A STRUCTURAL MODEL OF PRE-COMMITTED DEMAND: THE CASE OF FOOD DEMAND IN CHINA

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Abstract

The Exact Affine Stone Index (EASI) model of Lewbel and Pendakur (2009) is a new state-of-the-art demand system that resolves two fundamental issues plaguing previous literature. However, the EASI model does not account for committed quantities. This paper derives a generalized EASI (GEASI) demand system that incorporates committed quantities into the EASI model. Empirical evidence from Chinese household food consumption survey data indicates that the GEASI specification is superior to the EASI model.

Keywords: Committed demand, EASI model, generalized EASI system.

JEL Code: D11, D12.

1. Introduction

Lewbel and Pendakur (2009) develop an Exact Affine Stone Index (EASI) demand system that addresses two major challenges affecting the previous theory-plausible demand models such as the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) and its variants. Specifically, the EASI specification relieves Gorman's (1981) rank restriction on Engel curves, and allows for arbitrary curvilinear effects with the shape of the Engel curve determined by the data. Further, the EASI model accounts for unobserved consumer heterogeneity; which is necessary because consumers vary not only in terms of their economic circumstances but also with respect to their tastes and preferences (Browning and Carro 2007). The importance of modelling flexible Engel curves and allowing for unobserved consumer heterogeneity cannot be overstated given the empirical evidence of highly nonlinear Engel curves and the finding that typical observables (e.g., income, prices, and demographics) can only explain half the variation in budget shares (Banks et al. 1997).

The EASI model, however, ignores *committed quantities* invented by Gorman (1976) and popularized by Pollak and Wales (1981), whereas empirical literature provides ample evidence of committed quantities (*e.g.*, Bollino 1987, Hovhannisyan and Gould 2012). This paper generalizes the EASI system to include potential committed quantities. Empirical findings from this generalized EASI (GEASI) specification estimated on Chinese household food expenditure survey data provide strong empirical evidence of committed quantities.

2. The generalized EASI demand model

Consider the following cost function underlying the EASI demand system (Lewbel and Pendakur 2009):

(1) $$\ln C\left(p,u,\varepsilon\right)=u+\sum_{j=1}^{J}m_j\left(u\right)\ln p_j+\sum_{j=1}^{J}\sum_{k=1}^{J}\alpha_{jk}\ln p_j\ln p_k+\sum_{j=1}^{J}\varepsilon_j\ln p_j$$

where C represents cost, u is utility, $m_j(u)$ is a general function of u, p_j expresses the j^{th} product's price, ε_j reflects unobserved preference heterogeneity, and α_{jk} are parameters. Lewbel and Pendakur (2009) derive a linear approximate EASI demand system by applying Shephard's Lemma $\left(i.e., \frac{\partial \ln C}{\partial \ln p_i} = w_i \right)$ to the cost function in (1), which is subject to the usual adding-up, symmetry, and homogeneity demand restrictions:

(2) $$w_i(p,u,\varepsilon) = m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i$$

To incorporate committed quantities into the EASI system, I follow an approach offered by Bollino (1987) in generalizing the underlying cost function. Specifically, the cost function in (1) is modified to also include overhead costs as follows:[1]

(3) $$\ln\left(C - \sum_{j=1}^{J} t_j p_j \right) = u + \sum_{j=1}^{J} m_j(u) \ln p_j + \sum_{j=1}^{J}\sum_{k=1}^{J} \alpha_{jk} \ln p_j \ln p_k + \sum_{j=1}^{J} \varepsilon_j \ln p_j$$

where t_j is a parameter representing committed quantity of the j^{th} product.

The GEASI model is derived via the application of Sheppard's Lemma to the more general cost function in (3). More specifically, differentiating both sides of the cost function with respect to $(\ln p_i)$ generates the following budget share equation for the i^{th} product:

(4) $$\frac{\partial \ln\left(C - \sum_{j=1}^{J} t_j p_j \right)}{\partial \ln p_i} = m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i$$

[1] The approach used by Bollino (1987) to derive the generalized AIDS model from the indirect utility function cannot be applied here since $m_j(u)$ is in general an unknown function of utility.

Further simplification of the left hand side of the equation (4) yields:

$$\frac{\partial \ln\left(C-\sum_{j=1}^{J} t_j p_j\right)}{\partial \ln p_i} = \frac{\partial \ln\left(C-\sum_{j=1}^{J} t_j p_j\right)}{\partial p_i} \frac{\partial p_i}{\partial \ln p_i}$$

(5)

$$= \left(\frac{1}{\left(C-\sum_{j=1}^{J} t_j p_j\right)} \frac{\partial\left(C-\sum_{j=1}^{J} t_j p_j\right)}{\partial p_i} \right) p_i$$

$$= \left(\frac{\frac{\partial C}{\partial p_i} - t_i}{C-\sum_{j=1}^{J} t_j p_j} \right) p_i$$

Substituting (5) into (4) results in:

(6)

$$\left(\frac{\frac{\partial C}{\partial p_i} - t_i}{C-\sum_{j=1}^{J} t_j p_j} \right) p_i = m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i$$

Rearranging (6) yields the following expression for $\frac{\partial C}{\partial p_i}$:

(7)

$$\left(\frac{\partial C}{\partial p_i} - t_i\right) p_i = \left(C-\sum_{j=1}^{J} t_j p_j\right)\left(m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i\right)$$

$$\frac{\partial C}{\partial p_i} p_i - t_i p_i = \left(C-\sum_{j=1}^{J} t_j p_j\right)\left(m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i\right)$$

$$\frac{\partial C}{\partial p_i} p_i = t_i p_i + \left(C-\sum_{j=1}^{J} t_j p_j\right)\left(m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i\right)$$

$$\frac{\partial C}{\partial p_i} = t_i + \frac{1}{p_i}\left(C-\sum_{j=1}^{J} t_j p_j\right)\left(m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i\right)$$

Next, both sides of (7) are multiplied by $\left(\frac{p_i}{C}\right)$ to generate Hicksian budget share equations since

$w_i = \left(\frac{\partial C}{\partial p_i}\right)\left(\frac{p_i}{C}\right) = \left(\frac{q_i p_i}{C}\right)$:

$$
\begin{aligned}
w_i &= \frac{p_i}{C}\left(t_i + \frac{1}{p_i}\left(C - \sum_{j=1}^{J} t_j p_j\right)\left(m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i\right)\right) \\
&= \frac{t_i p_i}{C} + \left(\frac{C - \sum_{j=1}^{J} t_j p_j}{C}\right)\left(m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i\right) \qquad (8) \\
&= \frac{t_i p_i}{C} + \left(1 - \frac{\sum_{j=1}^{J} t_j p_j}{C}\right)\left(m_i(u) + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i\right)
\end{aligned}
$$

Finally, the implicit GEASI Marshallian demand system is obtained by: (*i*) substituting consumer total expenditure *X* for *C* given a utility maximizing consumer, and (*ii*) replacing $m_i(u)$ with a particular function offered by Lewbel and Pendakur (2009) as follows:

$$
(9) \qquad w_i = \frac{t_i p_i}{X} + \left(1 - \frac{\sum_{j=1}^{J} t_j p_j}{X}\right)\left(\sum_{r=0}^{L} \beta_{ir}\left(\ln\left(X - \sum_{j=1}^{J} t_j p_j\right) - \sum_{j=1}^{J} w_{ir} \ln p_{ik}\right)^{r} + \sum_{i=1}^{J} \alpha_{ik} \ln p_k + \varepsilon_i\right)
$$

where $m_i(u)$ is replaced by $\sum_{r=1}^{L} \beta_{ir} y^r$ with $y = \ln\left(X - \sum_{j=1}^{J} t_j p_j\right) - \sum_{j=1}^{J} w_{ir} \ln p_{ik}$ and r denotes the order of the polynomial function of real income that provides a flexible representation of Engel curves. Note that the system in (9) is subject to the theoretical restrictions of adding-up $\left(\sum_{i=1}^{J} \beta_{i0} = 1;\ \sum_{i=1}^{J} \beta_{ir} = 0, \forall r = 1,\ldots,L; \sum_{i=1}^{J} \alpha_{ik} = 0, \forall k = 1,\ldots,J; \sum_{i=1}^{J} t_i = 0\right)$ and symmetry

$\left(\alpha_{ik} = \alpha_{ki}, \forall i, k = 1,..., J\right)$. Further, the EASI model is nested in the GEASI specification and can be obtained via the joint restriction of $t_i = 0, \forall i = 1,..., J$ on the GEASI model.

I estimate the EASI and the GEASI budget share equations using provincial-level household food expenditure survey data from China (Table 1). These data cover 30 provinces/administrative districts over the span 2003-2012, and include seven widely consumed food commodity groups (*i.e.*, meats, seafood, vegetables, fruit, grains, eggs, and fats/oils). Based on the Likelihood ratio test outcome, the GEASI model is empirically superior to the EASI model (Table 1, upper part). The results are robust to the inclusion of provincial fixed effects, which account for unobserved time-invariant characteristics of Chinese provinces that can influence food consumption patterns through deeply rooted local food customs and traditions (Table 1, lower part).[2]

3. Summary

This paper derives a generalized EASI (GEASI) demand model via the incorporation of committed quantities into the EASI model. An econometric application of the GEASI to the Chinese household food expenditure survey data provides strong empirical evidence of superiority of the GEASI to the EASI specification. Studying the importance of *committed demand* in developing economies such as China and India in the face of improving living standards is one area where the GEASI system may prove invaluable.

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[2] Parameter values are not included for limited space but are available upon request.

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Table 1. Summary of the Model Diagnostic Tests

Hypothesis		Likelihood ratio value	df.	p-value
Food commodities are not consumed in pre-committed quantities ($t_j = 0, \forall j = 1,...,n$), that is, GEASI and EASI models are equivalent				
Without Provincial Fixed-Effects				
(i)	Linear Engel Curve (i.e., *r*=1)	928.5	7	0.00
(ii)	Quadratic Engel Curve (i.e., *r*=2)	935.4	7	0.00
(iii)	Cubic Engel Curve (i.e., *r*=3)	835.1	7	0.00
(iv)	Quartic Engel Curve (i.e., *r*=4)	866.4	7	0.00
(v)	Quintic Engel Curve (i.e., *r*=5)	880.0	7	0.00
(vi)	Sextic Engel Curve (i.e., *r*=6)	1278.4	7	0.00
With Provincial Fixed-Effects				
(vii)	Linear Engel Curve (i.e., *r*=1)	1,403.8	7	0.00
(viii)	Quadratic Engel Curve (i.e., *r*=2)	1,421.1	7	0.00
(ix)	Cubic Engel Curve (i.e., *r*=3)	1,423.0	7	0.00
(x)	Quartic Engel Curve (i.e., *r*=4)	1,370.2	7	0.00
(xi)	Quintic Engel Curve (i.e., *r*=5)	1,394.0	7	0.00
(xii)	Sextic Engel Curve (i.e., *r*=6)	811.9	7	0.00

Note 1: The EASI and GEASI specifications are estimated on household food expenditure panel data obtained from the National Bureau of Statistics of China. The data cover 30 provinces/administrative districts over the span 2003-2012, and include seven widely consumed food commodity groups (*i.e.*, meats, seafood, vegetables, fruit, grains, eggs, and fats/oils). A total of 2,100 observations have been utilized in the demand system estimation.

Note 2: The degree of polynomial functions estimated cannot exceed 6 (i.e., R<J), otherwise the resulting Engel curves will be arbitrarily complex (Lewbel and Pendakur 2009).

Appendix. ***Derivation of the Expenditure, Hicksian, and Marshallian Elasticity Formulas for the GEASI Model***

Expenditure Elasticities

To develop the expenditure elasticities for the GEASI model, we first derive the general formula for the expenditure elasticity using the definition of expenditure shares $w_i = \frac{p_i q_i}{X}$, which is rearranged to $q_i = \frac{w_i X}{p_i}$

(10) $$\frac{\partial q_i}{\partial \ln X} = \frac{1}{p_i}\left[\frac{\partial X}{\partial \ln X} w_i + X \frac{\partial w_i}{\partial \ln X}\right] = \frac{1}{p_i}\left[X w_i + X \frac{\partial w_i}{\partial \ln X}\right]$$

(11) $$\frac{\partial q_i}{\partial \ln X} = \frac{\partial e^{\ln q_i}}{\partial \ln X} = q_i \frac{\partial \ln q_i}{\partial \ln X}$$

(12) $$\begin{aligned}\frac{\partial \ln q_i}{\partial \ln X} &= \frac{1}{q_i}\frac{\partial q_i}{\partial \ln X} = \frac{1}{q_i}\left[\frac{1}{p_i}\left[X w_i + X \frac{\partial w_i}{\partial \ln X}\right]\right] = \frac{1}{p_i q_i}\left[X w_i + X \frac{\partial w_i}{\partial \ln X}\right] = \frac{w_i}{p_i q_i} X + \frac{X}{p_i q_i}\frac{\partial w_i}{\partial \ln X} \\ &= \frac{1}{X} X + \frac{1}{w_i}\frac{\partial w_i}{\partial \ln X} = \frac{1}{w_i}\frac{\partial w_i}{\partial \ln X} + 1\end{aligned}$$

where use is made of the fact that $\frac{w_i}{p_i q_i} = \frac{p_i q_i}{X}\frac{1}{p_i q_i} = \frac{1}{X}$ and $\frac{X}{p_i q_i} = \frac{1}{w_i}$.

The GEASI expenditure elasticities are then obtained by substituting $\frac{\partial w_i}{\partial \ln X}$ derived from the GEASI model into (12). To this end, we utilize the respective GEASI expenditure share equations provided below (see equation (9)):

$$w_i = \frac{t_i p_i}{X} + \left[1 - \frac{t' p}{X}\right]\left(\sum_{r=0}^{L} \beta_{ir}\left(\ln\left(X - t' p\right) - w' \ln p\right)^r + \sum_{k=1}^{J} \alpha_{ik} \ln p_k\right) + \varepsilon_i$$

Let $A_1 = \frac{t_i p_i}{X}$, $A_2 = \left[1 - \frac{t' p}{X}\right]$, $A_3 = \left(\sum_{r=0}^{L} \beta_{ir}\left(\ln\left(X - t' p\right) - w' \ln p\right)^r + \sum_{k=1}^{J} \alpha_{ik} \ln p_k\right)$. The derivative of the expenditure shares with respect to log expenditure $\left(i.e., \frac{\partial w_i}{\partial \ln X}\right)$ is as follows:

(13) $$\frac{\partial w_i}{\partial \ln X} = \frac{\partial A_1}{\partial \ln X} + \frac{\partial A_2}{\partial \ln X} A_3 + \frac{\partial A_3}{\partial \ln X} A_2$$

(14) $$\frac{\partial(A_1)}{\partial \ln X} = \frac{\partial(t_i p_i / X)}{\partial \ln X} = t_i p_i \frac{\partial(1/X)}{\partial \ln X} = t_i p_i \left(-X^{-2} X\right) = -\frac{t_i p_i}{X}$$

(15) $$\frac{\partial(A_2)}{\partial \ln X} = \frac{\partial\left(1 - \frac{t' p}{X}\right)}{\partial \ln X} = \frac{t' p}{X}$$

(16) $$\frac{\partial(A_3)}{\partial \ln X} = \left[\sum_{r=0}^{L} r\beta_{ir}\left(\ln(X - t' p) - w' \ln p\right)^{r-1} + \sum_{k=1}^{J} \alpha_{ik} \ln p_k\right]\left[\frac{X}{X - t' p} - \left(\frac{\partial w}{\partial \ln X}\right)' \ln p\right]$$

where $\left(\frac{\partial w}{\partial \ln X}\right)' = \left(\frac{\partial w_1}{\partial \ln X}, ..., \frac{\partial w_i}{\partial \ln X}, ..., \frac{\partial w_N}{\partial \ln X}\right)'$.

Substituting (14)--(16) into (13) results in:

(17) $$\begin{aligned}\frac{\partial w_i}{\partial \ln X} &= -\frac{t_i p_i}{X} + \frac{t' p}{X} A_3 \\ &+ \left[\sum_{r=0}^{L} r\beta_{ir}\left(\ln(X - t' p) - w' \ln p\right)^{r-1} + \sum_{k=1}^{J} \alpha_{ik} \ln p_k\right]\left[\frac{X}{X - t' p} - \left(\frac{\partial w}{\partial \ln X}\right)' \ln p\right] A_2 \\ &= -\frac{t_i p_i}{X} + \frac{t' p}{X} A_3 + A_4\left[\frac{X}{X - t' p} - \left(\frac{\partial w}{\partial \ln X}\right)' \ln p\right] A_2,\end{aligned}$$

where $A_4 = \left[\sum_{r=0}^{L} r\beta_{ir}\left(\ln(X - t' p) - w' \ln p\right)^{r-1} + \sum_{k=1}^{J} \alpha_{ik} \ln p_k\right]$.

Note that the equation in (17) represents a (*J x J*) system of implicit equations with

$\frac{\partial w_i}{\partial \ln X}$, $\forall i = 1, ..., J$ appearing on both sides of each of these equations. Using matrix algebra,

we solve the system in (17) for $\frac{\partial w_i}{\partial \ln X}$ as follows:

(18) $$\frac{\partial w}{\partial \ln X} = \left[I_J + \left(\left(\frac{X - t' p}{X}\right) * B\right)(\ln p)'\right]^{-1}\left[\frac{t \circ p}{X} + \frac{t' p}{X} A_3 + B\right]$$

where B is a (J x 1) vector with its i^{th} element equaling $\left(\sum_{r=1}^{L} r\beta_{il} y^{r-1}\right)$, and A_3 is as previously

defined, *i.e.*, $A_3 = \left(\sum_{r=0}^{L} \beta_{ir} \left(\ln\left(X - t'p\right) - w'\ln p\right)^r + \sum_{k=1}^{J} \alpha_{ik} \ln p_k\right)$.

Finally, we obtain the GEASI expenditure elasticity formula by substituting (18) into (12):

$$E = \left(diag\left(W\right)\right)^{-1} \left[\left[I_J + \left(\left(\frac{X - t'p}{X}\right) * B\right)\left(\ln p\right)'\right]^{-1} \left[\frac{t \circ p}{X} + \frac{t'p}{X} A_3 + B\right]\right] + 1_J, \tag{19}$$

where E is the (J x 1) expenditure elasticity vector with e_i denoting its i^{th} element, W is represents the (J x 1) vector of observed commodity budget shares, $\ln p$ is the (J x 1) vector of log prices, and 1_J is a (J x 1) vector of ones.

<u>Hicksian and Marshallian Elasticities</u>

We derive the GEASI Hicksian elasticities by deriving $\frac{\partial w_i}{\partial \ln p_j}$ for our more general model and substituting back into the Hicksian elasticity formula provided in general terms:

$$e_{ij}^H = \frac{1}{w_i}\left[\frac{\partial w_i}{\partial \ln p_j}\right] + w_j - \delta_{ij}, \;\; \forall i, j = 1, ..., J, \tag{20}$$

Using the GEASI expenditure share equations in (9), we obtain:

$$\frac{\partial w_i}{\partial \ln p_j} = -\frac{t_j p_j}{X} A_3 + \left[1 - \frac{t'p}{X}\right]\alpha_{ij}, \quad \forall i \neq j \tag{21}$$

$$\frac{\partial w_i}{\partial \ln p_i} = \frac{t_i p_i}{X} - \frac{t_i p_i}{X} A_3 + \left[1 - \frac{t'p}{X}\right]\alpha_{ii}, \tag{22}$$

Equations (21) and (22) are substituted into (20) to yield the GEASI Hicksian Elasticity formulas:

$$e_{ij}^H = \frac{1}{w_i}\left[\frac{t_i p_i}{X} - \frac{t_i p_i}{X} A_3 + \left[1 - \frac{t'p}{X}\right]\alpha_{ii}\right] + w_j - \delta_{ij}, \;\; \forall i, j = 1, ..., J, \tag{23}$$

Marshallian price elasticities $\left(e_{ij}^{M}\right)$ are obtained from the Slutsky equation: $e_{ij}^{M} = e_{ij}^{H} \dfrac{\alpha_{ij}}{w_i} - w_j \, \mathrm{e}_i$.

(24) $$e_{ij}^{M} = \left[\left[\frac{t_i p_i}{X} - \frac{t_i p_i}{X} A_3 + \left[1 - \frac{t' p}{X}\right]\alpha_{ii}\right] + w_j - \delta_{ij}\right]\frac{\alpha_{ij}}{w_i^2} - w_j \, \mathrm{e}_i \, .$$