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ENDOGENEITY TESTING OF WHEAT PRICE DETERMINATION IN A NONLINEAR SIMULTANEOUS STRUCTURAL MODEL

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ABSTRACT

Alternative specifications of price determination in a structural model are examined. Endogeneity testing of a nonlinear simultaneous wheat model to exogenous shock is evaluated by two different numerical solution methods for four alternative price specifications: Gauss-Seidel for a price-dependent (price -explicit) model and Newton for three quantity-dependent (price-implicit) models. Significantly different effects of the 1988 drought on wheat price and related supply and demand variables are found. Price impacts are considerably higher for quantity-dependent specifications than for the price-dependent specification; they are invariant among quantity-dependent, price-implicit models.

KEYWORDS

Price determination, structural model specifications, wheat, supply shock.

INTRODUCTION

Price determination in the structural model has been a subject of much theoretical research and empirical investigations. In the early stage of statistical price analysis, almost all demand, supply, and price relations were estimated using single equation methods (Fox). The estimated relations, as pointed out by Working, cannot be considered a true simultaneous model, especially, when simultaneous shifts of supply and demand occur. Due largely to the Cowles Commission's contribution, simultaneous equation methods were introduced to estimate price and quantity as mutually interdependent relationships (Haavelmo).

The analytical appeal of the simultaneous approach lies not only in the statistical properties of the estimated parameters, but also in the structure of the simultaneous equations in which price is solved jointly with supply and demand to achieve market equilibrium.

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Despite a rich collection of theoretical and empirical studies, the body of literature on price determination processes in the structural model is quite limited (Heien). More recently, Wu-Hausman tests were employed to evaluate structural model specifications of supply and demand functions regarding the normalization procedure and consistency properties of least squares estimators (Thurman). Of all the developments leading to improvements in structural model performance and application, none is more important than the behavioral response pattern of the simultaneous system to exogenous shocks. An investigation of alternative structural model specifications using numerical solution methods should provide useful insights into the price determination process of a farm commodity model under conditions of exogenous shocks. This is particularly true for large econometric models that emphasize intracommodity and intercommodity relationships, and that have important implications to farm commodity price analysis.

The primary purpose of this paper is to empirically examine alternative approaches of price determination in a structural model. A nonlinear simultaneous equations model for wheat is tested in four different price specifications: quantity (Q)-dependent, price (P)-implicit domestic demand; Q-dependent, P-implicit export demand; Q-dependent, P-implicit stock demand; and P-dependent, P-explicit stock demand functions. The structural model performance, is evaluated across the models, with particular emphasis on the sensitivity of wheat price to an exogenous shock induced by the 1988 drought. Numerical solution methods used for endogeneity testing include Gauss-Seidel for the P-dependent, P-explicit model and Newton (Newton-Ralphson) for Q-dependent, P-implicit models.

This paper is organized as follows. A discussion of price determination in a simultaneous structural model is presented in the next section. Then, two numerical solution methods used in nonlinear models with alternative price specifications are described, assumptions and procedures of endogeneity testing of a wheat model are presented, and empirical results of estimated models and impact simulations of an external shock induced by the 1988 drought are analyzed. The concluding section considers the implications of this study and offers recommendations for further research.

PRICE DETERMINATION IN STRUCTURAL MODELS

To model farm commodity markets one needs to specify and estimate a system of equations (*a structural model*) that captures all important demand, supply, and inventory stocks and their interrelationships. The basic form of components for such a structural system (as described by Just for a particular commodity) consists of a system of seven equations:[1]

$$Q_s = Q_s(\Pi; X_s) \tag{1}$$

$$Q_o = Q_o(P; X_o) \tag{2}$$

$$Q_e = Q_e(P; X_e) \tag{3}$$

$$Q_d = Q_d(P; X_d) \tag{4}$$

$$Q_x = Q_x(P; X_x) \tag{5}$$

$$Q_h = Q_h(P; X_h) \tag{6}$$

$$Q_s + Q_{h-1} = Q_o + Q_e + Q_d + Q_x + Q_h \tag{7}$$

where Q_s refers to quantity supplied, and Q with subscripts o,e,d,x, and h refers to demands for food, feed, seed, exports, and inventory stocks, respectively. While P,Π, and X refers to price, profit, and relevant exogenous variables, respectively. Equation 7 is the market clearing identity.

Price determination in structural models is an outcome of the solution of the simultaneous equation system. This procedure is directly related to the functional specification of the model. Models can be linear, nonlinear in variables, or nonlinear in variables and in parameters. However, structural models which are linear in variables, or nonlinear in parameters are almost nonexistent in empirical work due to unrealistic abstraction (linear models) and estimation and solution difficulties. A discussion on how price is determined in nonlinear structural models follows.

Nonlinear Simultaneous Systems and Numerical Methods

In structural models which are nonlinear in variables, how do we solve for prices? Obviously, reduced form expressions cannot be obtained, as matrix operations are infeasible due to nonlinearities of the model. Under such circumstances, several numerical methods for solving nonlinear systems can

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[1] Just specified a six-equation system. We also include a demand for seed.

be used using iteration procedures in search of equilibrium points. Such numerical methods conform to the structure of the model. Structural models can be specified in either (1) quantity-dependent, price implicit form, or (2) price-dependent, price-explicit form.

Q-dependent, P-implicit Structural Models

When supply and demand relationships are expressed in Q-dependent, P-implicit form, all demand and quantity relationships are expressed as quantity-dependent specifications. In such specifications price is determined simultaneously within the model where supply equals demands i.e., at market clearance. Thus, such specifications represent "pure" simultaneous structural models. Yet, the theoretical appeal of such specifications is often shadowed by the difficulties encountered in the solution process for price determination. To circumvent this problem, many alternative approaches have been adopted (Subotnik and Houck; Bailey). The basis of these approaches is to define a "price adjustment mechanism" which can be represented in the general form

$$P = (P_r, \Delta Q/k) \tag{8}$$

where P_r can be either lagged price, futures price, or some other price expectation; ΔQ represents a disequilibrium situation such as stock-domestic demand difference, or stock-aggregate demand difference which drives price; and k is a operator or a constant.

Although these types of specifications provide a convenient solution to the problem, they do not satisfy the requirement for a simultaneous solution where prices are determined at market clearance. To solve for price without compromising the simultaneity of prices in the specification , the only known method is the Newton (Newton-Ralphson) method, which is used for simulations analysis in this study.

P-dependent, P-explicit Structural Models

Due to difficulties in solving for prices in Q-dependent, P-implicit structural models, P-dependent, P-explicit structural models are becoming more popular in real-world modelling situations. The crux of price-dependent specification is to normalize a particular demand relationship in the structural model in order to present it in price dependent form. Conditions under which such normalization could be theoretically justified are given by Fox (1957) and also by Heien. Yet, price-dependent specifications are generally perceived as ad hoc mainly due to the notion that price-dependent specifications are not based on explicitly defined theoretical foundations. However, according to Shonkwiler and Taylor, they are consistent with utility (indirect) maximization hypotheses. Also, works of Houck and Anderson have contributed greatly in revamping estimation requirements of price-dependent demand specifications, augmenting their applicability in empirical work. As there are several demand equations in a structural model, theoretically one of these demand functions may be chosen to renormalize on price. Of the many possible alternative specifications the three most popular are (i) price-dependent stock demand, (ii) price-dependent domestic demand, and (iii) price-dependent export demand. A brief introduction to each of these specifications is given below.

P-dependent Stock Demand

For most major agricultural commodities, accumulation of stocks for such purposes as speculation, buffer stocks, and private storage has become an integral part of the market. Gardner mentioned that analyses which concentrate on explaining production and consumption behavior to the neglect of storage are prone to generate seriously misleading results. Stocks are usually accumulated by farmers, merchants, speculators, and government agents. In such cases, inventory holdings and their changes with respect to demand conditions may be the major determinant of price of the commodity. Such a price dependent-inventory demand can be obtained by renormalizing[2] equation (6) on price as

$$P = P(Q_h, X_h). \tag{9}$$

to obtain a price-dependent stock demand function. This basic specification has been respecified in many ways based on different theoretical hypotheses (Subotnik and Houck) and functional relationships (Adams and Behrman; Chen).

[2] Renormalization does not imply that the estimated quantity-dependent demand function is manipulated to put price on the left hand side of the equation. It refers to the theoretical derivation of the inverse demand function starting from a utility function (Huang), or from a profit function (for input demand functions) and estimation. Often this is necessary due to lack of symmetry between price elasticities and price flexibilities (Waugh).

P-dependent Domestic Demand

In the basic model, equation (2) could be renormalized on price to obtain a price-dependent domestic demand as

$$P = P(Q_o, X_o). \tag{10}$$

In an aggregated basis for all domestic demand components this specification is the demand for current utilization (Meilke and Young); in a disaggregated basis, this may represent demand for food, industry, feed, etc. Such disaggregation is based on data availability and on the relative importance of each sector in the total utilization of the commodity. Price-dependent demand functions are often specified in cases where supply is predetermined or highly inelastic in the short run, an underlying assumption for quarterly models for crops (Meilke and Young) or quarterly models for livestock (Stillman).

P-dependent Export Demand

In cases where exports plays an important role in the market, then short-run export demand is logically expected to be the major determinant of price. Under this hypothesis, equation (4) can be renormalized on price to obtain a price-dependent export demand function as

$$P = P(Q_x, X_x). \tag{11}$$

For most major export commodities such as wheat, soybeans, feed grains, etc., export demand may well constitute a crucial price determination relationship.

As in all price-dependent specifications, price appears on the left-hand side of the equation; thus price-dependent structural models could be solved using the Gauss-Seidel method, as this method is computationally less complex than the Newton method.

SOLUTION ALGORITHMS FOR NONLINEAR STRUCTURAL MODELS

Given necessary and sufficient conditions are satisfied for identification (Koopmans), then an appropriate estimation technique could be used for estimation of the structural model, depending on the degree of identification (Shephard). After the estimation, the next step is to solve the model to obtain solution values for prices (and other endogenous variables). Such solution techniques depend mainly on the functional form of the equations in the structural model.

Linear Simultaneous System and Reduced Form

The simultaneous equations system such as (8) can be expressed in compact matrix notation as

$$BY_t + \Gamma X_t = U_t \tag{12}$$

where Y_t is an mx1 vector of endogenous variables, X_t is an nx1 vector of exogenous variables, B is an mxm matrix of coefficients of endogenous variables, Γ is an mxn matrix of coefficients of exogenous variables, U_t is an mx1 vector of stochastic errors, and t=1,2,...,T observations. Then the system can be analytically solved to obtain reduced form equations for price (and all other endogenous variables) in terms of exogenous variables as

$$Y_t = \Pi X_t + V_t \tag{13}$$

where $\Pi=-B^{-1}\Gamma$, and $V_t=B^{-1}U_t$.

A necessary condition for obtaining analytically reduced form equations is that all equations in the model be linear. Yet, linear simultaneous systems are often limited to textbook examples. Even simple and essential transformations and specifications such as deflated prices and annual crop production (acreage x yield per acre) render the system nonlinear. Since nonlinear specifications are essential to specify structural models that represent real world situations, almost all existing structural models are nonlinear. Solution methods used in nonlinear structural models depend on the nature of the specification of price within the structural model. When price is implicitly specified (Q-dependent case) price is determined at market clearance. The resultant price should simultaneously satisfy the structural system, which can be represented as

$$F(Y,X;\theta)=0. \tag{14}$$

When the system is specified as in (14) the Newton method needs be invoked to solve for implicit prices.

<u>Newton Method</u>

Newton algorithm assumes the model is in the implicit form as (14), where F is a differentiable vector-function with as many coefficients as Y, which is the vector of solution (endogenous) variables (Drud). The method uses a derivative-based iterative procedure to go from

$(n)^{th}$ approximation (iteration) of the solution value for endogenous variables (Y) to the next $(n+1)^{th}$ approximation using the formula[3]

$$Y_{(n+1)} = Y_{(n)} - F(Y,X;\theta)/F'(Y_n,X;\theta). \qquad (15)$$

For a specified starting value algorithm approximate the implicit function $F(Y,X,\theta)=0$ by the tangent (derivative matrix) of the function. Since the solution from the n^{th} iteration is the starting value for the $(n+1)^{th}$ iteration, as we move towards the true value of Y (where $F(Y,X,\theta)=0$ is satisfied) the ratio $F(y_n)/F'(y_n)$ progressively becomes smaller (as F(.) approaches 0). The algorithm stops iteration when $|Y_{n+1} - Y_n| < \delta$, where δ is a small number close to zero which is internally set by the algorithm (or which can be externally specified by the user).

When price is explicitly specified (P-dependent) in the model the Gauss-Seidel method could be used to solve the system for price instead of Newton method. This algorithm is less complex in terms of solution technique and does not require the level of sophistication that is necessary for the Newton method. However, the Gauss-Seidel method can only be used when all endogenous variables are expressed by a unique dependent variable for each equation (Heien et al).

Gauss-Seidel Method

If the model is specified with a set of linearly independent equations as

$$\begin{aligned}
y_1 &= g_1(y_2,\ldots,y_j;x_1,\ldots,x_k;\theta)\\
y_2 &= g_2(y_1,\ldots,y_j;x_1,\ldots,x_k;\theta)\\
&\vdots\\
y_j &= g_j(y_1,\ldots,y_{j-1};x_1,\ldots,x_k;\theta) \qquad (16)
\end{aligned}$$

and if we consider a particular equation – say, j=1 – Gauss-Seidel solves the equation by iterating on

[3] Om the case of more than one variable, the term F(.) is the Jacobian of the solution variables (Chiang, p. 195).

$$y_{1,1} = g_1(y_{2,0},\dots y_{j,0},x_1,\dots,x_k,\theta)$$
$$y_{1,2} = g_1(y_{2,1},\dots y_{j,1},x_1,\dots,x_k,\theta)$$
$$\vdots \qquad \vdots$$
$$y_{1,n+1} = g_1(y_{2,n},\dots y_{j,n},x_1,\dots,x_k,\theta). \tag{17}$$

The first subscript refers to the variable while the second subscript refers to the iteration. The algorithm stops iteration when a specified tolerance level is reached such that

$$\mathrm{ABS}((y_{i,n+1} - y_{i,n})/y_{i,n}) < \delta. \tag{18}$$

As these two solution procedures employ different techniques to solve for price, the outcome depends on the specification of the model. Although price is endogenously determined in both P-dependent and Q-dependent cases, different specifications may generate different outcomes for price as specification of the model, and thus the solution algorithm use, differ.

ENDOGENEITY TESTING PROCEDURES

Endogeneity testing procedures consist of testing the price outcome from solutions of different price determination relationships in structural models. In the P-dependent case, three alternative structural models can be specified based on an alternative hypothesis on price-dependent demand functions in the model. These alternative models are given in Table 1.

In price-dependent specifications, price is explicitly determined by a particular demand function. In quantity-dependent specifications, on the other hand, it is possible only to implicitly determine price in the model. In the Q-dependent case it is also possible to implicitly specify price in alternative demand functions. The Solution algorithm in SAS (SAS/ETS, p. 51) suggests that if price is to be implicitly determined by a specific equation–for example in quantity-dependent stock demand–we need to respecify the demand function in price-implicit form as

Table 1. Alternative Price-dependent, Price-explicit Structural Models[4].

	Structural Model Specification		
	P-dependent P-explicit Stock Demand	P-dependent P-explicit Domestic Demand	P-dependent P-explicit Export Demand
Inventory Demand	$P=P(Q_h,X_h)$	$Q_h=Q_h(P,X_h)$	$Q_h=Q_h(P,X_h)$
Domestic Food Demand	$Q_o=Q_o(P,X_o)$	$P=P(Q_o,X_o)$	$Q_o=Q_o(P,X_o)$
Export Demand	$Q_x=Q_x(P,X_x)$	$Q_x=Q_x(P,X_x)$	$P=P(Q_x,X_x)$
Market Clearing Identity	$Q_h=Q_s-Q_{h-1}-Q_o-Q_e$ Q_x-Q_d	$Q_o=Q_s-Q_{h-1}-Q_h-Q_e$ $-Q_x-Q_d$	$Q_x=Q_s-Q_{h-1}-Q_o-Q_d$ $-Q_x-Q_d$

$$P = P + Q_h(Q_h,P;X_h). \tag{19}$$

Thus, for the Q-dependent case it is also possible to define alternative price-implicit models using the corresponding demand functions as in the P-dependent case. Thus, three alternative Q-dependent models were also defined (Table 2).

In P-dependent cases a unique price determination relationship is continuously used to generate new prices. Thus, how close P converges to price depends mainly on how well the price-dependent demand function explains the price determination process subject to the constraint of market clearing identity. In other words, the convergence of the solution algorithm is constrained by the price-dependent demand specification used. As Heien et al. pointed out the normalization decision (i.e., the type of price-dependent demand function) affects the convergence of the system. Normalization also affects how realistic the solution value of price is due to varying predictive abilities of different P-dependent demand functions.

In Q-dependent structural models price may appear (implicitly) in one equation or in several equations in the model. Yet, regardless of the demand functions used to solve for implicit price, the

[4] Only those demand functions that change across models are specified here. The rest of the model remain the same as in the basic model, i.e., equations (1)-(7).

solution should generate only one price. This can be shown easily in linear specifications, since, when the system is exactly identified only one relationship can be derived when solving for reduced form equations. In cases of nonlinear structural model specifications, this unique price is the price which simultaneously satisfies the system $F(X,Y;\theta)=0$. Just as in the linear case, there is only one unique price which satisfies the above system. This can be illustrated by following the operational sequence of computer algorithms used in the Newton method in SAS.

For simplicity, we select a particular demand function instead of the complete system for demonstration purposes. First, assume that the quantity-dependent stock demand specification was used to determine implicit price. Then the model is specified with the following relationship for stock demand within the complete model:

$$P_{(h)}=P_{(h)} + Q_h(Q_h,P_{(h)};\theta) \tag{20}$$

where subscript (h) is an index which identifies that price is implicitly determined using the quantity-dependent stock demand (h). If we represent the price-implicit stock demand function as

Table 2. Alternative Quantity-dependent, Price-implicit Structural Models.

	Structural Model Specification		
	Q-dependent P-implicit Stock Demand	Q-dependent P-implicit Domestic Demand	Q-dependent P-implicit Export Demand
Inventory Demand	$P=P+Q_h(Q_h;X_h)$	$Q_h=Q_h(P;X_h)$	$Q_h=Q_h(P;X_h)$
Domestic Food Demand	$Q_o=Q_o(P;X_o)$	$P=P+Q_o(Q_o;X_o)$	$Q_o=Q_o(P;X_o)$
Export Demand	$Q_x=Q_x(P;X_x)$	$Q_x=Q_x(P;X_x)$	$P=P+Q_x(Q_x;X_x)$
Market Clearing Identity	$Q_h=Q_s-Q_{h-1}-Q_o-Q_e-Q_x-Q_d$	$Q_o=Q_s-Q_{h-1}-Q_h-Q_e-Q_x-Q_d$	$Q_x=Q_s-Q_{h-1}-Q_o-Q_e-Q_h-Q_d$

$Q_h(Q_h,P_{(h)};\theta) = e$, where e is the random error in the solution process (20) changes to

$$\hat{P}_{(h)} = P_{(h)} + Q_h(Q_h,\hat{P}_{(h)};\theta). \tag{21}$$

In the solution process, $P_{(h)}$ is given the value of actual price (P) from data. Therefore, residual of price can be given as

$$\tilde{P}_{(h)} = \hat{P}_{(h)} - P_{(h)} \tag{22}$$

where $\tilde{P}_{(h)}$ is the residual created in determining price. Thus, $\tilde{P}_{(h)}$ in (22) becomes

$$\tilde{P}_{(h)} = Q_h(Q_h,\hat{P}_{(h)};\theta) \tag{23}$$

The algorithm then takes the derivative of $\tilde{P}_{(h)}$ with respect to endogenous variables (price and other endogenous variables), and uses the Newton method to iterate price to solve the system using the formula

$$P_{(h),(n+1)} = P_{(h),(n)} - f(Q_h,\hat{P}_{(h)};\theta)/f'(Q_h,\hat{P}_{(h)};\theta) \tag{24}$$

where subscript n refers to the n^{th} iteration.

Since, we are solving for price and other endogenous variables simultaneously, the denominator of the second term on the right-hand side is the inverse Jacobian of the $F(Y,X;\theta)$, which can be given as

$$J = \begin{bmatrix} \frac{\vartheta f_1(.)}{\vartheta Y_1} & \frac{\vartheta f_1(.)}{\vartheta Y_2}, & \cdots, & \frac{\vartheta f_1(.)}{\vartheta Y_j} & \frac{\vartheta f_1(.)}{\vartheta Y_m} \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \frac{\vartheta f_m(.)}{\vartheta Y_1} & \frac{\vartheta f_m(.)}{\vartheta Y_2}, & \cdots, & \frac{\vartheta f_m(.)}{\vartheta Y_j} & \frac{\vartheta f_m(.)}{\vartheta Y_m} \end{bmatrix} \tag{25}$$

However, as price is the only endogenous variable considered for illustrative purposes, let $Y_j = P$. Thus, under this assumption J reduces to $J_{(h)}$, which can be represented as a column vector:

$$J_{(h)} = \begin{bmatrix} \dfrac{\vartheta f_1(.)}{\vartheta P_{(h)}} \\ . \\ . \\ . \\ \dfrac{f_{(m)}(.)}{\vartheta P_{(h)}} \end{bmatrix} . \tag{26}$$

Now, if we take some other Q-dependent demand function, say, domestic demand for implicit price determination, then (24) and (26) becomes respectively,

$$P_{(o),(n+1)} = P_{(o),(n)} - f(Q_o,\hat{P}_{(o)}\ ;\theta)/f'(Q_o,\hat{P}_{(o)}\ ;\theta) \tag{27}$$

and

$$J_{(o)} = \begin{bmatrix} \dfrac{\vartheta f_1(.)}{\vartheta P_{(o)}} \\ . \\ . \\ . \\ \dfrac{f_{(m)}(.)}{\vartheta P_{(o)}} \end{bmatrix} \tag{28}$$

Both specifications start iteration from different points because of different starting values. At each subsequent iteration, elements of the inverse of $J_{(h)}$ and $J_{(o)}$ take different values, or are constant if in all demand functions price is in linear form. Thus, at each iteration the second term on the right-hand side of (24) and (27) takes different values while at the same time converging to 0 as $f(Q_h)$ and $f(Q_o)$ approach zero. Denote

$$\Delta f_{(n)}(.) = f_{(n)}(Q_h) - f_n(Q_o) . \tag{29}$$

Both models keep searching the price P at which $F(Y,X;\theta)=0$ is satisfied, with different specifications moving towards P along different demand functions. As both searchers get closer to P, at each successive iteration the Q-dependent stock demand and Q-dependent domestic demand approach zero simultaneously. Therefore,

$$\Delta f_{n+1}(.) < \Delta f_n(.) \ . \tag{30}$$

When iteration stops at iteration p for stock demand and iteration q for domestic demand, then

$$(Q_h,p) \approx f(Q_o,q) \approx 0 \ . \tag{31}$$

Because of (31) the second term on the right hand side of (24) and (27) approaches zero. Therefore,

$$P_{(h),p} = P \pm \delta \tag{32}$$

and

$$P_{(o),q} = P \pm \delta \tag{33}$$

where δ is a small number which is the convergence error set either by the algorithm or by the user. As $P \in \Re^n_+$ then always[5]

$$P_{(h),p} \approx P_{(o),q} \tag{34}$$

where $\Re^n_+$ refers to the positive real number line.

Thus, for all practical purposes, the price-implicit stock demand function and price-implicit domestic demand function (or any other quantity-dependent demand function) generates almost the same solution for price. On the other hand the Gauss-Seidel method generates different solutions for price depending on the type of price-dependent demand function used. This is because each P-dependent demand hypothesis is analogous to different model specifications.

EMPIRICAL RESULTS

The wheat model used for simulation is a complete sectoral model of 61 equations and 5 major blocks. For purposes of the present study, however, only the related demands and price equations are described here. (A general description is given by equations 1-7). Also, estimation of price dependent demand functions was limited to price-dependent stock demand, as this is a common

[5] When there are extensive nonlinearities in a function (when the function crosses the x-axis at more than one place), it is possible to get different solution outcomes. However, as far as demand functions are concerned, such a functional form violates demand theory.

specification for major agricultural commodities (Adams and Behrman).

Estimated Models

The estimated price-dependent and quantity-dependent demand functions are summarized in Table 3[6]. For estimation purposes annual data for wheat from 1973 to 1987 was used. Model specifications and estimated demand functions generally are consistent with other empirical studies for agricultural commodities. In this study, all demand functions show a good fit with high R^2. All variables have expected signs. Although some coefficients have low t-values, this does not pose a serious problem for the objective of this study. Variables need not be excluded from specifications based on their individual t-values, when such equations are used for predictive purposes or for impact analysis.

1988 Drought Impacts On Wheat

The specific objective of this paper is to trace the performance of alternative structural model specifications (i.e., P-dependent and Q-dependent functions in a structure model) in the face of a supply shock. Supply shocks are of particular interest since supply is highly variant to exogenous conditions such as weather, technology, etc. For empirical analysis, a U.S. Wheat Model is used to determine the effects of 1988 drought-induced production cutback in 1988. A complete sectoral model for wheat was used because it contains a comprehensive set of supply, demand, and inventory stocks relationships. However, instead of the complete model (with 61 structural equations), the sensitivity analysis was performed using only a small portion of the model, including the system of seven equations described earlier.

Due to adverse weather conditions in 1988, there was a significant decline in yield per acre for wheat. In conducting our simulation experiment, 1988 wheat yield per acre was shocked by using a normal weather condition of trend yield against the actual level to analyze the magnitude of impacts and the transmission mechanism of the supply shock. To derive the counterfactual scenario of normal wheat yield, a trend analysis was performed. An estimated trend equation is used to project a wheat

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[6] A description of the models are available upon request from the authors.

Table 3. Estimated Quantity-dependent and Price-dependent Demand Functions.

Quantity-Dependent Domestic Demand

$$Q_i = \underset{(3.181)}{-280.000} - \underset{(1.334)}{14.630}\, P_m + \underset{(6.118)}{0.447}\, Y$$

$R^2 = 0.94$ R Bar Sq. = 0.92 D.W. = 1.36

Quantity-Dependent Export Demand

$$Q_x = \underset{(0.238)}{92.063} - \underset{(1.247)}{103.918}\, P_m + \underset{(6.945)}{0.759}\, LQ_m + \underset{(0.537)}{267.189}X + \underset{(1.594)}{1.753}\, P^W_m$$

$R^2 = 0.80$ R Bar Sq. = 0.77 D.W. = 2.14

Quantity-Dependent Stock Demand

$$Q_m = \underset{(4.005)}{2631.320} - \underset{(3.785)}{801.395}\, P_m - \underset{(0.778)}{517.878}\, (Q_m/Q_t)^e$$

$R^2 = 0.83$ R Bar Sq = 0.80 D.W. = 1.34

Price-Dependent Stock Demand

$$P_m = \underset{(18.100)}{3.191} - \underset{(2.646)}{1.005}\, (Q_m/Q_t) - \underset{(5.643)}{2.195}\, (Q_m/Q_t)^e$$

$R^2 = 0.89$ R Bar Sq = 0.88 D.W. = 2.01

t-statistic is given in parentheses.
P_m=farm wheat price, Y=deflated disposable income, LQ_m=lagged Q_m, X=exchange rate,

P^W_m=world wheat prices, $(Q_m/Q_t)^e$=expected stock demand ratio, and $Q_t=(Q_i+Q_f+Q_x+Q_p)$.

yield of 38.04 bu/acre for 1988 in comparison to the actual level of 34.10[7]. First, a baseline solution was obtained with actual yield per acre for 1988. Then a simulation was done by shocking 1988 yield per acre with 38.04 bu/acre to obtain the simulated values of wheat prices and related supply and demand variables. Results are given in Table 4.

The impact of the supply shock induced by an increase in yield per acre (from 34.1 bu./ac

[7] The estimated trend function for wheat yield per acre with t-values in parenthesis is

$$Y_a = \underset{(-5.894)}{-1299.22} + \underset{(5.744)}{0.673}\, TREND$$

$$R^2 = 0.73 \quad \text{R bar sq} = 0.71 \quad \text{D.W.} = 1.62$$

Table 4. Impact Analysis of 1988 Drought on Wheat.

Wheat Yield Assumption

Actual (Drought): 34.10 bu/ac

Trend (Normal): 38.00 bu/ac

	Q-dependent Domestic Demand	Q-dependent Export Demand	Q-dependent Stock Demand	P-dependent Stock Demand
Variable				
Wheat Farm Price ($/bu)				
Actual	3.72	3.72	3.72	3.72
Impact	-0.64	-0.60	-0.62	-0.42
Domestic Demand (mil. bu)				
Actual	735	735	735	735
Impact	3	6	3	2
Stock Demand (mil. bu)				
Actual	702	702	702	702
Impact	143	129	129	130
Export Demand (mil. bu)				
Actual	1419	1419	1419	1419
Impact	19	19	19	13
Production (mil. bu)				
Actual	1812	1812	1812	1812
Impact	210	210	210	210

Impact = Change from baseline.

to 38.04 bu/ac) does affect all supply and demands in the model. Total production increase by 210 mil. bu. This increase generate a price reduction (due to the supply increase) of 42 cents in P-dependent stock demand model, 64 cents in Q-dependent domestic demand model, 60 cents in Q-dependent stock demand model, and 62 cents in Q-dependent stock demand model. For domestic demand, P-dependent stock demand model show the smallest increase with 2 mil. bu., with Q-dependent domestic demand model and Q-dependent stock demand model indicating a 3 mil. bu. increase in domestic demand. The largest impact on domestic demand is on q-dependent export demand model with a 6 mil. bu. increase. In case of export demand the impact is constant for all q-dependent models with a 19 mil. bu. increase with P-dependent stock demand model showing a lower 13 mil. bu. increase in stocks. Stock demand show the largest impact. Stocks in Q-dependent domestic demand model increase by 143 mil. bu., while the rest of the models generate a comparable impact of around 130 mil. bu.

Q-dependent specifications show an almost constant change in price across all demand specifications indicating the algorithm is invariant as to which demand function is implicitly specified in price. Also, they indicate a stronger (greater) impact on price than in price-dependent specifications. This discrepancy may be attributed to the different transmission mechanisms of shock based on the type of specification.

Comparison of P-Dependent and Q-Dependent Specifications

Which specification is appropriate in a particular situation? Often, due to computational difficulties and cost factors, Q-dependent specifications are not widely adopted in empirical studies. Even when price-dependent specifications are satisfied on theoretical grounds, they may not be unambiguously justified. On the other hand, even if technical aspects of quantity-dependent specifications can be overcome, they may not generate more realistic outcomes than price-dependent specifications under some situations.

Figure 1 illustrates price determination in the P-dependent case. When the supply shifts from S_1 to S_2 (due to increased yield per acre), it affects price-dependent and quantity-dependent specifications in different ways. In price-dependent specifications the supply shock first affects Q_s, which influences the market clearing identity. The-left-hand side variable of the market clearing identity (7) in each specification drives the price determination process. Thus, the shock first

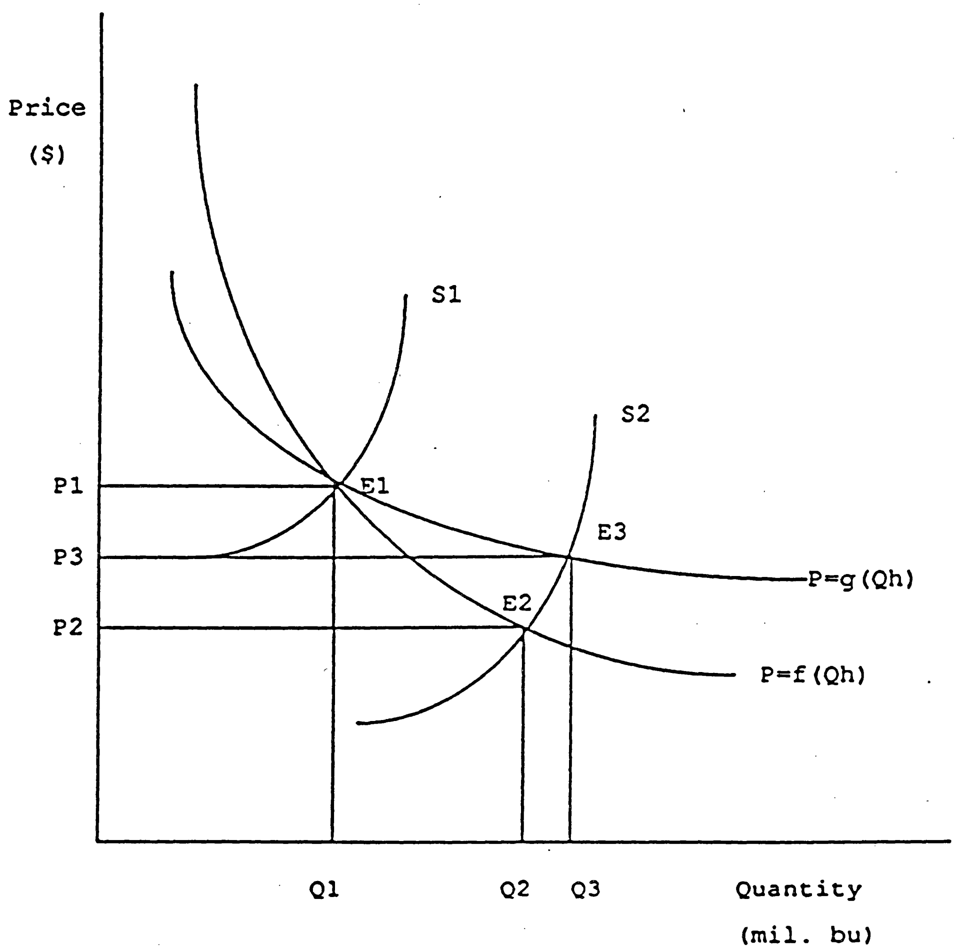

Figure 1. Price Determination in Quantity Dependent Stock Demand

influences the price through the specific P-dependent demand functions. Once price is initially changed, it in turn affects all related demand functions since price is an argument on the right-hand-side of each demand specification.

Changes in quantities (left hand side) of the other demand functions induced by this initial price change inturn affect the quantity of the particular P-dependent demand functions through the market clearing identity. This is the first iteration. This type of feedback (iterative process) continue until price satisfies a convergence criterion such as defined by (18). At each iteration the price is shocked by continuously diminishing amounts along the particular P-dependent demand function until it reaches a point E_2 (Figure 1). The key issue here is that each iteration impact on price is determined by the price flexibility of the P-dependent demand function, while the impact on the quantity of P-dependent demand functions is influenced by the price elasticities of other demand functions in the model (Table 5). Through different transmission mechanisms, all the demand functions play a role in the final outcome of price, and thus, simultaneity of the model is satisfied.

In Q-dependent specifications, the shock transmission mechanism takes a different path. If we look at the elements of $J_{(h)}$ and $J_{(i)}$, they are price derivatives of demand functions in the

Table 5. Price Elasticities and Price Flexibilities of Demand Functions

	Q-dependent	P-dependent
Demand Function	Price Elasticity	Price Flexibility
Stock Demand (Q_m)	-1.08	-0.34
Domestic Demand (Q_i)	-0.29	-4.56
Export Demand (Q_x)	-1.25	-0.41

system. As this complete coefficient vector is inverted at each iteration in the search, the elasticity[8] of all demand functions in the model plays a role in the initial impact of supply shock. Here the Q-dependent demand function in which price is implicitly defined does not play any particular important role in the price determination. This is different from the P-dependent case where the price flexibility of the demand function plays a more important role in affecting the price. Thus, it is possible that the outcome of price may be dominated more by some other demand function which has a relatively larger price elasticity than that of other demand functions in the system. This particular issue has some important implications in determining the selection of a suitable structure for the model. If the reliability of specification and estimation of a particular demand function is more than the reliability of the overall specification of the model, and if the condition for price dependency (Fox; Heien) is satisfied, then use of a price dependent demand function may be justified. On the other hand, if the confidence in the specification and estimation of all demand functions is more than the confidence on one particular demand function, then price should probably be implicitly determined.

Another issue that needs to be considered in selecting a particular specification is the sensitivity of the specification to structural changes in the model. Since price-dependent specifications use a unique demand function, the outcome for price or the impact on price to exogenous changes may be more sensitive to structural changes in that particular demand function than in the other demand functions in the model. This condition is illustrated in Figure 1. For the initial demand function, supply shock may generate a price difference of $P^1 - P^2$. If the structure of the demand function changes (i.e., if demand function becomes less price flexible, such as $P=g(Q_h)$ where, $f_1 > g_1$), then for the same supply shift the resultant price difference is $P^1 - P^3 < P^1 - P^2$. An elasticity change of similar magnitude of some other demand function may result in a price impact which is less than $P^1 - P^3$. On the contrary, as quantity-dependent specifications use all price-coefficients in the solution process "directly", structural changes in any of the demand functions

[8] Technically. it is the price coefficient vector which is inverted at each iteration. But as elasticities are functions of these coefficients, we prefer to address in terms of elasticities as they have more economic relevance.

affect the outcome for price "directly". Therefore, in the case of price-dependent specifications, if one used, say, price dependent export demand. If the export market changes due to conditions such as import restrictions, or more countries entering the export market such a situation would make the price-dependent export demand obsolete. Thus, a new specification needs to be searched. In Q-dependent specification, however, this problem is less serious as no specific demand function dominates the price determination. Therefore, reestimation of quantity-dependent export demand is sufficient to maintain the previous validity of the model.

Yet, even in a such a situation, there is no guarantee that Q-dependent specification would do a better job. In Q-dependent specifications, outcome is dominated by price coefficients of all the demand functions. Thus, any incorrectly specified or estimated demand function would greatly affect the outcome, especially if the price coefficient of that particular demand function, is relatively, of greater magnitude compared with price coefficients of other demand functions. The bottom line in P-dependent and Q-dependent specification is that while in P-dependent specification, the price flexibility of one particular demand function largely dominates the price outcome, in Q-dependent specifications the complete elasticity vector of all demand functions affects the price. Also, more elastic demand functions play a more important role in Q-dependent specifications.

CONCLUSIONS

This paper examines the endogeneity of the price determination in structural models. Price may either be explicitly determined (in price-dependent specifications), or implicitly determined (in quantity-dependent specifications). Results indicate that the alternative specifications have important implications to the sensitivity of the structural model to exogenous shock induced by the 1988 drought. In general quantity-dependent structural models give higher price responses as compared to price-dependent structural model for the same supply shock. Also, the price impact is comparable across quantity-dependent models. A critical evaluation of the differences in prices and demand variables needs to be done, in particular, the specification of individual demand functions and the market clearing identity. A useful source of information is to trace the differential price impacts through a comparative analysis of price elasticities of the relevant demand equations.

Our knowledge of the alternative price specification can be greatly expanded by testing the other price-dependent demand functions, especially for domestic demand, which is inelastic, and for export demand which is a high elastic case. The impact simulation analysis of the weather-induced production cutback provides useful information in terms of structural model price response to external shocks. The power of endogeneity tests can be further extended to include internal shocks in terms of the impact of changes in export demand and domestic consumption.

These preliminary results indicate that a redirection of research efforts is needed to explore the endogenous price determination process in structural models. For example, alternative price-dependent specifications need be tested to examine the outcomes under external shocks. To draw more concrete conclusions of the outcome, tests of internal shocks (endogenous shocks) are required. Tests on stochastic distribution of parameters, elasticities, and flexibilities may also provide useful information on convergence limits and sensitivity of prices to obtain "confidence limits" on parameters and elasticities, where realistic outcomes are possible. Further, as the effect of an external shock on price-dependent specifications is greatly constrained by their price flexibilities, its effect on the solution outcome also needs to be researched.

REFERENCES

Adams, F. Gerard, and Jere R. Behrman. *Econometric Models of World Agricultural Commodity Markets*. Cambridge: Ballinger Publishing Company.

Anderson, R. W. "Some Theory of Inverse Demand For Applied Demand Analysis." *Euro. Econ. Rev.* 14(1980):281-290.

Bailey, Kenneth W. "A Structural Econometric Model of the World Wheat Market." Technical Bulletin No. 1763. United States Department of Agriculture. 1989.

Chen, Dean T. "The Wharton Agicultural Model: Structure, Specification, and Some Simulation Results." *Amer. J. Agr. Econ.* 59(1977):107-116.

Chiang, Alpha C. *Fundamental Methods of Mathematical Economics*. New York: McGraw-Hill Book Company. 1974.

Drud, Arne. "A Survey of Model Representations and Simulation Algorithms in Some Existing Modelling Systems." *Journal of Economic Dynamics and Control* 5(1983):5-35.

Fox, K. A. *Econometric Analysis of Public Policy.* Ames: Iowa State College Press. 1957.

Haavelmo, T. "The Statistical Implications of a System of Simultaneous Equations." *Econometrica* 11(1943):1-12.

Gardner, Bruce. "Public Stocks of Grain Storage and the Market for Grain Storage." *New Directions in Econometric Modeling and Forecasting in U.S. Agiculture.* Gordon C. Rausser, ed. New York: North-Holland. 1982.

Heien, D. M. "Price Determination Processes for Agricultural Sector Models. *Amer. J. Agr. Econ.* 59(1977):126-132.

Heien, D., Jim Matthews, and Abner Womack. "A Methods Note on the Gauss-Seidel Algorithm for Solving Econometric Models." *Agr. Econ. Res.* 25(1973):71-80.

Houck, J. P. "A Look At Flexibilities and Elasticities." *J. of Farm Econ.* 48(1966):225-232.

Huang, Kuo S. "An Inverse Demand System for U.S. Composite Foods." *Amer. J. Agr. Econ.* 70(1988):902-909.

Just, Richard E. "Modelling the Interactive Effect of Alternative Sets of Policies on Agricultural Prices." *Primary Commodity Prices, Economic Model and Economic Policy.* L. Alan Winters, and David Sampsford, eds. Center for Economic Policy Research. London. 1990.

Koopmans, T. C. "Identification Problems in Economic Model Construction." *Econometrica* 17(1949):125-132.

Meilke, Karl D., and Larry Young. "A Quarterly North American Soybean Model." Working Papers, No. 4. Policy, Planning and Economics Branch. Agriculture Canada. 1979.

Shephard, Geoffrey S. *Agricultural Price Analysis.* Ames: The Iowa State College Press. 1957.

Shonkwiler, S., and T. G. Taylor. "The Impact of Estimating Market Demand Curves by Least Squares Regression." *Euro. R. Agr. Econ.* 11(1984):107-118.

Statistical Analysis System (SAS). SAS/ETS Users Guide. Version 5 Edition. Cary: SAS Institute, Inc. 1984.

Stillman, Richard P. "A Quarterly Model of the Livestock Industry." United States Department of Agriculture. Economic Research Service, Technical Bulletin No. 1711. 1985.

Subotnik, Abrahm, and James P. Houck. "A Quarterly Econometric Model For Corn: A Simultaneous Approach to Cash and Futures Markets." Technical Bulletin No. 318. Agriculture Experiment Station. University of Minnesota. 1979.

Thomas, George B., Jr., and Ross L. Finney. *Calculus and Analytical Geometry*. Seventh edition. New York: Addison-Wesley Publishing Company. 1988.

Thurman, Walter N. "Endogeneity Testing In A Supply and Demand Framework." *The Review of Economics and Statistics*. 1986. pp. 638-646.

Working, E. J. "What Do Statistical 'Demand Curves' Show?" *Quarterly Journal of Economics* 41(1927):35.

Waugh, F. V. *Demand and Price Analysis*. Washington, DC: U.S. Department of Agriculture. Tech. Bul. No. 1316, 1964.