

The World's Largest Open Access Agricultural & Applied Economics Digital Library

# This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search http://ageconsearch.umn.edu aesearch@umn.edu

Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.



5



DIR86-1

-

# **STAFF PAPER SERIES**

DIR 86-1 SP-4 June 1986

Integration of Dynamic Programming and Simulation Models To Value Lead Time Of Information Forecasting Systems

> James W. Mjelde Steven T. Sonka Bruce L. Dixon Peter J. Lamb

### **Departmental Information Report**

The Texas Agricultural Experiment Station Neville P. Clarke, Director Texas A&M University System, College Station, Texas 77843 DIR 86-1 SP-4 June 1986

Integration of Dynamic Programming and Simulation Models To Value Lead Time Of Information Forecasting Systems

> James W. Mjelde Steven T. Sonka Bruce L. Dixon Peter J. Lamb

Department of Agricultural Economics Texas A&M University Texas Agricultural Experiment Station

Staff papers are available for distribution without formal review by the Department of Agricultural Economics.

All programs and information of the Texas Agricultural Experiment Station are available without regard to race, ethnic origin, religion, sex and age.

The authors are, respectively, Assistant Professor of Agricultural Economics, Texas A&M University, Professor, University of Illinois, Associate Professor, University of Aarkansas, and Head of Climate and Meteorology Section, Illinois State Water Survey, Champaign Illinois. This research was partially supported by NSF Grant ATM 82-13734.

#### ABSTRACT

#### Integration of Dynamic Programming and Simulation Models to Value Lead Time of Information Forecasting Systems

Issues pertaining to lead time of information forecasting systems are presented. A methodological procedure is developed which values lead time. The procedure utilizes dynamic programming and simulation models. An application of this approach to corn production indicates lead time is important in climate forecasting and corn production. Integration of Dynamic Programming and Simulation Models to Value Lead Time of Information Forecasting Systems 1

Information forecasting systems are characterized by many dimensions. The usefulness of these dimensions ultimately determines the expected value of the information system to a decision maker. One of these dimensions is the timeliness of the forecast. For example, consider a production process in which input X is only applied in the current time period and final output is determined by an interaction between a stochastic event  $\theta$  and X which occurs in some future time period. Knowledge of  $\theta$  in the current period allows the decision maker to select the optimal level of X to apply. An information system which provides perfect knowledge of  $\theta$  in the current period is in all likelihood more valuable than a system which provides the same information in a time period between the current period and the time period when the interaction occurs. The first information system is potentially more valuable because the decision maker can use the information to select the level of X which is optimal given  $\theta$ . The dimension of timeliness illustrated in this example is referred to as lead time and is defined as the time interval between the release of a forecast and the beginning of the period in which the stochastic event being forecasted occurs.

Lead time is a dimension which is important in a variety of forecasting systems and decision making situations. Easterling surveyed subscribers of the NOAA Climate Analysis Center's *Monthly and Seasonal Weather Outlook.* Respondents of this survey were grouped into users and nonusers of the climate forecasts. Discriminant analysis was applied to identify variables that differentiated the two populations, users and nonusers. The need for lead time was found to be the most important factor differentiating nonusers of climate forecasts from users. (Nonusers required greater lead time than currently offered by the NOAA system.) The respondents were from a wide variety of industries including agriculture, energy, government and education, construction, transportation, business services, recreation, and communications. Another finding by Easterling was that industry type was not a significant discriminating factor, suggesting that a wide variety of industries place a value on lead time of climate forecasts. Although Easterling's results are for climate forecasting, it is easy to visualize the importance of lead time in other forecasting systems, e.g. price forecasting (market conditions) in which production decisions are made in advance of the actual selling period.

The present study considers the valuation of lead time from a decision theoretic approach. A methodological procedure to value lead time which utilizes the output from a dynamic programming decision model and a simulation model is developed. Finally, an example which utilizes the procedure is briefly presented.

#### LEAD TIME VALUATION

Decision theoretic approach to valuing information provides the basis for the methodology developed to value lead time. In this approach information can be defined as a message which alters probabilistic perceptions of random events. Under the decision theoretic framework, information has value only when the altered probabilities change the optimal decisions of the decision maker (over the prior or less information scenario). The altered optimal decisions are the observable

effects of using the new information. Therefore, modeling the decision making process and observing changes in the objective function caused by changes in the optimal decisions as information varies provides one effective means to value information.

Under the decision theoretic approach, a necessary condition for forecasts of stochastic events to have value to a decision maker is that there must be an interaction between management controlled factors and the stochastic event. The extent of this interaction determines the inherent flexibility of the decision process with respect to information. For forecasts to have value to a decision maker, the decision process must possess the flexibility to vary the management decision pertaining to input usage in response to varying forecasts.

In order to value lead time, the decision maker's objective function and prior knowledge of the stochastic events must be ascertained. Also, it is necessary to specify what the decision maker's beliefs are about the probability of receiving all the possible forecasts prior to receiving a particular forecast. That is, the decision maker must have a belief (probability distribution) about what the forecast will contain prior to receiving a specific forecast. For ease of exposition and manipulation of probability distributions, it is assumed in this study that: (1) the decision maker's objective is to maximize expected net returns, and (2) the decision maker's prior knowledge of the stochastic event and belief about any future forecast is the historical probability distribution (p.d.f.) of this event.

Finally, any procedure which values lead time must incorporate not only the effect of the new information on the optimal management

decisions, but also the effect on the objective function on the time period when the information is received. Therefore, lead time is only pertinent in dynamic stochastic decision making settings. A dynamic decision setting is defined as a process in which input decisions are made and implemented at different points in time than when the final product is realized.

#### Valuation Methodology

Given the above assumptions the expected value of any information system is given by

 $V = \int \max_{X} \int u(\theta, X) p(\theta | k) d\theta p(k) dk - \max_{X} \int u(\theta, X) p(\theta) d\theta$ (1) X X

were  $u(\theta, X)$  represents the decision makers utility function,  $\theta$  a stochastic event which can take on various values, X the management decision set,  $p(\theta|k)$  the probability of  $\theta$  occurring given forecast k, p(k)the probability of receiving forecast k and  $p(\theta)$  the historical p.d.f. of  $\theta$ . The gain from information is the difference between the expected utility when the information is used optimally and the expected utility of the best decision that would be made without the additional information.

A dynamic stochastic decision process is needed for lead time to be a relevant forecast parameter. Dynamic programming (DP) is chosen as the technique to evaluate the decision process for three reasons. First, DP is a powerful analytical and computational method for handling stochastic multi-period decision processes (Burt). Secondly, DP gives the optimal decision and expected value for all possible states of the decision process for each decision point in time as a by product of solving the decision problem. Finally, DP is a problem-solving approach or strategy rather than a specific mathematical technique. Therefore the scope of

potential applications is quite broad. Before proceeding to the valuation methodology, a short digression on DP is provided.

In order to be cast into a DP framework, a multi-period process must be divided into time intervals or stages, with a management decision being made at each stage. At the different stages of the process, state variables are used to describe the state or current status of the process. Markovian relationships, or transition equations, deterministically or probabilistically give the state of the process in the next stage given the current state, exogenous factors, and the decision alternative choosen. Usually, DP algorithms solve the decision process backwards, that is the algorithm starts at the final period of the decision problem and proceeds to the first period of the decision horizon. This leads to a backward numbering of the stages, i.e. stage one is the last stage or terminal period, the next to the last stage is stage two etc. With this numbering scheme, which will be used throughout this discussion, the beginning period of an N-stage decision process is stage N.

When determining the optimal decisions, a DP algorithm examines each possible state variable combination (state of the system) at each stage. The expected payoff from the current stage to stage one is calculated for each state variable combination for each management decision. Therefore, the expected payoff for following the optimal management strategy from the current stage to stage one for every possible state of the system at each stage is calculated by Bellman's Principle of Optimality (Nemhauser) with a single computer run of the decison model. These expected payoffs for each possible state of the system are required to calculate the expected value of lead time of forecasts. DP allows for these expected payoffs to

be calculated with a single decision model solution, whereas other efficient optimization techniques do not explicitly perform these calculations.

As an example of lead time valuation methodology consider the value of obtaining forecast k for stage N-i (stage with interaction between a stochastic event  $\theta$  and a management input applied earlier in the production process) at the beginning of the decision process (stage N) versus obtaining the forecast at stage N-j, where j<i. It will be assumed for generality that the decision process is subject to stochastic events at every stage. The probability of the various stochastic events will be set at there historical levels (prior knowledge) and the forecast will be concerned only with forecasting  $\theta$  at stage N-i. Also, it is assumed that the forecasts are exogenous to the decision process, i.e. the decision maker's actions do not affect the probability of receiving a forecast.

The expected value of obtaining forecast k at stage N is given by  $Z_N(k) = \max \int w(\theta, X) p(\theta | k) d\theta$  (2)

where w( $\theta$ ,X) represents the net returns or utility at the various stages in the DP decision model from stage N to stage one (the entire decision process). Then Z<sub>N</sub>(k) is the expected value of the decision process given that the exogenous forecast is received in stage N. As noted above, when determining Z<sub>N</sub>(k) the expected net returns for every possible state in stage N-j (denoted as a vector  $W_{N-j}$ ) are determined. Therefore, vector  $W_{N-j}$  contains the expected net returns from following an optimal decision policy from stage N-j to stage one for each of the possible states at stage N-j.

To determine the expected net returns when the forecast is received

in stage N-j requires the decision process to be simulated from stage N to stage N-j using optimal policy based on historical or prior probabilities,  $X^*_{H}$ , and historical probabilities for the stochastic events in stages N to N-j. The optimal policy,  $X^*_{H}$ , is defined as the optimal management decision for each stage and state derived from the DP decision model when using historical probabilities for all stochastic events. Recall it is assumed that the historical p.d.f. for  $\theta$  is the best proxy for any forecast and the stochastic events for the remaining stages are also set at their historical levels. Therefore,  $X^*_{H}$  is used as the decision policy to simulate the process to N-j.

In simulating the decision process forward, the probability of being in each state at each stage can be determined. These probability vectors (denoted as  $PR_{N-\ell}(X^*_H)$ ,  $\ell=0, 1, ..., j$ ) are used to obtain the expected net returns given the forecast is obtained in stage N-j. The probability of being in any state is a function of the optimal policy used to simulate the process and the historical probabilities of  $\theta$ . The expected net returns given that forecast k is received in stage N-j is given by

 $Q_{N-j}(k) = PR_{N-j}(X^*_H) \cdot W_{N-j} + \sum_{\ell=0}^{j-1} PR_{N-\ell}(X^*_H) \cdot C(X^*_H)_{N-\ell}$  (3) where  $PR_{N-j}(X^*_H) \cdot W_{N-j}$  gives the expected net returns from stage N-j to stage one,  $C(X^*_H)_{N-\ell}$  is the immediate net returns for each optimal decision at stage N- $\ell$ , and  $\cdot$  denotes the dot (inner) product between the various probability and monetary vectors such that a single monetary value is obtained (assuming only one possible state at stage N. If more than one initial state is to be considered, the procedure is adjusted such that one monetary value is obtained for each initial state.) The expected net returns of receiving the forecast in stage N versus N-j is

$$E(V_{L}) = Z_{N}(k) - Q_{N-1}(k)$$
.

Further clarification of equation (3) is useful at this point.  $W_{N-j}$ is the vector of expected net returns of being in a given state in stage N-j. Because these values are obtained from the DP model, the components of  $W_{N-j}$  are the optimal net returns for only the N-j remaining stages and do not include the net returns of decisions and events taken in stages N to N-j+1. Because  $W_{N-j}$  does not include stages N to N-j +1, since  $W_{N-j}$  is computed using backwards recursion, the immediate net returns from decisions in stages prior to N-j must be added to  $W_{N-j}$ . The vectors  $C(X^*_H)_{N-\ell}$  add these net returns. The probability vectors  $PR_{N-\ell}(X^*_H)$  ( $\ell =$ 0,1,...,j) are included because stochastic events occur at all stages and the exact state is not known, only the probability of being in each state. It should be noted that  $\Sigma PR_{N-\ell}(X^*_H) = 1.0$  for each  $\ell$  where the summation is over all possible states that may occur at a particular stage.

The approach outlined above can be generalized so that it is not restricted to valuing forecasts received solely in stage N. In general, the value of lead time from receiving the forecast in stage n-j instead of stage n is given by

$$\mathbf{E}(\mathbf{V}_{\mathrm{L}}) = \mathbf{Q}_{\mathrm{n}}(\mathbf{k}) - \mathbf{Q}_{\mathrm{n}-\mathrm{j}}(\mathbf{k}).$$
<sup>(5)</sup>

The values  $Q_n(k)$  and  $Q_{n-1}(k)$  are given by

$$Q_{n}(\mathbf{k}) = PR_{n}(\mathbf{X}_{1}) \cdot W_{n} + \sum_{\ell=0}^{N-n-1} (PR_{N-\ell}(\mathbf{X}_{1}) \cdot C(\mathbf{X}_{1})_{N-\ell})$$
(6)

 $\begin{aligned} & \mathbb{Q}_{n-j}(k) = \mathbb{PR}_{n-j}(X_2) \cdot \mathbb{W}_{n-j} + \sum_{\ell=0}^{N-(n-j)-1} (\mathbb{PR}_{N-\ell}(X_2) \cdot \mathbb{C}(X_2)_{N-\ell}) \end{aligned} \tag{7} \\ & \text{where } X_1 \text{ and } X_2 \text{ denote the appropriate decision policies with which to} \\ & \text{simulate the decision process, and the other notation is as defined earlier.} \end{aligned}$ 

(4)

Forecast k is only one possible forecast that could arise from the information system. The expected value of lead time of the information system is

$$E(E(V_{I_{i}})) = \int (Z_{N}(k) - Q_{N-1}(k)) p(k) dk$$
(8)

or

$$E(E(V_L)) = \int (Q_n(k) - Q_{n-1}(k)) p(k) dk$$
 (9)

for equations (4) and (5), respectively. A term which corresponds to the second term in equation (1) does not have to be calculated. This term is subtracted from both the Z and Q terms in equation (8) or is subtracted from both Q terms in equation (9), therefore cancels itself out in these equations.

#### LEADTIME VALUATION EXAMPLE

The above methodology has been applied to a corn production model for east-central Illinois (Mjelde). This model has eight stages of production; fall before planting, early spring, late spring, early summer, midsummer, late summer, early harvest and late harvest. At each stage of production the decision maker determines the optimal input usage. In this model, inputs under the decision maker's control are seed density at planting, hybrid to be planted, when to plant, when and at what level to apply nitrogen and when to harvest. The model specifies stochastic climatic conditions between each of the eight stages of production.

The expected value of receiving forecasts of early summer climatic conditions at fall, early spring or late spring are computed on a per acre basis. Early summer is a stage that contains a direct interaction between climatic conditions and applied nitrogen (Hollinger and Hoeft). In this example there are only three possible perfect forecasts, good, fair, and

poor climatic conditions with respect to growing conditions. A perfect forecast is defined as a forecast in which the forecasted climatic condition occurs with probability 1.0 and the remaining two conditions occur with probability 0.0.

Table 1 presents the net returns and probabilities of receiving each forecast, along with the expected values. The expected values show that perfect knowledge of the early summer climatic conditions received in the fall or early spring is worth approximately three dollars more than when received in late spring. The difference in expected values is explained by the variations in the optimal applied nitrogen levels given the different perfect forecasts. When using either only prior knowledge or when the forecast is received in the fall, the optimal fall decision is to apply no nitrogen. The fact that the optimal decision does not change is reflected in the low expected value of lead time for fall versus early spring. When using only prior knowledge, 267 lbs/acre of nitrogen are applied in early spring. The same level of nitrogen is applied for predictions of good early summer climatic conditions received in the fall or early spring. This is reflected in the low value of the good forecast for good climatic conditions in Table 1. For forecasts of fair (poor) received in the fall or early spring 150 (50) lbs/acre of nitrogen are applied in early spring.

The decrease in expected value of not knowing the early summer climatic conditions until late spring results because the optimal decision policy without the forecast recommends too much nitrogen if fair or poor climatic conditions occur. Therefore, knowing the early summer climatic conditions by early spring alters the decision policy when fair and poor perfect forecasts are received. This change in optimal policies is reflected in the expected values of each forecast in Table 1.

#### CONCLUSIONS

This study developed a methodological procedure which utilizes a DP and a simulation model to calculate the expected value of lead time. Dynamic programming provides a flexible optimization strategy to operationalize the procedure. This flexibility allows the methodology to be applied to a wide range of dynamic, stochastic decision processes. Also, with appropriate modifications of the procedure presented, the assumptions made for this study can be modified making the methodology applicable to a wide variation in the decision maker's prior knowledge, accuracy of forecasts, etc.

Numerical results presented here indicate what an individual farmer would be willing to pay per acre for perfect early summer forecasts received in various production periods. Current climatological forecast are far from perfect and possess virtually no lead time. Results computed for imperfect forecasts (but not recorded here) indicate that there is a tradeoff between the accuracy of forecasts and lead time. Under certain economic and forecast scenarios, a less accurate forecast received earlier in the production process may be of more value to the decision maker than a more accurate forecast received later in the production process. Tradeoffs such as the accuracy-lead time tradeoff should be taken into account when designing information sytems. The methodology presented here allows such tradeoffs to be examined.

Forecast	Prob. <sup>1</sup>	Net Returns Stage Received <sup>2</sup>			Exp01	Expected Value of Lead Time		
	•	<u>F</u>	ESp	LSp	Fvs.ESp	Fvs.LSp	ESpvs.LSp	
Good	6/14	309.59	309.55	309.55	.04	.04	.00	
Fair	3/14	267.09	267.06	264.51	.03	2.68	2.55	
Poor	5/14	212.92	212.89	205.94	.03	6.98	6.95	
Expected								
Value <sup>3</sup>	•	265.96	265.92	262.90	.04	3.06	3.02	

## Table 1. Calculation of the Expected Value of Lead Time Per Acre for Early Summer Forecasts.

- Probability of receiving each forecast. These probabilities are the historical probabilities of each climatic condition occurring because a perfect predictor is valued (Mjelde).
- 2) Abbreviations: F is for fall, ESp for early spring and LSp for late spring.
- 3) Expected value of the information system which gives rise to the three forecasts. Calculated as the sum of the net returns associated with each forecast multiplied by the probability of receiving that forecast.

