.0	5,630.3	7,052.0	6,014.6
III	100	7 7,237.1	18 7,163.9	29 6,655.1	40 8,035.3	51 7,432.1	62 6,701.9
III	200	8,080.3	8,491.8	8,570.7	9,015.0	6,529.1	6,970.8
IV	50	4,856.1		6,323.1		5,615.8	
IV	150	6,549.8		6,599.9		6,174.0	
V	0	4,506.4	5,559.7	5,469.8	4,294.1	5,568.1	2,137.7
V	100	12 6,926.0	21 5,770.7	34 5,741.1	43 6,228.7	56 3,923.8	65 5,487.9
V	200	5,502.1	5,616.4	5,131.7	5,802.8	4,828.2	5,433.7

TABLE 5. *Forage yields — pounds per acre (oven-dry weight).**

Irr	N	BLOCK I		BLOCK II		BLOCK III	
		Repl. 1	Repl. 2	Repl. 1	Repl. 2	Repl. 1	Repl. 2
I	0	9,967	11,199	11,236	11,012	12,132	12,282
I	100	11,572	10,900	12,394	11,236	14,598	13,737
I	200	8,847	12,020	13,177	13,103	12,692	12,543
II	50	12,618		13,252		11,535	
II	150	14,484		12,282		8,138	
III	0	6,831	10,900	10,452	9,407	10,826	9,071
III	100	10,602	12,095	11,684	11,236	12,319	10,527
III	200	9,556	12,916	9,482	12,244	11,759	10,639
IV	50	7,018		10,378		8,922	
IV	150	9,631		10,826		9,556	
V	0	7,391	8,325	9,258	7,839	6,383	4,218
V	100	9,370	9,594	9,706	9,519	8,586	7,167
V	200	8,698	9,967	8,586	8,623	7,765	8,138

*Corn forage, when harvested contains about 75% water. To convert these oven-dry weights to field weight estimates, multiply by 4.

Weather in Crop Year

Table 6 contains information which compares weather in the crop year to 70-year average of precipitation and temperature variables. The crop year, 1968, was near normal. Temperatures were about normal.

TABLE 6. *Temperature and precipitation comparisons between the growing seasons of 1968 and the 70-year averages from 1887 to 1957.*

Date	Mean Temperature °F		Precipitation (inches)	
	1968	1887-1957	1968	1887-1957
May	53.3	54.5	2.48	2.81
June	66.9	63.8	0.80	1.66
July	70.6	69.5	0.32	1.53
August	66.6	68.0	2.32	1.45
Total	64.4	64.0	5.90	7.45

Source: Temperatures for 1968 are from records of the Colorado State University Weather Station. Precipitation records for 1968 were recorded at the Colorado State University Agronomy Research Center. The 1887-1957 averages are taken from the Colorado State University Agricultural Experiment Station Bulletin 509-S.

Limitations of the Analysis

When analyzing only one year's experimental data, as we do here, it is important to recognize possible between-year variations in the results. Because of this limitation of data from only one year, the results of the analysis and the conclusions reached must be interpreted accordingly.

Further, in the analysis of the data relating to the response of forage yields to water and fertilizer treatments, the physical and economic optima derived lie far beyond the "safe" limits for extrapolation of our data. Just the same, we have presented the results of these analyses to demonstrate the techniques and to reveal some problems that can arise.

In interpreting the results the reader also should be aware of the relative efficiencies that can exist in the application of irrigation water. Under the experimental conditions, the water distribution system was virtually 100 percent efficient. Application efficiencies vary a great deal; consequently, the physical and economic optima derived must be conditioned by the efficiency of the particular irrigation system.

Production Functions

After collection of the yield data for grain and forage production, the production or yield functions were derived. Knowledge of these production functions is essential to the efficient use of water and fertilizer resources in corn production. The various economic relationships which are based upon the production function will become evident in subsequent sections of this report.

Several algebraic forms of production functions are available, some of which are eliminated because of the design of the experiment from which the basic data were generated. The results from fitting three different forms are presented here. They are represented in equations (1), (2), and (3), i.e., the quadratic, square root, and three-halves functions, respectively.

$$(1)\quad Y = a + b_1W + b_2N - b_3W^2 - b_4N^2 \pm b_5WN$$

$$(2)\quad Y = a + b_1W - b_2N + b_3W^{.5} + b_4N^{.5} + b_5W^{.5}N^{.5}$$

$$(3)\quad Y = a + b_1W + b_2N - b_3W^{1.5} - b_4N^{1.5} \pm b_5WN$$

where:

Y = yield of corn grain or corn forage, measured in pounds,
a = intercept value,
b_i = regression coefficients,
W = amount of water applied in inches, and
N = amount of fertilizer applied, measured in pounds per acre of available nitrogen.

Each of these equations was fitted to the data to estimate the response of corn grain to varied amounts of water and fertilizer applied and similarly, to the corn forage-input relation. The surfaces estimated then are only a portion of the total production surfaces since the responses estimated are to the varied amounts of water and fertilizer subjected to experimental control.

The estimating procedure used was that of multiple regression or least squares analysis. While the regression results are presented for all three of the above mentioned algebraic forms, only the quadratic production function was selected for the analyses which follow.[1]

The estimated production functions for corn grain (G) corresponding to equations (1), (2), and (3) are as in equations (4), (5), and (6). All three of these forms provide about the same degree of "goodness of fit" as measured by the magnitudes of the coefficients of determination, i.e., R^2.

$$(4)\quad G = 4{,}579.51740 + 549.23452W + 10.94143N - 29.96168W^2 - 0.03143N^2 + 0.06265WN$$

$$R^2 = 0.6345 \quad ; \quad F = 20.8319$$

$$(5)\quad G = 4{,}617.05470 + 65.00643W - 3.19034N + 655.78036W^{.5} + 110.71942N^{.5} + 3.832W^{.5}N^{.5}$$

$$R^2 = 0.6225 \quad ; \quad F = 19.7870$$

$$(6)\quad G = 4{,}577.24118 + 714.32909W + 15.88969N - 144.02976W^{1.5} - 0.79106N^{1.5} + 0.06158WN$$

$$R^2 = 0.6313 \quad ; \quad F = 20.5503$$

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[1] The Cobb-Douglas form can be fit to the response function by adding the measurements of the levels of the inputs prior to treatment to the input data. When available nitrogen in the first foot of soil (59 pounds) and the available water in the first 2 feet of soil plus the natural precipitation recorded (7.44 inches) are added to the input levels, the results are: $Y = aW^{0.66249}N^{0.08592}$ for grain and $Y = aW^{0.5976L}\,F^{0.07495}$ for forage. Coefficients of determination are 0.52 and 0.50, for grain and forage, respectively. All regression coefficients are significant at the 0.95 confidence level.

In each case about 63 percent of the variation in grain yields is explained by the variation in amounts of water and fertilizer applied. The F ratios for all three regressions indicate a high degree of statistical significance.

Tables 7, 8, and 9 summarize the results for the regression analyses used to derive equations (4), (5), and (6).

Of particular interest to the analysis in subsequent sections is equation (4). The signs of the regression coefficients indicate that grain yields respond to water and fertilizer according to the theory of response. That is, the negative signs on the squared terms indicate that diminishing returns prevail.

When fitting the algebraic forms of equations (1), (2), and (3) to the corn forage (S) yield data, equations (7), (8), and (9) result.

$$(7)\quad S = 7{,}114.75570 + 606.50705W + 26.72734N - 15.33467W^2 - 0.09494N^2 - 0.41555WN$$

$$R^2 = 0.5921 \quad ; \quad F = 17.4187$$

$$(8)\quad S = 7{,}160.63859 + 376.20802W - 15.85844N + 260.21405W^{.5} + 333.06347N^{.5} - 11.40857W^{.5}N^{.5}$$

$$R^2 = 0.5888 \quad ; \quad F = 17.1802$$

$$(9)\quad S = 7{,}109.04270 + 669.74768W + 40.59296N - 66.38056W^{1.5} - 2.31335N^{1.5} - 0.41870WN$$

$$R^2 = 0.5921 \quad ; \quad F = 17.4204$$

Again, all three equations explain about the same percent of variation in forage production, i.e., about 59 percent in each case. The F-ratios for the overall regressions are highly significant in each case. Tables 10, 11, and 12 summarize the regression results. The quadratic equation, (7), reveals a concave surface in that the coefficients on the squared terms are again negative; consequently, diminishing returns to the two variables are again found.

TABLE 7. *Summary of quadratic regression analysis for corn grain.*

Variable	Mean	Std. Dev.	Reg. Coeff.	Std. Error of Reg. Coeff.	Computed T Value	Level of Significance
Water	4.59818	3.37078	549.23452	117.14248	4.68860	0.001
Fertilizer	100.00000	77.45967	10.94143	5.37447	2.03581	0.050
Water Sq.	32.33329	30.53573	—29.96168	12.16268	—2.46341	0.025
Fert. Sq.	15909.09091	16158.09135	— 0.03143	0.02418	—1.29974	0.200
Water x Fert.	459.81818	562.37488	0.06265	0.40328	0.15536	—
Corn Grain	6759.14697	1403.75239	—	—	—	—

Intercept (a value) = 4579.51740
$R^2 = 0.6345$
Standard error of the estimate = 883.31177

TABLE 8. *Summary of square-root regression analysis for corn grain.*

Variable	Mean	Std. Dev.	Reg. Coeff.	Std. Error of Reg. Coeff.	Computed T Value	Level of Significance
Water	4.59818	3.37078	65.00643	138.98383	0.46773	—
Fertilizer	100.00000	77.45967	—3.19034	5.62984	—0.56668	—
$(\text{Water})^{.5}$	1.80081	1.17308	655.78036	421.16451	1.55706	0.200
$(\text{Fert.})^{.5}$	8.34045	5.55924	110.71942	83.45458	1.32670	0.200
$(\text{Water})^{.5}$ x $(\text{Fert.})^{.5}$	15.09054	15.35136	3.83232	16.16287	0.23711	—
Corn Grain	6759.14697	1403.75239	—	—	—	—

Intercept (a value) = 4617.05470
R2 = 0.6225
Standard error of the estimate = 897.71218

TABLE 9. *Summary of three-halves regression analysis for corn grain.*

Variable	Mean	Std. Dev.	Reg. Coeff.	Std. Error of Reg. Coeff.	Computed T Value	Level of Significance
Water	4.59818	3.37078	714.32909	192.50085	3.71078	0.001
Fertilizer	100.00000	77.45967	15.88969	8.60528	1.84650	0.010
$(\text{Water})^{1.5}$	12.05645	10.07916	—144.02976	63.00045	—2.28617	0.005
$(\text{Fert.})^{1.5}$	1243.26837	1116.19359	— 0.79106	0.58282	—1.35730	0.200
Water x Fert.	459.81818	562.37488	0.06158	0.40502	0.15204	—
Corn Grain	6759.14697	1403.75239	—	—	—	—

Intercept (a value) = 4577.24118
$R^2 = 0.6313$
Standard error of the estimate = 887.12513

TABLE 10. *Summary of quadratic regression analysis for corn forage.*

Variable	Mean	Std. Dev.	Reg. Coeff.	Std. Error of Reg. Coeff.	Computed T Value	Level of Significance
Water	4.59818	3.37078	606.50705	183.01024	3.31406	0.005
Fertilizer	100.00000	77.45967	26.72734	8.39647	3.18316	0.005
Water Sq.	32.33329	30.53573	—15.33467	19.00160	—0.80702	0.500
Fert. Sq.	15909.09091	16158.09135	— 0.09494	0.03778	—2.51296	0.025
Water x Fert.	459.81818	562.37488	— 0.41555	0.63004	—0.65957	—
Corn Forage	10379.00000	2075.94183		—	—	—

Intercept (a value) = 7114.75570
$R^2 = 0.5921$
Standard error of the estimate = 1,379.98696

TABLE 11. *Summary of square-root regression analysis for corn forage.*

Variable	Mean	Std. Dev.	Reg. Coeff.	Std. Error of Reg. Coeff.	Computed T Value	Level of Significance
Water	4.59818	3.37078	376.20802	214.52099	1.75371	0.100
Fertilizer	100.00000	77.45967	—15.85844	8.68963	—1.82498	0.100
$(\text{Water})^{.5}$	1.80081	1.17308	260.21405	650.06576	0.40029	—
$(\text{Fert.})^{.5}$	8.34045	5.55924	333.06347	128.81181	2.58566	0.025
$(\text{Water})^{.5} \times (\text{Fert.})^{.5}$	15.09054	15.35136	—11.40857	24.94733	—0.45731	—
Corn Forage	10379.00000	2075.94183	—	—	—	—

Intercept (a value) = 7160.63859
$R^2 = 0.5888$
Standard error of the estimate = 1,385.61522

TABLE 12. *Summary of three-halves regression analysis for corn forage.*

Variable	Mean	Std. Dev.	Reg. Coeff.	Std. Error of Reg. Coeff.	Computed T Value	Level of Significance
Water	4.59818	3.37078	669.74768	299.44016	2.23667	0.050
Fertilizer	100.00000	77.45967	40.59296	13.38574	3.03255	0.005
$(\text{Water})^{1.5}$	12.05645	10.07916	—66.38056	97.99886	—0.67736	0.500
$(\text{Fert.})^{1.5}$	1243.26837	1116.19359	— 2.31335	0.90659	—2.55170	0.025
Water x Fert.	459.81818	562.37488	— 0.41870	0.63002	—0.66459	—
Corn Forage	10379.00000	2075.94183	—	—	—	—

Intercept (a value) = 7109.04270
$R^2 = 0.5921$
Standard error of the estimate = 1,379.94662

Predicted Yields

Using the two quadratic production functions, it is possible to predict the yields of grain and forage that can be expected from varying applications of water and fertilizer. These predictions are arranged in a rectangular array in Tables 13 and 14 for grain and forage, respectively. By reading down a column of these tables one can see the response of corn to water as fertilizer remains constant at the level indicated by the column heading. Similarly, by reading across a row of the tables, the fertilizer response as water is held constant at the level indicated in the row caption can be seen. The values in Tables 13 and 14 are found by inserting the values for the two inputs into the predictive equations and solving for the yield estimate. In that an infinite number of values for these tables could be generated (the levels for the two input variables could be infinitely divided), a program was written to calculate a large number of predictions. This program and the programs for deriving data for the subsequent sections are presented in the Appendices. To show more clearly the individual input response curves that can be read from Tables 13 and 14, Figures 1 and 2 reveal the nature of the response of grain and forage to water as fertilizer is held constant at four alternative levels.

TABLE 13. *Predicted corn grain production for specified water and fertilizer applications (lbs.).*

Irrigation Water[*] (inches)	Nitrogen Fertilizer Added (lbs./acre)[*]								
	0	25	50	75	100	125	150	175	200
0	4579.5	4833.4	5048.0	5223.3	5359.4	5456.1	5513.6	5531.7	5510.6
1	5098.8	5354.2	5570.4	5747.3	5884.9	5983.2	6042.2	6062.0	6042.4
2	5558.1	5815.2	6032.9	6211.4	6350.5	6450.4	6511.0	6532.3	6514.3
3	5957.6	6216.2	6435.5	6615.5	6756.2	6857.6	6919.8	6942.7	6926.2
4	6297.1	6557.2	6778.1	6959.7	7102.0	7205.0	7268.7	7293.1	7278.3
5	6576.6	6838.4	7060.8	7244.0	7387.8	7492.4	7557.7	7583.7	7570.4
6	6796.3	7059.6	7283.6	7468.3	7613.7	7719.9	7786.7	7814.3	7802.6
7	6956.0	7220.9	7446.5	7632.7	7779.7	7887.4	7955.9	7985.0	7974.8
8	7055.8	7322.3	7549.4	7737.3	7885.8	7995.1	8065.1	8095.8	8087.2
9	7095.7	7363.7	7592.4	7781.8	7932.0	8042.8	8114.3	8146.6	8139.6
10	7075.7	7345.2	7575.5	7766.5	7918.2	8030.6	8103.7	8137.5	8132.1

Source: Appendix I

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[*] Here and in subsequent tables the amounts of the two inputs are the amounts added or subjected to experimental control. One could add to these supplemental amounts the quantity of water in the soil and the natural precipitation to the irrigation water, and the amount of nitrogen in the soil to the nitrogen applications.

TABLE 14. *Predicted corn forage production for specified water and fertilizer applications (lbs.).*

Irrigation Water (inches)	Nitrogen Fertilizer Added (lbs./acre) 0	25	50	75	100	125	150	175	200
0	7114.8	7723.6	8213.8	8585.3	8838.1	8972.2	8987.7	8884.5	8662.6
1	7705.9	8304.4	8784.2	9145.3	9387.7	9511.5	9516.5	9403.0	9170.7
2	8266.4	8854.5	9323.9	9674.6	9906.7	10020.0	10014.7	9890.7	9648.1
3	8796.3	9373.9	9832.9	10173.3	10394.9	10497.9	10482.2	10347.9	10094.8
4	9295.4	9862.7	10311.3	10641.3	10852.5	10945.1	10919.1	10774.3	10510.9
5	9763.9	10320.8	10759.1	11078.6	11279.5	11361.7	11325.2	11170.1	10896.2
6	10201.8	10748.3	11176.1	11485.3	11675.8	11747.6	11700.7	11535.2	11251.0
7	10608.9	11145.0	11562.5	11861.3	12041.4	12102.8	12045.5	11869.6	11575.0
8	10985.4	11511.1	11918.2	12206.6	12376.3	12427.3	12359.7	12173.4	11868.4
9	11331.2	11846.6	12243.2	12521.2	12680.6	12721.2	12643.2	12446.5	12131.1
10	11646.4	12151.3	12537.6	12805.2	12954.1	12984.4	12896.0	12688.9	12363.1

Source: Appendix I.

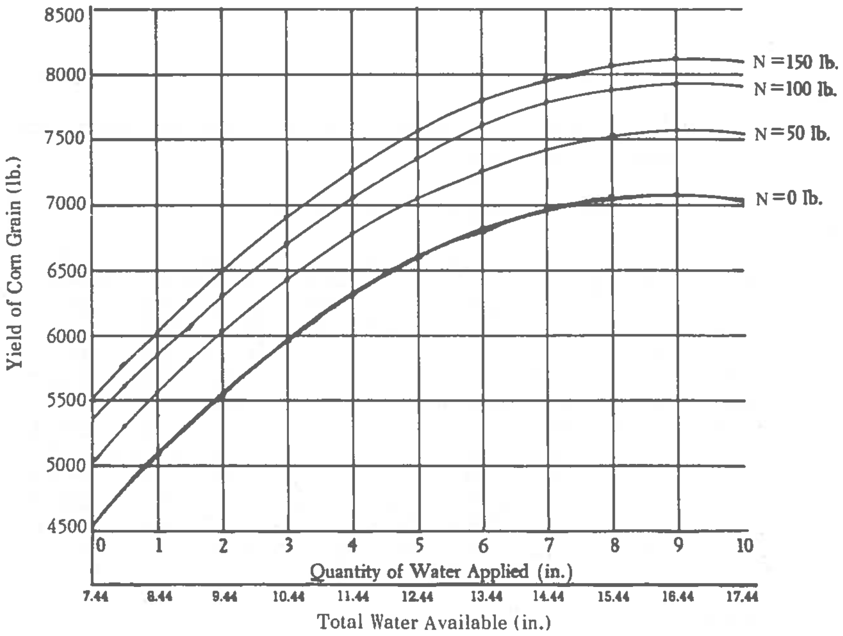

FIGURE 1. *Selected corn grain production curves for varying applications of water and four fertilizer levels.*

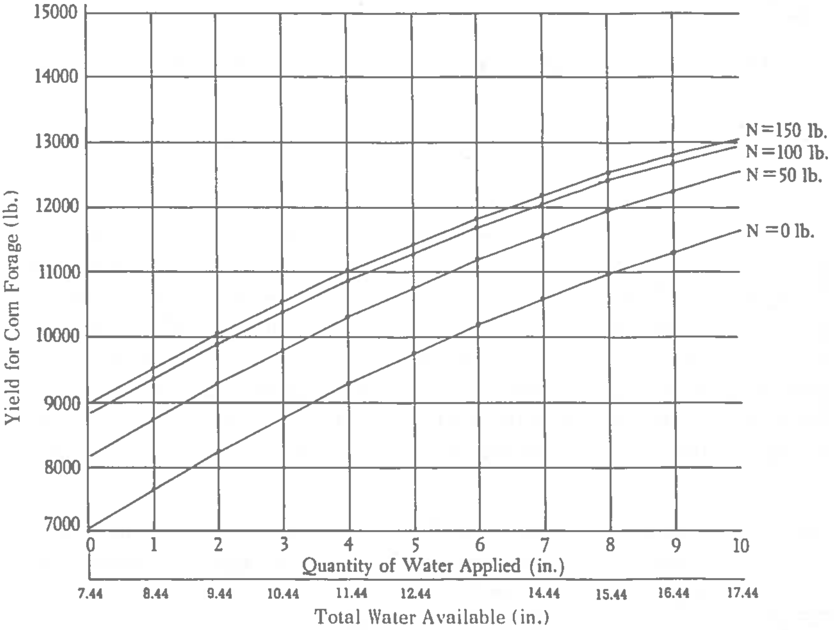

FIGURE 2. *Selected corn forage production curves for varying applications of water and four fertilizer levels.*

Marginal Physical Products

Essential to determining the economic optima of input use are estimates of the marginal physical products of the inputs. The marginal physical product is the measure of the increment to yield attributable to a one unit increment of an input. In short, the marginal physical products for each input can be derived by taking the first partial derivative of the production functions. The resulting equations are indicative of the slope of the production curve. Equations (10) and (11), respectively, are the marginal physical product equations for grain with respect to water and fertilizer.

$$(10) \quad \frac{\partial G}{\partial W} = 549.23452 - 59.92336W + 0.06265N$$

$$\frac{\partial G}{\partial N} = 10.94143 + 0.06265W - 0.06286N \tag{11}$$

In equations (12) and (13) the similar expressions for the forage equations are shown.

$$\frac{\partial S}{\partial W} = 606.50705 - 30.66934W - 0.41555N \tag{12}$$

$$\frac{\partial S}{\partial N} = 26.72734 - 0.41555W - 0.18988N \tag{13}$$

Equations (10) and (12) represent the marginal physical products of water and equations (11) and (13) are the marginal product expressions for fertilizer. One can see that the magnitude of the marginal physical product of each input depends upon the level of the other input. The concave response surfaces or diminishing (marginal) returns are also more clearly evidenced. For example, the negative sign associated with the water variable in the water expressions indicates that as W increases, the size of the marginal product will decline. Thus, while the total yield

TABLE 15. *Marginal physical product of corn grain for combinations indicated in rows and columns; upper figure for water; lower figure for fertilizer.**

Irrigation Water (inches)	Nitrogen Fertilizer Applied (lbs./acre)								
	0	25	50	75	100	125	150	175	200
0	549.2	550.8	552.4	553.9	555.5	557.1	558.6	560.2	561.8
	10.9	9.4	7.8	6.2	4.7	3.1	1.5	—0.1	—1.6
1	489.3	490.9	492.4	494.0	495.6	497.1	498.7	500.3	501.8
	11.0	9.4	7.9	6.3	4.7	3.1	1.6	0	—1.6
2	429.4	431.0	432.5	434.1	435.7	437.2	438.8	440.4	441.9
	11.1	9.5	7.9	6.4	4.8	3.2	1.6	0.1	—1.5
3	369.5	371.0	372.6	374.2	375.7	377.3	378.9	380.4	382.0
	11.1	9.6	8.0	6.4	4.8	3.3	1.7	0.1	—1.4
4	309.5	311.1	312.7	314.2	315.8	317.4	318.9	320.5	322.1
	11.2	9.6	8.0	6.5	4.9	3.3	1.8	0.2	—1.4
5	249.6	251.2	252.8	254.3	255.9	257.4	259.0	260.6	262.1
	11.3	9.7	8.1	6.5	5.0	3.4	1.8	0.3	—1.3
6	189.7	191.3	192.8	194.4	196.0	197.5	199.1	200.7	202.2
	11.3	9.7	8.2	6.6	5.0	3.5	1.9	0.3	—1.3
7	129.8	131.3	132.9	134.5	136.0	137.6	139.2	140.7	142.3
	11.4	9.8	8.2	6.7	5.1	3.5	2.0	0.4	—1.2
8	69.8	71.4	73.0	74.5	76.1	77.7	79.2	80.8	82.4
	11.4	9.9	8.3	6.7	5.2	3.6	2.0	0.4	—1.1
9	9.9	11.5	13.1	14.6	16.2	17.8	19.3	20.9	22.5
	11.5	9.9	8.4	6.8	5.2	3.6	2.1	0.5	—1.1
10	—50.0	—48.4	—46.9	—45.3	—43.7	—42.2	—40.6	—39.0	—37.5
	11.6	10.0	8.4	6.9	5.3	3.7	2.1	0.6	—1.0

*These figures are the derivatives of grain yield with respect to water (fertilizer) while fertilizer (water) is fixed.

Source: Appendix II.

increases with increasing amounts of water applied (up to a point), the increment to total yield resulting for successive additions to the amount of water applied decreases.

Each of the marginal product equations contain both the water and fertilizer term because of the interactions between these two inputs. Consequently, the marginal product of each of the variable inputs also depends upon the level at which the other input is applied.

By inserting alternative values for W and N into equations (10) through (13), the marginal product Tables 15 and 16 are found. Again, reading down any one column or across any one row, one can see the evidence of diminishing marginal returns to each factor. The negative marginal product values indicate diminishing total yields.

TABLE 16. *Marginal physical product of corn forage for combinations indicated in rows and columns; upper figure for water, lower figure for fertilizer.**

Irrigation Water (inches)	Nitrogen Fertilizer Applied (lbs./acre)								
	0	25	50	75	100	125	150	175	200
0	606.5	596.1	585.7	575.3	565.0	554.6	544.2	533.8	523.4
	26.7	22.0	17.2	12.5	7.7	3.0	—1.8	—6.5	—11.2
1	575.8	565.4	555.1	544.7	534.3	523.9	513.5	503.1	492.7
	26.3	21.6	16.8	12.1	7.3	2.6	—2.2	—6.9	—11.7
2	545.2	534.8	524.4	514.0	503.6	493.2	482.8	472.4	462.1
	25.9	21.1	16.4	11.7	6.9	2.2	—2.6	—7.3	—12.1
3	514.5	504.1	493.7	483.3	472.9	462.6	452.2	441.8	431.4
	25.5	20.7	16.0	11.2	6.5	1.7	—3.0	—7.7	—12.5
4	483.8	473.4	463.1	452.7	442.3	431.9	421.5	411.1	400.7
	25.1	20.3	15.6	10.8	6.1	1.3	—3.4	—8.2	—12.9
5	453.2	442.8	432.4	422.0	411.6	401.2	390.8	380.4	370.1
	24.6	19.9	15.2	10.4	5.7	0.9	—3.8	—8.6	—13.3
6	422.5	412.1	401.7	391.3	380.9	370.5	360.2	349.8	339.4
	24.2	19.5	14.7	10.0	5.2	0.5	—4.2	—9.0	—13.7
7	391.8	381.4	371.0	360.7	350.3	339.9	329.5	319.1	308.7
	23.8	19.1	14.3	9.6	4.8	0.1	—4.7	—9.4	—14.2
8	361.2	350.8	340.4	330.0	319.6	309.2	298.8	288.4	278.0
	23.4	18.7	13.9	9.2	4.4	—0.3	—5.1	—9.8	—14.6
9	330.5	320.1	309.7	299.3	288.9	278.5	268.2	257.8	247.4
	23.0	18.2	13.5	8.7	4.0	—0.7	—5.5	—10.2	—15.0
10	299.8	289.4	279.0	268.6	258.3	247.9	237.5	227.1	216.7
	22.6	17.8	13.1	8.3	3.6	—1.2	—5.9	—10.7	—15.4

*These figures are the derivatives of forage yield with respect to water (fertilizer) while fertilizer (water) is fixed.

Source: Appendix II.

Yield Maxima

Since the marginal physical product equations measure the slope of the production function, they can be used to determine the maximum yield obtainable. Together with the estimate of the maximum yield, one derives the level of the two inputs which correspond to this yield maxima. Taking the marginal product expression for either water or fertilizer and assuming a value for the other input variable, the maximum yield of any single variable response curve is found. This is accomplished by setting the partial derivative equation equal to zero, and solving for the quantity of the remaining variable. The assumed value of the one variable and the derived value of the other variable can then be substituted back into the production function equation, and the maximum yield is obtained.

Alternatively, the maximum yield on the production surface when both factors are allowed to vary can be determined. Setting the grain equations (10) and (11) equal to zero results in equations (14) and (15).

$$(14)\quad 549.23452 - 59.92336W + 0.06265N = 0$$

$$(15)\quad 10.94143 + 0.06265W - 0.06286N = 0$$

Solving these equations simultaneously for W and N we find that 9.3 inches of water and 183.4 pounds of fertilizer is the input combination that results in the maximum yield. Inserting these values into equation (4), the maximum yield is 8,152.4 pounds of grain. Performing similar operations on equations (12) and (13) and utilizing equation (7) results in a water use requirement at the maximum yield level that is far beyond the "safe" extrapolatable range of our data. The maximum yield of forage is 14,041.5 pounds and the input levels are 18.4 inches of water and 100.5 pounds of fertilizer.

Yield Isoquants

Any yield level on the production surface is achieved by a particular combination of inputs. There exists, then, for a given yield level many combinations of inputs that can be used to achieve this yield. The equation or the curve which depicts the alternative combination of inputs to achieve a particular yield is called an isoquant. Thus, if we take equation (4) and rearrange the terms so as to express one input as a function of the other input and the yield; e.g., $W = f\ (G, N)$ as in equation (16):

$$(16)\quad W = \left\{ 549.23452 + 0.06265N - \left[(549.23452 + 0.06265N)^2 - 4(29.96168)(G + 0.03143N^2 - 10.94143N - 4579.51740) \right]^{.5} \right\} \div \left\{ 2\,(29.96168) \right\}$$

we have the isoquant equation for grain.[2] By setting the grain yield at any level on the surface, say $G = G^*$, the combination of W and N to achieve G^* can be found.

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[2] Equations are not simplified in order to make it clear to the reader how coefficients are derived.

Taking equation (7) for forage and making the same transformations, we have:

$$(17)\quad W = \Big\{ 606.50705 - 0.41555N - [(606.50705 - 0.41555N)^2 - 4(15.3346\,33467)(S + 0.0949N^2 - 26.72734N - 7{,}114.75570)]^{.5} \Big\} \div \Big\{ 2(15.33467) \Big\}$$

the yield isoquant equation for forage.

Selected isoquants predicted by equations (16) and (17) are graphed in Figures 3 and 4, and tabulated in Tables 17 and 18, respectively.

Scale analysis. It has been noted before that both factors exhibit diminishing marginal returns. The same can be observed from the isoquant diagrams. Moving along any straight line through the origin, notice that the isoquants are successively further apart. Thus, it is taking increasing amounts of both water and fertilizer to obtain equal increments to yield as we move to higher and higher yield levels.

Marginal rates of substiution. All of the isoquants in Figure 3 and 4 are convex to the origin. This indicates that these two inputs are less than perfect substitutes for each other, in fact, decreasing marginal rates of substitution exist. The marginal rate of substitution of fertilizer for water would be defined as the amount of water that could be replaced by one pound of fertilizer with the yield remaining constant. As we continue to substitute fertilizer for water, a pound of fertilizer will replace a smaller and smaller quantity of water; hence, the decreasing marginal rate of substitution. For example at point "a" in Figure 3, the marginal rate of substitution of fertilizer for water is 0.30 which indicates that one pound of fertilizer will substitute for 0.30 inches of water. At point "b" in the same diagram, one pound of fertilizer will replace only 0.03 inches of water.

The marginal rate of substitution measures the slope of the isoquant. It can be evaluated at any point desired by taking the first partial derivative of the isoquant equation with respect to the other input, $\partial W/\partial N$, where $G = G^*$ or $S = S^*$. Alternatively, the marginal rate of substitution can be evaluated as the ratio of the marginal physical products since:

$$(18)\quad \frac{\partial W}{\partial N} = \frac{MPP_N}{MPP_W} = \frac{\partial G/\partial N}{\partial G/\partial W} = \frac{\partial G}{\partial N} \cdot \frac{\partial W}{\partial G} .$$

TABLE 17. *Values of selected isoquants showing combinations of water and fertilizer required to produce specified yields of corn grain predicted from production function (4).*

Corn Grain Yield (lbs.)	Fertilizer Applied (lbs./acre)	Irrigation Water (inches)	Corn Grain Yield (lbs.)	Fertilizer Applied (lbs./acre)	Irrigation Water (inches)
5600	0	2.10	7280	0	*
	50	1.06		50	5.98
	100	0.44		100	4.60
	150	0.16		150	4.04
	200	0.16		200	4.01
6160	0	3.58	7840	0	*
	50	2.30		50	*
	100	1.58		100	7.50
	150	1.24		150	6.28
	200	1.24		200	6.19
6720	0	5.62			
	50	3.82			
	100	2.90			
	150	2.49			
	200	2.48			

*Indeterminate
Source: Appendix III.

TABLE 18. *Values of selected isoquants showing combinations of water and fertilizer required to produce specified yields of corn forage predicted from production function (7).*

Corn Forage Yield (lbs.)	Fertilizer Applied (lbs./acre)	Irrigation Water (inches)	Corn Forage Yield (lbs.)	Fertilizer Applied (lbs./acre)	Irrigation Water (inches)
9,000	0	3.40	12,000	0	11.26
	50	1.39		50	8.24
	100	0.29		100	6.88
	150	0.02		150	6.86
	200	0.66		200	8.49
10,000	0	5.53	13,000	0	17.08
	50	3.34		50	11.84
	100	2.19		100	10.18
	150	1.97		150	10.45
	200	2.78		200	14.17
11,000	0	8.04	14,000	0	*
	50	5.57		50	*
	100	4.34		100	16.78
	150	4.19		150	*
	200	5.28		200	*

*Indeterminate
Source: Appendix III.

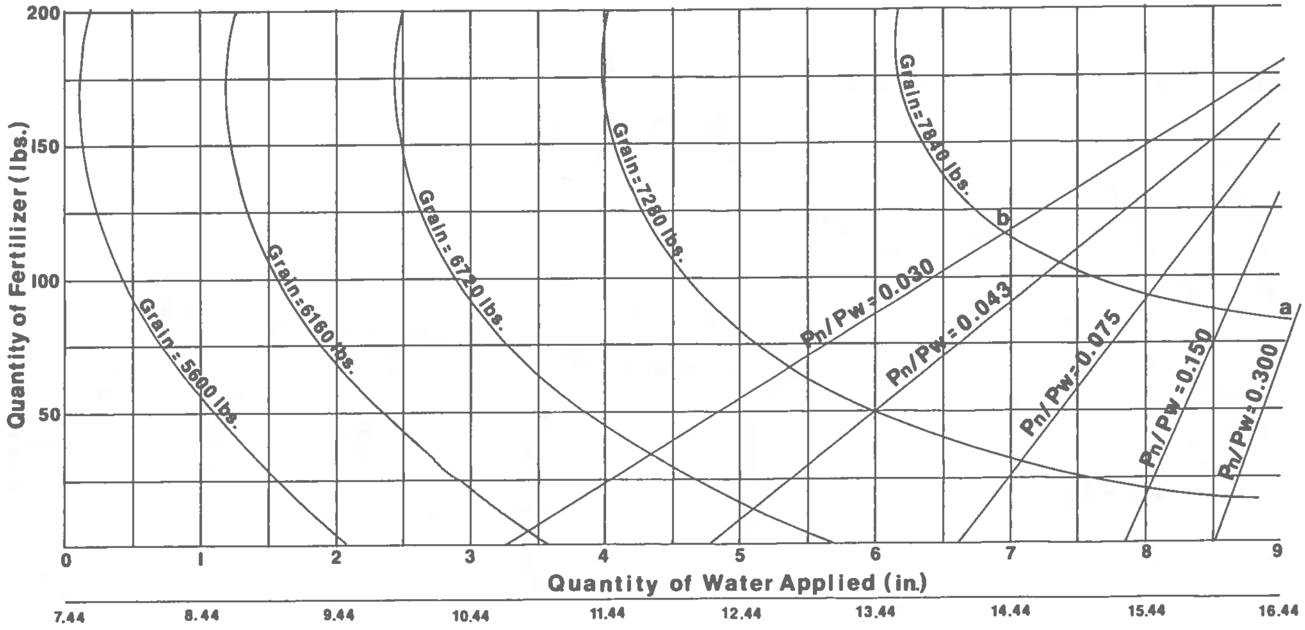

FIGURE 3. *Isoquant and isocline curves for corn grain.*

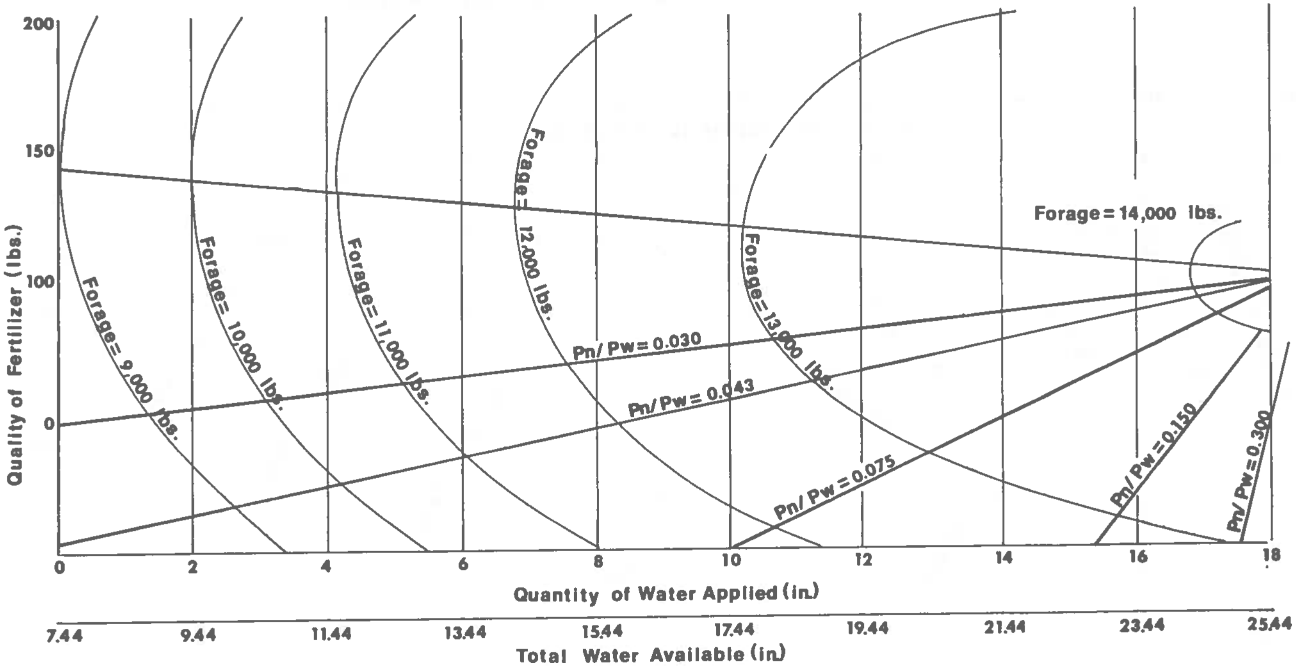

FIGURE 4. *Isoquant and isocline curves for corn forage.**
*The reader should be cautioned that the water applications suggested by figure are extrapolated far beyond the safe range of our data.

Technical complementarity. Since the isoquants tend to slope downward and to the right, the marginal rates of substitution will be negative when evaluated. In Figure 4, however, we notice that the isoquants include portions that bend around to include segments with positive slopes. This implies that both inputs must be increased to maintain the same yield level. To combine the input's in the ways suggested by these positively sloping isoquants would be irrational, since the same yield could be obtained by using a lesser amount of at least one input. The line rr in Figure 4 separates the area of irrational input use from the area of rational input combinations. Above the line rr the inputs are known as *technical complements* and below the line the inputs are *technical substitutes*.

Least cost combinations: While it is possible to achieve a given yield with many combinations of water and fertilizer, if the prices of these two inputs are known, one can determine the least cost combination of the inputs to obtain the yield desired. The least cost combination of inputs is found by equating the marginal rate of substitution to the inverse price ratio of the factors, as:

$$(19) \qquad \frac{\partial W}{\partial N} = \frac{P_N}{P_W},$$

where P_N represents the price per unit of nitrogen and P_W represents the price per unit of water. If the price of nitrogen is 6 cents per pound and the price of water is 20 cents per acre inch, then point "a" in Figure 3 is the least cost combination of water and fertilizer to obtain the yield of 7,840 pounds of grain. Similarly, if nitrogen is 6 cents per pound and water increases to 2 dollars per acre inch, point "b" represents the least cost combination to reach the same yields.[3]

The price of nitrogen has remained relatively constant in recent years. The 6 cents per pound is approximately equal to the current price paid by farmers for nitrogen fertilizer. The price of water at 20 cents per acre inch ($2.40 per acre foot) approximates the cost of water in the area of this experiment. Consequently, the P_N/P_W price ratio of 0.30 as represented by point "a" reflects the current situation. Several analyses have used a price of about 2 dollars per acre inch ($24.00 per acre foot) representing the value of water in agriculture. If this price is charged for water and the fertilizer price remains constant, the 0.03 price ratio, as represented by point "b," is in effect.

Yield Isoclines

With this analysis before us, we need not limit ourselves to pointing out one or two least cost input combinations along a given isoquant. For any given factor price ratio, a best combination of inputs to achieve any

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[3] Alternatively, at point "b," the price of fertilizer could be $0.04 per pound and the price of water $1.33 per acre inch; or any combination of prices of the two inputs, such that the price ratio is 0.03, will suffice.

yield on the production surface exists. As in equation (19), the least cost input combination on any isoquant is determined. Consequently, by setting the first partial derivative of the isoquant equations, (16) and (17), (i.e., the marginal rate of substitution of fertilizer for water) equal to the fertilizer-water price ratio and rearranging the expression we obtain (20) for grain.

$$(20)\quad W = \frac{549.23452\left(\frac{P_N}{P_W}\right) - 10.94143}{0.06265 + 2(29.96168)\,(P_N/P_W)} + \left[\frac{0.06265\,(P_N/P_W) + 2(0.03143)}{0.06265 + 2(29.96168)\,(P_N/P_W)}\right] N$$

Equation (20) shows the amount of water to use in conjunction with varying quantities of fertilizer for a particular fertilizer—water price ratio. Equation (21) is a similar expression for the forage relationships.

$$(21)\quad W = \frac{606.50705\,(P_N/P_W) - 26.72734}{2(15.33467)\,(P_N/P_W) - 0.41555} + \left[\frac{2(0.09494) - 0.41555(P_N/P_W)}{2(15.33467)\,(P_N/P_W) - 0.4155}\right] N$$

With a given price ratio the alternative values of N and W can be inserted into the production function for grain (4) and forage (7) to determine the resulting yield level. The expressions such as (20) and (21) are known as *isocline* equations. An isocline would exist for every conceivable factor price ratio since an isocline connects points of equal marginal rates of substitution on successive isoquants. Isoclines for several selected prices ratios are shown in Figures 3 and 4. Also, the line rr in Figure 4 represents another isocline (a ridge line) for which the marginal rate of substitution of fertilizer for water is zero.

Moving up any one isocline shows us the least cost combination of inputs to use to obtain successively higher yields. The isoclines converge at the point of maximum yield predicted earlier. Tables 19 and 20 show values for the variables for selected points along the isoclines depicted in Figures 3 and 4.

One can see at this point the ease at which irrigation systems of varied efficiency can be accounted for. For example, if the irrigation system is 50 percent efficient in the application of water and water costs \$0.20 per acre inch, the cost per inch of water delivered is \$0.20 ÷ 0.5 = \$0.40. Hence, the \$0.40 price of water would be the appropriate price in determining the economic optima under these circumstances.

Profit Maximization

The isocline analysis is a method to determine the least cost combination of water and fertilizer to obtain any specified yield of corn, but not the combination which will result in the maximum return. By equating equations (10) through (13) to the inverse of the price ratios of the output to the input and solving each set simultaneously, the combination of water and fertilizer yielding the maximum profit can be determined. That is, equations (22) and (23) are solved simultaneously and equations (24) and (25) are solved simultaneously.

$$(22) \quad \frac{\partial G}{\partial W} = \frac{P_W}{P_G} \qquad (24) \quad \frac{\partial S}{\partial W} = \frac{P_W}{P_S}$$

$$(23) \quad \frac{\partial G}{\partial N} = \frac{P_N}{P_G} \qquad (25) \quad \frac{\partial S}{\partial N} = \frac{P_N}{P_S}$$

where P_G is the price of grain and P_S is the price of forage and the other variables are as defined previously.

TABLE 19. *Values of selected isoclines showing the optimum water and fertilizer combination to produce corn grain for varying water prices and a fertilizer price of $0.06 per pound.*

Price of Water ($/in.)	Price Ratio (P_N/P_W)	Nitrogen Fertilizer (lbs./acre)	Water (inches)
0.20	0.300	0	8.53
		50	8.75
		100	8.98
		150	9.21
		200	9.43
0.40	0.150	0	7.89
		50	8.29
		100	8.69
		150	9.09
		200	9.49
0.80	0.075	0	6.64
		50	7.38
		100	8.12
		150	8.86
		200	9.61
1.40	0.043	0	4.79
		50	6.03
		100	7.28
		150	8.55
		200	9.80
2.00	0.030	0	3.32
		50	4.72
		100	6.46
		150	8.23
		200	9.98

Source: Appendix IV.

TABLE 20. *Values of selected isoclines showing the optimum water fertilizer combinations to produce corn forage for varying water prices and a fertilizer price of \$0.06 per pound.*

Price of Water (\$/in.)	Price Ratio $(P_F\ P_W)$	Nitrogen Fertilizer (lbs./acre)	Water (inches)
0.20	0.300	0	17.67
		50	18.04
		100	18.41
		150	18.78
		200	19.15
0.40	0.150	0	15.35
		50	16.88
		100	18.40
		150	19.93
		200	21.45
0.80	0.075	0	9.95
		50	14.17
		100	18.38
		150	22.58
		200	26.80
1.40	0.043	0	*
		50	8.76
		100	18.33
		150	27.85
		200	37.37
2.00	0.030	0	*
		50	0.67
		100	18.25
		150	35.83
		200	53.42

*Indeterminate
Source: Appendix IV.

Equations (26) and (27) are the results for corn grain and equations (28) and (29) are for corn forage. By inserting the prices of fertilizer, water, grain, and forage into these equations, one can find the most profitable level of input use.

$$(26)\quad W = [\,2(0.03143)\,(P_W/P_G) + 0.06265\,(P_N/P_G) - 2(549.23452)(0.03143) - (10.94143)(0.06265)\,] \div [\,(0.06265)^2 - 4(29.96168)(0.03143)\,]$$

$$(27)\quad N = [\,10.94143 + 0.06265W - (P_N/P_G)\,] \div [\,2(0.03143)\,]$$

$$(28)\quad W = [\,2(0.09494)(P_W/P_S) - 0.41555(P_N/P_S) - 2(606.50705)(0.09494) + (26.72734)(0.41555)\,] \div [\,(.41555)^2 - 4(15.33467)(0.09494)\,]$$

$$(29)\quad N = [\,26.72734 - 0.41555W - (P_N/P_S)\,] \div [\,2(0.09494)\,]$$

If the same prices of water and fertilizer are used in the above equations as were used in the isocline calculations, then the most profitable level of production (and the corresponding least cost combination of inputs) along a specified isocline can be found. In order to do this, however, the prices of water and fertilizer must be specified as well as the price of the product since varying prices of the inputs relative to the product price will shift the optimum combination along the isocline. One can insert the values for W and N into equations (4) and (7) to solve for the grain and forage yields associated with the optimum input combination. Values are given in Tables 21 and 22 for grain and forage, respectively.

Water Demand

Given the physical input-output relationships as defined by the production functions, a static, short-run demand function for the water or fertilizer inputs can be derived. Of particular interest to this study is the nature of the demand function for water.

Upon equating the marginal product expressions, (10) and (12) to the water-grain or forage prices ratios (i.e., $\frac{\partial G}{\partial W} = \frac{P_W}{P_G}$ and $\frac{\partial S}{\partial W} = \frac{P_W}{P_S}$), the most profitable level of water application is determined for a given level of fertilization. Solving these profit maximizing equations for water we find:

$$(30)\quad W = [\,549.23452 + 0.06265N - (P_W / P_G) \div [\,2(29.96168)]$$

$$(31)\quad W = [\,606.50705 - 0.41555N - (P_W / P_S) \div [\,2(15.33467)]$$

for grain and forage, respectively.

These equations place the demand for water as a function of the amount of fertilizer applied, the price of water and the price of the product in question. By specifying values for N and the product price, the static factor demand curve for water is obtained. Several demand curves for water are graphed in Figures 5 and 6 and are shown in Tables 23 and 24. Again, one can acoount for alternative efficiences by adjusting the price to a price per unit of water delivered.

The water demand curves are quite inelastic. If the price of water increaes from \$.80 per acre inch to \$1.00 per acre inch, fertilizer is held constant at 100 pounds per acre, and the price of grain is 1.8 cents per pound, the simple arc elasticity of demand is:

$$d = \frac{\dfrac{\Delta W}{W}}{\dfrac{\Delta P_W}{P_W}} = \frac{\dfrac{8.528 - 8.343}{8.528}}{\dfrac{0.80 - 1.00}{0.80}} = \frac{-.0217}{.25} = -0.087$$

Thus, in this price range, if the price of water increases by 1 percent, the demand for water will decrease by about 0.087 percent.

TABLE 21. *Optimum water and fertilizer combinations to produce corn grain at maximum profit for varying prices of corn grain and water, and a fertilizer price of $0.06 per pound.*

Price of Grain ($/lb.)	Price of Water ($/in.)	Price Ratio (P_N/P_W)	Water (inches)	Nitrogen Fertilizer (lbs./acre)	Yield of Grain (lbs.)
0.018	0.20	0.300	9.12	130.12	8062.52
	0.40	0.150	8.93	129.93	8058.63
	0.60	0.100	8.75	129.75	8052.86
	0.80	0.075	8.56	129.56	8045.02
	1.00	0.060	8.37	129.38	8035.12
	1.20	0.050	8.19	129.19	8023.17
	1.40	0.043	8.00	129.01	8009.14
	1.60	0.038	7.82	128.82	7993.06
	1.80	0.033	7.63	128.64	7974.91
	2.00	0.030	7.45	128.45	7954.70
0.020	0.20	0.300	9.14	135.49	8079.57
	0.40	0.150	8.97	135.28	8076.46
	0.60	0.100	8.81	135.11	8071.78
	0.80	0.075	8.64	134.95	8065.44
	1.00	0.060	8.47	134.78	8057.42
	1.20	0.050	8.31	134.61	8047.73
	1.40	0.043	8.14	134.44	8036.37
	1.60	0.038	7.97	134.28	8023.35
	1.80	0.033	7.80	134.11	8008.65
	2.00	0.030	7.64	133.95	7992.28
0.024	0.20	0.300	9.18	143.44	8100.89
	0.40	0.150	9.04	143.30	8099.68
	0.60	0.100	8.90	143.16	8096.43
	0.80	0.075	8.76	143.02	8092.03
	1.00	0.060	8.62	142.88	8086.46
	1.20	0.050	8.48	142.74	8079.73
	1.40	0.043	8.34	142.60	8071.85
	1.60	0.038	8.20	142.46	8062.80
	1.80	0.033	8.06	142.33	8052.59
	2.00	0.030	7.92	142.19	8041.22
0.026	0.20	0.300	9.19	146.51	8109.16
	0.40	0.150	9.06	146.38	8107.49
	0.60	0.100	8.93	146.25	8104.72
	0.80	0.075	8.81	146.12	8100.97
	1.00	0.060	8.68	146.00	8096.22
	1.20	0.050	8.55	145.87	8090.49
	1.40	0.043	8.42	145.74	8083.77
	1.60	0.038	8.29	145.61	8076.06
	1.80	0.033	8.16	145.48	8067.36
	2.00	0.030	8.03	145.36	8057.68

*Multiply by 56 to get price per bushel

Source: Appendix V.

TABLE 22. *Optimum water and fertilizer combinations to produce corn forage at maximum profit for varying prices of corn forage and water, and a fertilizer price of $0.06 per pound.*

Price of Forage ($/lb.)	Price of Water ($/in.)	Price Ratio (P_N/P_W)	Water (inches)	Nitrogen Fertilizer (lbs./acre)	Yield of Forage (lbs.)
0.010*	0.20	0.300	18.27	69.18	14026.89
	0.40	0.150	17.51	70.84	13934.60
	0.60	0.100	16.84	72.31	13909.82
	0.80	0.075	16.17	73.78	13871.61
	1.00	0.060	15.50	75.25	13819.94
	1.20	0.050	14.82	76.72	13754.84
	1.40	0.043	14.15	78.19	13676.30
	1.60	0.038	13.48	79.66	13584.32
	1.80	0.033	12.80	81.13	13478.90
	2.00	0.030	12.14	82.60	13360.04
0.012	0.20	0.300	18.31	74.36	13975.51
	0.40	0.150	17.66	75.77	13967.27
	0.60	0.100	17.10	77.00	13950.07
	0.80	0.075	16.54	78.22	13923.52
	1.00	0.060	15.98	79.45	13887.65
	1.20	0.050	15.42	80.68	13842.44
	1.40	0.043	14.86	81.90	13787.90
	1.60	0.038	14.30	83.13	13724.02
	1.80	0.033	13.74	84.35	13650.81
	2.00	0.030	13.18	85.58	13568.27
0.014	0.20	0.300	18.33	78.07	13993.02
	0.40	0.150	17.77	79.30	13986.97
	0.60	0.100	17.29	80.35	13974.33
	0.80	0.075	16.81	81.40	13954.83
	1.00	0.060	16.33	82.45	13928.48
	1.20	0.050	15.85	83.50	13895.26
	1.40	0.043	15.37	84.55	13855.19
	1.60	0.038	14.89	85.60	13808.25
	1.80	0.033	14.41	86.65	13754.47
	2.00	0.030	13.93	87.70	13693.82
0.016	0.20	0.300	18.35	80.84	14004.44
	0.40	0.150	17.85	81.95	13999.76
	0.60	0.100	17.43	82.86	13990.08
	0.80	0.075	17.01	83.78	13975.15
	1.00	0.060	16.59	84.70	13954.97
	1.20	0.050	16.17	85.62	13929.54
	1.40	0.043	15.75	86.54	13898.86
	1.60	0.038	15.33	87.46	13862.93
	1.80	0.033	14.91	88.38	13821.75
	2.00	0.030	14.49	89.30	13775.32

*Multiply by 2000 to get price per ton.

Source: Appendix V.

Summary

This report has presented the functional relationships found to exist in a controlled experiment involving water and nitrogen fertilizer variables and corn production. A 5 x 5 incomplete factorial design was used to generate the basic yield response data. Statistical production functions were fitted to the input-output data. The quadratic, square root and three-halves algebraic forms of functions were fitted to the data and the quadratic form was selected for more detailed analyses.

Analyses were made to derive the single variable response functions, yield maxima and product isoquants. Marginal rates of input substitution, economic optima, and isoclines were found.

Finally, the most profitable point of operation for alternative price relations was presented and a static short run demand curve for water was estimated.

Information from analyses such as this are necessary to achieve an efficient allocation of resources. Within the agricultural sector, knowledge of the most profitable use of resources on a particular crop is important to farmer decision-making. To obtain the most efficient allocation of resources between crops, knowledge of these production relationships for competing crops is necessary. Some data of this type are being generated under the combination of this project.

In a policy setting, production function analysis can serve to guide decisions relating to the interregional allocation of inputs as well as the intersectoral input allocations. Studies such as this for other soils and different geographic locations would generate the data necessary to determine the value of water associated with different conditions. Irrigation development projects could follow the guidelines of developing those projects in regions or on soil types for which the estimated value productivities of water is the greatest.

As water becomes increasingly scarce, as is occurring in the western United States, the reallocation of water from agriculture to urban uses is important. Estimates of the demand for water in agriculture at alternative prices is useful information to planners contemplating water pricing policies to bring about this reallocation.

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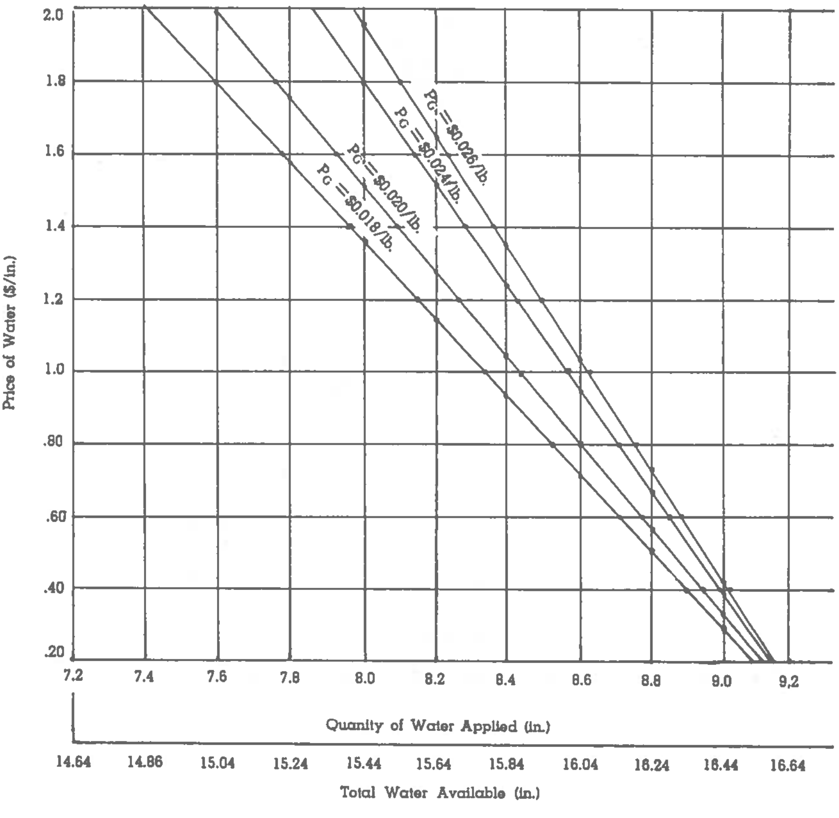

FIGURE 5. *Estimated water demand curves at a fixed fertilizer level (100 lbs.) and four corn grain prices.*

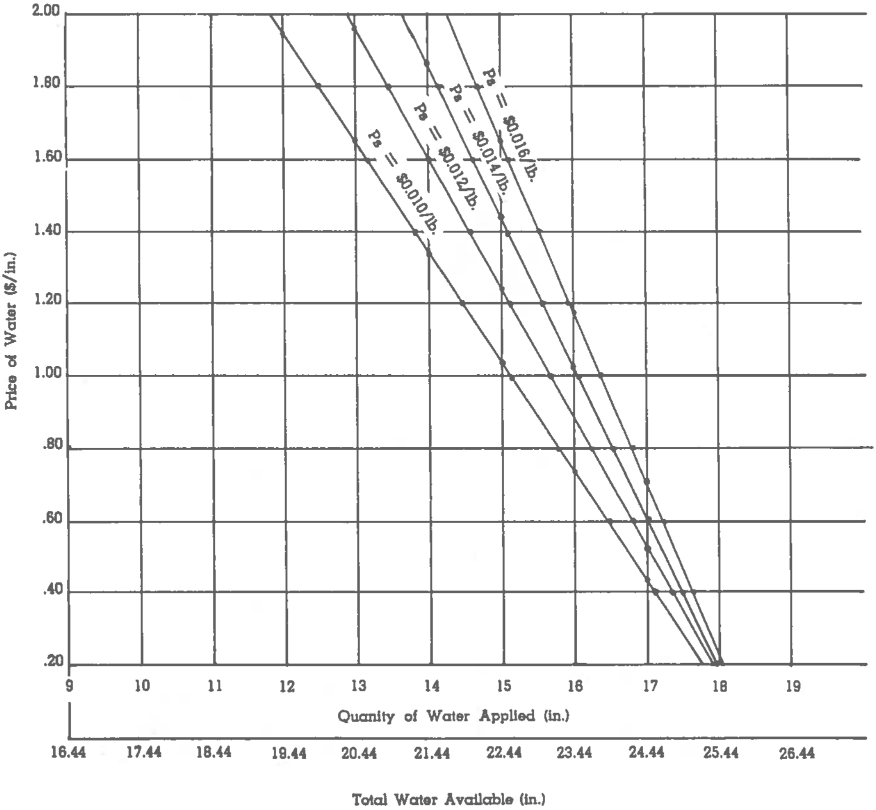

FIGURE 6. *Estimated water demand curves at a fixed fertilizer level (100 lbs.) and four forage prices.**

*The reader should be cautioned that the water applications suggested by this figure are extrapolated far beyond the safe range of our data.

TABLE 23. *Values of selected static factor demand functions for water and nitrogen fertilizer to produce corn grain.*

Price of Water ($/in.)	Nitrogen Fertilizer (lbs./acre)	Inches of water when the price of grain is: $0.018/lb.	$0.020/lb.	$0.024/lb.	$0.026/lb.
0.20	0	8.98	9.00	9.03	9.04
	50	9.03	9.05	9.08	9.09
	100	9.08	9.10	9.13	9.14
	150	9.14	9.16	9.18	9.19
	200	9.19	9.21	9.24	9.25
0.40	0	8.80	8.83	8.89	8.91
	50	8.85	8.88	8.94	8.96
	100	8.90	8.94	8.99	9.01
	150	8.95	8.99	9.04	9.07
	200	9.00	9.04	9.10	9.12
0.60	0	8.61	8.67	8.75	8.78
	50	8.66	8.71	8.80	8.83
	100	8.71	8.77	8.85	8.89
	150	8.77	8.82	8.91	8.94
	200	8.82	8.87	8.96	8.99
0.80	0	8.42	8.50	8.61	8.65
	50	8.48	8.55	8.66	8.70
	100	8.53	8.60	8.71	8.76
	150	8.58	8.66	8.77	8.81
	200	8.63	8.71	8.82	8.86
1.00	0	8.24	8.33	8.47	8.52
	50	8.29	8.38	8.52	8.58
	100	8.34	8.44	8.58	8.63
	150	8.40	8.49	8.63	8.68
	200	8.45	8.54	8.68	8.73
1.20	0	8.05	8.16	8.33	8.40
	50	8.10	8.22	8.38	8.45
	100	8.11	8.27	8.44	8.50
	150	8.21	8.32	8.49	8.55
	200	8.26	8.37	8.54	8.61
1.40	0	7.87	8.00	8.19	8.27
	50	7.92	8.05	8.24	8.32
	100	7.97	8.10	8.30	8.37
	150	8.02	8.15	8.35	8.42
	200	8.08	8.21	8.40	8.48
1.60	0	7.68	7.83	8.05	8.14
	50	7.74	7.88	8.11	8.19
	100	7.79	7.94	8.16	8.24
	150	7.84	7.99	8.21	8.30
	200	7.89	8.04	8.26	8.35
1.80	0	7.50	7.66	7.91	8.01
	50	7.55	7.72	7.97	8.06
	100	7.60	7.77	8.02	8.12
	150	7.65	7.82	8.07	8.17
	200	7.71	7.87	8.12	8.22
2.00	0	7.31	7.50	7.78	7.88
	50	7.36	7.55	7.83	7.93
	100	7.42	7.60	7.88	7.99
	150	7.47	7.65	7.93	8.04
	200	7.52	7.71	7.98	8.09

Source: Appendix VI.

TABLE 24. *Values of selected static factor demand functions for water and nitrogen fertilizer to produce corn forage.*

Price of Water ($/in.)	Nitrogen Fertilizer (lbs./acre)	Inches of water when the price of forage is:			
		$0.010/lb.	$0.012/lb.	$0.014/lb.	$0.016/lb.
0.20	0	19.12	19.23	19.31	19.37
	50	18.45	18.54	19.63	18.69
	100	17.77	17.88	17.95	18.01
	150	17.09	17.20	17.28	17.34
	200	16.41	16.52	16.60	16.66
0.40	0	18.47	18.69	18.84	18.96
	50	17.79	18.01	18.17	18.28
	100	17.12	17.33	17.49	17.61
	150	16.44	16.66	16.81	16.93
	200	15.76	15.98	16.13	16.25
0.60	0	17.82	18.15	18.38	18.55
	50	17.14	17.47	17.70	17.88
	100	16.46	16.79	17.02	17.20
	150	15.79	16.11	16.35	16.52
	200	15.11	15.44	15.67	15.84
0.80	0	17.17	17.60	17.91	18.15
	50	16.49	16.92	17.24	17.47
	100	15.81	16.25	16.56	16.79
	150	15.14	15.57	15.88	16.11
	200	14.46	14.89	15.20	15.44
1.00	0	16.52	17.06	17.45	17.74
	50	15.84	16.38	16.77	17.06
	100	15.16	15.70	16.09	16.38
	150	14.48	15.03	15.41	15.71
	200	13.81	14.35	14.74	15.03
1.20	0	15.86	16.52	16.98	17.33
	50	15.19	15.84	16.30	16.65
	100	14.51	15.16	15.63	15.98
	150	13.83	14.48	14.95	15.30
	200	13.15	13.81	14.27	14.62
1.40	0	15.21	15.97	16.52	16.92
	50	14.53	15.29	15.84	16.25
	100	13.86	14.61	15.16	15.57
	150	13.18	13.94	14.48	14.89
	200	12.50	13.26	13.81	14.21
1.60	0	14.56	15.43	16.05	16.52
	50	13.88	14.75	15.37	15.84
	100	13.20	14.07	14.69	15.16
	150	12.53	13.40	14.02	14.48
	200	11.85	12.72	13.34	13.81
1.80	0	13.91	14.89	15.58	16.11
	50	13.23	14.21	14.91	15.43
	100	12.55	13.53	14.23	14.75
	150	11.87	12.85	13.55	14.08
	200	11.20	12.18	12.87	13.40
2.00	0	13.26	14.34	15.12	15.70
	50	12.58	13.66	14.44	15.02
	100	11.90	12.99	13.76	14.35
	150	11.22	12.31	13.09	13.67
	200	10.55	11.63	12.41	12.99

Source: Appendix VI.

Appendices

APPENDIX I. *Computer Program for Calculating Predicted Corn Grain and Forage Production Yields for Specified Water and Fertilizer Applications.*

```
*FORTRAN          PRODUCTION MATRIX
      PROGRAM
      DIMENSION YGA (11,9),YFA(11,9),YGB(11,9),YFB(11,9)
      DO 200 I=1.11
      F=—25
      W=I—1
      DO 100 J=1,9
      F=F+25
      YGA(I,J)=4579.51740+(549.23452*W)+(10.94143*F)
      1—(29.96168*(W**2)) — (0.03143*(F**2)) + (0.06265*W*F)
      YFA(I,J)=7114.7557 + (606.50705*W) + (26.72734*F) — (15.33467*
         (W**2) )
      1—(0.09494*(F**2)) — (0.41555*W*F)
      YGB(I,J) = 10.**3.83129*W**0.03214*F**0.01039
      YFB(I,J) = 10.**4.01701*W**0.02752*F**0.00938
100   CONTINUE
200   CONTINUE
      DO 300 I=1,11
      PRINT 1000,(YFA(I,J), J=1,9)
1000  FORMAT (/,9(1x,E13.6) )
 300  CONTINUE
      PRINT 2000
2000  FORMAT (/ / / / /)
      DO 310 I=1,11
      PRINT 1000, (YGA(I,J), J=1,9)
 310  CONTINUE
      PRINT 2000
      DO 320 I=1,11
      PRINT 1000, (YFB(I,J), J=1,9)
 320  CONTINUE
      PRINT 2000
      DO 330 I=1,11
      PRINT 1000, (YGB(I,J), J=1,9)
 330  CONTINUE
      END
```

APPENDIX II. *Computer Program for Computing Marginal Physical Products of Corn Grain and Forage for Various Combinations of Water and Fertilizer.*

```
*FORTRAN
      PROGRAM MARGINAL PRODUCT MATRIX
      DIMENSION SGW (11,9), SGF(11,9), SFW(11,9), SFF(11,9)
      DO 200 I=1,11
      F=-25
      W=I-1
      DO 100 J=1,9
      F=F+25
      SGW(I,J)=10.94143-0.06286*F+0.06265*W
      SGF(I,J)=549.23452-59.92336*W+0.06265*F
      SFW(I,J)=26.72734-0.18988*F-0.41555*W
      SFF(I,J)=606.50705-30.66934*W-0.41555*F
  100 CONTINUE
  200 CONTINUE
      DO 300 I=1,11
      PRINT 1000,(SGW(I,J), J=1,9)
 1000 FORMAT(/,9(1X,F12.8))
  300 CONTINUE
      PRINT 2000
 2000 FORMAT (/ / / / /)
      DO 310 I=1,11
      PRINT 1000,(SFG(I,J), J=1,9)
  310 CONTINUE
      PRINT 2000
      DO 320 I=1,11
      PRINT 1000,(SFW(I,J), J=1,9)
  320 CONTINUE
      PRINT 2000
      DO 330 I=1,11
      PRINT 1000,(SFF(I,J), J=1,9)
  320 CONTINUE
      END
```

APPENDIX III. *Computer Program for Calculating Tabular Values of Isoquants for Corn Grain and Forage.*

```
*FORTRAN
      PROGRAM ISOQUANT
      DO 50 I=1,2
      WRITE (6,400)
 400 FORMAT (*1          F          Y                W               W*)
      READ (5,100) A,B1,B2,B3,B4,B5,B6,B7,B8
 100  FORMAT (6F10.0)
      READ (5,200) IY1,IY2,INC
 200  FORMAT (311O)
      DO 60 K=IY1, IY2, INC
      DO 60 J=1,201,5
      Y = K
      F = J — 1
      W = (B1+B5*F)**2—4.* B3 * (Y + B4*F*F—B2*F—A)
      IF (W .GF. 0) GO TO 1
      W =99999999
    1 W = (B1 + B5*F—W**.5) / (2*B3)
      W1 = 99999.
      IF (F  .NE. 0) W1 = (Y/(10.**B6*F**B7) )**(1./B8)
   60 WRITE (6,300) F, Y, W, W1
  300 FORMAT (2F10.0,2F20.10)
   50 CONTINUE
      CAL L EXIT
      END
```

APPENDIX IV. *Computer Program for Calculating Tabular Values of Isoclines for Corn Grain and Forage.*

```
*FORTRAN
      PROGRAM  ISOCLINE
      DO 50 I=1,2
      WRITE (6,400)
 400  FORMAT  (*1*,9x,1HF,18X,2HPF,18X,2HPW,18X,2HPR,19X,1HW,
        18X,2HW1)
      READ  (5,100),  A,B1,B2,B3,B4,B5,B6,B7,B8
 100  FORMAT (6F10.0)
      DO 60 J=500,1000,100
      DO 60 K=400,2000,200
      DO 60 L=50,250,50
      PF=J
      PF=PF/10000
      PW=K
      PW=PW/1000
      F=L—50
      W=((B1*(PF/PW)—B2)/(B5+2*B3*(PF/PW)))
     1+((B5*(PF/PW)+2*B4)/(B5+2*B3*(PF/PW)))*F
      W1=(B8*(PF/PW)*F )/B7
      PR=PF/PW
  60  WRITE(6,300) F,PF,PW,PR,W,W1
 300  FORMAT (6F20.8)
  50  CONTINUE
      CAL L EXIT
      END
```

APPENDIX V. *Computer Program for Calculating Optimum Combinations of Water and Fertilizer on a Given Isocline for Corn Grain and Forage.*

```
*FORTRAN
     PROGRAM OPTIMUM INPUTS
     READ (5,100) A,B1,B2,B3,B4,B5,B6,B7, B8
 100 FORMAT (6F10.0)
     DO 60 K=14,26,2
     DO 60 J=500,1000,100
     DO 60 L=400,2000,200
     PG=K
     PG=PG/1000
     PF=J
     PF=PF/10000
     PW=L
     PW=PW/1000
     W=(2*B4*(PW/PG)—B5*(PF/PG)—2*B1*B4*+B2*B5)/
       (B5**2-4*B3*B4)
     F=(B2—B5*W-(PF/PG))/(2*B4)
     YG=A+B1*W+B2*F-B3*W**2-B4*F**2—B5*W*F
     PR=PF/PW
  60 WRITE (6,300) PG,PF,PW,PR,W,F,YG
 300 FORMAT (7F17.8)
     CAL L EXIT
     END
```

APPENDIX VI. *Computer Program for Calculating Tabular Values of Static Factor Demand Functions for Water and Fertilizer to Produce Corn Grain and Forage.*

```
*FORTRAN
      PROGRAM STATIC FACTOR DEMAND FUNCTION
      WRITE (6,400)
 400  FORMAT (*1*,9X,2HPG,18X,2HPW,18X,1HF,19X,1HW)
      READ (5,100)A,B1,B2,B3,B4,B5
 100  FORMAT (6F10.0)
      DO 60 K=14,26,2
      DO 60 J=400,2000,200
      DO 60 L=10,210,10
      PG=K
      PG=PG/1000
      PW=J
      PW=PW/1000
      F=L-10
      W=(B1+B5*F—(PW/PG)/(2*B3)
  60  WRITE(6,300) PG,PW,F,W
 300  FORMAT (6F20.8)
      CAL L EXIT
      END
```