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A Modified Armington Procedure

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-- A Modified Armington Procedure --

Abstract

This study attempts to develop the original Armington procedure into a multi-CES non-homothetic model. The results using rice trade data indicate that the new approach is superior to the original. Elasticities of substitution for rice imported from individual suppliers appear to vary, and demand for imported rice is not homothetic.

Key words: Armington procedure, trade, rice, multi-CES, non-homotheticity.

Introduction

The Armington procedure (AP) has been increasingly popular for analyzing agricultural trade. The procedure attempts to differentiate products from different suppliers (Armington). Grennes *et al.* applied the AP in their study of world grain trade in several commodities in 1978. Subsequently, Honma and Heady in 1984 employed the procedure for their wheat trade analysis, Babula in 1986 for U.S. exports of wheat, corn, and cotton, Figueroa and Webb in 1986 also for wheat and corn trade analyses, and Haniotis and Ames in 1988 for soybean imports to the EC. Despite its popularity, however, the assumptions of the AP are suspect. Thompson, while advocating the AP in his extensive research review, questions the single CES assumption, in particular. He notes that "there seems to be a logical inconsistency between assuming a commodity is differentiated by country of origin and then assuming the same parameters," (p. 44). None of the studies cited above tested whether or not imposing the assumption of a single CES is appropriate for agricultural trade analyses.

Further, Winters and Carter *et al.* attack another assumption in the AP, homotheticity. Winters and Carter *et al.*, using trade data of manufactured goods and wheat and cotton, respectively, compared the AP with the AIDS models and concluded that Armington's assumptions are not acceptable. In using an AIDS model for this type of trade analysis, however, multicollinearity may be a serious problem because the price of products in the same market tend to move together.[1] Ito *et al.* found that different importers have distinct preferences for rice from different sources so that shares varied as the budget allocated to imported rice

[1] Both Winters and Carter *et al.* did not report detailed results of their price coefficients.

changed. The purpose of this paper is to develop a modified Armington model that removes the single-CES and homotheticity assumptions and compare this modified version with a model based on the original procedure to determine whether or not the original assumptions are appropriate.

A Modified Armington Approach

Armington attempts to differentiate products from different suppliers in a market. He employs a two step procedure, assuming that, at the first stage, a "buyer" decides on the total volume to purchase, and at the second stage, allocates portions of the total volume to individual suppliers in order to minimize the costs. For the first stage equation, he specifies the total demand for both foreign and domestic products as the dependent variable. Assuming that a "buyer" maximizes utility, U, subject to available income, the problem is to:

$$(1)\quad \text{Max} \quad U = (Q_1, Q_2 \ldots\ldots Q_n), \quad \text{subject to} \quad Y = \Sigma Q_i P_i,$$

where, Q_i is the i-th good or market consisting of a group of products, P_i is a price index for the i-th market, and Y is income. Forming a Lagrangean equation and solving the first order conditions, a Marshallian demand function for Q_i can be derived:

$$(2)\quad Q_i = f(Y, P_1, P_2, \ldots\ldots P_n).$$

For the second stage equation, Armington makes two major assumptions: 1) the elasticity of substitution is constant (CES) regardless of the share of a product; 2) there is a single elasticity of substitution between any pair of products in the group. The two assumptions, which are together regarded as the single CES assumption, allow us to reduce the number of coefficients to be estimated and make the estimation process easier. Under these assumptions, Armington specifies the generalized CES form for Q_i:

(3) $Q_i = (\Sigma_j b_{ij} q_{ij}^{-\rho_i})^{-1/\rho_i}$, (j = 1, 2, ... n),

where $\Sigma_j b_{ij} = 1$, q_{ij} is a product from the j-th supplier to the i-th market, and ρ_i is a constant for the i-th market. Rewriting ρ_i as $(1/\sigma_i - 1)$, he derives a CES demand function for q_{ij}:[2]

(4) $q_{ij} = b_{ij}^{\sigma_i} Q_i (P_{ij}/P_i)^{-\sigma_i}$,

where, σ_i is the constant elasticity of substitution for the products in the i-th market, and P_{ij} is the price of q_{ij}. Equation(4) is expressed as a quantity dependent equation. To specify the equation as a market-share dependent equation, divide the both sides by Q_i:

(5) $q_{ij}/Q_i = b_{ij}^{\sigma_i} (P_{ij}/P_i)^{-\sigma_i}$,

Linearizing Equation(5) using the double-log form for time-series analysis gives:

(6) $\ln(q_{ij}/Q_i)_t = \sigma_i \ln b_{ij} - \sigma_i \ln(P_{ij}/P_i)_t$.

As written, Equation(6) includes the single CES assumption (only one CES for the entire market shared by all n suppliers) and the homotheticity assumption (no budget expenditure independent variable to explain the market-share).

In order to develop a multi-CES and non-homothetic demand function

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[2] See Armington (pp. 172-173) for the detailed mathematical derivation of Equation(4).

for q_{ij}, the q_{ij} in Equation(3) is replaced by m_{ij}, where m_{ij} is equal to q_{ij} to the power of γ_{ij}:

$$(7)\quad Q_i = (\Sigma_j b_{ij} m_{ij}^{-\rho_i})^{-1/\rho_i},$$

$$\text{where,}\quad m_{ij} = q_{ij}^{\gamma_{ij}}.$$

Following the same derivation process, a quantity dependent equation for m_{ij} can be expressed:

$$(8)\quad m_{ij} = b_{ij}^{\sigma_i}\, Q_i\, (P_{ij}/P_i)^{-\sigma_i}.$$

Replacing m_{ij} by $q_{ij}^{\gamma_{ij}}$, Equation(8) would be:

$$(9)\quad q_{ij}^{\gamma_{ij}} = b_{ij}^{\sigma_i}\, Q_i\, (P_{ij}/P_i)^{-\sigma_i},$$

therefore,

$$(10)\quad q_{ij} = b_{ij}^{\sigma_i/\gamma_{ij}}\, Q_i^{1/\gamma_{ij}}\, (P_{ij}/P_i)^{-\sigma_i/\gamma_{ij}}.$$

To specify the demand function as a market-share dependent equation, divide both sides by Q_i:

$$(11)\quad q_{ij}/Q_i = b_{ij}^{\sigma_i/\gamma_{ij}}\, Q_i^{(1/\gamma_{ij}-1)}\, (P_{ij}/P_i)^{-\sigma_i/\gamma_{ij}}.$$

In this equation, the market share is a function of total demand, Q_i, and the price ratio variable, P_{ij}/P_i. In the real world, the share of a specific product is not necessarily a function of total demand. If the quality of the product is inferior to the other products, its market share may decrease despite of its low price as buyers allocate more to the budget for that class of products and total demand grows. It is also possible that the share of the low quality product may increase because of its low price, if a buyer needs to purchase the same amount of the good with a reduced budget. Therefore, it is important to introduce a variable for the budget allocated to the good consisting of individual products in the market. Once a budget, V_i, for the products in the i-th market is determined, actual expenditures can be less than or equal to this budget allocation. V_i^{μ} is the amount actually spent on good i (P_i times Q_i), and μ is a factor that allows less than the total budget allocation to be spent. If the budget is fully spent, μ is equal to 1 otherwise smaller than 1; *i.e.* $0 \leq \mu \leq 1$. Accordingly, the relationship between V_i and Q_i is expressed as follows:

$$(12) \quad Q_i = V_i^{\mu}/P_i.$$

Replacing Q_i on the right-hand side in Equation(11) with V_i^{μ}/P_i in Equation(12):

$$(13) \quad q_{ij}/Q_i = b_{ij}^{\sigma_i/\gamma_{ij}} (V_i^{\mu}/P_i)^{(1/\gamma_{ij}-1)} (P_{ij}/P_i)^{-\sigma_i/\gamma_{ij}}.$$

$$= b_{ij}^{\sigma_i/\gamma_{ij}} V_i^{\mu(1/\gamma_{ij}-1)} P_{ij}^{-\sigma_i/\gamma_{ij}} (1/P_i)^{(-\sigma_i/\gamma_{ij} + 1/\gamma_{ij} -1)}$$

To simplify, Equation(13) is rewritten:

$$(14) \quad q_{ij}/Q_i = b_{ij}^{\alpha_{ij}} V_i^{\theta_{ij}} (P_{ij}/P_i)^{-\sigma_{ij}},$$

where, $\alpha_{ij} = \sigma_i/\gamma_{ij}$,

$\theta_{ij} = \mu(1/\gamma_{ij} - 1)$, and

σ_{ij} = nonlinear combination of $(-\sigma_i/\gamma_{ij})$ and $(-\sigma_i/\gamma_{ij} + 1/\gamma_{ij} -1)$.

Using the double-log form and adding a subscription for the time-series analysis, Equation(14) can be rewritten:

$$(15) \quad \ln(q_{ij}/Q_i)_t = \alpha_{ij}\ln b_{ij} + \theta_{ij}\ln(V_i)_t - \sigma_{ij}\ln(P_{ij}/P_i)_t.$$

To avoid money illusion, V_i is filtered by a price index, P^*, and Equation(15) is rewritten:

(16) $\ln(q_{ij}/Q_i)_t = \alpha_{ij}\ln b_{ij} + \beta_{ij}\ln(V_i/P^*)_t - \sigma_{ij}\ln(P_{ij}/P_i)_t$.

Equation(16) has to be specified for all suppliers to the i-th market; $\{j = 1, 2, ... n\}$. Because Equation(16) is expressed in the double-log form, the coefficients, β_{ij} and $(-)\sigma_{ij}$, are elasticities. The σ_{ij}, in particular, is regarded as a CES for the j-th supplier to the i-th market. Accordingly, the number of estimated CES is n, the number of suppliers.

The β_{ij}, on the other hand, is regarded as an elasticity of budget expenditure for products from the j-th country to the i-th market. Thus, there exist the same number of estimated β as the number of suppliers. If all β_{ij}'s are found to be not significantly different from zero, it is concluded that rice imports are homothetic and that imports of rice from a specific supplier are independent of the level of budget allocated to imported rice. If, on the other hand, at least one of β_{ij}'s is found to be different from zero, this implies that homotheticity may not hold. A positive estimated β_{ij} indicates that market share of the j-th supplier increases as the allocated budget in the i-th market (or importing region) increases. This can also be interpreted to mean that the importing region tends to consume more of the products from the j-th country at the expense of other suppliers' shares as the allocated budget to imports increases. The larger the absolute value of β, the more elastic the preference for the products in the importing region.

The analysis is for all the suppliers to a particular market in which the suppliers are competing with one another and behavior of a specific supplier is *not* independent of the behaviors of other suppliers in the market. Accordingly, the demand equations for all individual suppliers have to be estimated simultaneously in a system. The seemingly unrelated regression analysis originally developed by Zellner is appropriate.

Finally, it is necessary to test whether or not the multi-CES and non-homothetic approach is superior to the original AP. The system of equations in Model I, the original Armington single-CES and homotheticity assumptions with market-share dependent specification expressed in Equation(6), was compared with that of Model II, multi-CES and non-homotheticity assumptions specified in Equation(16). This is to jointly test whether or not a specification based on the multi-CES and non-homotheticity assumptions is statistically superior to a specification based on the single-CES and homotheticity assumptions. A test of a set of linear restrictions was performed using F-statistics estimated from the Lagrange multiplier (LM) (Judge *et al.*, 1982, pp. 326-328 and Judge *et al.*, 1985, pp. 472-477). The set of restrictions are expressed by:

(17) $\mathbf{R}\mathbf{B} - \mathbf{r} = \mathbf{0},$

where $\mathbf{R}$ and $\mathbf{r}$ are known matrices of dimensions (JxK) and (Jx1), respectively. The hypothesis is:

Ho: $\mathbf{R}\mathbf{B} - \mathbf{r} = \mathbf{0},$

Ha: $\mathbf{R}\mathbf{B} - \mathbf{r} \neq \mathbf{0}.$

If Ho is rejected, it is concluded that restrictions under the assumption of single-CES and homotheticity are inappropriate. The system of equations in Model I is such that restrictions on price coefficients (the single-CES estimate) and budget coefficients (the coefficients being equal to 0 under homotheticity) are imposed. On the other hand, the Model II system has no restrictions.

The Lagrange multiplier test is expressed as follows:

(18) $\lambda = A_J \,/\, B_{MT-K} \sim F_{J,MT-K}\,,$

where,

$$A_J = \{(y - X\hat{B}^*)'(\Sigma^{-1} \times I)(y - X\hat{B}^*) - (y - X\hat{B})'(\Sigma^{-1} \times I)(y - X\hat{B})\} / J,$$

$$B_{MT-K} = (y - X\hat{B})'(\Sigma^{-1} \times I)(y - X\hat{B}) / (MT - K),$$

$\hat{B}^*$ represents estimated coefficients under restrictions, and $\hat{B}$ represents the estimated coefficients under no restrictions. Σ is the covariance matrix, J is a number of restrictions, M is a number of equations in each system, K is a number of explanatory variables including the intercepts in the system with no restrictions, and T is the number of observations in each equation.

Data

Trade flow data on rice were collected from the *Commodity Trade Statistics* (United Nations) for twenty-five years, 1962-1986, based on calendar years. Imports by an individual country not only vary but are often zero in certain years. This makes econometric analysis more difficult. As a result, all the importing countries and regions were aggregated into one region. Seven exporting countries are identified: Thailand, the U.S., Argentina, Australia, Burma, Italy, and Pakistan.[3] Exporters that have not reported their data to the United Nations were excluded.[4] Shipments by the U.S. Government under concessional government-financed programs such as P.L. 480, foreign donations (Section 416), and AID mutual security programs are excluded, because actual prices for these types of shipments deviate considerably from the market prices. These U.S. Government shipment data were collected from *FATUS* (U.S.

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[3] To avoid the singularity problem in the SUR, the equation for Argentina, which is the smallest rice exporter among the seven nations, is deleted.

[4] The People's Republic of China, a major rice exporter, is excluded because of this problem.

Department of Agriculture). Export prices were calculated from the total export value divided by total quantity for each exporting country and all expressed in U.S. dollars. Data on budget allocated to imports are not available; therefore, X_i was approximated by total expenditure for imports of the products from individual suppliers. The consumer price index in the U.S. was employed as price index, P*, because X_i was expressed in U.S. dollars.

Empirical Results

The empirical results of Model I, based on the original Armington single-CES and homotheticity assumptions specified in Equation(6), and Model II, based on a multi-CES and non-homothetic approach specified in Equation(16), are reported in Table 1. While the estimated single-CES, $-\sigma_i$, for Model I was statistically significant, multi-CES for all individual suppliers in Model II were also significant. In addition, the estimated multi-CES appeared to be vary among the suppliers. The CES estimated for Pakistan was the largest at -1.726, and the one for Burma was the smallest at -0.984. The estimated coefficients for budget expenditures in Model II were significant for two countries; Burma and Pakistan. Interestingly, the budget expenditure coefficients were negative for Burma and positive for Pakistan.[5] This indicates that market shares of individual exporters are not always independent of the change in level of budget expenditures and that an assumption of homotheticity in the AP may be erroneous. Further, R-squares for equations in Model II are

[5] This may reflect the importer's preferences for quality related to different sources. Rice from Burma is generally considered to be inferior to rice from other exporters. On the other hand, rice from Pakistan is mainly aromatic rice called "basmati," and is more expensive than rice from the other exporters.

all greater than those in Model I.

It is necessary to perform the Lagrange multiplier test in order to determine whether or not Model II is superior to Model I. The Lagrange multiplier test resulted in an estimated F-value being equal to 3.451, which is greater than 2.18, the critical value at the 1% significance level for degrees of freedom of $v_1=12$ and $v_2=121$ to ∞ (Table 2). Accordingly, Armington's original assumptions of the single-CES and homotheticity are statistically rejected at 1% significance level. This indicates that the system of equations in Model II is statistically superior to those in Model I. These results suggest that Armington's original assumptions of the single-CES and homotheticity may not be appropriate for this particular market.

Conclusion

The Armington procedure is becoming more popular for agricultural trade analyses. However, the relevance of Armington's original assumptions are suspect. In this paper, Armington's original assumptions were tested against a modified version containing less restrictive assumptions. The empirical results show that the assumption of the single CES appear to be inconsistent with the data for world rice markets. In addition, homotheticity is not an appropriate assumption for this market. The results in this research are basically consistent with those found by Winters and Carter *et al*. However, these results do not imply that the Armington's basic concept should be totally rejected. Rather, they suggest that it may be useful to test the single-CES and homotheticity assumptions for the particular market being studied. If these assumptions are inappropriate, the Armington procedure can be modified as illustrated in this example. The modified version estimated for this study provides more information on the world rice market than the original Armington procedure and is a powerful method for analyzing agricultural trade.

Table 1. Comparison of different specifications among original and modified Armington procedures.[1]

	Original Armington procedure (Model I)		Modified Armington procedure (Model II)		
Assumption:	Single-CES and homotheticity		Multi-CES and non-homotheticity		
	R^2	$-\sigma_i$	R^2	$-\sigma_{ij}$	β_{ij}
Country:					
Thailand	0.210	-1.689	0.264	-1.586	-0.100
		(0.111)		(0.320)	(0.094)
U.S.	0.553	"	0.555	-1.519	0.082
		(")		(0.304)	(0.145)
Australia	0.546	"	0.576	-1.851	0.122
		(")		(0.340)	(0.185)
Burma	0.182	"	0.444	-0.984	-1.291
		(")		(0.489)	(0.386)
Italy	0.058	"	0.197	-1.284	0.388
		(")		(0.288)	(0.261)
Pakistan	0.624	"	0.724	-1.726	0.546
		(")		(0.301)	(0.229)

Standard errors are in parentheses.
Those " indicate being identical to the number above.
1: The seemingly unrelated regression (SUR) is used in each model.

Table 2. Results of hypothesis testing for Armington's original assumptions.

$\Sigma_j SSE_{ij}$		J	MT-K	F-value	$F_{1\%,12,\infty}$
Model I	Model II				
31.966	24.333	12	132	3.451	2.18

References

Armington, Paul S., "A Theory of Demand for Products Distinguished by Place of Production," *International Monetary Fund Staff Papers,* Vol. 16, No. 1, 1969.

Babula, Ronald A., *Development of a Multi-Region, Multi-Crop International Trade Sector: An Armington Approach Within a Macroeconomic Context,* Unpublished Ph.D. dissertation, Department of Agricultural Economics, Texas A&M University, College Station, 1986.

Carter, Colin A., Richard Green, and Daniel Pick, "Two-stage Agricultural Import Demand Models, Theory and Applications," unpublished paper, University of California, Davis, June 1988.

Figueroa, Enrique E., and Alan Webb, "An Analysis of the U.S. Grain Embargo Using a Quarterly Armington-Type Model," in *Embargoes, Surplus Disposal, and U.S. Agriculture,* Agricultural Economic Report Number 564, Economic Research Service, United States Department of Agriculture, 1986.

Grennes, Thomas, Paul R. Johnson, and Marie Thursby, *The Economics of World Grain Trade,* Praeger Publishers, New York, 1978.

Haniotis, Tassos and Glenn C. W. Ames, "The 'Oilseed Tax' and U.S. Soybean Exports to the Enlarged European Community," *European Review of Agricultural Economics,* pp. 39-54, Vol. 15, 1988.

Honma, Masayoshi, and Earl O. Heady, *An Econometric Model for International Wheat Trade: Exports, Imports, and Trade Flows,* CARD Report 124, The Center for Agricultural and Rural Development, Iowa State University, Ames, Iowa, 1984.

Ito, Shoichi, E. Wesley F. Peterson, and Dean T. Chen, "Analysis of Competitiveness Between U.S. and Thai Rice Exports," Texas Agricultural Experiment Station Technical Article No. 24174, Texas A&M University, College Station, Texas, 1988.

Judge, George G., R. Carter Hill, William E. Griffiths, Helmut Lutkepohl, and Tsoung-Chao Lee, *Introduction to the Theory and Practice of Econometrics,* John Wiley & Sons, New York, 1982.

Judge, George G., W. E. Griffiths, R. Carter Hill, Helmut Lutkepohl, and Tsoung-Chao Lee, *The Theory and Practice of Econometrics,* 2nd edition, John Wiley & Sons, New York, 1985.

Thompson, Robert L., *A Survey of Recent U.S. Developments in International Agricultural Trade Models,* Bibliographies and Literature of Agriculture Number 21, Economic Research Service, U.S. Department of Agriculture, 1981.

United Nations, *Commodity Trade Statistics,* several issues.

U.S. Department of Agriculture, *FATUS: Foreign Agricultural Trade of the United States,* several issues, Economic Research Service, USDA.

Winters, L. Alan, "Separability and the Specification of Foreign Trade Functions," *Journal of International Economics,* pp.239-263, Vol.17, 1984.

Zellner, A., "Estimaters of Seemingly Unrelated Regressions: Some Exact Finite Sample Results," *Journal of American Statistical Association,* Vol. 58, 1963, pp. 977-992.