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ALLOCATABLE FIXED FACTORS AND JOINTNESS
IN AGRICULTURAL PRODUCTION:
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Abstract

Allocatable fixed factors, e.g., land, must be added to the traditionally-regarded causes of jointness in agriculture. Their presence also necessitates multiple-product systems for modeling product supply and factor demand. In other important ways, however, their analytical implications are very different from other causes of jointness.

ALLOCATABLE FIXED FACTORS AND JOINTNESS IN AGRICULTURAL PRODUCTION: IMPLICATIONS FOR ECONOMIC MODELING

One of the most neglected subjects in production theory is jointness. After mentioning the mutton and wool example and alluding to possible interdependencies in production, the subject is typically dropped by most theorists. Few texts even provide a formal defintion of jointness. Thus, it is small wonder that considerable confusion exists concerning the extent of jointness and its relevance for modeling agricultural production.

The purpose of this paper is to: (a) define and give a visual interpretation to jointness, (b) identify its causes, (c) demonstrate why different causes have different implications for modeling production, and (d) identify limitations of dual production models when allocatable fixed factors are the sole cause of jointness.[1]

Jointness defined

Little consensus is evident among economists concerning the extent to which jointness occurs in agricultural production. Part of the problem is due to the frequent failure to define the type of jointness being discussed. Fixed-proportion jointness in inputs means that two or more products are always produced in the same proportions. Few examples of truly fixed-proportion jointness exist in agriculture. However, even if there were many instances, this type of joint production would not be interesting analytically since any problems to be examined always reduce to the single-product case; only the definition of the product needs to be altered.

Primal Definition (Single Variable Input)

The interesting and much more prevalent type of jointness in inputs involves variable proportions production.[2] Jointness thus encompasses all cases of production of two or more products which are technically interdependent (Henderson and Quandt, p. 89). When there is one variable input (or input aggregate), technical independence or nonjointness then requires

(1) $\partial^2 x/\partial y_i \partial y_j = 0, \; i \neq j,$

where $y_{i(j)}$ is output level of product i (j) and x is the input requirement (Carlson, p. 79). This means that the marginal factor requirement to produce a given unit of one product is independent of the amount of a second product produced. Or, for a given level of y_i, the marginal product of x in y_i is independent of the level of y_j. Jointness is simply the converse, i.e., that the marginal factor requirement (or the marginal productivity of the input) for a given unit of i is dependent on the quantity of j produced.

A visual interpretation of a nonjoint technology is given in panel A of figure 1. If a change in the quantity produced of product j either has no effect or causes a horizontally parallel shift in the production function for product i, the technology is nonjoint. In either situation, the marginal factor requirement for a given unit of i is independent of the quantity of j produced. A joint technology occurs when a change in j causes either a vertically parallel shift (e.g., from $TPP_{y_i|y_j^o}$ to $TPP_{y_i|y_j'}$ in panel B or a nonparallel shift (e.g., from $TPP_{y_i|y_j^o}$ to $TPP_{y_i|y_j''}$) in the production function for i.

Equation (1) is both necessary and sufficient for nonjointness. The other condition commonly associated with nonjointness, i.e., that independent production functions can be written for each product, is sufficient but not necessary when there are no allocatable fixed factors. It is a more restrictive condition than equation (1) in that case since it does not admit even parallel shifts in the production function for i when the quantity of j is changed.

Dual Definition

Under appropriate regularity conditions on technology and for price-taking firms seeking to maximize short-run profits (or minimize costs), a duality exists (Lau, 1978, pp. 170-72) between the transformation or production function and the indirect profit (or cost) function. Several authors have proposed tests for nonjointness using the dual models. Hall (pp. 884-7) asserts that for a cost-minimizing firm, a technology is nonjoint if the product- product cross derivatives of the joint cost function are zero, i.e.,

(2) $\partial^2 C/\partial y_i \partial y_j = 0, \quad i \neq j,$

where C is cost and $y_{i(j)}$ is output of i (j). Lau (1972, pp. 287-89) presents a similar test for a profit-maximizing competitive firm using the indirect profit function,

(3) $\partial^2 \pi^* / \partial p_i \partial p_j = 0, \quad i \neq j,$

where π^* is profit and $p_{i(j)}$ is price of product i (j). Equation (3) also applies to cross derivatives of the normalized profit function where profit and prices are each divided by the price of one variable input (Lau, 1978, p. 183).

Implications for Primal (Multiple Variable Inputs)

Both dual tests for nonjointness are consistent with Carlson's primal definition when only one input is variable. When multiple inputs are variable, the dual definitions retain their intuitive meaning for price-taking firms but imply more complex restrictions on the primal.

The relevant restrictions can be derived by noting that the Hessian matrix of the transformation function, i.e.,

(4) $x_1 = f\,(y_1,\ldots,y_m,x_2,\ldots,x_n),$

is the inverse of the Hessian of the normalized profit function, i.e.,

$$(5) \qquad \pi^*/r_1 = g(p_1/r_1,\ldots,p_m/r_1,r_2/r_1,\ldots,r_n/r_1),$$

where r_i is the price of input i. Lau (1976, pp. 148-49) shows further that when some inputs and/or outputs are fixed, the Hessian submatrix for variable inputs and outputs of the transformation function is the inverse of the Hessian submatrix for variable input and output prices of the normalized restricted profit function. Thus, if an element of the profit Hessian is zero, e.g., $\partial^2 (\pi^*/r_1)/\partial(p_1/r_1)\partial(p_2/r_1) = 0$, the corresponding minor of the transformation Hessian submatrix is singular; e.g., with two variable outputs (y_1,y_2), two variable inputs (x_1,x_2), and any number of fixed outputs or inputs,

$$(6) \qquad \begin{vmatrix} \partial^2 x_1/\partial y_2 \partial y_1 & \partial^2 x_1/\partial y_2 \partial x_2 \\ \partial^2 x_1/\partial x_2 \partial y_1 & \partial^2 x_1/\partial x_2^2 \end{vmatrix} = 0.$$

This result is a straightforward application of the inverse theorem of matrix algebra (Hadley, p. 103), apparently is the underlying point in Samuelson's singularity theorem for non-joint production, and is consistent with Huenemann's recent derivation based on the inverse relationship between the cost and transformation functions.

Causes of Jointness [3]

Interdependent Production Processes

Interdependent production processes are often viewed as the equivalent of a joint technology. Certainly they imply jointness. By the very descriptiveness of the term, they convey the intuitive impression that the marginal factor requirement (or marginal cost when there is more than one variable input) for one product depends somehow on the quantity produced of one or more other products.

There are several reasons why production processes are often interdependent. Four examples are given that often occur in agricultural production. Some of the interdependence is intrinsic and some is due to practical limitations in disaggregating and classifying data.

a. Even if weather, location, and initial yield per acre of corn were held constant, the marginal cost of another bushel of corn may be greatly affected by the crop planted the previous year. If the preceding crop were nitrogen fixing, e.g., a legume, more nitrogen would be available in the soil at planting time than if the previous crop were nitrogen using. This jointness in grain and legume production is due to an unmeasured change in soil quality.

b. A similar effect is observed when crops which are not hosts to certain insects are planted in rotation, or sometimes in adjoining fields, with the host crop. In this case the marginal cost curve for the host crop is shifted downward because of rotation with the nonhost crop.

c. Timing of equipment requirements creates additional opportunities for jointness in production. A piece of equipment may be required to produce 100 acres of one crop, but it may only be used for 30 days. The same equipment may be needed at a different time to produce another crop. This apparent cause of jointness may disappear at the micro level when adequate equipment rental markets exist, but it does not disappear at the aggregate level. The stock of equipment that must be maintained strictly for use in the second crop depends on how much of the first crop is produced. Thus, the marginal cost of the second crop depends on the amount of the first crop produced.

d. The typically-fixed inputs of operator labor and facilities are an additional cause of jointness, if only in the fact that it is often difficult to accurately allocate services provided by them to different products. Part of the operator's labor is always spent in general management which can be allocated only arbitrarily to specific products. It is likewise difficult to fully allocate the services of barns, sheds, and other facilities. Although possibly planned primarily to enhance production of one commodity, improvements in quantity and/or quality of such inputs may lead to a shifting of the marginal cost curves of other products.

Allocatable Fixed Factors

It is clear that interdependent production processes, of which there are many forms, imply a joint technology. There is another cause of jointness which is not so intuitively obvious but which often causes both primal and dual tests for nonjointness to be rejected. This cause is the presence of allocatable fixed factors. Since many agricultural firms produce more than one product and since quantity (although not necessarily quality) of land is virtually always distinguishable as to the product to which it is allocated, this case is particularly representative of agriculture. At the firm level, available agricultural land is largely fixed over short to intermediate adjustment periods. At the regional level, it is essentially fixed over very long adjustment periods, except as land is removed due mainly to forces external to the agricultural sector.

To show that the presence of allocatable fixed factors generally results in jointness, consider for simplicity the production of only two products by independent production processes using two factors, one of which is fixed and allocatable. The production system is

(7) $y_i = f_i(x_i, z_i)$, $i = 1,2$,

(8) $z_1 + z_2 \leq \bar{z}$,

where y_i is quantity of product i, x_i is quantity of the variable factor used in the production of i, z_i is quantity of the fixed factor used in the production of i, and $\bar{z}$ is quantity of the fixed factor available. Assuming that inverses exist, the following transformations of this system can be made:

(9) $x_1 = f^{-1}_{1x_1}(y_1, z_1)$,

(10) $z_2 = f^{-1}_{2z_2}(y_2, x_2, \bar{z})$,

(11) $z_1 = \bar{z} - z_2 = \bar{z} - f^{-1}_{2z_2}(y_2, x_2, \bar{z})$.

From equations (9) and (11), the cross partial derivative of the variable factor x_1 with respect to products is:

$$(12)\quad \partial^2 x_1/\partial y_1 \partial y_2 = \partial(\partial x_1/\partial y_1)/\partial y_2$$
$$= [\partial(\partial x_1/\partial y_1)/\partial z_1](\partial z_1/\partial y_2)$$
$$= [\partial(\partial x_1/\partial y_1)/\partial z_1](-\partial f^{-1}_{2z_2}/\partial y_2)$$
$$= [\partial(\partial x_1/\partial y_1)/\partial z_1](-\partial z_2/\partial y_2).$$

Since there is only one variable input, the primal test in equation (1) is applicable. For this test to indicate a nonjoint technology, one of the right-hand terms of equation (12) must be zero. Since the existence of an inverse assures that the latter term is nonzero, the first term, $\partial(\partial x_1/\partial y_1)/\partial z_1$, must be zero. This would occur only if x_1 and z_1 were additively separable.

The dual tests, equations (2) and (3), are consistent with the primal test under this circumstance:

a. Only when $\partial(\partial x_1/\partial y_1)/\partial z_1 = 0$, i.e., when the marginal requirement of the variable factor is independent of the allocation of the fixed factor, is the marginal cost of y_1 independent of y_2 (equation 2) for a price-taking firm with no other causes of jointness.

b. If the marginal requirement for a variable factor is independent of the allocation of a fixed factor, it is also independent of the quantities of other products when there are no other causes of jointness. Since a price-taking firm maximizes profit by equating marginal factor requirements to the ratio of product and factor prices, the optimal quantity of the variable factor used to produce y_1, and the quantity of y_1 itself, are independent of the fixed factors and consequently independent of the prices of other products. From McFadden's lemma,

$$(13)\quad \partial\pi^*/\partial p_i = y_i^*,$$

where y_i^* is the optimal quantity (or the supply function) of product i. In this case it is a function only of its own price and the variable input price, so

$$(14)\quad \partial^2\pi^*/\partial p_i \partial p_j = \partial y_i^*/\partial p_j = 0.$$

If y_i^* is independent of p_j, equation (3) also implies nonjointness, and all three tests are consistent when allocatable fixed factors are present.

It will be shown below that since allocatable fixed factors have different implications for modeling production than other causes of jointness, it will be important in some situations to be able to test whether a factor is fixed and allocatable. From equation (11), a test for allocatable fixed factors among two or more products is

$$(15) \qquad \sum_j \partial z_j / \partial y_i = 0 \ .$$

Implications of Alternative Causes of Jointness for Modeling Product Supply and Factor Demand

If a firm produces a single product or if its production of multiple products is nonjoint in inputs, a separate production function, cost function, and profit function can generally be written for each product.[4] The quantities of other products can be excluded *a priori* from the cost functions and the prices of other products from the indirect profit functions. Product supply and demand for factors used in the production of each product can be modeled independent of other products.

If production is joint due to interdependent production processes, there is no way to write separate production, cost, or profit functions for individual products. Traditionally, demand functions for a factor used in the production of a single product have not been defined and applied. However, a supply function can still easily be formulated for each product, and a total demand function can be derived for each factor. Each is a function of all product prices, variable factor prices, and fixed factor quantities. Estimation requires that production, product supplies, and/or factor demands be modeled as part of a multiple-product system. When there are no allocatable fixed factors, the existence of jointness is generally independent of the length of the adjustment period but not necessarily independent of aggregation level.

If production is joint only because of the presence of allocatable fixed factors, separate production, cost, and profit functions can still be written for each product. This point is contrary to typical references that imply that the ability to write separate production functions is limited to nonjoint technologies. Separate functions can be written, but they are part of a system with availability constraints on the allocatable fixed factors. The separate profit (cost) functions can be maximized (minimized) only as a system subject to the constraint. As with other causes of jointness, factor demand and product supply equations are thus functions of all product prices and must be modeled within a multiple-product system. This source of jointness is dependent on the length of the adjustment period and tends to disappear, at least at the firm level, in the long run.

Limitations of Dual Models

When allocatable fixed factors are the only cause of jointness, it is in principle still possible to derive all demand functions for factors used in the production of individual products. The problem is one of constrained optimization. For the case of two products, n variable factors, and one allocatable fixed factor, the Lagrangean problem is

$$(16) \qquad L = p_1 f_1(X_1, z_1) + p_2 f_2(X_2, z_2) - \sum_{i=1}^{2} \sum_{j=1}^{n} r_j x_{ij} + \lambda(\bar{z} - \sum_{i=1}^{2} z_i),$$

where X_i is the vector of variable factor quantities used to produce product i, x_{ij} is the quantity of factor j used in the production of i, and r_j is the price of factor j. Taking first derivatives with respect to each x_{ij}, z_j, and λ gives $2n + 3$ equations in the same number of unknowns. Solving this system of equations gives each x_{ij}, z_j, and λ as a function of all product prices, variable factor prices, and total quantity available of the fixed factor.

It is not possible, however, to derive factor demand allocations from the dual specification of the same problem. This is because the partial derivative with respect to r_j of the constrained profit maximization problem is $-x_j^*$ rather than $-x_{ij}^*$:

(17) $$L^* = P_1 f_1[X_1^*(P,R,\bar{z}),\ z_1^*(P,R,\bar{z})] + P_2 f_2[X_2^*(P,R,\bar{z}),\ z_2^*(P,R,\bar{z})] - \sum_{i=1}^{2} \sum_{j=1}^{n} r_j x_{ij}^*(P,R,\bar{z}) + \lambda^*(P,R,\bar{z})\ [\bar{z} - \sum_{i=1}^{2} z_i^*(P,R,\bar{z})],$$

(18) $$\partial L^*/\partial r_1 = p_1(\sum_j f_{1j} \partial x_{1j}^*/\partial r_1 + f_{1z_1} \partial z_1^*/\partial r_1) + p_2(\sum_j f_{2j} \partial x_{2j}^*/\partial r_1 + f_{2z_2} \partial z_2^*/\partial r_1) - \sum_j r_j \sum_i \partial x_{ij}^*/\partial r_1 - \sum_i x_{i1}^* - \lambda \sum_i \partial z_i^*/\partial r_1 + (\bar{z} - \sum_i z_i^*) \partial \lambda^*/\partial r_1,$$

(18a) $$\partial L^*/\partial r_1 = \sum_j (p_1 f_{1j} - r_j) \partial x_{1j}^*/\partial r_1 + \sum_j (P_2 f_{2j} - r_j) \partial x_{2j}^*/\partial r_1 + \sum_i (P_1 f_{iz_i} - \lambda) \partial z_i^*/\partial r_1 + (\bar{z} - \sum_i z_i^*)\ \partial \lambda^*/\partial r_1 - \sum_i x_{i1}^*)$$

(18b) $$\partial L^*/\partial r_1 = - \sum_i x_{i1}^* \equiv - x_1^*$$

where $*(P,R,\bar{z})$ is the optimized value of the variable preceding it which is a function of all product prices, P, variable factor prices, R, and quantity available of the fixed factor, $\bar{z}$. The first three parenthetical terms in equation (18a) are presumed zero because of the first-order conditions for an interior profit-maximizing solution. The fourth is zero if the fixed factor is fully employed. This leaves the partial derivative of the Lagrangean equal to the total quantity of the factor used in all products.

What is true for the first factor is also true for each of the others. Further, demand functions for constrained allocatable factors cannot be recovered from the dual approach. With one exception neither can the correct variable factor demand allocation equations be recovered by taking the partial derivatives of the unconstrained profit function for each product. The exception is this: the first derivative of a single profit function with respect to a factor price is the demand function for the factor used in that particular product only if a price is attached to the allocatable fixed factor which coincides exactly with it's VMP. Then the fixed factor is treated the same as a variable factor and constrained optimization is unnecessary.

Since it is generally not possible to recover the factor allocation equations from the dual when allocatable fixed factors are present, a further problem occurs in dual modeling. There is no incentive to utilize available allocation data for estimation. It is true that the prices implicitly contain the allocation information in an efficient market. However, since random errors undoubtedly occur due to mistakes made in the allocation decision, useful information for econometric modeling may be discarded by this approach. Since inferences based on such models assume we make use of all relevant information, it is of considerable practical concern to simply discard available data on allocations. It is of some comfort that we don't have to have complete allocation information on all inputs to carry out econometric estimation, but that is true for primal as well as dual specifications. The fundamental concern is that we must say in principle that the allocations are of no value whatsoever when we model the multiple-product firm or industry that has allocatable fixed factors present. Since much attention in our mathematical programming and econometric models focuses on acreage allocations, this conclusion is unfortunate.

Conclusions

Because so many agricultural firms produce multiple products and operate subject to at least one allocatable fixed factor in the short to intermediate-run, jointness appears to be a much more pervasive problem in agriculture than previously supposed. Depending on the objective of the study, it may be important to try to discern the cause of jointness when it occurs since not all causes produce the same implications for modeling. The maximum information that can conceivably be extracted, the appropriate modeling approach (i.e., primal or dual), the selection of data, and the theoretical impact of aggregation and length of run are not independent of the cause of jointness.

When firms produce multiple products and allocatable fixed factors are present, jointness is especially likely to occur. Consequently, it is important to model production and the corresponding economic relations as part of a multiple-product system. Strong assumptions must be verified or departures assumed inconsequential to justify single-product analyses under such conditions. Only when firms produce a single product or it is apparent that little jointness of any type exists can firm-level production analysis of one product be safely conducted in isolation from other products. However, jointness is less likely to be a problem for long-run firm analyses than for short to intermediate-run regional studies.

Footnotes

1. See Nash for further elaboration of the modeling implications of allocatable fixed factors.

2. Jointness in output is another concept, but it is not the traditional one most economists associate with jointness. Since it is not directly relevant to the focus of this paper, the interested reader is refered to Lau (1972, p. 287). Only jointness in inputs is addressed here.

3. For a more comprehensive discussion of interdependent production processes in agriculture, see Heady, chapter 7.

4. The only exception occurs for multiple-product firms when changes in the production of one commodity causes a horizontal shift in the production function of another commodity.

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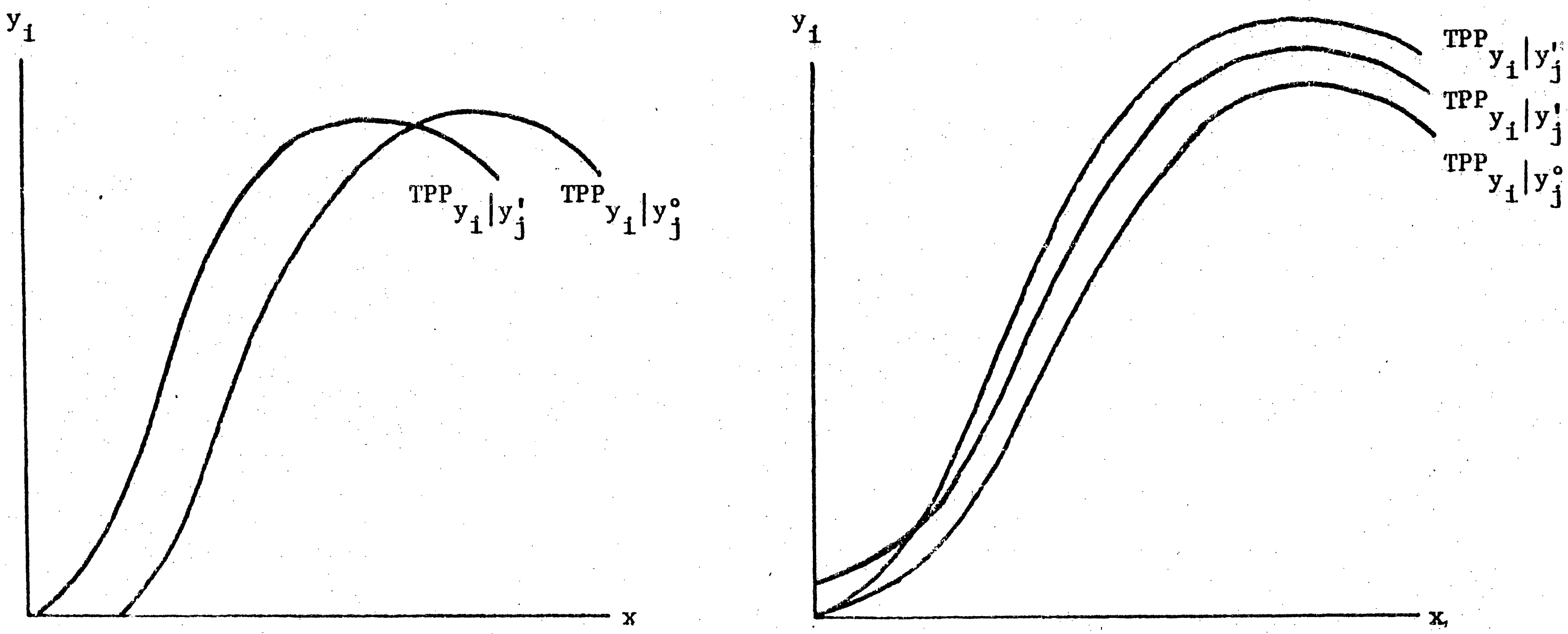

A. Nonjointness

B. Jointness

Figure 1. Nonjoint and Joint Technologies