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FARM PRICE DETERMINATION IN STRUCTURAL MODELS: IMPLICATIONS OF QUANTITY-DEPENDENT AND PRICE-DEPENDENT SPECIFICATIONS

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ABSTRACT

Alternative farm price specifications in a structural model are examined. Differential price impacts of various specifications are analyzed with respect to an exogenous shock. Four propositions are formulated to explain the price variations in relation to structural coefficients, price elasticities, and price flexibilities of the models. A wheat model is used for empirical evaluation of price response properties of different specifications using simultaneous solution methods: Gauss-Seidel for a price-dependent model and Newton (Newton-Ralphson) for three quantity-dependent models. Wheat price impacts induced by the 1988 drought generally conform with the propositions. The analytical framework and procedures are useful for comparing structural model performance and identifying inappropriate and unsatisfactory specifications.

KEYWORDS

Price determination specifications, structural models, wheat, supply shock.

INTRODUCTION

Structural econometric models have been criticized for lack of uniformity or standard in assessing price impacts of exogenous shocks. As models often generate substantially different prices and price impacts, the choice of an appropriate specification has crucial implications for agricultural price analysis and policy simulation. These differential impacts are largely due to variations in model structure and estimated parameters. This suggests the need of a theoretical basis for evaluating price response properties in various structural models. In applied econometric literature, important progress has been made concerning price determination behavior in a structural model: Working in the identification of supply-demand structure, Cowles Commission in simultaneous estimation methods (Haavelmo), and more recently, Wu-Hausman in specification tests regarding normalization procedure and consistency properties of price and quantity endogeneity relations (Thurman). However, the literature has seldom addressed the crucial question of the simulation capability of a model under alternative price specifications. A prior knowledge of price response behavior should help enhance structural model performance and selection of appropriate price determination specifications.

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Numerous alternative specifications have been developed for farm commodity price determination in structural models. The two major categories are quantity dependent and price dependent models. With each category containing several different specifications, numerous admissible models can be constructed. Selection of major types of specifications for simulation analysis should provide useful insight into the price response behavior of a farm commodity model. This is particularly true for large econometric models that emphasize intracommodity and intercommodity relationships, and that have important implications to farm commodity price analysis.

The primary purpose of this paper is to empirically examine the implications of alternative farm price specifications in structural models. A theoretical framework is proposed to evaluate the model performance under conditions of exogenous shocks. A nonlinear simultaneous model for wheat is tested under four different price specifications: quantity (Q)-dependent, price (P)-implicit domestic demand; Q-dependent, P-implicit export demand; Q-dependent, P-implicit stock demand; and P-dependent, P-explicit stock demand functions. A comparison of price impacts induced by 1988 drought are evaluated across these models that are identical except for the price determination equation, and the market clearance identity. Numerical solution methods used for simulation experiment include Gauss-Seidel for the P-dependent, P-explicit model and Newton (Newton-Ralphson) for Q-dependent, P-implicit models. This paper is organized as follows. In the next section we describe alternative specifications of Q-dependent and P-dependent models. Then, four major propositions relating differential price impacts to an exogenous shock are formulated. Theoretical aspects of model solutions are analyzed with respect to Newton and Gauss-Seidel methods. Then, assumptions and procedures of impact simulation of a wheat model, and empirical results of the shock induced by the 1988 drought, are examined. Finally, some important implications of the study are discussed. The concluding section offers a few recommendations for further research.

Price Determination in Structural Models

In formulating a structural model for farm commodity markets, one needs to specify and estimate a system of equations (*a structural model*) that captures all important demand, supply, and inventory stocks and their interrelationships. The basic form of components for such a simultaneous system (as described by Just for a particular commodity) consists of a system of seven equations: [1]

$$Q_h = Q_h(P; X_h) \tag{1}$$

$$Q_o = Q_o(P; X_o) \tag{2}$$

$$Q_x = Q_x(P; X_x) \tag{3}$$

$$Q_e = Q_e(P; X_e) \tag{4}$$

$$Q_d = Q_d(P; X_d) \tag{5}$$

$$Q_s = Q_s(\Pi; X_s) \tag{6}$$

$$Q_s + Q_{h-1} = Q_o + Q_e + Q_d + Q_x + Q_h \tag{7}$$

where Q_s refers to quantity supplied, and Q with subscripts o, e, d, x, and h refers to demands for food, feed, seed, exports, and inventory stocks, respectively. P, Π, and X refer to price, profit, and relevant exogenous variables, respectively. Equation 7 is the supply demand balance identity for market clearance.

In linear models, price can be explicitly determined by analytical reduced form equations[2]. However, with nonlinear models, the most common form of specifications, numerical methods need to be used to solve for price. Such numerical methods conform to the structure of the model which can be either (1) quantity-dependent (price-implicit), or (2) price-dependent (price-explicit).

Alternative Q-dependent and P-dependent Specifications

In Q-dependent models, all demand relationships are expressed in quantity-dependent form, i.e., with quantity on the left-hand side of the equation. In such models, price can be solved only by an implicitly defined

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[1] Just specified a six-equation system. We also include a demand function for feed.

[2] If the simultaneous equation system (1)-(7) is in linear form, it can be expressed in compact matrix notation as

$$BY_t + \Gamma X_t = U_t$$

where Y_t is a mx1 vector of endogenous variables, X_t is a nx1 vector of exogenous variables, B is an mxm matrix of coefficients of endogenous variables, Γ is a mxn matrix of coefficients of exogenous variables, U_t is a mx1 vector of stochastic errors, and t=1,2,...,T observations. Then the system can be analytically solved to obtain reduced form equations for price (and all other endogenous variables) in terms of exogenous variables as

$$Y_t = \Pi X_t + V_t$$

where, $\Pi = -B^{-1}\Gamma$, and $V_t = B^{-1}U_t$.

price equation in the simultaneous system. If price is implicitly determined by a specific demand function, for example the stock demand, we need to express it in price-implicit form as

$$P = P + Q_h(Q_h, P_h; X_h) \quad . \tag{8}$$

Thus, for the Q-dependent case it is possible to obtain different Q-dependent specifications by using various price-implicit demand functions. In the seven-equation system described in the previous section, at least three major demand functions can be used: Q-dependent domestic demand, Q-dependent export demand, and Q-dependent stock demand. Each of these demand functions can be expressed as a price implicit equation, to obtain three different structural model specifications.

In P-dependent models the approach is to normalize a certain demand function, i.e., an inverse demand, for price determination. Conditions under which such normalization could be theoretically justified are given by Fox, Heien, and Waugh. In empirical work, however, P-dependent models are generally viewed as direct transformations of Q-dependent structural models, without an explicitly defined theoretical foundation. Several studies have helped to develop theoretical justification of P-dependent specifications (Huang; Shonkwiler and Taylor), and to provide conditions for estimation of inverse demand functions (Houck; Anderson). Among the various demand functions that could be normalized on price, those most commonly used are (i) P-dependent stock demand, (ii) P-dependent export demand, and (iii) P-dependent domestic demand. In these models, price is explicitly determined by a selected inverse demand function in the system. Table 1 presents these six different specifications: three Q-dependent models and three P-dependent models.

On Differential Price Impacts: Some Propositions

Q-dependent specifications are considered "pure" structural models. However, the theoretical appeal of these models is often shadowed by the difficulties encountered in the solution process. Apart from modifying the model to include a "price adjustment" mechanism (Subotnik and Houck; Bailey), to solve the model in Q-dependent form numerical solution methods need to be invoked. When the model is price implicit, the Newton (Newton-Ralphson) method is the most widely used. The Newton method requires inversion of the coefficient matrix of the endogenous variables, i.e., inversion of the Jacobian (Chiang). This Jacobian consists of the structural coefficients of price of demand functions. Since structural coefficients of price are related to price

Table 1. Alternative P-dependent and Q-dependent Structural Models.

	Structural Model Specifications[1]					
	P-dependent Models			Q-dependent Models[2]		
	P-dependent Domestic Demand	P-dependent Export Demand	P-dependent Stock Demand	Q-dependent Domestic Demand	Export Demand	Q-dependent Stock Demand
Inventory Demand	$Q_h=Q_h(P;Z_h)$	$Q_h=Q_h(P;Z_h)$	$P=P(Q_h;Z_h)$	$Q_h=Q_h(P;Z_h)$	$Q_h=Q_h(P;Z_h)$	$P=P+Q_h(P;Z_h)$
Domestic Demand	$P=P(Q_o;Z_o)$	$Q_o=Q_o(P;Z_o)$	$Q_o=Q_o(P;Z_o)$	$P=P+Q_o(P;Z_o)$	$Q_o=Q_o(P;Z_o)$	$Q_o=Q_o(P;Z_o)$
Export Demand	$Q_x=Q_x(P;Z_x)$	$P=P(Q_x;Z_x)$	$Q_x=Q_x(P;Z_x)$	$Q_x=Q_x(P;Z_x)$	$P=P+Q_x(P;Z_x)$	$Q_x=Q_x(P;Z_x)$
Feed Demand	$Q_e=Q_e(P;Z_e)$	$Q_e=Q_e(P;Z_e)$	$Q_e=Q_e(P;Z_e)$	$Q_e=Q_e(P;Z_e)$	$Q_e=Q_e(P;Z_e)$	$Q_e=Q_e(P;Z_e)$
Seed Demand	$Q_d=Q_d(P;Z_d)$	$Q_d=Q_d(P;Z_d)$	$Q_d=Q_d(P;Z_d)$	$Q_d=Q_d(P;Z_d)$	$Q_d=Q_d(P;Z_d)$	$Q_d=Q_d(P;Z_d)$
Supply	$Q_s=Q_s(\Pi;Z_s)$	$Q_s=Q_s(\Pi;Z_s)$	$Q_s=Q_s(\Pi;Z_s)$	$Q_s=Q_s(\Pi;Z_s)$	$Q_s=Q_s(\Pi;Z_s)$	$Q_s=Q_s(\Pi;Z_s)$
Market Clearing Identity	$Q_o=Q_{h-1}+Q_s -Q_e-Q_d-Q_x-Q_h$	$Q_x=Q_{h-1}+Q_s -Q_e-Q_d-Q_x-Q_h$	$Q_h=Q_{h-1}+Q_s -Q_e-Q_d-Q_x-Q_h$	$Q_o=Q_{h-1}+Q_s -Q_e-Q_d-Q_x-Q_h$	$Q_x=Q_{h-1}+Q_s -Q_e-Q_d-Q_o-Q_h$	$Q_h=Q_{h-1}+Q_s -Q_e-Q_d-Q_o-Q_x$

1. Q_s refers to quantity supplied, and Q with subscripts o,e,d,x and h refers to demands for domestic consumption, feed, seed, export, and inventory stocks respectively. P and Π refer to farm price of wheat, and profits from wheat respectively. Z refers to the vector of relevant exogenous variables identified by the subscripts described above.

2. Q-dependent, P-implicit stock demand model is solved by solving for implicit prices using Newton method. The computer code for specifying an implicit solution for prices using Q-dependent stock demand is $P=P+Q_h(Q_h;Z_h)$ (SAS/ETS, p. 51).

elasticities by a weight factor of relevant P and Q, this implies that price elasticities of *all* demand functions have a direct impact on the price outcome. Thus, we formulate the first proposition on the price impact of an exogenous shock.

Proposition One. *In Q-dependent models, the magnitude of price impact of an exogenous shock is determined by the summation of structural coefficients of price of the demand functions, and hence the price elasticities of demands in the model. This implies that the higher (lower) the summation of structural coefficients of the demand functions, the lower (higher) the price impact.*

In P-dependent models, impacts of an exogenous shock on price may vary depending on the demand function normalized. Price impact is thus directly determined by the inverse demand function, and indirectly by the other demand functions in the model. Therefore, this suggests the second proposition.

Proposition Two: *In P-dependent models the magnitude of price impact of an exogenous shock is determined directly by the structural coefficient of quantity, and hence the price flexibility of the inverse demand function in the model; and indirectly by the price elasticities of the other demand functions in the system. This implies that the higher (lower) the price flexibility, the higher (lower) the price impact; and the higher (lower) the price elasticities of other demand functions, then the higher (lower) the offsetting effect on price.*

In application for price analysis, the differential effects of an exogenous shock between Q-dependent and P-dependent models can be anticipated a priori by the knowledge of structural coefficients (price elasticities and price flexibilities). As magnitude of price elasticities and flexibilities are crucial indicators of solution outcomes, we formulate two additional propositions for intermodel comparison of price impact.

Proposition Three: *When the summation of structural coefficients of price of demand functions (the value of the inverse Jacobian) in a Q-dependent model is lower (higher) than the value of the structural coefficient of quantity of the inverse demand (price flexibility) in a P-dependent model, then the price impact in the Q-dependent model is lower (higher) as compared to the P-dependent model.*

In Q-dependent models, price is determined implicitly in the simultaneous system. Regardless of the demand function chosen for price determination, the value of the Jacobian is the same across models. However, in P-dependent cases, different solutions are generated from different inverse demand functions. This distinction is the basis for the final proposition.

Proposition Four: *In Q-dependent models, the price impact of an exogenous shock is invariant to the selection of the specific demand function as the implicit price equation. In P-dependent models, price impacts vary across models, depending on the specific demand function normalized on price.*

These four propositions provide a theoretical framework for empirical evaluation of alternative structural model performance regarding the price impact of an exogenous shock.

Model Solutions: Theoretical Considerations

In the previous section we formulated four propositions with reference to solution outcomes from Q-dependent and P-dependent models. The distinction between these specifications is not only on the model structure, but also the model responses to an exogenous shock in relation to the estimated structural coefficients, price elasticities, and flexibilities of the model. In this section, model solutions are analyzed with respect to the choice of numerical methods, i.e., Newton, and Gauss-Seidel solution algorithms.

Solution of Q-dependent Models (Newton method)

In Q-dependent models, price is implicitly determined in the supply/demand framework to satisfy market equilibrium conditions. This structural model can be represented as

$$F(Y, X; \theta) = 0 \tag{9}$$

where F is a differentiable vector-function of endogenous variables Y, exogenous variables X, and estimated parameters θ (Drud). The Newton method uses a derivative-based iterative procedure to generate $(n+1)^{th}$ solution from $(n)^{th}$ solution using

$$Y_{n+1} = Y_n - F(Y, X; \theta)/F'(Y_n, X; \theta). \tag{10}$$

Consider the solution for price using stock demand as the implicit price equation in the model. Then (10) can be respecified as,

$$P_{(h),(n+1)} = P_{(h),(n)} - F(Q_h, \hat{P}_{(h)}\ ; \theta)/F'(Q_h, \hat{P}_{(h)}\ ; \theta) \tag{11}$$

where subscript h and n are for stock demand and number of iterations, respectively.

If we expand the denominator of the second term on the right hand side of (11), it is the matrix of structural coefficients of endogenous variables, i.e. , the Jacobian, of the model.

$$J = \begin{bmatrix} \frac{\partial f_1(.)}{\partial Y_1} & \frac{\partial f_1(.)}{\partial Y_2}, \ldots, & \frac{\partial f_1(.)}{\partial Y_j} & \frac{\partial f_1(.)}{\partial Y_m} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial f_m(.)}{\partial Y_1} & \frac{\partial f_m(.)}{\partial Y_2}, \ldots, & \frac{\partial f_m(.)}{\partial Y_j} & \frac{\partial f_m(.)}{\partial Y_m} \end{bmatrix} . \tag{12}$$

For a particular demand function in the model, price and quantity usually are the only endogenous variables in the function. Thus, the Jacobian consists basically of a column vector of 1's, and a vector of structural coefficients of price. Inversion of this Jacobian thus amounts to the inversion of the summation of the structural coefficients of price.

If we isolate the structural coefficient vector of price in the Jacobian, it can be given as a column vector:

$$J_{(h)} = \begin{bmatrix} \frac{\partial f_1(.)}{\partial P_{(h)}} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial f_{(m)}(.)}{\partial P_{(h)}} \end{bmatrix} . \tag{13}$$

These structural coefficients, if weighted by relevant P and Q terms, are price elasticities of demand functions. For a particular demand function, price elasticity is given as

$$\xi = \frac{\partial f_i(.)}{\partial P_{(h)}} \frac{P}{Q_i} . \tag{14}$$

Equation (14) implies that the elasticities are nondecreasing functions of structural coefficients of price. This supports proposition one, that price impacts in Q-dependent models are directly determined by the summation of the structural coefficients, and hence the price elasticities of the demand functions.

Solution of P-dependent Models (Gauss-Seidel method)

In P-dependent models, price is explicitly expressed as an inverse demand function in the model. To illustrate the solution process in the transmission of an exogenous shock, we use a diagram to trace the price effect through the simultaneous system in a P-dependent stock demand model (Figure 1).

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
Y\uparrow \arrow[r] & Q_s\uparrow \arrow[r, "\text{Market Clearance}"] & Q_h\uparrow \arrow[r, "{P = f(Q_h),\ f'<0}"] & P\downarrow\ (\Delta P_{(1)}) \arrow[r, "{Q_j = g_j(P)\ ;\ j = \text{x,o,d,and f.}\ \ g'_j<0}"] & Q_j\uparrow \arrow[d, "\text{Market Clearance.}"] \\
 & & Q_j\uparrow \arrow[u, "\text{Market Clearance}"'] & \uparrow P\ (\Delta P_{(2)}) \arrow[l, "{Q_j = g_j(P),\ g'_j<0}"] & Q_h\downarrow \arrow[l, "{P = f(Q_h),\ f'<0}"]
\end{tikzcd}
$$

Figure 1. Supply Shock Transmission in a P-dependent Stock Demand Model.

The initial effect of an exogenous shock, e.g. , an increase in yield per acre (Y), increases the quantity of supply (Q_s), and through market clearance, increases stocks (Q_h). The direct impact of this stock increase is an initial decrease in price, $\Delta P_{(1)}$, because of the negative coefficient of quantity of stock ($f' < 0$) in the inverse demand function, $P = f(Q_h)$. This price impact is related to price flexibility ($\hat{\eta}$) of the inverse demand function as

$$\hat{\eta} = \frac{\partial f_i(.)}{\partial Q_{(I)}} \frac{Q_I}{P} \quad . \tag{15}$$

Equation (15) is reciprocal to equation (14) regarding the P and Q relationship. Price flexibility is also a nondecreasing function of the structural coefficient. Thus, as stated in proposition two the initial price impacts are directly related to the price flexibility of the inverse demand function in the model.

This initial price reduction is transmitted through the system, inducing quantity increases in all the other demand functions, $Q_j = g_j(P)$, where j = x, o, d, and f, because of the negative price coefficient ($g'_j < 0$). The effect of these quantity-of-demand increases, in turn, causes a stock reduction through market clearance adjustment. This constitutes a feedback relationship resulting in a second-round price increase ($\Delta P_{(2)}$) through the same inverse demand function. The second-round price increase is a function of the structural coefficients of price. This suggests that the larger the structural coefficients (price elasticities), the larger the reduction in stocks, and hence the greater the second-round price increase. The overall price impact of the supply shock is the net result of the iteration process, until convergency is achieved. The convergency condition needs to satisfy

$$|(P_{n+1} - P_n)/P_n| < \alpha \tag{16}$$

where α is a small positive number (Heien, Matthews, and Womack).

Comparison of Solution Outcomes

In P-dependent models the structural coefficient of the inverse demand equation sets the upper bound on the price impact, while the inverse Jacobian sets the upper bound in the Q-dependent model. If the inverse Jacobian is less than the value of the structural coefficient, lower price impacts are expected in Q-dependent models than in P-dependent models, as stated in proposition three.

Q-dependent models, regardless of the implicit price function specified, are expected to converge to the same solution. For example, consider a domestic demand specification. Then equation (11) becomes

$$P_{(o),(n+1)} = P_{(o),(n)} - F(Q_o, \hat{P}_{(o)}\ ; \theta)/F'(Q_o, \hat{P}_{(o)}\ ; \theta)\ . \tag{17}$$

In the solution process, the initial value of price in specifications (11) and (17) may differ. However, when iteration stops at, say, p^{th} iteration for stock demand and q^{th} iteration for domestic demand, this implies

$$F(Q_o, \hat{P}_{(o)}\ ; \theta) \approx F(Q_h, \hat{P}_{(h)}\ ; \theta) \approx 0\ . \tag{18}$$

Therefore, when the models have converged the following conditions should hold:

$$P_{(h),p} = P \pm \delta \tag{19}$$

and

$$P_{(o),q} = P \pm \delta \tag{20}$$

where δ is a small number which is the convergence error. As $P \in \Re^n_+$, then

$$P_{(h),p} \approx P_{(o),q} \qquad (21)$$

where $\Re^n_+$ refers to the positive real number.

Thus, for all practical purposes, Q-dependent (P-implicit) stock demand and Q-dependent (P-implicit) domestic demand (or any other Q-dependent (P-implicit) specifications) generate almost the same solution value for price. On the other hand, P-dependent models may generate different solutions for price depending on the type of inverse demand function used (proposition four).

Empirical Results

To provide empirical support for the propositions formed, a complete sectoral model for wheat was used. The model for simulation study consists of seven equations (equations 1-7). For the purpose of comparative analysis of Q-dependent and P-dependent models, only four of the six specifications described in the previous section were chosen. All three Q-dependent specifications are needed for inter-model comparison of price implicit specifications. For simplicity, only one of the three P-dependent models (a more commonly used P-dependent stock demand specification) was chosen (Adams and Behrman; Chen; Meilke and Young; Gardner). The estimated models are presented in Table 2. The models were estimated using annual data from 1973 to 1987. Specifications and estimated parameters of the models are generally consistent with those of previous modeling studies. Estimated price elasticities and price flexibilities are also similar to earlier work. Statistical results of all the demand functions in the models show a good fit, with high R^2, and expected signs across different specifications. Most of the estimated coefficients have statistically significant t-values at the 95% level.

1988 Drought Impacts On Wheat

Due to adverse weather conditions in 1988, there was a significant decline in yield per acre for wheat. In conducting our simulation experiment, 1988 wheat yield per acre was shocked by using a normal weather condition of trend yield against the actual[3]. To derive the normal weather projection, a trend analysis of wheat yield was performed. The projected wheat yield of 38.04 bushels per acre was used to simulate the weather

[3] The estimated trend function for wheat yield per acre with t-value in parentheses is

$$Y_a = -1299.22 + 0.673\ \text{TREND}$$
$$(-5.894) \quad (5.744)$$
$$R^2 = 0.73 \qquad \text{R bar Sq.} = 0.71 \qquad \text{D.W.} = 1.62$$

Table 2. Estimated Wheat Models: Alternative Q-dependent and P-dependent Specifications.

Supply and Demand Equations

Inventory Demand
$$Q_h = -801.94\,P - 517.88\,(Q_h / Q_t)^e - 2631.32$$
(3.79) (0.78) (4.0)

R. Sq. = 0.83 R. bar Sq = 0.80 D.W. = 1.34

Domestic Demand
$$Q_o = -14.63\,P + 0.45\,I - 280.00$$
(1.33) (6.12) (3.81)

R. Sq. = 0.94 R. bar Sq. = 0.92 D.W. = 1.36

Export Demand
$$Q_x = -103.98\,P + 0.76\,Q_{x,-1} + 267.19\,X + 1.75\,P^w + 92.06$$
(1.25) (6.95) (0.54) (1.59) (0.24)

R. Sq. = 0.80 R. bar Sq. = 0.77 D.W. 2.14

Seed Demand
$$Q_d = 3.53\,P + 1.33\,A - 12.17$$
(1.47) (3.56) 0.71)

R. Sq. = 0.51 R. bar Sq. 0.43 D.W. = 1.5

Supply
$$Q_s = Y \times A$$

Price Equation and Market Clearance Identity*

Q-dependent P-implicit Stock Demand Model

Price
$$P = P + (-Q_h - 801.94\,P - 517.88\,(Q_h / Q_t)^e - 2631.32)$$

Market Clearance
$$Q_h = Q_{h-1} + Q_s - Q_o - Q_d - Q_x$$

Q-dependent P-implicit Domestic Demand Model

Price
$$P = P + (-Q_o - 14.63\,P + 0.45\,Y - 280.00)$$

Market Clearance
$$Q_o = Q_{h-1} + Q_s - Q_d - Q_x - Q_h$$

Q-dependent P-implicit Export Demand Model

Price
$$P = P + (-Q_x - 103.92\,P + 0.76\,Q_{x,-1} + 267.19\,X + 1.75\,P^w + 92.06)$$

Market Clearance
$$Q_x = Q_{h-1} + Q_s - Q_d - Q_o - Q_h$$

P-dependent P-explicit Stock Demand Model

Price
$$P = -0.00068\,Q_h - 1.16\,(Q_h / Q_t)^e + 3.16$$
(3.78) (4.08) (21.93)

R. Sq. = 0.92 R. bar Sq. = 0.91 D.W. = 1.79

Market Clearance
$$Q_h = Q_{h-1} + Q_s - Q_o - Q_d - Q_x$$

t - statistic is given in parenthesis. D.W. is the Durbin-Watson Statistic.
P = farm wheat price (deflated), $(Q_h / Q_t)^e$ = expected stock/demand ratio, total demand $Q_t = (Q_o + Q_e + Q_d + Q_x)$, I = disposable income (deflated), $Q_{x,-1}$ = lagged exports, X = exchange rate, P^w = world wheat price, A = acreage, and Y = yield per acre.
* Price in Q-dependent models are implicit equations derived from the corresponding estimated demand functions given above. Price in P-dependent model is directly estimated.

impact against the actual. The supply shock induced by an increase in yield per acre (from 34.1 bushels peracre to 38.04 bushels per acre) affects price in each model (Table 3). Total wheat production is projected to increase by 210 mil. bu. This increase generates a price reduction (due to the supply increase) of 47 cents in the P-dependent stock demand model, 64 cents in the Q-dependent domestic demand model, 60 cents in the Q-dependent export demand model, and 62 cents in the Q-dependent stock demand model.

Table 3. Impact Analysis of 1988 Drought on Wheat.

	Q-dependent Domestic Demand	Q-dependent Export Demand	Q-dependent Stock Demand	P-dependent Stock Demand
Supply Shock Assumption				
Yield per acre (bushels)				
Actual (Drought)	34.10	34.10	34.10	34.10
Normal (Trend)	38.04	38.04	38.04	38.04
Variable				
Production (mil. bu)				
Actual	1812	1812	1812	1812
Impact	210	210	210	210
Wheat Farm Price ($/bu)				
Actual	3.72	3.72	3.72	3.72
Impact	-0.64	-0.60	-0.62	-0.47

Impact = Change from baseline.

Comparison of Price Impacts

Based upon the analytical framework and the propositions formulated, we empirically evaluate price impacts of an exogenous shock with respect to the Jacobian in the Q-dependent model, and the structural coefficients in the P-dependent model. Using the estimated structural coefficients of demand functions in Table 2, we obtain the Jacobian by taking the derivative of each equation with respect to each endogenous variable. However, feed demand was excluded as it does not represent a major demand component for wheat. The order in which derivatives are taken is P, Q_h, Q_o, Q_x, and Q_d. The Jacobian (J) is

$$J = \begin{bmatrix} -801.94 & -1 & 0 & 0 & 0 \\ -14.63 & 0 & -1 & 0 & 0 \\ -103.98 & 0 & 0 & -1 & 0 \\ 3.53 & 0 & 0 & 0 & -1 \\ 0 & -1 & -1 & -1 & -1 \end{bmatrix} . \tag{22}$$

To obtain the price impact of increased yield per acre (Y), the solution of a Q-dependent model can be derived by taking the total differential of the equation system

$$\begin{bmatrix} -801.94 & -1 & 0 & 0 & 0 \\ -14.63 & 0 & -1 & 0 & 0 \\ -103.98 & 0 & 0 & -1 & 0 \\ 3.53 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} dP/dY \\ dQ_h/dY \\ dQ_o/dY \\ dQ_x/dY \\ dQ_d/dY \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ A \times dY \end{bmatrix} \tag{23}$$

where dY is the assumption on yield increase, and A, the acreage, is constant.

Using Cramer's rule, dP/dY could be solved as

$$dP/dY = |K|/|J| \tag{24}$$

where $|K|$ is the determinant of the matrix produced by substituting column 1 of the Jacobian by the column vector on the right-hand side of (22), and $|J|$ is the determinant of the Jacobian.

The value of $|J|$ can be expressed as the summation of the elements in column 1 of J, i.e., the summation of price coefficients of all demand functions. Also, it is clear that regardless of the specific demand function used as the implicit equation, $|J|$ remains unchanged. The value of $|K|$ is equal to A x dY.

Equation (24) yields

$$dP/dY = - A \times dY / (801.94 + 14.63 + 103.98 - 3.53)$$

$$= - (0.0011) \times (A \times dY) . \tag{25}$$

The first term, - 0.0011 in equation (25) is the inverse of the summation of the structural coefficients of the demand functions. Thus, price impact of an exogenous shock induced by the increased supply is determined by structural coefficients of price in the model. Obviously, the price impact should be the same for all Q-dependent specifications regardless of the selection of the implicit price equation.

In the P-dependent case, the estimated inverse demand equation used for analysis is

$$P = 0.00068\, Q_h - 1.66\, (Q_h/Q_T)^e + 3.16 \ . \tag{26}$$

In determining price impact of a supply shock, wheat yield (Y) as well as quantity of supply (Q_s) is first increased by A x dY. This effect is transmitted through the market clearance identity to increase stocks (Q_h) by the same amount. Thus, the initial price impact is directly determined by the P-dependent stock demand function as

$$dP/dY = (\partial P / \partial Q_h) \times dQ_h \ . \tag{27}$$

As dQ_h is equal to (A x dY), then

$$dP/dY = - 0.00068 \times (A \times dY) \ . \tag{28}$$

The initial price impact sets an upper bound by equation (28), as described by proposition two. The final price outcome is determined by the iteration process in which the effect of the initial price decline increases quantities of other demands and reduces inventory stocks. This iteration can be viewed as a process generating the initial price decline by $\Delta P_{(1)}$, and subsequent price increases by $\Delta P_{(2)}$ due to a reduction in Q_h. The offsetting price increase is dependent upon the price elasticities of the demand functions. The iteration process continues until the system converges, given the following inequality condition[4].

[4] If the price impact of the $(n+1)^{th}$ iteration is greater than the impact of the n^{th} iteration, then such an iterative process will not converge. Setting an iteration limit will result in a large positive number of a large negative number. On the other hand, if iterations are all of the same magnitude, but alternate in signs, then such a model would not converge either (see Thomas and Finney, 7th ed., p. 157).

$$|\Delta P_{(n+1)}| < |\Delta P_{(n)}| \ . \tag{29}$$

Absolute price change of the $n+1^{th}$ iteration has to be less than the absolute change of the n^{th} iteration.

Under convergency condition, absolute value of price decreases are greater than the absolute value of the offsetting price increases, due to a supply increase. Therefore, the final price impact is directly related to the price flexibility of the inverse demand function, and indirectly related to the price elasticities of the other demands in the model.

The results in Table 4 support the proposition three that when P-dependent demand has a larger structural coefficient than the inverse Jacobian of the Q-dependent model, then the price impact is also larger. For example, P-dependent export demand has the smallest structural coefficient of - 0.0005, the smallest price flexibility of - 0.42, and hence has the smallest price impact of 0.29. When the value of the structural coefficient increases, so do the price flexibility and the price impact. The estimated coefficient of P-dependent stock demand is - 0.00068. Therefore, the price flexibility and price impact are larger than the P-dependent export demand model, at - 0.50, and - 0.47, respectively. P-dependent domestic demand has the largest structural coefficient of - 0.0113, and the largest price flexibility of - 4.56. As a result this model show the largest price impact of a supply shock, at $1.40 per bushel.

Table 4. The Relationship Among Structural Coefficients, Price Flexibilities, and Price Impacts.

Demand Specification	Structural Coefficient	Price Flexibility	Final Price Impact
P-dependent export demand	- 0.0005	-0.42	- 0.29
P-dependent stock demand	- 0.00068	-0.50	- 0.47
P-dependent domestic demand	- 0.0113	-4.56	- 1.40
Q-dependent stock demand	- 0.0011*	-	- 0.62

* Inverse Jacobian.

On the other hand, the inverse Jacobian of the Q-dependent stock demand is - 0.0011, and the final price impact is 0.62. A comparison of these four different specifications suggests that the price impact of the Q-dependent stock demand model is greater than the P-dependent export demand, P-dependent stock demand, but smaller than the P-dependent domestic demand. This supports proposition three regarding the price impact in relation to structural coefficients, and the inverse Jacobian.

Some Implications

The inferences drawn in this study have important implications for modeling farm commodity markets in general, and agricultural price analysis in particular. Simulation results indicate considerable discrepancies in price impacts among Q-dependent and P-dependent models under an exogenous shock. The study illustrates how structural coefficients, and hence price elasticities and price flexibilities of demand functions affect the solution outcome in different models. The findings are particularly applicable for identifying appropriate and inappropriate structural model specifications. The analytical framework developed is useful for evaluating price response behavior in structural models for policy simulation.

In practical applications, estimated price elasticities and price flexibilities from single equation models are more commonly used than impact simulation of structural models. The justification for single equation approach is often based on the assumption of invariability or inelastic nature of agricultural production in the short-run. There are three major drawbacks to this: first, inelasticity assumption fails to reflect the dynamic nature of agriculture production adjustment; second, the effect of changes in supply through inventory stock adjustment is not accounted for; third, the significant influence of the changes in other demand components are omitted. The limitation of this approach of price analysis is particularly evident in the study. The price response generated by an inverse demand equation measures only the initial price impact as described in equation (28). On the other hand, the final price impact of the structural model depends not only on the price flexibility of the demand function, but also on price elasticities and changes in the quantities of other demands. A clear implication is that the single equation approach does not suffice for price analysis. Such an approach is likely to overestimate price impacts and underestimate demand responses, leading to seriously biased conditions in impact simulations.

For modeling farm commodity markets, probably the most important implication of this study is on a prior information to identify appropriate and inappropriate specifications. Price response behavior across different specifications suggests a set of conditions can be obtained to evaluate structural model performance. How can this be achieved? In general it is possible to perceive a reasonable price range in response to an exogenous shock. Such a range may be obtained by using the expert knowledge of commodity analysts and policy researchers [5]. This generally can be a useful guideline in selection of structural model specifications. For example, if a reasonable price range is defined as $\Delta R = (\Delta P_{max} - \Delta P_{min})$, ΔP_{max} and ΔP_{min} are the maximum and minimum values of price impacts, respectively.

For Q-dependent models, estimated structural coefficients may be utilized to calculate the initial price using equation (25). For P-dependent models, on the other hand, using the estimated structural coefficient of the inverse demand equation, initial impact can also be calculated by equation (28). If,

$$\Delta P_{(i)} \leq \Delta P_{min} \tag{30}$$

where subscript i identifies the initial impact, the model may be considered as inappropriate or unsatisfactory. This is because as the initial impact imposes an upper bound, the final price outcome must fall outside the reasonable range of ΔR. In the Q-dependent case, $\Delta P_{(i)}$ is derived from the inverse Jacobian of all the demand functions in the system, while in the P-dependent case, it is calculated by the structural coefficient of a specific inverse demand equation. Taking into account the effect of the feedback, final price impact would be even lower than the initial impact. Therefore, in P-dependent models the inverse demand specification may be considered inappropriate, while in a Q-dependent model the overall structure of demand functions is not adequate.

On the other hand, if the initial price impact of an exogenous shock is greater than the maximum value of the reasonable range

$$\Delta P_{(i)} > \Delta P_{max} \tag{31}$$

there is the possibility that the final price outcome may be within the range, depending upon the magnitude of the offsetting price effect. This offsetting effect, which in the Q-dependent case represents iterative solution outcomes until convergency, and reflects induced changes of other demand components in response to initial

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[5] A survey of expert opinion from Food and Agriculture Policy Research Institute (FAPRI), and Wharton Econometric Forecasting Associates (WEFA) indicated a 210 mil. bu. increase of wheat production would lead to a 40-46 cents decrease in wheat price.

price shock in P-dependent models.

Implications of the study reach beyond the specification and solution issues, and extend to the estimation aspects of the demand functions. A priori knowledge of the final price impact provides useful guidelines in evaluating the magnitude of structural coefficients. For example, let us consider the P-dependent case. If the price impact is lower than the desired level, it may be the result of a smaller than the desired structural coefficient of the inverse demand, or larger than the desired structural coefficients of price in other demand functions. On the other hand, in Q-dependent models this may be due to relatively elastic demands in the model. In both cases respecification and reestimation of the demand functions can be explored to improve the price response behavior of the models.

In the search for a suitable structural model specification, an important consideration is a priori information on price response to an exogenous shock. An evaluation of the estimated parameters should help to anticipate the price response behavior of the model. The selection of P-dependent and Q-dependent specifications has long been a controversial methodological issue in farm commodity modeling work. Implications of this study suggest that if the price impact falls within a reasonable range, the decision on P-dependent or Q-dependent specifications is a matter of choice. However, for P-dependent models there are conditions for price dependency to be satisfied (Fox; Heien).

Another implication is on the sensitivity of the model to structural changes. The single equation approach fails to account for structural changes of other demand functions in the model. In the structural model framework, on the other hand, the effect of structural changes of any demand function must be fully reflected in the solution process for price determination. For example, in a P-dependent stock demand model, the effect of structural changes in export demand and domestic demand can be transmitted through the changes in their price elasticities. The same type of effect can be directly traced in a Q-dependent model from the changes in the inverse Jacobian of the model. Since single equation models have no such interdependent relationships, it is evident the model is not capable of accounting for impacts of changes of other demand components.

Conclusions

This paper examines alternative structural model specifications in farm price determination. Price may either be explicitly determined in price-dependent models, or implicitly determined in quantity-dependent models. Simulation of simultaneous equation systems reveals considerable price discrepancies among models. In P-dependent models, inverse demand function has a direct impact, while in Q-dependent models the overall demand structure plays a critical role. Price differentials between Q-dependent and P-dependent models can be analyzed with respect to price flexibility of the inverse demand function, and the summation of price elasticities of the Q-dependent models. Based on these findings, useful guidelines can be developed to evaluate price response behavior of structural models.

Our knowledge of farm price determination can be greatly expanded by simulation studies of other P-dependent specifications, especially domestic demand, which is inelastic, and export demand, which is elastic. The impact analysis of the weather-induced production cutback in 1988 provides useful information regarding structural model response to an external shock. Further study needs to address internal shocks induced by changes in export demand and domestic consumption. Testing stochastic distributions of parameters, elasticities, and flexibilities should help us to understand the sensitivity of the model. Further research on these topics can help improve our understanding of the structural model performance, and enhance the model's capability for forecasting, impact simulation, and policy analysis.

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