Calculating Willingness to Pay in Mixed Logit Models

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Background

- In calculating willingness to pay (WTP) for an amenity in mixed logit models, a common approach is to consider the ratio of the parameter on the amenity and the cost or price parameter.
- In some cases this is misleading since we show that it does not directly result from the expected maximum utility condition.

Objective

- **This paper briefly outlines an approach for calculating WTP under the maximum utility condition, which is necessary for the optimality of the welfare analysis.**

Model Setting

- **Define the utility of the two alternatives as**

$$U_1 = -f_E(\beta+\eta)p_1 + f_L(\gamma+\nu)x_1 + \varepsilon_1$$

$$U_2 = -f_E(\beta+\eta)p_2 + f_L(\gamma+\nu)x_2 + \varepsilon_2$$

where

- p denotes a price and x denotes an amenity
- ε_j (j=1,2) $\overset{iid}{\sim}$ EV(0,1)
- $\eta \sim N(0, \sigma^2_\eta)$, $\nu \sim (0, \sigma^2_\nu)$ *(Other distributions may be also concerned)*
- $f_L(.)$ is the link function defined for some arbitrary link
- $f_E(\beta+\eta)=\exp(\beta+\eta)$ *(to ensure expectation of the inverse of the random variable exits, Daly et al., 2012)*

- **Expected maximum utility at the base level x^0**
  - $Emax(U_1^0, U_2^0|\eta,\nu)$= constant+$\log\{\exp(-f_E(\beta+\eta)p_1+f_L(\gamma+\nu)x_1^0)+\exp(-f_E(\beta+\eta)p_2+f_L(\gamma+\nu)x_2^0)\}$

- **The change in utility from x0 to x1=x0+Δ**
  - $Emax(U_1^1, U_2^1|\eta,\nu) - Emax(U_1^0, U_2^0|\eta,\nu)$=

$$log\left\{\frac{\exp(-f_E(\beta+\eta)p_1 + f_L(\gamma+\nu)x_1^0)\exp(f_L(\gamma+\nu)\Delta) + \exp(-f_E(\beta+\eta)p_2 + f_L(\gamma+\nu)x_2^0)\exp(f_L(\gamma+\nu)\Delta)}{\exp(-f_E(\beta+\eta)p_1 + f_L(\gamma+\nu)x_1^0) + \exp(-f_E(\beta+\eta)p_2 + f_L(\gamma+\nu)x_2^0)}\right\}$$

$$= f_L(\gamma+\nu)\Delta$$

***key result**: it is **immaterial** whether the price coefficient is random because the statistical theory reveals that the change in expected maximum utility caused by a change in the amenity depends on the subject specific error ν—not on the subject specific error η.*

- **Population averaged maximum utility with the amenity change.**
  - Integrate ν (a nuisance variable) out of the expression $f_L(\gamma+\nu)\Delta$.

$$\Delta \int_{-\infty}^{\infty}(\gamma+\nu)\exp(-\nu^2/2\sigma_\nu^2)/(\sigma_\nu\sqrt{2\pi})d\nu = \gamma\Delta \quad \text{(linear link)}$$

$$\text{and} \quad \Delta \int_{-\infty}^{\infty}\exp(\gamma+\nu)\exp(-\nu^2/2\sigma_\nu^2)/(\sigma_\nu\sqrt{2\pi})d\nu = \exp(\gamma+\sigma_\nu^2/2)\Delta \quad \text{(exponential Link)}$$

Expected Population Averaged Willingness to Pay

- As no statistical theory generates the expected maximum WTP, ***an economic assumption of change in expected maximum utility:***
  - If the price coefficient is constant, expected WTP is obtained by dividing the above expressions by β
  - If price coefficient is random, WTP conditional on the subject specific error η=

$$E(WTP|\eta) = \frac{\gamma\Delta}{\exp(\beta+\eta)} \quad \text{when } \beta \text{ is log normal and } \gamma \text{ is normally distributed}$$

$$= \frac{\exp(\gamma+\sigma_\nu^2/2)\Delta}{\exp(\beta+\eta)} \quad \text{when } \beta \text{ and } \gamma \text{ are both log normally distributed}$$

- Integrating out the random variable η, **Expectations of the Population Averaged WTP**

$$= \boldsymbol{\gamma\, exp(-\beta + \sigma_\eta^2/2)\Delta} \text{ and } \boldsymbol{exp(\gamma - \beta + \sigma_\eta^2/2 + \sigma_\nu^2/2)\Delta}, \text{ respectively.}$$

Comparison with Other Approaches

- **Some approaches (eg, Greene et al., 2005)**, adopt $E(WTP|\eta,\nu) = \frac{f_L(\gamma+\nu)\Delta}{\exp(\beta+\eta)}$
  - This approach is correct given the subject specific values of η and ν.
  - However, when η and ν are not known, there are ***closed form solutions*** to the expectation of this ratio under the log-normal and normal/log-normal specification of the price and amenity parameters even when the parameters are correlated.

- $E[\frac{f_L(\gamma+\nu)\Delta}{\exp(\beta+\eta)}]$ =
- $\Delta \iint_{-\infty}^{\infty}(\gamma+\nu)/\exp(\beta+\eta)\cdot\exp[(-\eta^2/\sigma_\eta^2 - \nu^2/\sigma_\nu^2 + 2\rho\eta\nu)/2(1-\rho^2)]/(2\pi(1-\rho^2)^{1/2}\sigma_\eta\sigma_\nu)d\eta d\nu$
  $= \boldsymbol{exp(-\beta+\sigma_\eta^2/2)(\gamma-\sigma_{\eta\nu})\Delta}$ (normally distributed amenity coefficient)
- $\Delta \iint_{-\infty}^{\infty}\exp(-\beta-\eta+\gamma+\nu)\exp[(-\eta^2/\sigma_\eta^2 - \nu^2/\sigma_\nu^2 + 2\rho\eta\nu)/2(1-\rho^2)]/(2\pi(1-\rho^2)^{1/2}\sigma_\eta\sigma_\nu)d\eta d\nu$
  $= \boldsymbol{exp(-\beta+\sigma_\eta^2/2 + \gamma + \sigma_\nu^2/2 - \sigma_{\eta\nu})\Delta}$ (log-normally distributed amenity coefficient)

* Here ρ and $\sigma_{\eta\nu}$ denote the correlation and covariance of the error terms.

***Our contention:** These formulas **should not** be used when the parameters are correlated. Instead the population averaged WTP should be used.*

Application: The Mixed Logit Model in Willingness to Pay

- **Parameterized our model**: Specify the parameter on the amenity to be the product of the random price parameter and a parameter ω which is related to the WTP for the amenity.
  - $Emax(U_1^0, U_2^0|\eta,\nu)$=

constant + $\log\{\exp(-\exp(\beta+\eta)p_1 + \exp(\beta+\eta)f_L(\omega+\nu)x_1^0)+\exp(-\exp(\beta+\eta)p_2 + \exp(\beta+\eta)f_L(\omega+\nu)x_2^0)\}$

- **Expected population averaged differences in maximum utility**

$$\Delta \iint_{-\infty}^{\infty}\exp(\beta+\eta)(\omega+\nu)\exp[(-\eta^2/\sigma_\eta^2 - \nu^2/\sigma_\nu^2 + 2\rho\eta\nu)/2(1-\rho^2)]/(2\pi(1-\rho^2)^{1/2}\sigma_\eta\sigma_\nu)d\eta d\nu$$

$$= \boldsymbol{exp(\beta+\sigma_\eta^2/2)(\omega+\sigma_{\eta\nu})\Delta}$$

$$\Delta \iint_{-\infty}^{\infty}\exp(\beta+\eta+\omega+\nu)\exp[(-\eta^2/\sigma_\eta^2 - \nu^2/\sigma_\nu^2 + 2\rho\eta\nu)/2(1-\rho^2)]/(2\pi(1-\rho^2)^{1/2}\sigma_\eta\sigma_\nu)d\eta d\nu$$

$$= \boldsymbol{exp(\beta+\sigma_\eta^2/2 + \omega + \sigma_\nu^2/2 + \sigma_{\eta\nu})\Delta}$$

- **Expected Population Averaged WTP = $\exp(\sigma_\eta^2)(\omega+\sigma_{\eta\nu})\Delta$** and **$\exp(\sigma_\eta^2)\exp(\omega + \sigma_\nu^2/2 + \sigma_{\eta\nu})\Delta$**

*Applications in the literature (typically with the assumption that $\sigma_{\eta\nu}$=0) tend to find that **expected WTP is smaller** when estimating the model **in WTP space** than in preference space (Train and Weeks, 2005). Apparently this is due to the **neglect of the factor $\exp(\sigma_\eta^2)$**.*

Empirical Application

- **Data:** subset of data used by Kenneth Train (2006), 100 respondents regarding their stated preferences for gas, electric, or hybrid cars
- **Specification** Utility is specified as depending on
  - the negative of the price of the vehicle(Price);
  - range in hundreds of miles for electric vehicles, 0 otherwise (Range);
  - indicator of an electric vehicle (EV);
  - indicator of a hybrid vehicle (Hybrid); and
  - whether the vehicle is medium or high performance (Perf)
- **Method:** the panel model is estimated by using high order Gauss-Hermite integration
- **Result:** Using simulated maximum Likelihood (SML) with modified Latin hypercube sampling, we closely match the exact results using 40,000 draws per respondent

Table 1. Model of Vehicle Choice Using Gauss-Hermite Integration (6832 points)
Log-likelihood -896.13

Parameter	Estimate	Std.Error	z-Value
Price	-0.7831	0.1722	-4.548
Range	-0.3100	0.3768	-0.823
EV	-1.6704	0.3926	-4.255
Hybrid	0.8982	0.1638	5.484
Perf	0.6283	0.1058	5.939
SE-Price	1.0440	0.1608	6.493
SE-Range	0.5822	0.2609	2.232
SE-EV	1.1159	0.2975	3.751
SE-Hybrid	0.7713	0.1967	3.921

Table 2. Model of Vehicle Choice Using Train's SML Program (40,000 draws)
Log-likelihood -896.10

Parameter	Estimate	Std.Error	z-Value
Price	-0.7833	0.1716	-4.565
Range	-0.3014	0.3755	-0.803
EV	-1.6711	0.3937	-4.245
Hybrid	0.9000	0.1640	5.488
Perf	0.6294	0.1059	5.943
SE-Price	1.0411	0.1585	6.568
SE-Range	0.5710	0.2651	2.154
SE-EV	1.1252	0.2952	3.812
SE-Hybrid	0.7732	0.1959	3.947

Table 3. Model with Correlated Parameters (6832 points)
Log likelihood -884.10

Parameter	Estimate	Std.Error	z-Value
Price-log	-0.7816	0.1935	-4.039
Range-log	-0.3487	0.4576	-0.762
EV	-1.7403	0.4457	-3.905
Hybrid	1.1377	0.2237	5.087
Perf	0.6301	0.1082	5.821
SE-Price	1.2337	0.1884	6.549
SE-Range	0.9198	0.3248	2.832
SE-EV	1.2466	0.4664	2.673
SE-Hybrid	1.4293	0.2524	5.663
COV-Pr&Range	0.7170	0.4062	1.765
COV-Pr&Ev	-0.3593	0.6164	-0.583
COV-Pr&Hybrd	1.1420	0.5034	2.269
COV-Range&EV	0.0280	0.7009	0.040
COV-Range&Hyb	1.0339	0.6443	1.605
COV-EV&Hybrd	0.6213	0.7661	0.811

Table 4. Logit Maximum Likelihood Results
Log likelihood -946.3609

Parameter	Estimate	Std.Error[a]	Std.Error[b]	z-Value[c]
Price	0.4260	0.0443	0.0415	9.607
Range	0.7031	0.2214	0.2176	3.176
EV	-1.3289	0.3163	0.3178	-4.201
Hybrid	0.5961	0.1230	0.1197	4.846
Perf	0.4636	0.0911	0.0905	5.089

[a] Robust standard error
[b] Conventional standard error from empirical Hessian
[c] Estimated parameter divided by robust standard error

- **Test whether parameter covariance is 0:**
  H0: that all parameter covariances are zero;
  LR=24.06 ~ Chi-sauqre (df= 6 degrees) (p<.001).
- **Magnitude of the differences**

- **Population Averaged WTP** for Hybrid Vehicle = 5.32 (Median = 2.49)
- **Other Approach:** $E[\frac{(\gamma+\nu)}{\exp(\beta+\eta)}]$ for Hybrid Vehicle = -0.02
- **Conditional logit model** (Table 4), the coefficient on Hybrid is clearly statistically significantly *greater than zero* even when using robust standard errors.

Conclusion

- ***We show via an empirical application that some approaches to calculating expected maximum WTP in mixed logit models (e.g. Greene et al. 2005), both in preference and WTP space, do not produce the proper welfare measures. Generally this is caused by failing to compute expected maximum utility before denominating it in dollars.***

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