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Conceptual and Functional Form Issues in Estimating Demand Elasticities for Food

Michael K. Wohlgenant

In estimating demand elasticities for food, we must choose a functional form for the statistical model. Because this choice can have untoward consequences for the user (King), we seek a form which does not impose undue restrictions on the estimated demand elasticities. Statistically, we seek a functional form which gives close approximations to the elasticities of interest. In this connection, Gallant (1981, 1982) points out that the key consideration is the method of approximation. There are two: Taylor's series and Fourier series methods. The problem with Taylor's theorem is that it applies only locally on a region of unspecified size. This method can yield testable implications of the theory at the point of approximation; but this point is in general unknown and may not even be in the range of the data (Caves and Christensen, White). Moreover, the approximation error from Taylor's series can be large for small departures from the (unspecified) point of expansion (White). In short, there is no guarantee that Taylor's theorem will give close approximations at any point in the sample (Gallant 1981). Thus there is no assurance that even the recently popularized locally flexible functional forms (translog, generalized Leontief, almost ideal demand system) will give close approximations to the true demand system.

In contrast to Taylor's series, Fourier series methods have the capability in principle to approximate globally the true price and income elasticities (Gallant 1981). Moreover, El Badawi, Gallant, and Souza show that demand elasticities from the Fourier form can be estimated consistently regardless of the statistical method used. The purpose of this paper is to show that the Fourier demand system introduced in Gallant (1981) is a viable statistical model for estimating demand elasticities for food commodities. The model is fitted to U.S. postwar annual data on food and nonfood commodity aggregates. The viability of this system is then assessed on the conformity of the estimated demand elasticities with our prior beliefs. This paper begins with a discussion of some conceptual issues in choosing a functional form.

Conceptual Basis

The conceptual basis for specifying demand functions is the neoclassical theory of consumer behavior. In this approach, the representative consumer maximizes a twice-differentiable utility function subject to a linear budget constraint. The solution is a set of demand functions, expressing quantities consumed as functions of relative prices and total consumer expenditures. This demand system is assumed to satisfy the restrictions of homogeneity, symmetry, adding-up, and negativity. It is also assumed that the same restrictions hold for aggregate data, and that prices and consumer expenditures are predetermined. The question then becomes: Which functional form should we choose?

There are a number of criteria to be used in selecting a functional form. Statistically, we seek a function which performs well over the entire range of the data, in addition to some base point (Caves and Christensen). From an economic point of view, the function should be able to capture full price interactions (Barten). For food, Engel's law should hold and estimated own-price elasticities should be less than one in absolute value.

Two areas of concern by King are (*a*) changing elasticities over time and (*b*) relationships between income and price elasticities. With respect to condition (*a*), the linear expenditure system and Rotterdam model are too restrictive because they imply the demand elas-

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ticities for food increase as its expenditure share decreases. King claims, on the basis of Wold and Jureen's theorem 1 (p. 114), we should expect income elasticities for food to be smaller than their price elasticities. This expectation, however, is not implied by their analysis. All their theorem states is that the income elasticity of food will be less (greater) than its price elasticity (in absolute value) according as the price elasticity of nonfood is greater (less) than one in absolute value.

Three additional functional forms that have proved popular in empirical applications include (*a*) indirect translog model (TL), (*b*) generalized Leontief (GL), and (*c*) almost ideal demand system (AIDS). These three forms belong to the class of so-called locally flexible functional forms. The TL and GL can be rationalized as second-order approximations to an arbitrary, twice-differentiable indirect utility function. The TL (Christensen, Jorgenson, and Lau) uses as arguments of the approximation natural logarithms of the income normalized prices; the GL (Diewert) uses as arguments the square roots of the income normalized prices. The demand functions for these forms in the two-good case can be written:

$$(1)\quad w_i = (a_i + b_{i1}\ln x_1 + b_{i2}\ln x_2)/[a_1 + a_2 + (b_{11} + b_{12})\ln x_1 + (b_{12} + b_{22})\ln x_2], \text{ and}$$

$$(2)\quad w_i = (b_i x_i^{1/2} + c_{i1}x_i^{1/2}x_1^{1/2} + c_{i2}x_i^{1/2}x_2^{1/2})/(b_1 x_1^{1/2} + b_2 x_2^{1/2} + c_{11}x_1 + 2c_{12}x_1^{1/2}x_2^{1/2} + c_{22}x_2),$$

for $i = 1, 2$ where $b_{12} = b_{21}$ and $c_{12} = c_{21}$. Here w_i is the expenditure share and $x_i = p_i/y$, where p_i is price and y is total expenditures. The AIDS form (Deaton and Muellbauer) can be written

$$w_i = \alpha_i + \gamma_{i1}\ln p_1 + \gamma_{i2}\ln p_2 + \beta_i \ln(y/P),$$

for $i = 1, 2$, where

$$\ln P = \alpha_0 + \alpha_1 \ln p_1 + \alpha_2 \ln p_2 + 1/2\gamma_{11}(\ln p_1)^2 + \gamma_{12}(\ln p_1)(\ln p_2) + 1/2\gamma_{22}(\ln p_2)^2.$$

Despite their apparent flexibility, these three functional forms can impose rather tight restrictions on the behavior of own-price elasticity of food over time. In an expanded version of this paper (available upon request), I show that all three of these forms constrain the sign of the relationship between income and own-price elasticity to be the same at all data points. The AIDS is the most restrictive of the three, implying demand for food always becomes more inelastic with respect to price as real income rises. This agrees with the conventional view (Waugh, p. 81), but my attempts to apply this system to U.S. consumption data were unsuccessful, suggesting this form is inconsistent with U.S. consumption behavior. Moreover, Deaton and Muellbauer obtained positive own-price elasticities for food, suggesting this form is also inconsistent with British consumption behavior.

These limitations provide additional motivation for the use of the Fourier flexible form introduced in Gallant (1981). As indicated previously, the main attractiveness of this form is its capability to globally approximate demand elasticities. Intuitively, by finding the best trigonometric expansion, we are also finding elasticities which give the closest approximations (Gallant 1981; El Badawi, Gallant, and Souza). How one determines the best model depends on whether the problem is one of hypothesis testing or estimation (Gallant 1982, pp. 321–22). Here the main concern is consistent estimation of demand elasticities. This means we seek a specification that gives a smooth fit to the data (El Badawi, Gallant, and Souza). This specification could be determined adaptively by the downward or upward selection procedure described in Gallant (1982, pp. 321–22).

The particular model chosen for this application was determined by the downward selection procedure. This procedure resulted in the Fourier expansion with just main effects, i.e., $A = 2$, $J = 1$, $N = 2$ with multi-indexes $k_\alpha = (1\ 0), (0\ 1)$—see Wohlgenant for details. These demand functions can be written (Gallant 1981, sec. 5)

$$(3)\quad w_i = \{b_i x_i - 2u_{0i}x_i^2 - 2[u_{1i}\sin(x_i) + V_{1i}\cos(x_i)]x_i\} / \sum_{j=1}^{2} \{b_j x_j - 2u_{0j}x_j^2 - 2[u_{1j}\sin(x_j) + V_{1j}\cos(x_j)]x_j\}$$

for $i = 1, 2$. Note that both the numerator and denominator are linear functions of sine and cosine functions. Since $\partial\sin(x)/\partial x = \cos(x)$ and $\partial\cos(x)/\partial x = -\sin(x)$, derivatives of the numerator and denominator of (3) will be linear functions of the same sine and cosine functions as (3). This proportionality property of derivatives is one reason why the Fourier form confers important approximation properties on the elasticities (El Badawi, Gallant, Souza).

We now turn to an empirical comparison of the Fourier model with the TL and GL models.

Empirical Comparison of Fourier with Translog and Generalized Leontief

This section reports econometric estimates for the three functional forms—Fourier, TL, and GL. The data are annual postwar expenditures on food and nonfood from the national income and product accounts, U.S. Department of Commerce. Food includes tobacco and alcoholic beverages; nonfood is the sum of other nondurables, durables, services, and miscellaneous goods. Prices are implicit deflators, obtained by dividing current year by constant dollar expenditures. Because of the periodic nature of the Fourier form, income normalized prices (prices divided by per capita total expenditures) were rescaled to be between zero and six. Because there are only two goods and the shares sum to one, each statistical model consists of only one equation, food. The demand equations (1)–(3) were estimated by nonlinear least squares with correction for first-order serial correlation. Since each demand equation is homogenous of degree zero in its parameters, some normalization must be chosen. Without loss of generality, I chose $a_2 = -(1 + a_1)$ for the TL and $b_2 = -1$ for both the GL and Fourier models.

Parameter estimates for the three food demand specifications are reported in table 1. In the two-good case, symmetry is redundant given homogeneity and adding-up. Thus, the only conditions to check are monotonicity and quasi-convexity (see Caves and Christensen). These properties hold for each functional form at all data points.

Which demand estimates are most likely to have generated the observed data? On a Bayesian criterion, given the same (diffuse) prior for each functional form, we would choose the model with the smallest estimated residual variance (Berndt, Darrough, and Diewert). On this criterion, the Fourier model is preferred *a posteriori*. The strength of this choice can be measured by the posterior odds ratio, the product of the prior odds ratio and the likelihood ratio (Zellner, pp. 292–98). Assuming all three models are equally likely a priori, this ratio reduces to the likelihood ratio. This calculation gives posterior odds for the Fourier relative to the TL of 9.83:1, and 41.95:1 for the Fourier relative to the GL.

A more informed basis for choosing among these functional forms is the conformity of the estimated elasticities with our prior beliefs. Figures 1 and 2 show plots of own-price and income elasticities over the sample period. In general, the TL and GL give similar elasticities. However, the Fourier gives different estimates, particularly over the first half of the sample period.

Table 1. Parameter Estimates of Food Demand for Three Functional Forms, 1948–78

Fourier		TL		GL	
Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
b_1	−0.217	a_1	−0.141	b_1	−0.081
	(0.040)[a]		(0.017)		(0.052)
u_{01}	−0.014	b_{11}	−0.064	c_{11}	−0.037
	(0.011)		(0.026)		(0.028)
u_{02}	−0.117	b_{12}	0.072	c_{12}	−0.002
	(0.014)		(0.006)		(0.016)
u_{11}	0.003	b_{22}	0.250	c_{22}	0.248
	(0.003)		(0.028)		(0.031)
v_{11}	0.007				
	(0.009)				
u_{12}	0.029				
	(0.010)				
v_{12}	0.023				
	(0.017)				
ρ[b]	0.390	ρ	0.021	ρ	0.022
	(0.168)		(0.009)		(0.009)
σ^2	5.62×10^{-6}	σ^2	6.06×10^{-6}	σ^2	6.34×10^{-6}

[a] Values in parentheses are estimated standard errors adjusted for degrees of freedom.
[b] Estimated first-order autocorrelation parameter.

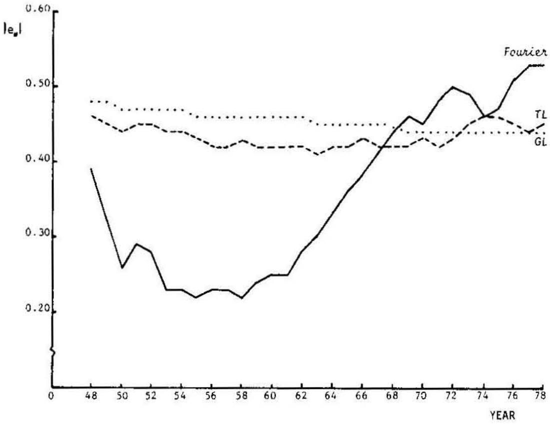

Figure 1. Own-price elasticities of demand for food for three functional forms, 1948–78

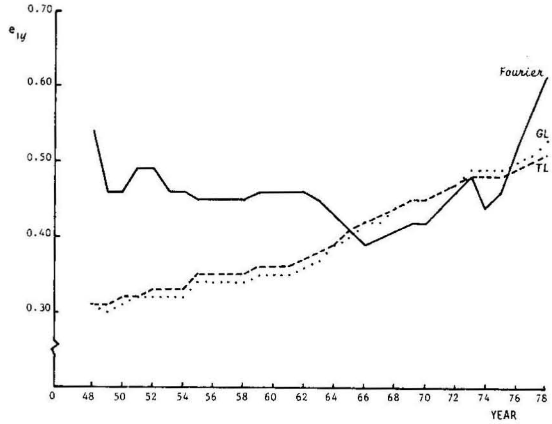

Figure 2. Income (expenditure) elasticities of demand for food for three functional forms, 1948–78

The earlier work of Brandow, Waugh, and George and King suggests the own-price elasticity for food is about −0.25 and income elasticity is about 0.20. The relevant time period for comparing these results is roughly 1948–65. The Fourier own-price elasticity estimates compare favorably with these earlier estimates, but the TL and GL price elasticities are about twice as large. All three functional forms suggest much higher income elasticities. The difference is likely due to the different income variables used. The earlier studies used disposable income; this study used personal consumption expenditures (PCE). Houthakker and Taylor (p. 33) suggest that income elasticity estimates using PCE will be closer to long-run elasticities. This is because consumers have better control over their expenditures than income receipts. If this is true, the two estimates might be reconciled through short-run estimates of the marginal propensity to consume (MPC). Because the estimates shown in figure 2 are roughly twice those reported in the earlier studies, a value for MPC of about 0.5 could account for this difference.

The most interesting results are the behavior of elasticities over time. The TL and GL suggest little change in price elasticities over time. The Fourier model suggests that demand for food first became less price elastic (until about 1955) but thereafter became increasingly more price elastic (figure 1). The pattern of change for the first part of the sample period is consistent with the analysis of Waugh (p. 18). But how can we explain the apparent tendency toward increased price responsiveness? One hypothesis, consistent with Allen and Bowley (p. 125), is that, as real income rises, increased expenditures are spread over a larger number of goods, which increases substitution possibilities and thereby price elasticity of demand for food. I show elsewhere (Wohlgenant) that the data are consistent with this hypothesis.

The Fourier model suggests little change in income elasticity over time (figure 2). (This is especially evident when we ignore the two extreme points in the sample.) The TL and GL indicate steadily rising income elasticities. This behavior, however, is inconsistent with cross-section estimates (e.g., George and King, p. 82), which suggest little change over time.

Summary

This paper has compared the Fourier flexible demand model introduced in Gallant (1981) with the two popular locally flexible forms—the translog and generalized Leontief. On theoretical and empirical grounds, the Fourier form was found to be superior. The main attractiveness of the Fourier form is its capability to globally approximate price and income elasticities. This property has not been shown to hold for either the translog or generalized Leontief. Moreover, these two locally flexible forms impose rather rigid restrictions on the behavior of price elasticities over time. The Fourier model gives results which conform well with our prior beliefs. It also gives new insights into the pattern of change of demand elasticities for food. In light of these considerations, we should expect the Fourier model to perform well in other applications.

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Estimating Demand Elasticities for Food*

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Estimating Demand Elasticities for Food

In estimating demand elasticities for food commodities, the analyst is confronted with a number of questions among which is selection of a functional form. King argues that the choice depends on the specific hypotheses to be tested and whether the chosen analytical model can provide answers to the questions asked. While these considerations are always of prime importance in empirical work, the conceptual basis from a statistical point of view is selection of a functional form which gives close approximations to the elasticities of interest.

In this connection, Gallant (1981, 1982) points out that the key consideration is the method of approximation of which there are two: Taylor's series approximations and Fourier series approximations. The problem with Taylor's theorem is that it only applies locally on a region of unspecified size. While this method can yield testable implications of the theory at the point of approximation, this point is in general unknown and may not even be in the range of the data (Caves and Christensen; White). Moreover, the approximation error from Taylor's series methods can be quite large for small departures from the (unspecified) point of expansion (Simmons and Weiserbs; White). In short, there is no guarantee that Taylor's theorem will give close approximations to the true elasticities at any point in the sample (Gallant 1981). This means we have no assurance that even the recently popularized locally flexible functional forms (translog, generalized Leontief, almost ideal demand system) will give close approximations to the true demand system.

In contrast to Taylor's series methods, Fourier series approximations, based on sine/cosine expansions, have the capability in principle to globally approximate the true price and income elasticities within arbitrarily close accuracy (Gallant 1981). Moreover, El Badawi, Gallant, and Souza show that demand elasticities derived from the Fourier demand system can be estimated consistently regardless of the statistical procedure used to estimate the parameters of the system. The purpose of this paper is to show that the Fourier demand system introduced in Gallant (1981) is a viable statistical model for estimating demand elasticities for food commodities. This model is fitted to U.S. postwar annual data on food and nonfood commodity aggregates. The estimated price and income elasticities are then used to reexamine some of the functional form questions raised by King. The main conclusions from this analysis are: (a) in comparison to the translog and generalized Leontief, the Fourier model is preferred a posteriori; (b) the Fourier model gives price and income elasticities which differ markedly from those of the translog and generalized Leontief; and (c) the Fourier model gives results which are consistent with some, but not all, prior notions regarding the relative magnitudes and behavior over time of demand elasticities for food. This paper begins with a discussion of some conceptual issues in estimating demand for food.

Conceptual Basis

The traditional starting point for specifying demand functions is the neoclassical theory of consumer behavior. In this approach, the representative consumer is presumed to maximize a twice-differentiable utility function subject to linear budget constraint. The solution to this constrained maximization problem is a set of demand functions, expressing quantities consumed as functions of relative prices and total consumer expenditures. This system of demand functions is assumed to satisfy the restrictions of homogeneity, symmetry, adding-up, and negativity. It is also typically assumed that the same restrictions hold for aggregate data, and that prices and consumer expenditures are predetermined (Barten).

There are three different approaches to functional form specification. These include starting from a specified direct utility function, starting from a specified indirect utility function, or specifying the demand functions directly (Barten). The advantage of the first two approaches is that the demand equations embody all the restrictions of the theory. In the third approach, the restrictions of the theory are imposed directly on the estimating equations. The second approach is generally more attractive in empirical applications, since through application of Roy's identity we obtain directly a set of estimable demand functions.

There are a number of criteria to be used in selecting a functional form. Statistically, the main criterion is the ability of the function to perform well over the entire range of the data, in addition to some base point (Caves and Christensen). From an economic point of view, the main criterion is the ability of the function to capture full price interactions (Barten). Two additional concerns with regard to food are Engel's law and estimated own-price elasticities being less than one in absolute value. Engel's law, which implies all income elasticities for food be less than unity, rules out demand systems derived from homothetic utility functions. The requirement that own-price elasticities be less than one in absolute value rules out certain types of additive demand systems, including specific variants of log-linear demand systems and the Rotterdam model, which result in the Cobb-Douglas system (Barten, p. 42).

Two areas of concern expressed by King are: (a) ability of function to show decreasing elasticities over time and (b) ability of the function to permit income elasticities to be smaller than price elasticities, i.e., Wold's rule (Wold and Jureen, pp. 114-15). With respect to condition (a), King shows that both the linear expenditure system and Rotterdam model can be ruled out, since these forms always imply that demand elasticities for food increase as its expenditure share decreases. He also argues that we can rule out the linear expenditure system on the basis of condition (b). However, as pointed out by Green and Hassan, this system can produce elasticity estimates consistent with Wold's rule if some of the minimum subsistence parameters are negative. In contrast to King, the position taken here is that _a priori_ there is no basis for expecting Wold's rule to hold because Wold and Jureen's definition of necessity relates to own-price elasticities, rather than income elasticities. All Wold and Jureen's theorem 1 (p. 114) says is that the income elasticity of food will be less (greater) than its price elasticity (in absolute value) according as the price elasticity of nonfood is greater (less) than one in absolute value. In principle, the own-price elasticity of demand for nonfood can be larger or smaller than one in absolute value. Thus we have no firm basis for expecting income elasticities for food to be less than their price elasticities. In fact, it is entirely possible that the relationship between price and income elasticities can change over the range of the data, depending on the relative magnitudes of income and substitution effects.

The most commonly used functional forms in empirical applications include: (a) linear expenditure system (LES), (b) indirect addilog model (IAL), (c) Rotterdam model (RM), (d) indirect translog model (TL), (e) generalized Leontief (GL), and (f) almost ideal demand system (AIDS). Limitations of the LES and RM have already been discussed. The IAL has similar limitations, which can be seen by examining the formulas for price and income elasticities (e.g., Green, Hassan, and Johnson, p. 95). What is apparently not known, though, is that the last three locally flexible functional forms can also impose undue restrictions on demand elasticities for food. In particular, as shown below, all three of these forms impose rather tight restrictions on the relationship between own-price elasticity and income. On the hypothesis that income is the main determinant of changing elasticities, this suggests that these three functional forms are not likely to provide insights into the issue whether price and income elasticities for food have been increasing or decreasing over time.

The TL and GL forms can be rationalized as second-order approximations to an arbitrary, twice-differentiable indirect utility function. The TL form (Christensen, Jorgenson, and Lau) uses as arguments of the approximation natural logarithms of the income normalized prices; The GL form (Diewert) uses as arguments the square-roots of the income normalized prices. These functional forms in the two-good case can be written

$$(1) \quad \ln V = a_1 \ln x_1 + a_2 \ln x_2 + 1/2 b_{11}(\ln x_1)^2 + b_{12}(\ln x_1)(\ln x_2) + 1/2 b_{22}(\ln x_2)^2 ,$$

$$(2) \quad V = b_1 x_1^{1/2} + b_2 x_2^{1/2} + C_{11} x_1 + 2c_{12} x_1^{1/2} x_2^{1/2} + c_{22} x_2 ,$$

where $x_i = p_i/y$, p_i is the price of the i^{th} good, and y is consumer expenditures. The demand functions for these functional forms can be obtained through application of Roy's identity. This formula in budget share form can be written (see Diewert, p. 125)

$$(3) \quad w_i = x_i \partial V/\partial x_i / \Sigma x_j \partial V/\partial x_j$$

where $w_i = p_i q_i / y$ is the budget share of the i^{th} good. Applying this formula to equations (1) and (2) we obtain

$$(4)\quad w_i = (a_i + b_{i1}\ln x_1 + b_{i2}\ln x_2)/[a_1 + a_2 + (b_{11} + b_{12})\ln x_1 + (b_{12} + b_{22})\ln x_2] ,$$

$$(5)\quad w_i = (b_i x_i^{1/2} + c_{i1}x_i^{1/2}x_1^{1/2} + c_{i2}x_i^{1/2}x_2^{1/2}) /(b_1 x_1^{1/2} + b_2 x_2^{1/2} + c_{11}x_1 + 2c_{12}x_1^{1/2}x_2^{1/2} + c_{22}x_2)$$

for i=1,2 where $b_{12}=b_{21}$ and $c_{12}=c_{21}$. Finally, we have the AIDS form (Deaton and Muellbauer), which relates budget shares to natural logarithms in relative prices and real income. This form can be written

$$(6)\quad w_i = \alpha_i + \gamma_{i1}\ln p_1 + \gamma_{i2}\ln p_2 + \beta_i \ln(y/P)$$

for i=1,2, where

$$\ln P = \alpha_0 + \alpha_1 \ln p_1 + \alpha_2 \ln p_2 + 1/2\ \gamma_{11}(\ln p_1)^2 + \gamma_{12}(\ln p_1)(\ln p_2) + 1/2\gamma_{22}(\ln p_2)^2 .$$

Price elasticities for all these forms can be obtained through use of the formula

$$(7)\quad e_{ij} = \partial \ln w_i/\partial \ln p_j - \delta_{ij}$$

where e_{ij} is the price elasticity of good i with respect to price j and δ_{ij} equals 1 when i=j but zero otherwise. Income (expenditure) elasticities, e_{iy}, can be obtained through the homogeneity restraint

(8) $e_{iy} = -\sum_j e_{ij}$.

Applying (3) to equations (5) through (7) the own-price elasticities for good 1, say food, can be characterized as

(9) $e_{11} = [b_1 - w_1(b_{11} + b_{12})]/(a_1 + b_{11}\ln x_1 + b_{12}\ln x_2) - 1$,

(10) $e_{11} = [(1/2b_1x_1^{1/2} + c_{11}x_1 + 1/2c_{12}x_1^{1/2}x_2^{1/2})$

$-w_1(1/2\ b_1x_1^{1/2} + c_{11}x_1 + c_{12}x_1^{1/2}x_{1/2})]/$

$(b_1x_1^{1/2} + c_{11}x_1 + c_{12}x_1^{1/2}x_2^{1/2}) - 1$,

(11) $e_{11} = [\gamma_{11} - \beta w_1 + \beta_1^2\ln(y/P)]/w_1 - 1.$

To see how elasticities change as income changes, we partially differentiate these formulas with respect to y. For the TL form, equation (9), this derivative can be shown to equal

(12) $\partial e_{11}/\partial \ln y = (b_{11} + b_{12})[w_1(1 - e_1y) + (1 + e_{11})]/$
$(a_1 + b_{11}\ln x_1 + b_{12}\ln x_2)$.

First notice that the denominator of (12) is the first-order partial derivative of (1) with respect to $\ln p_1$. The neoclassical monotonicity requirement is that this derivative be strictly less than zero. Thus the sign of (12) is determined by the sign of the numerator. Secondly, we expect income and own-price elasticities for food to be strictly less than one in absolute value, implying the sign of (12) will be the negative of the sign of the term $(b_{11}+b_{12})$. In general, this term is unsigned (Caves and Christensan). However, for any given body of data, it will be either positive or negative. This means the sign of (12) will be negative (positive) at all data points according as $(b_{11}+b_{12})$ is positive (negative). Hence, the TL form implies a fixed relationship (in terms of sign) between own-price elasticity for food and real income. Moreover, when $(b_{11}+b_{12})$ is small the derivative in (12) can be quite small, suggesting little or no change over time in the price elasticity of demand for food.

Letting N_1 be the numerator of (5), the derivative of the elasticity for the GL form, (10), with respect to lny can be shown to equal

$$(13) \quad \partial e_{11}/\partial \ln y = w_1(1 - e_{1y}) - (1/2\ b_1 x_1^{1/2}/N_1)[(1 + e_{11}) + w_1(1 - e_{1y})].$$

Aside from the first term on the right-hand-side, this derivative has the same form as that for the TL, equation (12). For food, the first term will be positive at all data points. However, the sign of the second term depends on the sign of b_1. It will be negative (positive) according as b_1 is negative (positive). Thus, in general, the sign of (13) is ambiguous. Furthermore, in principle, it can change signs depending upon the point of evaluation. However, it seems likely that the first term will dominate, since the term premultiplying the bracketed expression in (13) will be very small. Also, when b_1 is negative, equation (13) will consist of two offsetting terms, suggesting little or no responsiveness of own-price elasticity to changes in real income. Moreover, by the analysis of Caves and Christensen we should expect similar behavior for both the TL and GL forms in the case of food.

Despite its name, the AIDS can actually impose more stringent conditions on changes in price elasticities than either the TL or GL forms. Upon differentiating equation (6) with respect to lny and simplifying, this derivative can be shown to equal

$$\partial e_{11}/\partial \ln y = (1 + e_{11})(1 - e_{1y}).$$

Thus, since for food $e_{1y}<1$ and $e_{11} >-1$, we see that the AIDS effectively constrains this relationship to be positive at all data points. While this agrees with the conventional view that demand for food becomes more inelastic with respect to its price as real income rises (Waugh, p.18), the AIDS is not flexible enough to let the data determine this relationship. In this context, it is interesting to note that in attempting to apply this system to British postwar data, Deaton and Muellbauer found positive own-price elasticities for food. My attempts to apply this system to U.S. postwar data on food and non-food aggregates were also unsuccessful, suggesting that this form is also incompatible with U.S. food consumption behavior. For these reasons, the empirical comparison among functional forms is limited to the TL, GL, and Fourier systems.

The Fourier Demand System

The foregoing analysis provides additional motivation for the use of a semi-parametric functional form such as the Fourier flexible form introduced in Gallant (1981). As indicated previously, the main attractiveness of this form is its capability, in principle, to give arbitrarily close approximations to the true price and income elasticities. Intuitively, the reason for this is that by finding the best fitting functional form we are also finding elasticities which give the closest approximations (Gallant 1981; El Badawi, Gallant, and Souza). From a technical point of view, a Fourier expansion does the job because it satisfies the Sobolov norm, a measure of distance which takes into account derivatives of the functional form (Gallant 1981). This derivative property has not been shown to hold for fixed parameter models such as the TL or GL forms.

In general, it is not possible to determine a priori which of the numerous Fourier expansions is best with respect to a given body of data. Rather, by analogy with time series analysis, we would fit different models for different orders of the trigonometric polynomial to determine which specification is best according to some criterion. As Gallant (1982,pp. 321-22) points out, the choice depends on whether the problem is one of hypothesis testing or estimation. Here the main concern is consistent estimation of price and income elasticities, meaning we are interested in the specification that gives the smoothest fits to the data (El Badawi, Gallant, and Souza). Again, by analogy with time series analysis, we seek a parsimonious representation, i.e., a model which gives close approximations with the fewest number of parameters. This specification could be determined either by the downward or upward selection procedure described in Gallant (1982,pp.321-22).

For discussion purposes, it is useful to consider a Fourier expansion of the same order comparable to that of the TL and GL forms. For A=3, J=1, N=2, and the multi-indices k_α = (1 0)'(0 1)'(1)' the Fourier representation of the indirect utility function may be written (Gallant 1981, sec.5)

$$\begin{aligned}(14)\quad V = \text{const.} &+ b_1x_1 + b_2x_2 - (U_{01} + U_{03})x_1^2 - (U_{02} + U_{03})x_2^2 \\ &- 2U_{03}x_1x_2 + 2[U_{11}\cos(x_1) - v_{11}\sin(x_1)] \\ &+ 2[u_{12}\cos(x_2) - v_{12}\sin(x_2)] \\ &+ 2[u_{13}\cos(x_1 + x_2) - v_{13}\sin(x_1 + x_2)].\end{aligned}$$

Recalling the trigonometric identities, $\cos(x_1 + x_2) = \cos(x_1) \cdot \cos(x_2) - \sin(x_1) \cdot \sin(x_2)$ and $\sin(x_1 + x_2) = \sin(x_1) \cdot \cos(x_2) + \cos(x_1) \cdot \sin(x_2)$, equation (14) can be viewed as a quadratic function in the x's augmented by first- and second-order terms involving sine and cosine functions of the x's. The quadratic component is included to impose convexity requirements of the indirect utility function. It needs to be emphasized again that equation (14) is only one of many possible representations of the Fourier form; the exact form will depend on the specific set of data under consideration, and whether the main interest is one of hypothesis testing or estimation.

Recalling that $\partial\sin(x)/\partial x=\cos(x)$ and $\partial\cos(x)/\partial x = -\sin(x)$, Roy's identity, (3), applied to equation (14) results in the budget share equations

$$(15)\quad w_i = \{b_ix_i - 2(u_{0i} + u_{03})x_i^2 - 2u_{03}x_1x_2 - 2[u_{1i}\sin(x_i) + v_{1i}\cos(x_i)]x_i - 2[u_{13}\sin(x_1 + x_2) + v_{13}\cos(x_1 + x_2)]x_i\}/ \{b_1x_1 + b_2x_2 - 2(u_{01} + u_{03})x_1^2 - 4u_{03}x_1x_2 - 2(u_{02} + u_{03})x_2^2 - 2[u_{11}\sin(x_1) + v_{11}\cos(x_1)]x_1 - 2[u_{12}\sin(x_2) + v_{12}\cos(x_2)]x_2 - 2[u_{13}\sin(x_1 + x_2) + v_{13}\cos(x_1 + x_2)9x_1 + x_2)\}$$

for i=1,2. Price and income elasticities can be obtained through application of the formulas (7) and (8). Letting N_i be the numerator and D the denominator of (15), the price elasticities can be shown to have the general form

$$e_{ij} = x_j(\partial N_i/\partial x_j - w_i\partial D/\partial x_j)/(w_iD) - \delta_{ij}$$

where w_i is determined by (15). This expression makes clear that elasticities may be viewed as functions of the derivatives of the numerator and denominator of (15) with respect to the x's. Since derivatives of sines are cosines and vice-versa, derivatives of the numerator and denominator of (15) will be linear combinations of the same sine and cosine functions as (15). This, in essence, explains why the Fourier form confers important approximation properties on the elasticities. In other words, by finding the best fitting demand model, we are simultaneously finding the derivatives (and therefore elasticities), which are proportional to the original demand function (El Badawi, Gallant, and Souza).

We now turn to an empirical comparison of the Fourier model with the TL and GL models.

Application to Demand for Food

This section reports econometric estimates for the three functional forms (Fourier, TL, and GL) applied to demand for food. The data are annual expenditures on food and nonfood over the period 1947-78. They come from the national income and product accounts, U.S. Department of Commerce. Food includes tobacco and alcoholic beverages. Thus nonfood is the sum of other nondurables, durables, services, and miscellaneous goods. The price data are implicit deflators, obtained by dividing current year expenditures by constant dollar expenditures. These prices were then divided by per capita total consumption expenditures to obtain income normalized prices. Due to the periodic nature of the Fourier form, these normalized prices were rescaled to be between zero and 6. All data are available from the author upon request.

In the present application, each demand system consists only of two goods: food and nonfood. Since the adding-up condition implies one of the equations can be deleted without any loss of information (Barten, p. 26), the nonfood equation was deleted from each system, leaving only the food demand specifications to estimate. (Parameter estimates for the nonfood equations can be obtained through the restrictions implied by the adding-up condition, $w_1+w_2=1$.) This means single-equation methods can be employed to estimate the first equations in (4),(5), and (15). I used the Nonlinear Least Squares method, with correction for first-order autocorrelation in the residuals. Since when correcting for serial correlation we lose one observation, the total number of observations was T=31(1948-78). Finally, notice that the budget share equations (4),(5), and (15) are each homogenous of degree zero in their parameters. This means some normalization must be chosen. Without loss of generality, I chose $a_2 = -(1+a_1)$ for the TL and $b_2 = -1$ for both the GL and Fourier models.

Parameter estimates for the three food demand specifications are reported in table 1. The estimates for the Fourier model correspond to the simple additive indirect utility function, i.e., equation (15) with the parameters u_{03}, u_{13}, and v_{13} set equal to zero. This model was chosen after estimating a number of alternative representations of the Fourier model and employing the downward selection procedure described in Gallant (1982,pp.321-22)--see Wohlgenant for details.

In a two-good world, the symmetry restriction is redundant given the homogeneity and adding-up conditions. This means the demand estimates in table 1 satisfy these three restrictions exactly. There is no assurance, however, that the demand estimates will satisfy the regularity conditions of monotonicity and quasi-convexity. The monotonicity requirement is that the first-order partial derivatives of the indirect utility function be strictly less than zero. In the two-good case, the quasi-convexity requirement of the indirect utility function is equivalent to requiring that the compensated own-price elasticities, $e_{ii}+w_i e_{iy}$(i=1,2), be nonpositive. Both of these regularity conditions were checked for the estimates reported in table 1 and were shown to hold for each functional form at all data points.

A priori we are unable to choose among these three models on econometric grounds because these forms are nonnested, i.e., no one model includes the others as special cases. We can, however, informally compare the models *a posteriori* by examining how the various parameter and elasticity estimates conform with our prior beliefs. As Berndt, Darrough, and Diewert point out, we could also formally discriminate among these functional forms by employing Bayesian methods. They show that, given the same (diffuse) prior for each functional form, we can choose *a posteriori* which model is most likely to have generated to observed data by simply comparing the values of the estimated log likelihood functions. In the present application, this is equivalent to choosing the model with the smallest estimated residual variance. On this criterion, we see from table 1 that the Fourier model is the preferred specification. A measure of the strength of this choice is the posterior odds ratio, which can be viewed as the revised prior odds ratio of one model relative to the other. Zellner (pp.292-98) shows that the posterior odds ratio can be calculated simply as the product of the prior odds ratio and the likelihood ratio. On the assumption that all three models are equally likely *a priori*, this ratio can be computed simply as the likelihood ratio. In the present application, the posterior odds of the Fourier relative to the TL are 9.83:1, and the posterior odds of the Fourier relative to the GL are 41.95:1. Thus, even on this somewhat pessimistic view of the Fourier form, the sample information strongly favors this specification.

A more informed basis for choosing among these functional forms is the conformity of the estimated elasticities with our prior beliefs. Own-price and income elasticities for each model were computed using the formulas in (7) and (8). Demand elasticities for selected years are reported in table 2. Figures 1 and 2 show plots of these elasticities over the sample period. In general, the TL and GL forms give similar own-price and income elasticities. However, the Fourier form suggests quite different elasticity estimates, particularly over the first half of the sample period. The last two rows in table 2 gives estimates of two commonly used measures of mean elasticities. The first is a simple average of the elasticity estimates for each form; the second evaluates the elasticities at the sample mean values of the income normalized prices.

How do the elasticities from these models compare with previous studies? Based on the earlier work of Brandow, Waugh, and George and King, we might expect the own-price elasticity for food to be about -0.25 and income elasticity to be about 0.20. The relevant time period for comparison is 1948-65. Over this period, both the TL and GL models give own-price elasticity estimates about twice the magnitude of those reported in these earlier studies. On the other hand, the Fourier model gives own-price elasticity estimates which are in close agreement with the earlier studies. What appears to be a puzzle, though, is that all three functional forms (including the Fourier) suggest income elasticities substantially larger than those reported in the earlier studies. One hypothesis is that the difference is due to the different income variables used. Brandow, Waugh, and George and King used disposable income (DI), whereas the systems approach employed here used personal consumption expenditures (PCE). Houthakker and Taylor (chap. 6) suggest that income elasticity estimates using PCE will be closer to long-run elasticities. This is because consumers have better control over their expenditures than income receipts, so PCE should be a better measure of the true income of consumers (Houthakker and Taylor, p.33). In other words, in the context of the permanent income hypothesis, there will be measurement error from using DI as a proxy for permanent income, implying the estimates derived from DI will be closer to short-run elasticities. If this is true, then these two income elasticity estimates can be reconciled through the short-run relationship between PCE and DI,i.e., through short-run estimates of the marginal propensity to consume (MPC). A plausible value for the MPC is 0.5. This suggests that income elasticity estimates using PCE should be roughly twice those using DI, which is in fact what we observe. To sum up, the Fourier model gives both price and income elasticities which can be reconciled with those reported in the earlier studies. Income elasticity estimates from the TL and GL forms can also be reconciled with the previous studies; however, these two functional forms suggest implausible price elasticity estimates over the relevant time period for comparison.

The most interesting results relate to changing elasticities over time. Both the TL and GL suggest virtually no change in price elasticities over time. However, the Fourier model suggests that demand for food first became less price elastic (until about 1955), but thereafter became increasingly more price elastic (figure 1). The pattern of change for the first part of the sample period is entirely consistent with the observations by Waugh (p.18). But how can we explain the apparent tendency toward increased price responsiveness? One hypothesis, consistent with Allen and Bowley (p.125), is that, as real income rises, increased expenditures are spread over a larger number of goods, which increases substitution possibilities and thereby price elasticity of demand for food. I have shown elsewhere (Wohlgenant) that the data are consistent with this hypothesis. The estimated relationship between own-price elasticity and real income, $\partial e_{11}\partial \ln y$, is shown to have a pattern entirely consistent with the observed behavior of own-price elasticity over time. This suggests income is the main determinant of changing price elasticities. Moreover, the main factor contributing to this variability is shown to be changes in the rate of substitutability between food and nonfood, which is consistent with the stated hypothesis.

In contrast to the behavior of own-price elasticity over time, the Fourier model suggests little or no change in income elasticity over time (figure 2). (This is especially evident when we ignore the two extreme points in the sample.) On the other hand, both the TL and GL forms suggest steadily rising income elasticities over time. Since this behavior is inconsistent with estimates derived from cross-section data(George and King, p. 82), we prefer the Fourier estimates.

Summary

This paper has compared the Fourier flexible demand model introduced in Gallant (1981) with the two popular locally flexible forms--the translog and generalized Leontief. On the basis of theoretical considerations and empirical application to aggregate demand for food, the Fourier form was shown to be superior. The main attractiveness of the Fourier form is its capability, in principle, to globally approximate price and income elasticities. This approximating property has not been shown to hold for fixed parameter models such as the translog or generalized Leontief forms. Moreover, these two locally flexible forms impose rather rigid restrictions on the behavior of price elasticities over time. The Fourier model give results which conform well with our prior beliefs. It also gives new insights into the pattern of change of demand elasticities for food. In light of these considerations, we should expect the Fourier model to give superior results in other applications.

Table 1. Parameter Estimates of Food Demand for Three Functional Forms, 1948-78

Fourier		TL		GL	
Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
b_1	-0.217	a_1	-0.141	b_1	-0.081
	(0.040)[a]		(0.017)		(0.052)
u_{01}	-0.014	b_{11}	-0.064	c_{11}	-0.037
	(0.011)		(0.026)		(0.028)
u_{02}	-0.117	b_{12}	0.072	c_{12}	-0.002
	(0.014)		(0.006)		(0.016)
u_{11}	0.003	b_{22}	0.250	c_{22}	0.248
	(0.003)		(0.028)		(0.031)
v_{11}	(0.007)				
	(0.009)				
u_{12}	0.029				
	(0.010)				
v_{12}	0.023				
	(0.017)				
ρ[b]	0.390	ρ	0.021	ρ	0.022
	(0.168)		(0.009)		(0.009)
σ^2	5.62×10^{-6}	σ^2	6.06×10^{-6}	σ^2	6.34×10^{-6}

[a]Values in parentheses are estimated standard errors adjusted for degrees of freedom.

[b]Estimated first-order autocorrelation parameter.

Table 2. Estimated Demand Elasticities of Food for Three Functional Forms; Selected Years, 1948-78

	Own-Price			Income		
Year	Fourier	TL	GL	Fourier	TL	GL
1948	-0.39	-0.46	-0.48	0.54	0.31	0.31
1951	-0.29	-0.45	-0.47	0.49	0.32	0.32
1954	-0.23	-0.44	-0.47	0.46	0.33	0.32
1957	-0.23	-0.42	-0.46	0.45	0.35	0.34
1960	-0.25	-0.42	-0.46	0.46	0.36	0.35
1963	-0.30	-0.41	-0.45	0.45	0.38	0.37
1966	-0.38	-0.43	-0.45	0.39	0.42	0.42
1969	-0.46	-0.42	-0.44	0.42	0.45	0.45
1972	-0.50	-0.43	-0.44	0.46	0.47	0.47
1975	-0.47	-0.46	-0.44	0.46	0.48	0.49
1978	-0.53	-0.45	-0.44	0.61	0.51	0.52
Simple Average	-0.36	-0.43	-0.45	0.46	0.40	0.39
At Sample Means	-0.29	-0.44	-0.45	0.38	0.40	0.39

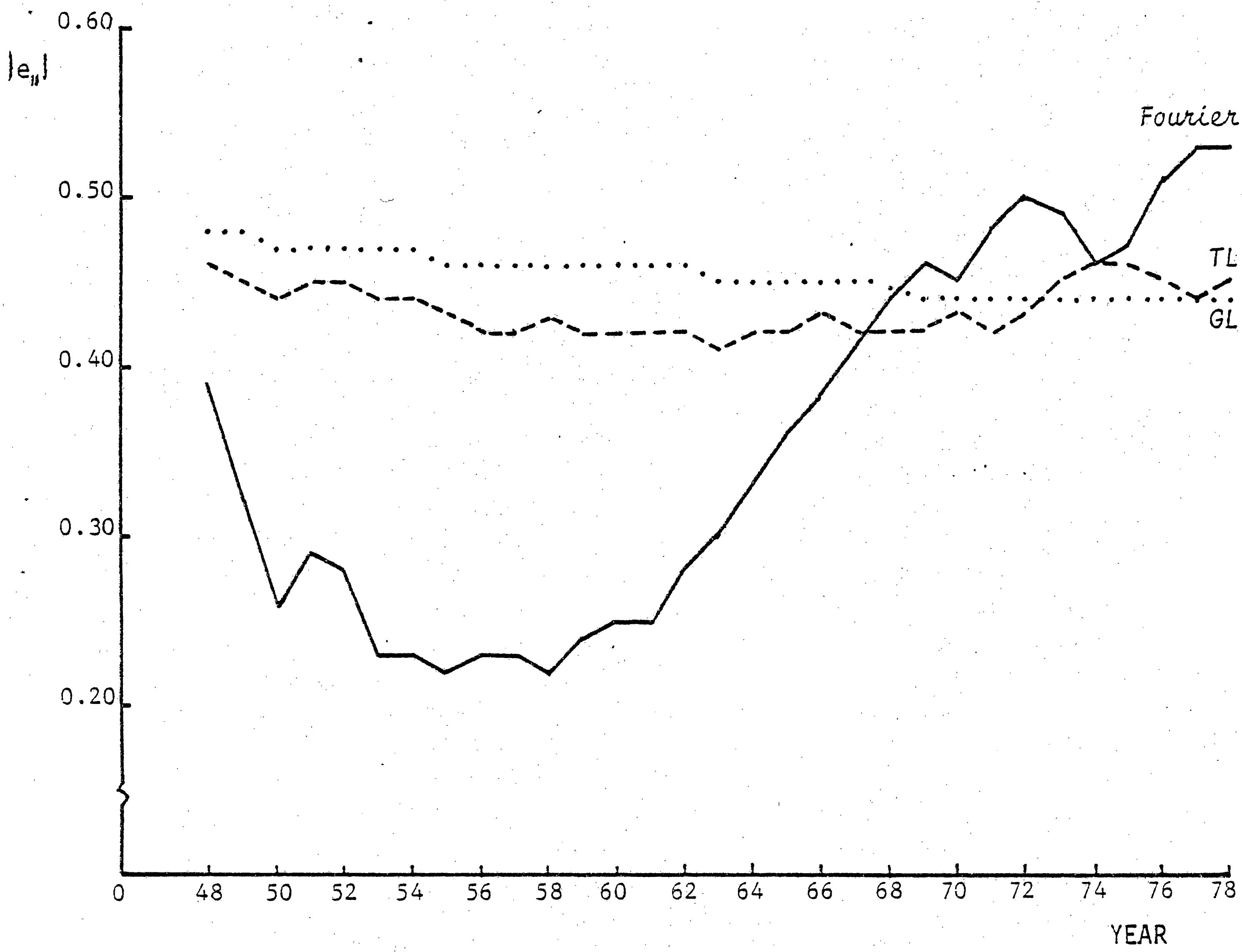

Figure 1. Own-Price Elasticities of Demand for Food for Three Functional Forms, 1948-78.

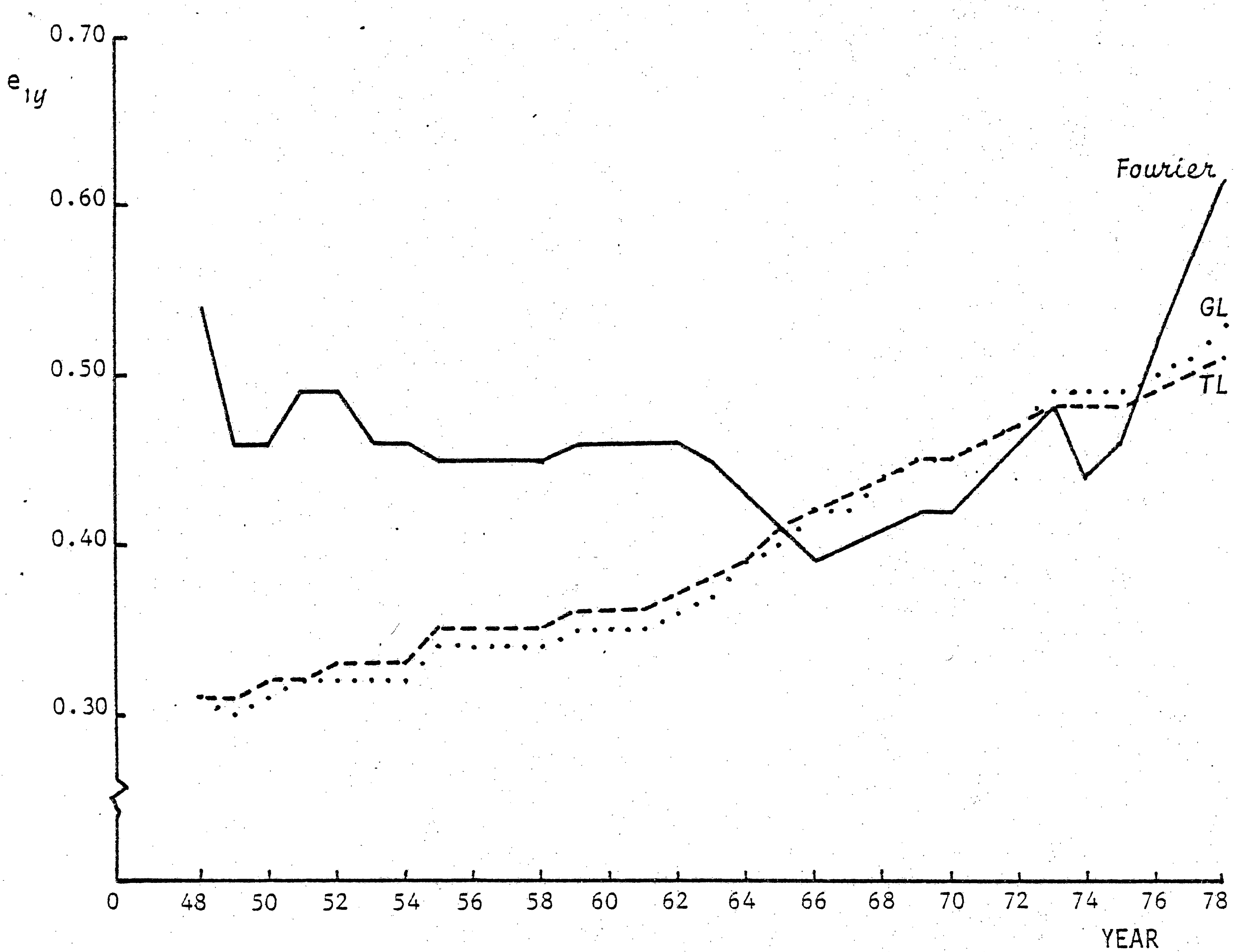

Figure 2. Income (Expenditure) Elasticities of Demand for Food for Three Functional Forms, 1948-78.

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