Cattle, Cutouts, and the Drop: How much information is in disaggregated cattle prices?

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Abstract: I estimate and test a vector autoregression (VAR) model for 12 cattle prices and 3 beef packer prices to see (1) if some or all of these cattle prices can be averaged without affecting forecasts and (2) exactly how similar these cattle prices are. I demonstrate that prices do not have to similar to be averaged.

There are 6 steer-heifer pairs in this data set. These 6 pairs are the terms that can be averaged with the least effect on the forecasts. While 3 of these pairs are statistically insignificant and in theory could be averaged, 3 are not and should not be averaged.

All 15 prices are cointegrated with a single root equal to 1. This tends to make cattle prices "stick together" over the long run. All 3 of the prices that can be averaged have statistically significant differences in their shortrun and longrun behaviors. I find some similarities in other groups of prices.

Keywords

Cattle prices, beef prices, cointegration

Summary and Introduction

The first goal of this research was to see if reported cattle prices could be averaged without loss of information to the market. Over the years, a smaller and smaller proportion of cattle have been sold through auctions or are priced by packer-feeder negotiation. A common alternative pricing method involves using a formula on some reported auction or negotiated price. People are now concerned that the small proportion of negotiated prices makes them less reflective of market conditions and more vulnerable to manipulation by packers. See Mathews et al (2015) for a discussion of these issues.

A number of analysts have proposed solutions for the thinning of negotiated cattle markets. One that Koontz (2015) proposed was the averaging of cattle prices: that the USDA might want to aggregate its reports. The USDA would report fewer average prices but with more cattle in each of the reported averages. I test a set of negotiated cattle prices to see if some types of averaging these prices is appropriate.[2]

In Mathews et al, we compared cattle prices under a range of pricing methods and found that they looked remarkably similar. I also test to see exactly how similar negotiated cattle prices are. To test both of these types of hypotheses I estimate and test a vector autoregression (VAR) model. It turns out that there is a mathematical difference between "I can average these two prices without affecting my forecasts" and "These two prices' forecasts are pretty much the same."

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[1] The views expressed in this paper are the author's and do not represent those of the Economic Research Service or the U.S. Department of Agriculture. To contact the author email him at whahn@ers.usda.gov.

[2] The weekly cattle price data is from the weekly 5-area cattle report and is already a set of national averages. Koontz analyzed a series of regional reports.

The VAR is an expansion of the one found in Mathews et al. The one here uses 12 negotiated fed cattle prices and 3 prices for beef-packer outputs. (The fed cattle are purchased by beef packers.) The VAR I estimated for Mathews et al had 1 of the 12 cattle prices in it.

I looked at a range of cattle-price averaging approaches. Cattle price averaging will impose linear restrictions on the VAR. The 6 least statistically significant averages involve averaging steer-heifer pairs. Three of the six pairs are statistically insignificant—meaning that for 3 of these 6 pairs cannot be averaged without affecting forecasts. It appears that generically averaging cattle prices is not a great idea.

I also looked at a range of tests for cattle price similarities. The most extreme version of "similarity" makes a pair of price forecasts the same. This most extreme type was rejected for all cattle-price pairs. I did find some less extreme versions of price similarities when I tested the cointegration relationships between the cattle prices.

Some "Issues" with the General Approach

I am working with national average weekly data: it is already fairly aggregated, and, more importantly, consistently reported. Koontz' analysis focused on regional prices. This could make it somewhat less likely that I am going to find averaging valuable. However, as noted above, negotiated cattle sales are a declining part of the market. If the decline continues, even these weekly averages could become spottily-reported. It would be useful to examine averaging before we have problems.

My general approach is to build a forecasting model, specifically a VAR. Price averaging and similarities imply restrictions on the VAR coefficients. I test these restrictions. The basic assumption underlying this approach is that the value of this data is in forecasting. This approach ignores other potential uses of the data. As I note in the summary I find that I can average 3 of the 6 steer-heifer pairs. However, steers and heifers in each of these pairs show some statistically significant differences that averaging covers up.

Data

In Mathews et al I used a vector autoregression (VAR) with weekly data. I related 1 cattle price and 3 packer prices; I expand on this approach by including 11 additional cattle prices.

The cattle prices used in this report come from the AMS report "(LM CT 105)5 Area Weekly Weighted Average Direct Slaughter Cattle". All the cattle's' prices in this report are negotiated between the packer and the producer. I use 6 of the Dressed Delivered prices and 6 of the live-weight FOB prices. (These are the most consistently reported series in this report.)

"Dressed" cattle are priced based on the weights of their carcasses as they enter the carcass cooler. Live-weight cattle are weighed when they are alive. The packer arranges and pays for the shipping of FOB (free on board) cattle. The feedlot arranges and pays for the shipping of delivered cattle. The report also has live-delivered and dressed FOB cattle prices. Relatively few cattle are sold these two ways.

USDA AMS separates these cattle prices by sex: prices are reported for both steers and heifers. They also separate the reports into 4 different grade levels: 0-35% Choice, 35-65% Choice, 65-80% Choice, and over 80% Choice. Choice is the second-highest USDA cattle/beef grade and it is the most common grade for grainfed steers and heifers. In recent years around 70% of the steers and heifers that are graded are Choice. Most of the rest of the cattle in these lots will grade Select, the 3rd-highest grade. The highest grade is Prime; around 4-6.5% of steers and heifers will that are graded are Prime.

Few lots of cattle grade 0-35% Choice, so USDA AMS seldom has prices to report in this range. The 12 cattle prices I use in this report come from 2 types of pricing (Live FOB and Dressed Delivered) by 2 sexes (heifers and steers) and 3 grade level (35-65% Choice, 65-80% Choice, and over 80% Choice.) In Mathews et al, I just used the Live, FOB Steer, 35-65% Choice.

I also use the weekly Choice and Select cutouts from the AMS report (LM_XB459) "National Weekly Boxed Beef Cutout and Boxed Beef Cuts - Negotiated Sales." The Choice and Select cutouts are carcass-weighted prices for the Choice and Select beef cuts sold by packers. The last variable in the model is the weekly steer, FOB Central U.S. By-Product Drop Value (Steer) from the *Weekly National Carlot Meat Report*. All the data in this study except for the Drop[3] value can be download from USDA AMS MPR Data Mart site. The weekly data runs from the first week in April 2001 to the last week of 2016. The first 3 weeks of October 2013 are missing due to the Sequester.

I will use abbreviated names to refer to the 15 endogenous variables that I analyzed. These can be found in Table 1. In Mathews et al, I analyzed SLF35, Choice, Select, and Drop. USDA ERS uses SLF35, Choice, and Drop in its calculation of Choice beef values and price spreads.

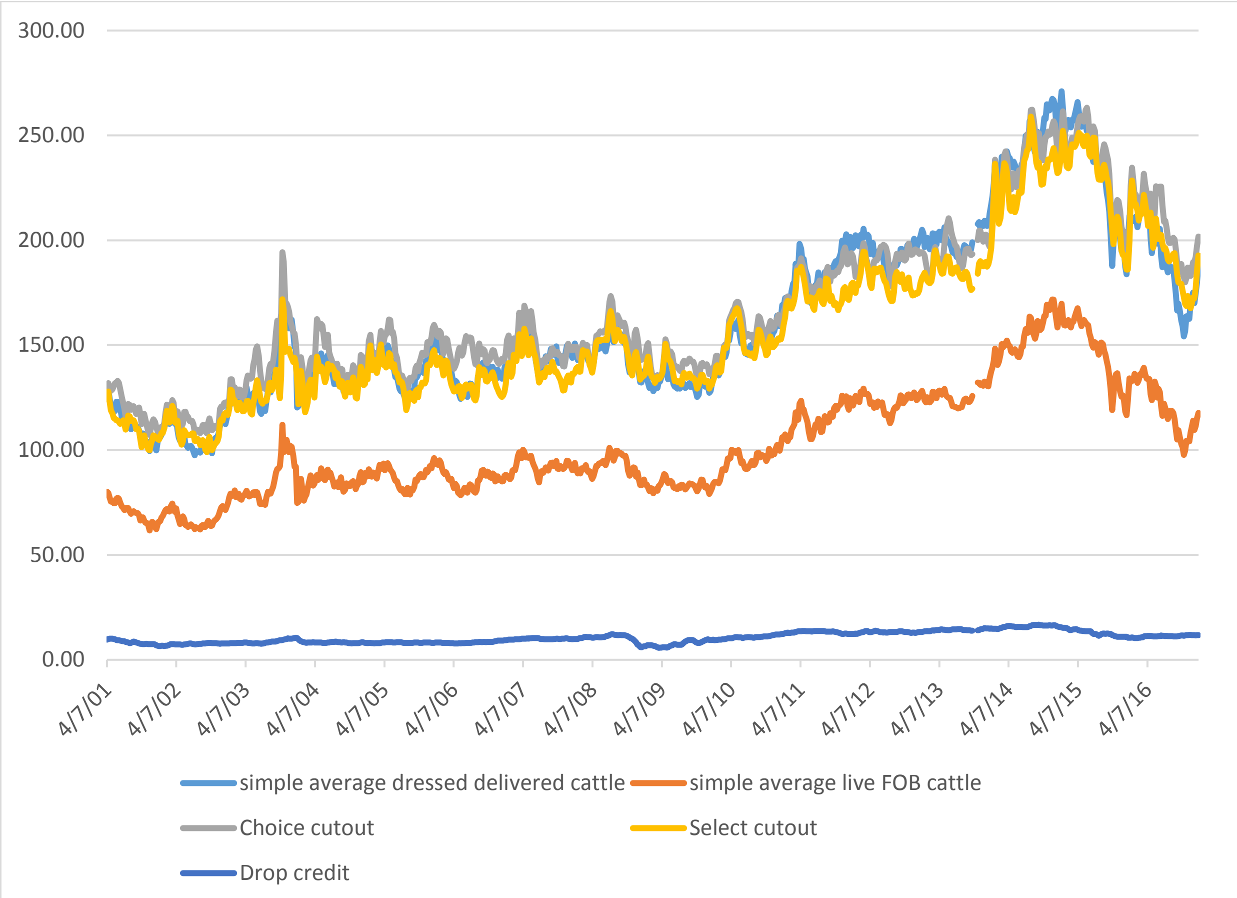
Table 1—endogenous variable key

cattle prices			
name	***Sex***	***Marketing type***	***grade range***
HDD35	Heifers	dressed delivered	35-65% Choice
HDD65	Heifers	dressed delivered	65-80% Choice
HDD80	Heifers	dressed delivered	80%+ Choice
HLF35	Heifers	live, FOB	35-65% Choice
HLF65	Heifers	live, FOB	65-80% Choice
HLF80	Heifers	live, FOB	80%+ Choice
SDD35	Steers	dressed delivered	35-65% Choice
SDD65	Steers	dressed delivered	65-80% Choice
SDD80	Steers	dressed delivered	80%+ Choice
SLF35	Steers	live, FOB	35-65% Choice
SLF65	Steers	live, FOB	65-80% Choice
SLF80	Steers	live, FOB	80%+ Choice
packer prices			
Choice	Choice cutout		
Select	Select cutout		
Drop	FOB central U.S. by-product drop value (steer)		

[3] USDA ERS collects the weekly drop value to calculate the price spreads for Choice beef. I have it going back before 2000 on my hard drive.

[Figure 1](#) shows how the data evolves over time. Rather than show all 12 cattle prices, Figure 1 has the simple average of all the dressed delivered and live FOB cattle. It also shows, the Choice, Select, and Drop values.

Figure 1—selected times series of the data used in the VAR



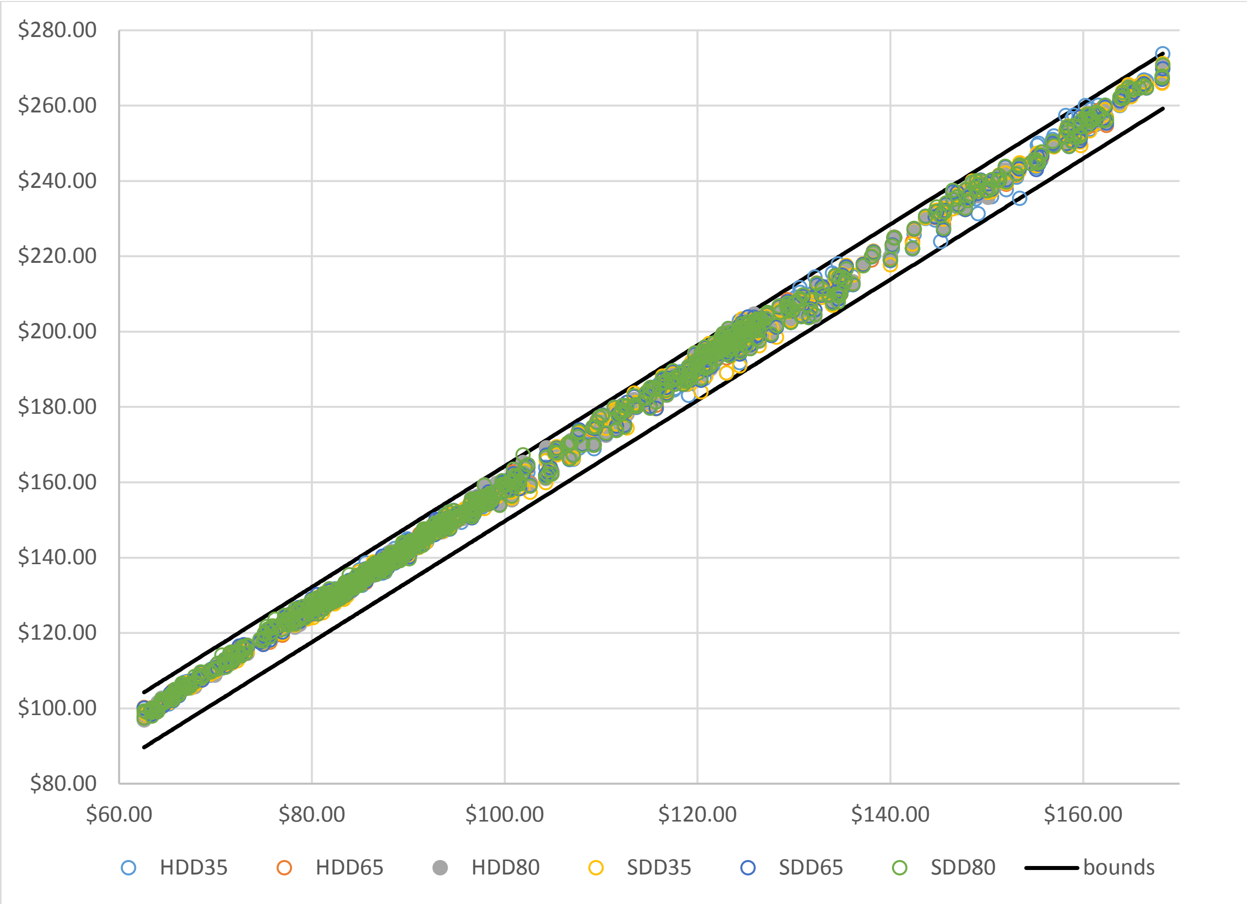
Note all prices in dollars per hundred pounds

Dressed delivered cattle prices are always higher than the live FOB cattle prices. Fed steers and heifers have carcass-to-live yields or dressing percentages ranging 60-65%. Dressed cattle are priced on a carcass-weight basis and consequently have a higher price. You will note that the average dressed delivered price tracks the two cutouts closely. It is sometimes a bit higher, sometimes a bit lower. Packers sell both meat and byproducts and there is a piece of beef packing folklore that meat sales cover the costs of the animal and that the byproducts cover packing costs and profits.

As noted above, Choice is a higher grade than Select. Figure 1 shows that Choice cutout is almost always higher than the Select cutout. In this data set, the Select cutout is higher than Choice only once: for the week ending March 28, 2009 when the Select cutout is 13 cents higher than Choice.

[Figure 2](#) shows all 6 prices for the dressed delivered cattle. Figure 2 is a scatter plot where I have graphed the cattle prices against a sort of weighted average price for all 12 cattle and the 2 cutouts. When I made this average, I multiplied the 6 dressed delivered cattle prices and 2 cutouts by a carcass yield factor of 0.63 to transfer the carcass prices to an approximate live-weight basis. Figures 3, 4, and 5 are scatter plots for the 6 live FOB cattle, 2 cutouts, and the drop values.

Figure 2—dressed-delivered cattle prices plotted against the live-weight average price of the 12 cattle and 2 cutouts

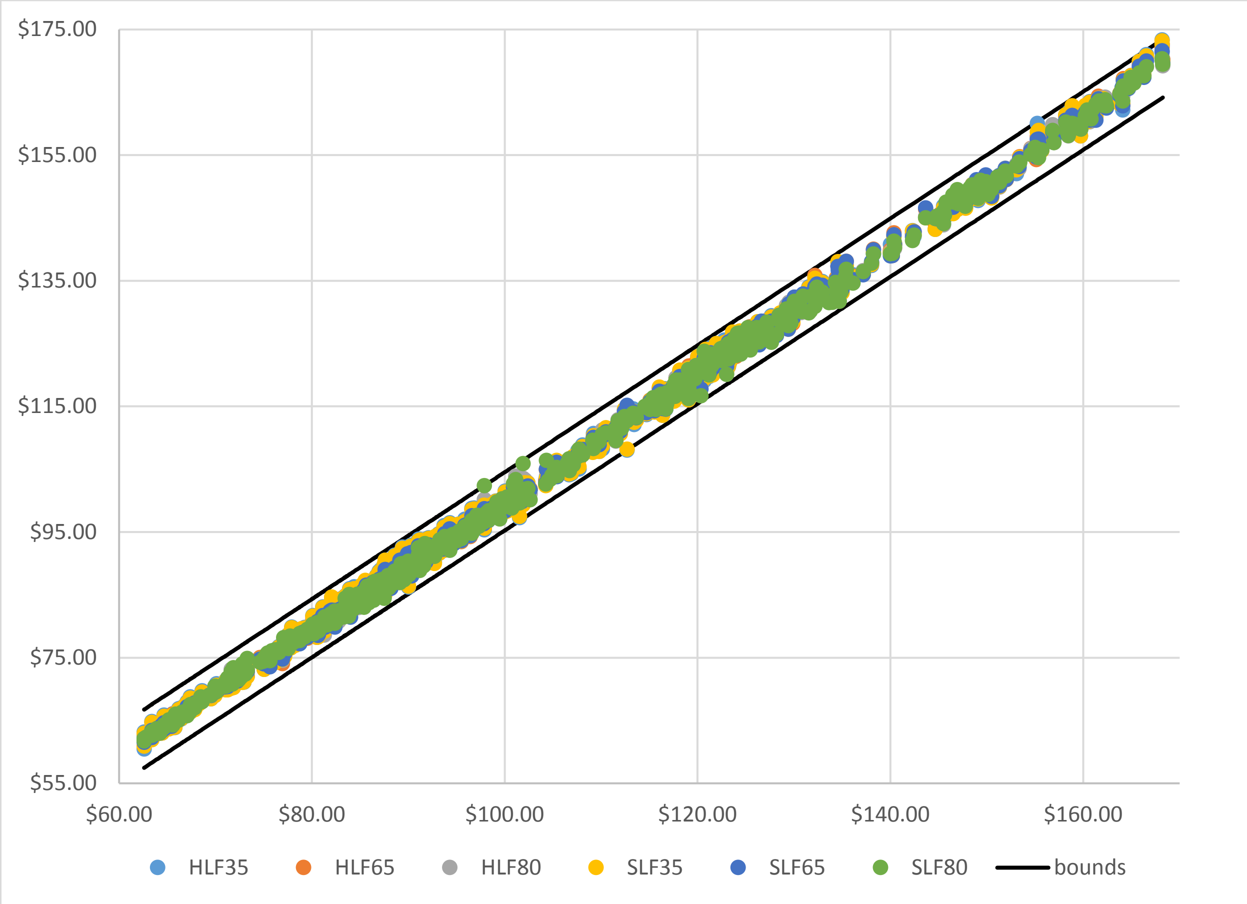


Notes All prices in dollars per hundred pounds. The average values are the averages of the live FOB cattle prices, the dressed delivered cattle prices time 0.63, and the two cutouts also times 0.63. 0.63 is the USDA ERS standard yield used to calculate beef price spreads. See Table 1 for the cattle name abbreviations.

In Mathews et al I found that the prices were *cointegrated*. All the prices are statically non-stationary but have a tendency to move with one another. Basically what I found is that there is a single, common "thing" that made all 4 prices unstable. The scatter plots provide evidence of the cointegration of this data. Figure 1 shows that the data used in this analysis varies widely over time. Figures 2, 3, 4, and 5 show how closely the data tends to stick together. The relationships between the cattle prices and the averages are particularly tight.

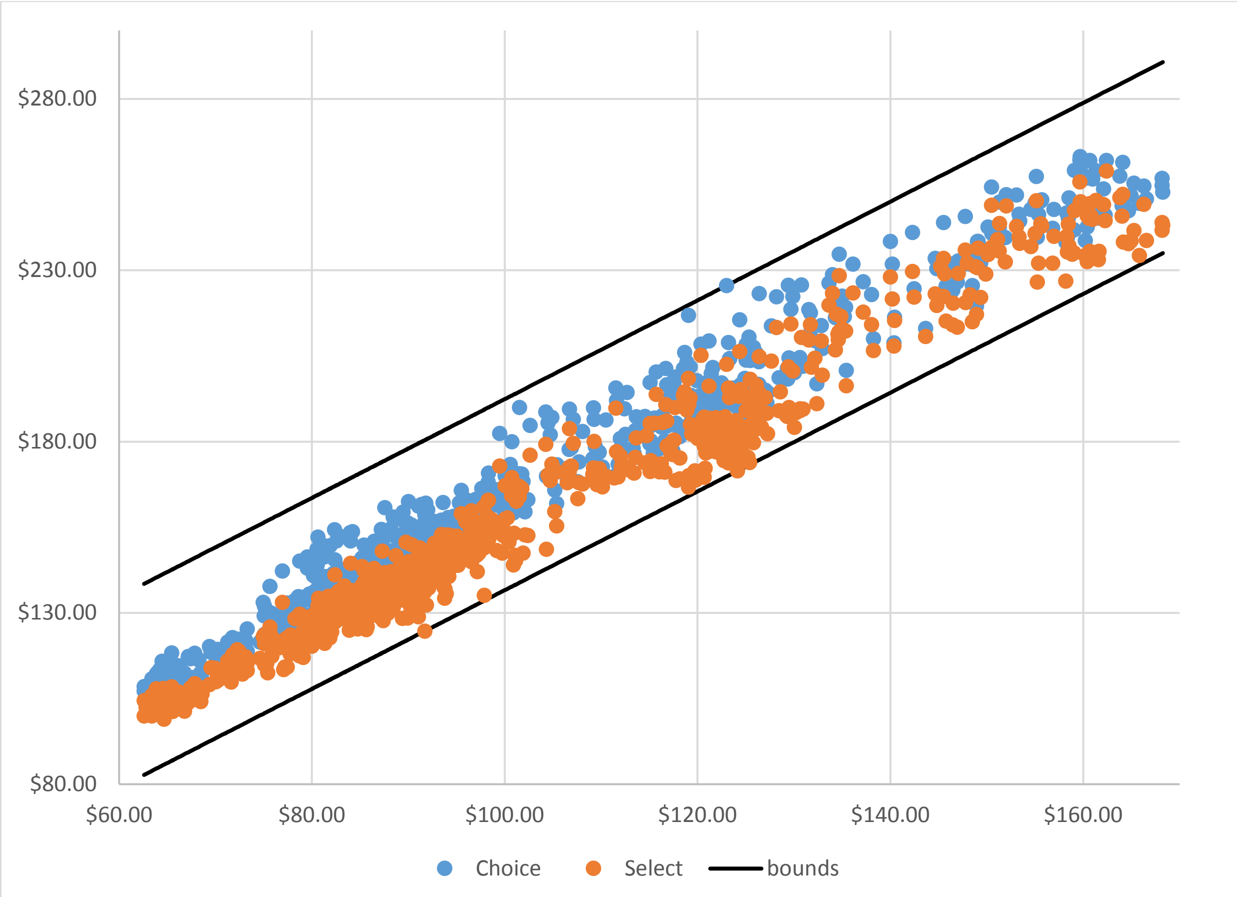
The highest price in either the dressed, delivered or live FOB class varies over time. The most important factor differentiating Choice from Select grade beef is marbling, small flecks of fat in the meat. Feeding cattle to produce more marbling tends to produce more subcutaneous fat that needs to be trimmed off the beef cuts. Subcutaneous fat is less valuable than meat. Higher grading cattle will have more valuable meat; the higher meat value may be offset by higher level of fat in their carcasses.

Figure 3—live FOB cattle prices plotted against the live-weight average price of the 12 cattle and 2 cutouts



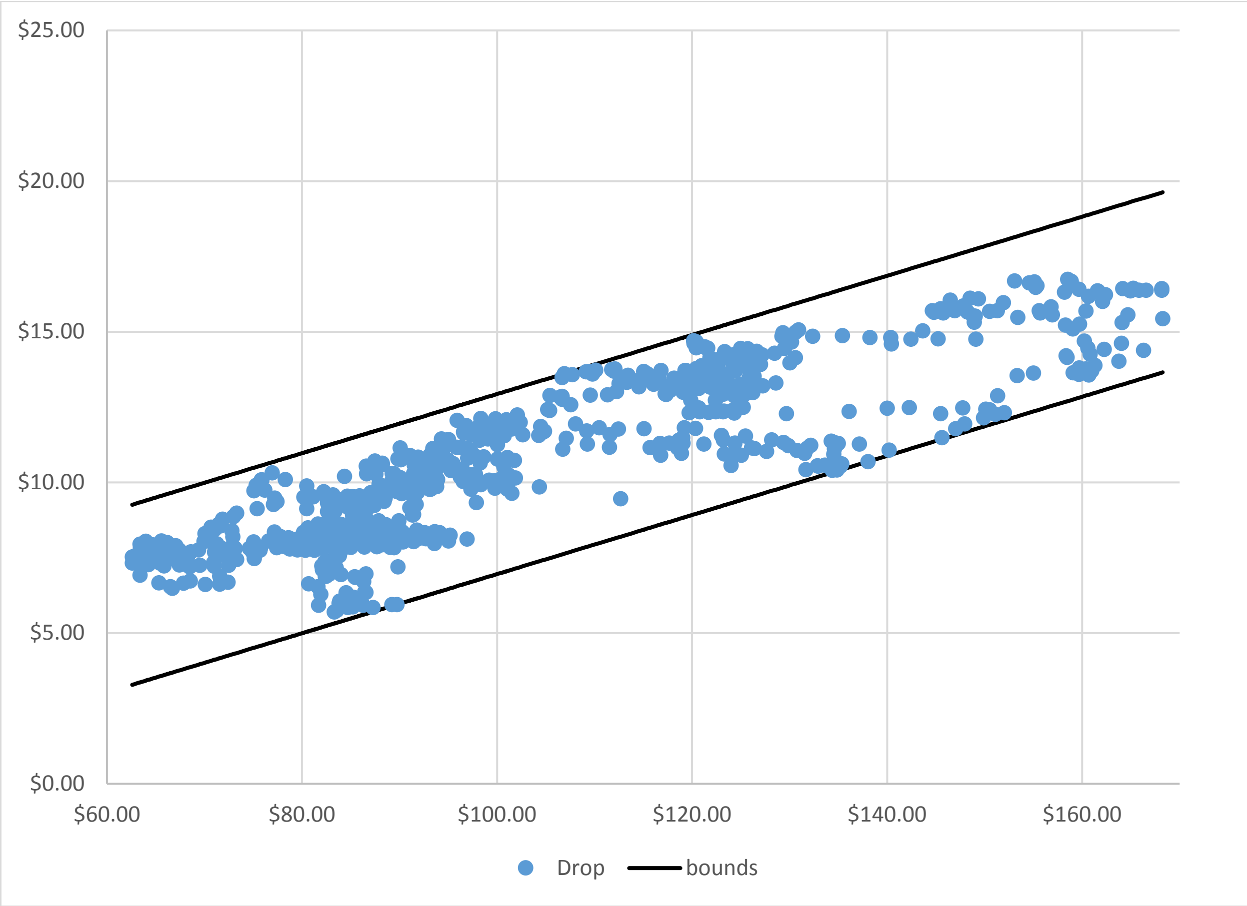
Notes All prices in dollars per hundred pounds. The average values are the averages of the live FOB cattle prices, the dressed delivered cattle prices time 0.63, and the two cutouts also times 0.63. 0.63 is the USDA ERS standard yield used to calculate beef price spreads. See Table 1 for the cattle name abbreviations.

Figure 4—Choice and Select cutouts plotted against the live-weight average price of the 12 cattle and 2 cutouts



Notes All prices in dollars per hundred pounds. The average values are the averages of the live FOB cattle prices, the dressed delivered cattle prices time 0.63, and the two cutouts also times 0.63. 0.63 is the USDA ERS standard yield used to calculate beef price spreads. See Table 1 for the cattle name abbreviations.

Figure 5—Drop value plotted against the live-weight average price of the 12 cattle and 2 cutouts



Notes All prices in dollars per hundred pounds. The average values are the averages of the live FOB cattle prices, the dressed delivered cattle prices time 0.63, and the two cutouts also times 0.63. 0.63 is the USDA ERS standard yield used to calculate beef price spreads. See Table 1 for the cattle name abbreviations.

The VAR

The model that I estimated can be written:

(1) $Y_t = V_1 Y_{t-1} + CX_t + E_t$, or

(2) $y_{it} = \sum_j v_{ij} y_{jt-1} + \sum_k c_{ik} x_{kt} + e_{it}$

(1) is the vector version of the VAR, (2) a scalar version. I switch between versions based on which is more convenient. In (1) Y_t is a [15 by 1] column vector of prices for the period numbered t; Y_{t-1} is that vector's value in the previous week. I write individual elements of the endogenous-viable vector as y_{it}. The subscript "i" refers to a specific price. In my estimation program, the endogenous variable sets are defined using the names in Table 1.

The term V_1 is a [15 by 15] matrix of coefficients that I have to estimate. I will denote individual elements of this matrix with the symbol v_{ij}. The "i" subscript is for the equation determining the i^{th} endogenous variable. The "j" subscript refers to the j^{th} lagged price.

C is a matrix of coefficients that I estimate, and X_t a vector of exogenous variables. The exogenous variables in the model are listed in Table 2. Finally E_t is a vector of error terms with a mean of 0. I assume that is independently and identically distributed over time. (I allow the error terms to have cross-equation covariance.)

Table 2—exogenous variable key

names		***Explanation***
X0		Intercept
		seasonal variables
COS1	SIN1	Cosine and sine terms making 1 revolution per year assuming (52+1.25/7) weeks per year. Week 0 is week ending Jan. 7, 2006/week starting Jan. 1, 2006.
COS2	SIN2	cosine and sine terms making 2 revolutions
COS3	SIN3	cosine and sine terms making 3 revolutions
COS4	SIN4	cosine and sine terms making 4 revolutions
COS5	SIN5	cosine and sine terms making 5 revolutions
COS6	SIN6	cosine and sine terms making 6 revolutions
		disease outbreak effects
BSE1		dummy for the week the U.S. Government announced the first U.S. case of BSE (week ending Dec. 27, 2003)
BSE2		dummy for the week after the BSE announcement
BSE3		dummy for 2 weeks after the BSE announcement

Notes: The cells with yellow highlighting were dropped in the early phases of model development as they were statistically insignificant for the VAR as a whole.

The VAR that I specified in (1) has only 1 lag in it. This makes it consistent with the model that I ran in Mathews et al. I started with a 6-lag model and tested it in the preliminary phases of this study. The 2^{nd}-6^{th} lags were statistically insignificant.

I started with more exogenous variables than I ended up using. In the preliminary phase of model development I also tested the exogenous. I started with 16 and ended up dropping 5 of them. See Table 2.

I use cosines and sines to get seasonal variation in the VAR. When people estimate monthly or quarterly models, they typically use dummy variables. The problem with weekly data is that there are not an even number of weeks in a year. "Regular" years have 365 days—52 weeks and 1 day. Leap years have 366 days. Years have 4 quarters or 12 months exactly, so dummy variables work perfectly in this case. One may use sines and cosines to put seasonal variation in quarterly and monthly data too. See Doran and Quilkey (1972). I used this same approach to imposing seasonality in Mathews et al.

Averaging constraints for the VAR

Suppose that we could use a simple average of some or all the cattle prices in our forecasting model. Let j' be a subset of the lagged cattle prices that I may average. There are $n' \leq 12$ cattle prices in this subset. The lagged wholesale prices and "unaveraged" cattle prices are in set i'. If I can average prices then I may rewrite (2) as:

(3) $y_{it} = \sum_{i'} v_{ii'} y_{i't-1} + d_i \sum_{j'} y_{j't-1}/n' + \sum_k c_{ik} x_{kt} + e_{it}$

Rather than explicitly average some of the lagged cattle prices, I could impose a linear constraint on some of the elements of the *v* coefficients as:

(4) $v_{ij'} = \frac{d_i}{n'} \; \forall i, j'$

Equation (4) also implies that:

(5) $v_{ij_1} = v_{ij_2} \forall i, and\ j_1, j_2 \in j'$

Equation (5) is the basic form that I used to impose averaging on the VAR. One may expand upon (5) by including more than one averaging subset.

Similarity constraints for the VAR

I test and impose price similarity using different levels of similarity. The most extreme will be that the forecast for price 1 and price 2 are the same. This will be true if:

(6) $v_{1j} = v_{2j} \; \forall j$ and

(7) $c_{1k} = c_{2k} \; \forall k$

Equations (6) and (7) imply that the only difference between price 1 and price 2 in period t is their error term. Note that we can impose (6) or (7) without imposing the averaging constraint, (5). I believe that the constraint on the VAR coefficients is the most important set of constraints for imposing price similarities. If (6) holds and (7) is true for all but the intercepts, then one of the two prices will end to be higher than the other. We may also allow the seasonal patterns to differ. My use of sines and cosines for the seasonal pattern insures that the seasonal pattern (roughly) averages to 0 over the course of a year. If (6) holds in general, and (7) holds for the intercepts but not the seasonal pattern, then price 1 and price 2 will be roughly the same over the course of a year, albeit with a different seasonal pattern.

Specific Solution and Cointegration Similarities

A less restrictive version of similarity allows the price forecasts to differ in statistically significant ways but allows the two prices to tend toward being similar. I start with the idea of a "specific solution." I take this idea from Baumol (1970). The intercept and seasonal variables are simple functions of time. The intercept is always 1. It turns out that the sines and cosines are first order functions of time too. What this means, for example, is that if you know COS1 and SIN1 for period "t" you can calculate their values for t+1, even if you do not know what "t" is.[4]

The intercepts and seasonal variables drive the current forecasts for the prices. Because last week's prices drive this week's prices in the VAR, last week's exogenous variables drive this week's prices too. The intercept and seasonal variables will induce a pattern in the prices. I can find specific solutions for the intercepts and seasonal variables by finding a set of coefficients *A* such that:

(8) $AX_t^s = V_1 AX_{t-1}^s + CX_t^s$

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[4] I found that SIN4 and SIN5 and excluded them from the VAR. However, I need their values to "forecast" next period's COS4 and COS5 respectively. These two terms show up in the specific solutions even though they are excluded for the VAR.

Equation (8) uses the matrix form of the model. I use the term $X^s{}_t$ because I use only a subset of the exogenous that excludes the BSE coefficients. We can generate the intercepts and seasonal variables using a non-stochastic first-order process:

(9) $$X_t^s = V_x X_{t-1}^s$$

We substitute (9) into (8):

(10) $$AV_x X_{t-1}^s = V_1 A X_{t-1}^s + CV_x X_{t-1}^s$$

Equation (10) has to work for all possible values of $X^s{}_t$ so the restriction I imposed in the estimation program is:

(11) $$AV_x = V_1 A + CV_x$$

The coefficients for V_x are known; V_1 and A are estimated. The specific solution in (11) represents a type of equilibrium relationship among the prices; the prices adjust over time toward CX_t^s. If a pair of prices have the same C coefficients their averages over time will tend to be the same. We can test for and impose this type of similarity on the VAR by imposing restrictions on the C.

There are, however, cases where solutions for C need to be modified. Things get more complicated when V_x and V_1 share a common root. This is where *cointegration* comes into play. In Mathews et al I found that 4 of these prices share a root equal to 1; that is they are cointegrated. The intercept also has a root equal to 1. In these cases the most general solutions for (11) will include a time trend that is driven by the intercept-coefficient (c_{ix0}) estimates.

Dicky and Fuller (1979) demonstrated that the test distribution for roots=1 in autoregression (AR) models varies depending on whether or not the model has an intercept. A non-0 intercept in an AR induces a trend in the data and makes the unit root test asymptotically normal. Putting an estimated intercept in an AR that does not actually need one changes the distribution of the unit root test. In their 1981 paper, they derived the distributions for intercepts and trends in AR models with roots equal to 1.

Sims, Stock, and Watson (1990) demonstrated that intercepts and trends in VAR models with roots equal to 1 could have non-normal asymptotic distributions. Johansen (1988, 1991) derived the test distributions for roots equal to 1 in the VAR. In his 1991 article, Johansen addressed the issue of intercepts and noted that while intercepts could induce non-stochastic trends in cointegrated data, there are cases where they will not. He noted how one could restrict what is essentially the specific solution for the intercept so that estimated VAR had non-0 estimates but no trends. One of the things that I tested in the development of the VAR is whether or not the intercepts induce trends in the data.

My approach to dealing with cointegration

This paper uses a unique approach to dealing with cointegration. Analysts generally deal with cointegration/roots equal to 1 by transforming a VAR into its error-correction, EC, form and approach pioneered by Engle and Granger (1987). Engle and Granger developed a 2-stage method for estimating the EC form. Johansen (1999) showed how the EC form could be estimated in a single step. His unit-root tests are a likelihood-ratio type test; one tests for roots equal to 1 by comparing the likelihood of an unrestricted model to one with roots imposed on it.

Baumol showed that the roots of a VAR or AR are the Eigen values of lagged endogenous variable matrix. Sims, Stock, and Watson used this approach in their analysis of the cointegrated VAR. My approach imposes roots equal to 1 on the VAR by imposing Eigen vector(s) on the VAR. The first example of my approach can be found in Taha and Hahn (2014). This is also the approach that I used in Mathews et al.

I could have used an EC for this approach; the problem with the EC form is that it would be more difficult to impose the averaging constraints on it. The typical symbol for an Eigen value is λ—in my case I want λ to be 1. If the VAR has an Eigen value equal to 1, then there exists a vector, I call it U such that:

(12) $V_1 U = U$

Equation (12) is trivially true if U is a vector of 0s. I need some type of arbitrary restriction on the U to keep it from being 0. In many mathematical applications, people impose a length of 1 on the Eigen vector. However, following my procedures in Taha and Hahn and Mathews et al, I normalized the U by fixing one of its elements to 1, specifically HDD35.

Error-Correction Models Versus the Eigen-Vector approach

Engle and Granger developed the idea of cointegration before Dicky and Fuller developed the distribution for the root=1 test. Data with a root equal to 1 is not stationary; differencing the data makes it stationary.

Engle and Granger noted that sometimes different, non-stationary variables seemed to moving together. They called data "cointegrated" when some linear combination of two or more variables was stationary when the variables themselves were not.[5] The scatter plots, figures 2, 3, 4 and 5, show evidence of cointegration in these 15 prices.

For data like ours, which shares a common root=1, we can eliminate the unit root from y_{it} using y_{jt} and some weight α_{ij}. While y_{it} and y_{jt} need to be differenced to be made stable, $y_{it} - \alpha_{ij} y_{jt}$ is stationary. Engle and Granger showed that you could estimate the α_{ij} using ordinary least squares. This was the first stage of the error-correction form. In the second stage, they would take the lagged, estimated first-stage error and estimate a model like:

(13) $\Delta Y_t = \Theta \widehat{\mathrm{U}}_{t-1} + \sum_l B_l \, \Delta Y_{t-l} + E_t$, where $\hat{u}_{i,t-1} = \ y_{it} - \hat{\alpha}_{ij} y_{jt} \ \forall i \ not \ j$

Johansen demonstrated that one could estimate the θ and α in a single step. Imposing a single, shared root equal to 1 on the EC form can be seen as restricting lagged price level terms[6]; adding more roots equal to 1 imposes more restrictions on the VAR.

Sims, Stock, and Watson used all the Eigen and Jordan vectors in the VAR to explore the properties of the estimates of the cointegrated VAR. Based on their results, it is possible to show

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[5] Technically, they defined cointegration as a case where each of y1, y_2.. need to be differenced K times to be made stationary, but the exists some linear combinations $\alpha_1 y_1 + \alpha_2 y_2$... with non-0 α that needs to be differenced strictly less than K times to be made stable.

[6] The lagged errors are functions of the lagged levels of the endogenous variables. If you have N endogenous variables, there are N^2 coefficients in lagged price level matrix. With 1 root=1, you can create N-1 lagged "error" terms and N-1 α. There are N(N-1) θ in that error correction form, so 1 root=1 is a 1 degree of freedom restriction. Imposing K≤N Eigen vectors for 1 on a VAR imposes K^2 restrictions.

that if I were to estimate this model in error correction form, and use HDD35 to create the U, that $\alpha_{1j} = u_j$, where u_j is the j[th] price's element in U. Here I explicitly make HDD35 the first price.

The unit root has the same effect on two cattle prices (i and j) if $u_i=u_j$. I can test pairs of cattle coefficients to see if they have a common u. The specific solutions effects are going to matter too. A pair of prices with the same u and specific solutions is going to tend to adjust toward the same value over time.

Tests and results

You will recall that I tested lag-lengths and exogenous variables in the preliminary stages of model development. Sims, Stock, and Watson and Johansen (1991) both demonstrated that most of the coefficients estimates in a cointegrated VAR were asymptotically normally distributed, with the possible exceptions of intercepts, trends, etc. (Unit root tests can have non-normal distributions.) Johansen specifically recommended testing for lag-lengths prior to testing for unit roots as these tests are asymptotically normal if the VAR or AR has stable roots.

I ran parallel tests in the next phase. I ran VAR were I tested for unit-roots and non-trending intercepts. I also ran VAR with averaging restrictions. Finally, I looked at imposing the "cattle prices are largely the same" either equations (6) or (6) and (7).

I then put together the insignificant unit-root, intercept, and averaging restrictions and tested cointegration and specific solutions similarities. All my tests are likelihood-ratio tests.

1—Unit root and intercept testing

The likelihood ratio test for imposing a single root=1 on the VAR 0.93. If the unit root test were normally distributed, this 1 degree of freedom, chi-square term would have an alpha of 33.4%; it is not significant at the 5% level. The test would be asymptotically chi-square if the intercept introduces a trend. If I impose a non-trending intercept on this model, the (1 degree-of-freedom) tests is 0.35 and has a chi-square 55.5%, also insignificant.[7]

Since I have that non-trending intercept, neither the unit root nor intercept tests are likely to be normal/chi-square. The Dicky and Fuller's unit root and intercept tests are fatter tailed than normal when they are not normal, so I will consider the tests insignificant.[8]

2—Price averaging tests

I decided to test for price averaging using what I thought were logical groups. For example, maybe we could average over all the dressed delivered cattle, or all the live FOB cattle. (Because the dressed cattle and live cattle are weighed at different points in the production I did not average over their prices—at least for the preliminary analysis.) Table 3 outlines the cattle-

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[7] I estimated the specific solution's intercept terms assuming no trend. In VAR without unit roots, the specific solution estimates are not restrictive. In models with roots=1, the intercepts are not identified. Johansen showed that you need as many arbitrary restrictions as roots=1. I tried several different arbitrary restrictions and they all produced the same test statistic. I settled on making Select's specific solution intercept 0. Later I found that making the byproduct intercept 0 also was statistically insignificant and improved convergence of the estimates.

[8] The likelihood-ratio test for adding 2 roots equal to 1 in the unrestricted VAR was 18.36. Johansen (1988) estimated that the 5% value of this test was 12.0. His calculations were for a VAR without intercepts. The worse-case scenario here is a model where I estimate intercepts when I do not need them. Using 1,000 Monte-Carlo analysis I found that 18.36 has an "alpha" 1.4%; 14 of the Monte-Carlo tests exceeded the actual one. The odds of 14 or fewer tests in 1,000 at or above the 5% level is less than 1e-9.

price groups that I averaged. I have these cattle price groups sorted from least to most statistically significant. The 6 least significant groups average over steer-heifer pairs. Three of the six pairs have tests under the 5% value.

Table 3—cattle-price averaging tests

narrative	*index*	*Test*	*degrees of freedom*	*chi-square alpha*
HLF65=SLF65	LF65	18.00	15	26.24%
HLF80=SLF80	LF80	19.72	15	18.29%
HDD80=SDD80	DD80	23.23	15	7.93%
HDD65=SDD65	DD65	26.56	15	3.25%
HLF35=SLF35	LF35	32.96	15	0.47%
HDD35=SDD35	DD35	38.44	15	0.08%
all the DD heifers are the same	HDD	64.50	30	0.03%
all the DD Steers are the same	SDD	70.72	30	0.00%
all the LF heifers are the same	HLF	89.49	30	0.00%
all the LF Steers are the same	SLF	99.71	30	0.00%
all the Dressed-Delivered cattle are the same	Dress	194.31	75	0.00%
all the live, FOB cattle are the same	LIVE	556.81	75	0.00%

Note: For cattle price index names see Table 1.

Table 4 shows what happens when I put the steer-heifer pair restrictions together. The 3 pairs that are insignificant at the 5% level "work" together. I found that I could average HLF56 & SLF65, HLF80 & SLF80, and HDD80 & SDD80.

In the next phase of testing, I checked to see if I could impose the steer-heifer pairs on subsets of the equations. First I looked at adding the significant steer-heifer pairs to the cattle-price equations. All these are rejected. The least significant pair to add to the cattle equations matches HDD65 & SDD65. The tests statistic for this 26.32, and has an alpha level with 12 degrees of freedom of 0.97%.

I can impose additional cattle-price restrictions of the 3 equations for packer outputs: Choice, Select, and Drop. I could impose the remaining 3 heifer-steer pairs on these equations. In fact, making all the live-FOB cattle average together works for the packer-output-price equations. Adding the 3 additional pairs and making all the live-FOB cattle average imposes 15 restrictions on the packer-price equations of the VAR. This test statistic is 17.65 and has an alpha level of 28.13%.

3—Testing for extreme price similarity

In this 3rd set of tests, I started again with the unconstrained VAR and imposed the "price forecasts are largely the same" restrictions, (6) and (7) above. Equation (6) requires two prices to have the same VAR or v_{ij} coefficients, (7) matches up the exogenous variable coefficients. For this set of tests I looked at restricting pairs of coefficients. I tested 15 pairs of dressed delivered cattle and 15 of the live, FOB cattle pairs. When I impose (6) and (7) all the tests are extremely significant—their alpha are 0 to 4 places or 0.00%. If I just impose (6), restrict the VAR but allow for differences in the intercepts and seasonality, my least significant test is 28.81 for the HLF35-SLF35 pair. This test statistic has a 15-degree-of-freedom, chi-square alpha level of 1.70%.

Table 4—putting together the steer-heifer pair averaging

		cumulative test[3]			*15 DF step tests*[4]	
Index[1]	***Step***[2]	***Test***	***degrees of freedom***	***alpha***	***Test***	***alpha***
LF65	1	18.00	15	26.24%		
LF80	2	36.94	30	17.89%	18.94	21.67%
DD80	3	59.82	45	6.86%	22.88	8.66%
DD65	4	88.69	60	0.94%	28.87	1.67%
LF35	5	121.98	75	0.05%	33.29	0.43%
DD35	6	164.56	90	0.00%	42.58	0.02%

Notes

[1] For index definitions see Table 3.

[2] I programed the computer to do a double loop over the pairs of prices. It checked all the pairs, selected the least significant, and then rechecked the remaining pairs. The step shows when each pair entered.

[3] The cumulative test tests the model against the free model with no pair restrictions. The test become significant with the 4th pair.

[4] The step test tests the added pair.

4—Imposing similarities in the cointegration and specific solutions

In the last phases of model testing, I impose similarities in the cointegration and specific solutions. I started with a VAR that incorporated the insignificant restrictions from part 1 and 2. Restricting the elements of *U* and specific solution's intercepts are all single degree off freedom restrictions. Recall that I identified *U* by making HDD35's coefficient equal to 1. HDD35 is a carcass weight price. USDA ERS uses 0.63 as its carcass yield when calculating the farm value of Choice cattle from SLF35. I also tested making the SLF35 and the other live, FOB cattle's *U* equal to 0.63.

For these single-degree-of-freedom restrictions, I programmed a large double loop. I had the software test all my 1- degree-of-freedom restrictions by themselves, and keep the smallest of the tests. In the next loop, I imposed the least significant restriction, and tested the remaining ones and so on. Table 5 shows the results of these sets of tests. I ended up imposing 14 single degrees of freedom restrictions on the VAR.

I have assumed that the chi-square distribution is relevant in Table 5. If I am interpreting him correctly, Johansen in his 1991 article noted that the cointegration relationships are asymptotically normal. I am not sure how the specific solution intercepts should be distributed. If either the *U* or intercepts in the *C* are not normally distributed, my use of a chi-square distribution is going to lead me to over-reject true hypotheses. The procedure I used in Table 5 is likely on the conservative side.[9]

Table 5—single degree of freedom tests for making cointegration and specific solution intercepts the same[1]

				cumulative test versus model with averaging and root = 1			*1 DF step tests*	
Type[2]	***First price***	***Second price***	***step***	***Test***	***degrees of freedom***	***alpha***	***Test***	***alpha***
SSX0	HLF65	SLF80	1	0.01	1	94.36%	0.01	94.36%
UFX	SDD35		2	0.03	2	98.75%	0.02	88.74%
US	SDD65	SDD80	3	0.06	3	99.58%	0.04	84.46%
SSX0	HDD35	HDD80	4	0.46	4	97.75%	0.39	53.02%
UFX	HDD80		5	0.64	5	98.62%	0.18	67.12%
US	Choice	Select	6	1.83	6	93.50%	1.19	27.56%
UFX	HLF35		7	2.90	7	89.38%	1.08	29.93%
US	HLF65	HLF80	8	3.65	8	88.69%	0.75	38.62%
SSX0	HLF80	SLF65	9	4.43	9	88.09%	0.78	37.85%
SSX0	HDD35	SDD35	10	7.76	10	65.20%	3.33	6.79%
US	HDD65	SDD65	11	11.01	11	44.21%	3.25	7.13%
SSX0	HDD65	SDD65	12	13.47	12	33.58%	2.46	11.71%
US	SLF35	SLF80	13	16.86	13	20.56%	3.39	6.54%
UFX	SLF35		14	20.35	14	11.95%	3.48	6.20%
SSX0	HDD65	SDD80	15	28.85	15	1.68%	8.50	0.35%
US	HLF65	SLF65	16	42.93	16	0.03%	14.08	0.02%
SSX0	HLF35	SLF35	17	59.19	17	0.00%	16.26	0.01%
SSX0	Choice	Select	18	64.34	18	0.00%	5.15	2.33%
SSX0	HLF65	HLF80	19	68.47	19	0.00%	4.13	4.22%
SSX0	HLF35	HLF65	20	81.09	20	0.00%	12.62	0.04%
UFX	HLF65		21	93.57	21	0.00%	12.48	0.04%
UFX	HDD65		23	101.94	23	0.00%	8.37	0.38%

Notes

[1]In many cases, restrictions became redundant. Imposing a restriction on one pair would impose the same restriction on other. The table shows only 1 of these restrictions.
Cells with yellows highlighting are statistically significant

[2] Type of restrictions are:
UFX makes live cattle's *U* 0.63 & dressed cattle's 1.00
SSX0 matches the specific solution intercepts
US   matches the *U* by pairs

[9] On the other hand, there is that whole "post-test estimator" issue. My testing of everything but the kitchen sink approach would be considered dubious at best by some statistical purists.

I used a slightly different procedure when testing for matching seasonality. I first checked all pairs of dressed delivered, and all pairs of live FOB cattle for common seasonality. (I also tested the two cutouts for a common seasonal pattern.) I had 6 pairs of cattle prices that had statistically insignificantly different seasonal variables. I then did my double-loop "thing" with this restricted set. Table 6 shows these pairs and how the looped procedure came out.

Table 6—testing the 6 pairs with insignificantly different seasonality

			cumulative test versus previous model		
first price[1]	***second price***	***Step***	***Test***	***degrees of freedom***	***alpha***
HLF35	SLF35	1	14.13	10	16.70%
HDD65	HDD80	2	28.97	20	8.84%
HDD65	SDD35[2]	3	42.10	30	7.02%
HDD80	SDD35[2]	3	42.10	30	7.02%
SDD65	SDD80	4	58.75	40	2.81%

Notes

1 For price index names see Table 1.

2 Given that HDD65 and HDD80 entered in the second step, making SDD35 equal to either makes it equal to both.

Cells with yellows highlighting are statistically significant at the 5% level.

Table 7 uses color coding to show price commonalities. The two most common prices are HDD80 and SDD35. They have the same *U* and the same values for all their specific solutions. One of the interesting results in Table 7 none of the pairs of prices that I found we can average over have either the *U* or specific solutions in common.

Table 7—graphical representation of the cattle price similarities

endogenous variable[1]	unit-root Eigen vector	Intercept	seasonal
HDD35			
HDD80			
SDD35			
HDD65			
SDD65			
SDD80			
HLF65			
HLF80			
HLF35			
SLF35			
SLF80			
SLF65			
Choice			
Select			
Drop			

color key

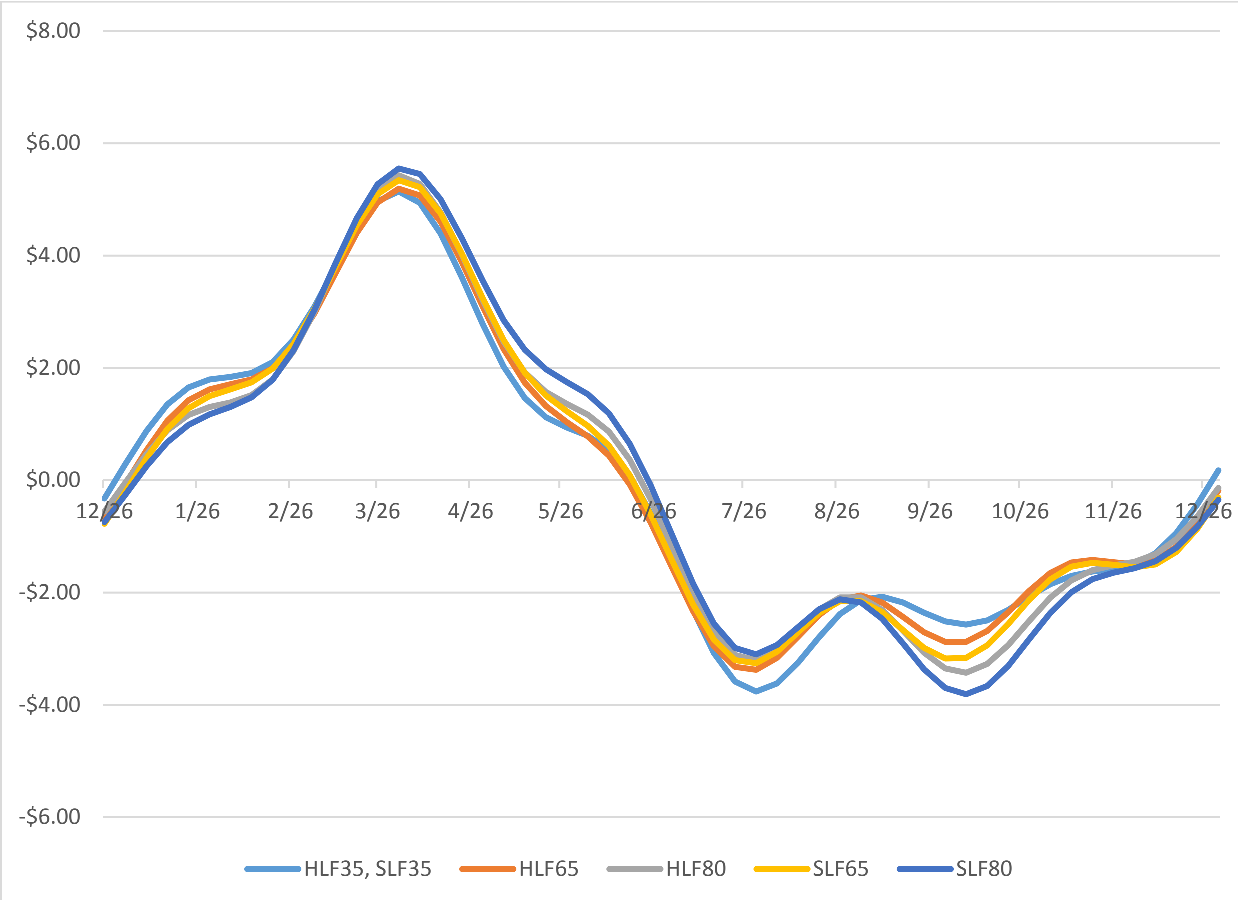
- 3 of the DD cattle have 1 in U_VEC and same intercept. 2 have the same seasonal pattern, which is shared with HDD65
- other 3 DD cattle with common U_VEC, 2 of these have a common intercept, 2 others common seasonal pattern
- Choice = Select in U_VEC
- 2 LF cattle match in U_VEC
- 3 LF cattle have U_VEC fixed to 0.63, 2 share a common seasonal pattern
- 2 LF cattle with same intercepts but different U_VEC and seasonal
- 2 more LF cattle with same intercepts but different U_VEC and seasonal

[1] For price index names see Table 1.

Figure 6—2016 seasonal pattern for dressed, delivered cattle

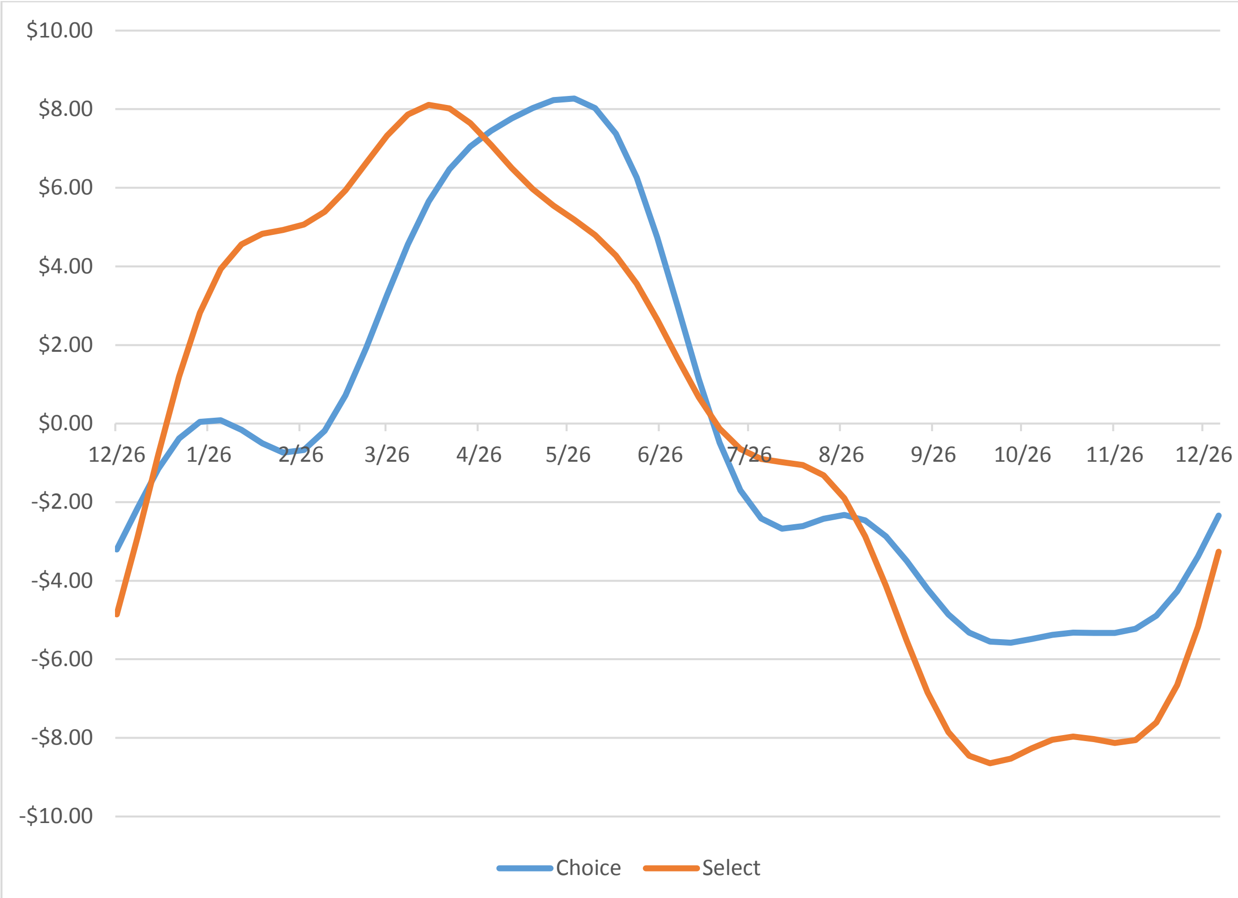
See Table 1 for the cattle name abbreviations. All prices in dollars per hundred pounds.

Figure 7—2016 seasonal pattern for live, FOB cattle



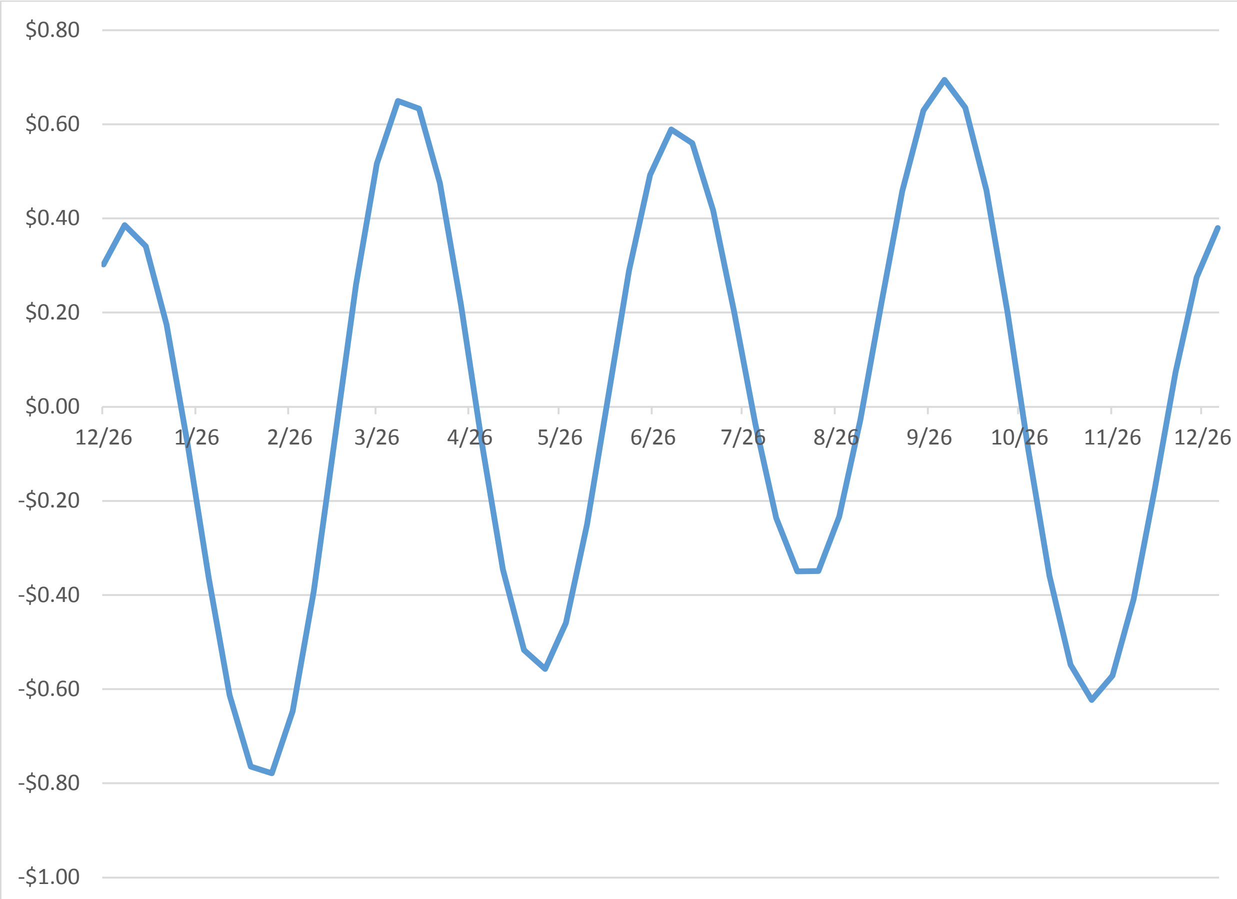
See Table 1 for the cattle name abbreviations. All prices in dollars per hundred pounds.

Figure 8—2016 seasonal pattern for carcass cutouts



All prices in dollars per hundred pounds.

Figure 9—2016 seasonal pattern for drop value



Drop value in dollars per hundred pounds of live steer.

Selected Estimates

Table 8 shows the estimates for the elements of *U*. These terms show the effect of the unit root on the endogenous variables. I have normalized *U* so that HDD35, SSD35, & HDD80 are fixed to 1. A $1 per hundred weight change in the unit root changes these 3 prices by $1/CWT. The other 3 dressed delivered cattle, HDD65, SDD65, & SDD80, have the same *u*: 0.9983. A $1/CWT increase in the unit root increases HDD65, SDD65, & SDD80 by $0.9983/CWT. The difference between 0.9983 and 1 is small for all practical purposes. It is, however, statistically significant.

Three of the live, FOB cattle have their *u* fixed to 0.63. The other live, FOB cattle have a higher value for their *u*: 0.6317 and 0.6321, like the dressed cattle, live cattle prices' unit root effect differ from other live cattle's' unit root effect only in the 3rd decimal place. These small differences are statistically significant. I found that Choice and Select had the same *u*; its estimate is 0.9081.

Table 8—unit root Eigen vector estimates and summary statistics

Endogenous variable[1]	*Estimate*[2]	*Z statistics*[3] *tested against*	*Z*
HDD35, SSD35, HDD80	**1**		
HDD65, SDD65, SDD80	0.9983	1.00	-2.52
HLF35, SLF35, SLF80	**0.63**		
HLF65, HLF80	0.6317	0.63	4.58
SLF65	0.6321	0.63	3.87
Choice, Select	0.9081	1.00	-5.12
Drop	0.0606	0.00	46.55

[1] For price index names see Table 1.
[2] Numbers in bold text were fixed to that value and have a standard deviation of 0.
[3] Standard deviations and Z statistics based on 10,000 Monte-Carlo iterations of the model estimates.

Table 9—specific solutions for the intercepts

Endogenous variable[1]	*Estimate*[2]	*Z statistic*[3] *versus 0*
HDD35, SSD35, HDD80	-$12.859	*-4.31*
HDD65, SDD65	-$12.477	*-4.22*
SDD80	-$12.417	*-4.19*
HLF35	-$7.913	*-4.20*
SLF35	-$7.962	*-4.23*
HLF65, SLF80	-$8.332	*-4.43*
HLF80, SLF65	-8.5434	*-4.55*
Choice	$7.552	*9.32*
Select	**0**	
Drop	**0**	

[1] For price index names see Table 1.
[2] Numbers in bold text were fixed to that value and have a standard deviation of 0.
[3] Standard deviations and Z statistics based on 10,000 Monte-Carlo iterations of the model estimates.

Table 9 shows the specific solutions for the intercepts. I set Select's intercept to 0 to identify these coefficients. Given that Select's coefficient is 0, the drop coefficient was also insignificantly different from 0 and I imposed that restriction too. All the cattle-price terms are negative, Choice's is positive. It makes more sense to compare the intercepts across groups. For example, both Choice and Select have the same unit root effect or *u*. That means the unit root has the same effect on both. Choice's intercept estimate ~$7.55 implies that the Choice cutout will over the course of a year average about $7.55 over the Select cutout. I found that Choice and Select have different seasonal patterns and this implies that the "typical" Choice-Select premium is going to vary over the course of a year.

The unit-root effects for the 6 dressed delivered cattle are close in value, as they are for the 6 live, FOB cattle. The intercept differences show sort of a baseline difference in the cattle prices. The dressed, delivered cattle have a baseline price range of $0.44/CWT; the live FOB range is $0.58/CWT.

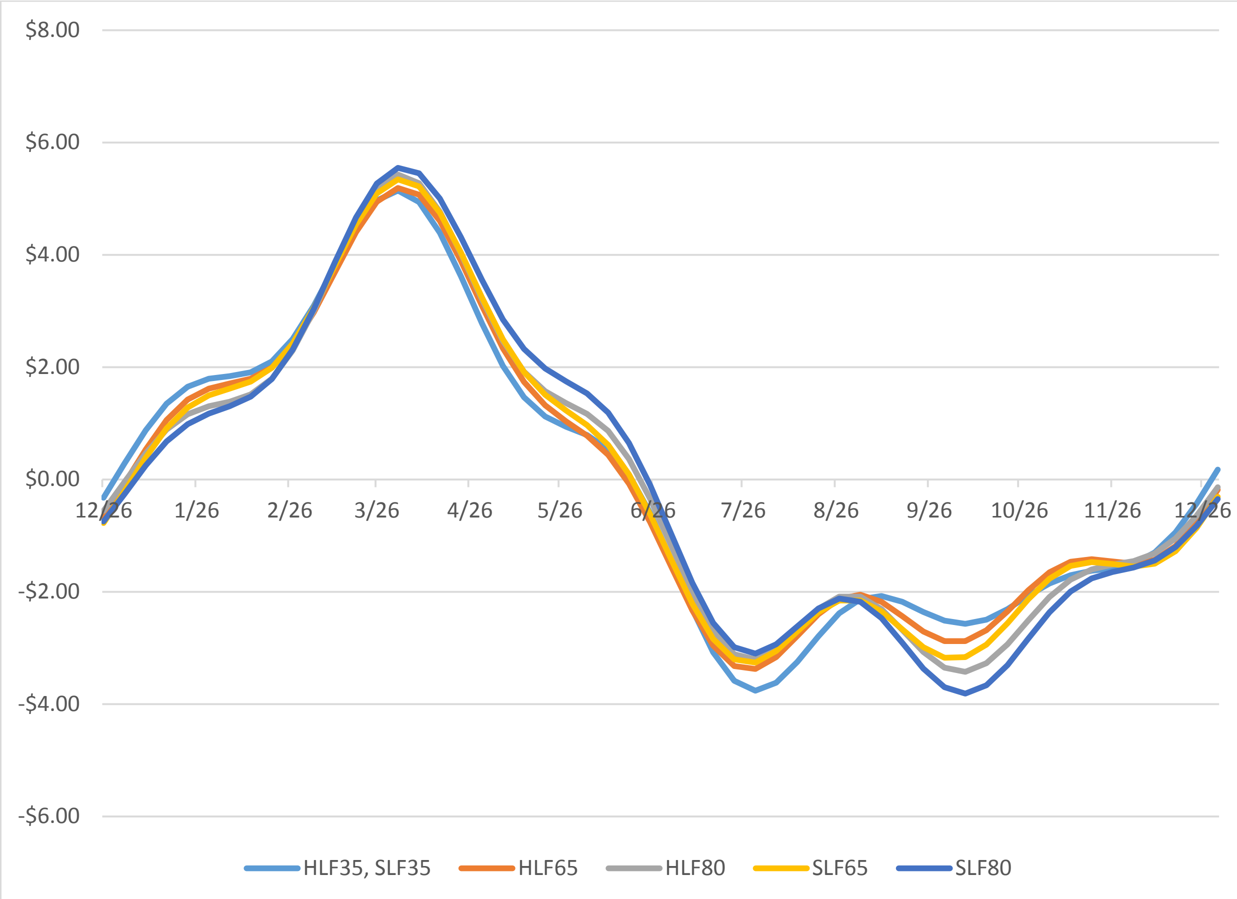
Interpreting the specific solution estimates for the seasonal variables is not straight-forward. Figures 6, 7, 8, and 9 show the seasonal patterns implied by the cosine and sine terms' specific solutions for the last 53 weeks in my data set.

Readers who are interested in the rest of the VAR parameter estimates can contact me and I will email you a spreadsheet with the rest of the model estimates and Z statistics in it.

Figure 6—2016 seasonal pattern for dressed, delivered cattle

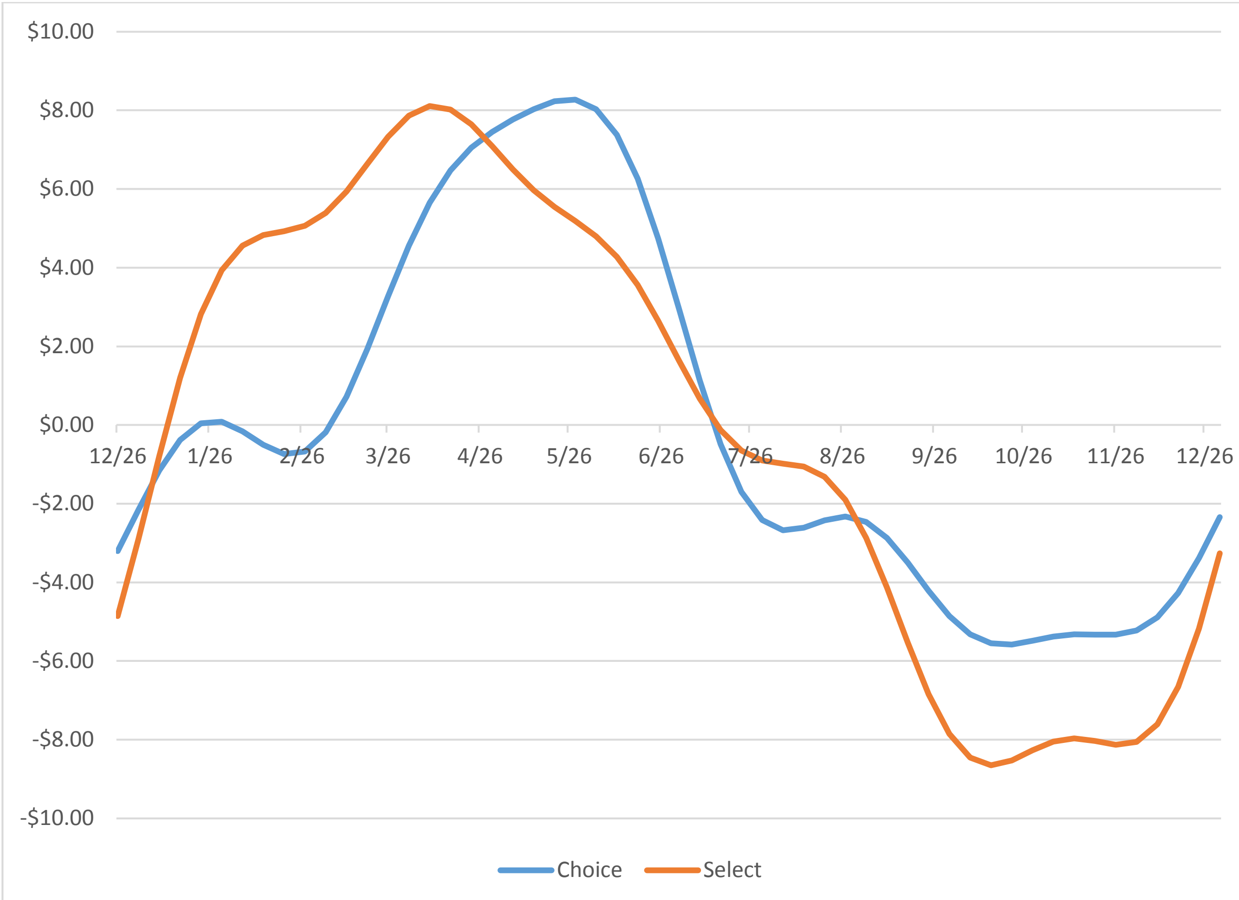
See Table 1 for the cattle name abbreviations. All prices in dollars per hundred pounds.

Figure 7—2016 seasonal pattern for live, FOB cattle



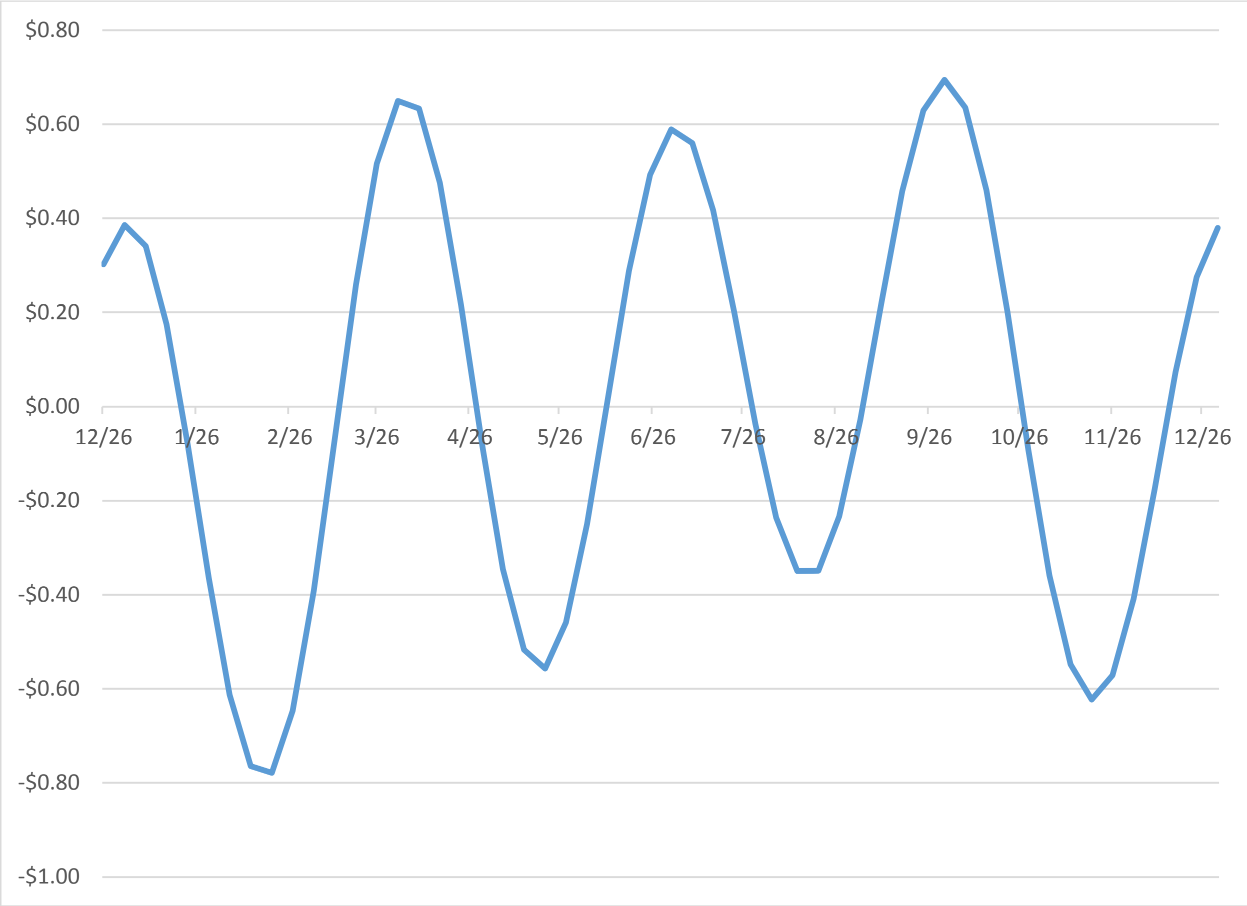
See Table 1 for the cattle name abbreviations. All prices in dollars per hundred pounds.

Figure 8—2016 seasonal pattern for carcass cutouts



All prices in dollars per hundred pounds.

Figure 9—2016 seasonal pattern for drop value



Drop value in dollars per hundred pounds of live steer.

Conclusions

My primary goal in estimating this model was to see if averaging cattle price for cattle price reporting is a good idea or not. I will conclude that it is not a great idea for these prices. One of the problems I noted with the idea of averaging is that it does limit the amount of information available to the market. I find that I can average 3 steer-heifer pairs. There are important differences between heifers and steers for all these pairs. Each has slightly different reactions to changes in the over-all market level as measured by the "unit root." Each has a different seasonal pattern and each pair has a different intercept, implying a more or less consistent premium/discount for one relative to the other. Price averaging covers up these differences.

I knew going in that price averaging could cause us to lose this type of information. I just find that price averaging does not hurt forecasting. Forecasting is not the only use for this data. An even bigger problem is the fact that I do not come up with a "generic" averaging procedure. I cannot average over **all** steer-heifer pairs, only half of them. Two of the pairs I can average over are live FOB, the other is dressed delivered.

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