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SUPPLY AND INPUT CHOICE RESPONSE
BY MULTIPLE PRODUCT FIRMS: NEW APPROACHES

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Abstract

Supply and Input Choice Response by Multiproduct Firms: New Approaches

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Economic theory provides an explicit basis for statistical measurement of choice response. Choice functions derived from marginalist theories must be linearly homogeneous in prices and have symmetric cross-derivatives. Furthermore, the characteristics of technology (homotheticity, homogeneity, jointness, separability) are reflected in the choice functions through the necessary conditions for optimal choice. Following a definition of "consistent" choice functions (Weaver), Whittaker reviewed the constant elasticity of transformation (C.E.T.) model and reported an application to regional acreage response. Shumway and Chang presented an application to Texas field crop supply emphasizing shifters of the C.E.T. function. Considerable discussion was given to the interpretation and sensitivity to specification of estimated price elasticities. Concluding, Weaver demonstrated a methodology based on duality which derives from a maintained choice hypothesis input and output choice functions which are consistent with any underlying technology (joint in inputs, non-homothetic and variable elasticities of substitution or transformation). Discussion focused on the applicability of the methodologies to choices of processing, marketing and other multiproduct firms. Data requirements and facility of implementation were also reviewed.

Introduction: Theoretical Issues

Robert D. Weaver*

In his well-known article "Conceptualizing the Supply Relation in Agriculture" in J.F.E. 1955, Cochrane argued that two types of supply relations might be distinghished: supply functions and supply response relations. Although the distinction seems semantically tenuous time seems to have led to the consideration of the latter as relations whose specification is to some degree ad hoc and the former as relations which have been derived from and are, therefore, consistent with some maintained hypothesis about the way in which decisions are made. Our title promises something new. We shall focus on methods of measuring the relationships between choices and their determinants which are consistent with underlying maintained hypotheses concerning the objectives employed by as well as the constraints and technology faced by decision-makers involved.

The present section shall present an overview of what from a theoretical standpoint should be expected of consistent choice relations. Next, three applications will be considered which involve estimation of such "consistent choice relations." In proceeding, an attempt will be made to assess the potential usefulness and outline problems involved with the particular methodologies. In so doing, it is hoped that further discussion of these same issues will be provoked.

Following the initial theoretical overview will be a paper by Dr. James K. Whittaker. Dr. Whittaker is Assistant Professor of Agricultural Economics at Oregon State University. He will review an approach which allows estimation of a linear system of supply functions which are consistent with production possibilities characterized by constant elasticities of transformation. He will introduce the model, summarize past

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efforts which have employed the model and briefly present results of an application to measurement of regional acreage response.

Next, a paper by Dr. Richard Shumway, Associate Professor of Agricultural Economics and Anne A. Chang at Texas A & M University will report an application of the C.E.T. to the measurement of Texas field crop response which focuses on factors which may shift the transformation function.

The final paper reviews work which I have completed which employs a convenient methodology to estimate the elasticities of input and output choice for multiproduct firms. These elasticities are consistent with a hypothesized characterization of technology. Although all three applications involve crop supply, it should be emphasized that they are broadly applicable.

A Definition of Consistent Choice Functions

As the parametric programmers of the 60's would remind us, analysis of decisions relies upon a careful specification of the technological and other constraints faced by the firm as well as the firm's objective. Agricultural production, processing and distribution firms as well as numerous other providers of goods and services studied by agricultural economists are safely classified as producing multiple products. In addition, it is frequently the case that a particular product is both produced by and utilized as an input in the same firm.

Although we will not attempt to explain why these firms diversify, their survival indicates an advantage due to diversification. To study the enterprises of these firms separately would ignore the important interactions which have led to diversification. Thus, to begin we shall pursue approaches which accommodate the existence of multiproducts and the possibility of interaction among enterprises.

Specification of the technology and analysis of the decisions of multiple product firms has proceeded at a theoretical level for a long time. Hick's appendices to Value and Capital (1939) and R. G. D. Allen's Mathematical Economics (1964) contain early considerations. It is here that we find the basis for our "new" approach.

If we suppose the existence of a convex transformation function relating efficient combinations of net outputs and net inputs and postulate a behavioral or decision objective for the firm (e.g., profit maximization), the necessary conditions for optimal choice define a set of rules for choosing outputs and inputs. They relate choices to their determinants which include prices and any net outputs or inputs that the firm cannot control within the decision period. However, in addition to specifying these determinants, the choice rules exactly specify the relation between the determinants and the choices made. Specifically, they state that if all prices are changed in equal proportion, thus leaving relative prices undisturbed, choices would not change. That is, the maintained hypothesis of profit maximization, in fact of any marginalist hedonic choice, implies that choices are linearly homogeneous in prices. If we estimate supply response functions which are not linearly homogeneous in prices can we rely upon their predictions? Certainly, there exist behavioral hypotheses with which such choice relations would be consistent, but they are not consistent with the marginalist theory that is typically relied upon by economists.

Going further, the link of choices and technology in the first order conditions implies that the form of the supply relation is clearly determined by the form of the production function. Thus, in specifying the form of the supply relation we specify implicitly the form of the production or transformation function and vice versa. One specific example illustrates the implications of this link. If we hypothesize that the production or transformation function faced by decision-makers is a continuous function, then the change in its derivatives as we move along the production surface must be symmetric. For example, the marginal productivity of input 1 measured in terms of output 1 changes when the utilization input 2 is changed. Symmetry requires that this change be equal to the change in the marginal product (measured in terms of output 1) of input 2 for a change in input 1. This being the case it is easy to show that cross-elasticities of choice should be constrained with this symmetry. If we estimate a supply response function for soybeans and one for corn, can we rely upon an estimated elasticity of corn supply to changes in soybean prices which is not based on such symmetry?

If we think production is Cobb-Douglas, as the repeated reliance on homogeneous Cobb-Douglas forms for estimation of agricultural production functions indicates, why are the typically estimated supply relations linear? In fact, for multiproduct technologies the Cobb-Douglas form is of limited usefulness, it has the wrong curvature. However, since Mundlak's initial effort in 1964 of extending Halter, Carter and Hocking's transcendental form several alternatives have emerged. At the least, when used in supply analysis these new functional forms allow for imposition of homogeneity in prices and measurement of choice elasticities which are consistent with symmetric production. At best, they allow for specification of choice functions which are consistent with alternative characteristics of the transformation function. In fact, by relying upon fundamental theorems concerning the duality of choice and technology the structural characteristics of production are easily explored by the testing of various linear and non-linear restrictions on the parameters of choice functions. As will be illustrated later [Weaver], these new forms allow the researcher freedom from having to specify the exact characteristics of production a priori. Instead, by employing functional forms which may be made consistent with many underlying production functions through hypothesized parameter restrictions, the specific characteristics of production may be explored through statistical tests rather than assumed.

Finally, issues of functional form are not only related to the form that is employed to represent the relation among variables involved in the estimation. A critical step that must be taken in moving from theoretical forms which relate vectors of net products or their prices to empirically estimable forms is the specification of groups of commodities which will be aggregated and the method of aggregation. As has been known at least since Leontief's work in 1947, the validity of the specification of such aggregates depends upon the separability of the production or transformation function. By duality of choice and technology this implies that price aggregation is implicitly based upon maintained hypotheses concerning the separability of technology. Furthermore, the validity of one aggregation procedure versus another depends upon the nature of technology. [See Weaver, 1978]

To summarize, a "consistent" model of choice is one which employs as restrictions all implications of the maintained hypothesis concerning the way in which decisions are made and the technology which constrains them.

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THE USE OF LINEAR RESTRICTIONS IN ESTIMATION OF CROP ACREAGE RESPONSE: HOMOGENEITY AND THE CONSTANT ELASTICITY OF TRANSFORMATION

James K. Whittaker*

Introduction

Most studies in crop supply response have contributed to the production literature primarily in one of two areas: models with improved variable formulations (e.g. Nerlove and Houck and Ryan) or models that give improved forecasts (e.g. Walker and Penn). Although these two types of contributions are not mutually exclusive, few studies have attempted to contribute in both areas. The goal of this research is to contribute to supply response knowledge in a third way by showing how prior information about the relationships that exist among the estimated coefficients can be used to obtain "better" parameter estimates. This research will rely upon existing variable formulations and no attempt will be made to improve on previous short run acreage forecasts.

In demand research, price and income elasticity relationships that are derived from economic theory have often been incorporated into the estimation of the parameters of the demand model (e.g. George and King). Incorporating prior knowledge about these elasticities into the parameter estimation techniques leads to estimated parameters with smaller variances than those of parameters estimated without a priori information. The use of prior information may also make parameter estimation possible when too few observations exist to estimate the parameters of a model without prior information.

Several theoretical relationships also exist among the supply price and cross price elasticities. To date these relationships have largely been ignored in crop supply response research.[1/] This paper demonstrates how a priori information on elasticity relationships can be incorporated into supply response

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research by the use of exact linear restrictions.

In this research, the coefficients of three regional crop acreage response models for the United States were estimated (Whittaker). The three models have varying levels of restrictions built into the estimation procedures. All three models were estimated using the same data and all contain the same equations and variables. The first model is a traditional type of crop supply response model with no restrictions. The parameters of the model were estimated using ordinary least squares (OLS model). The second model is identical to the first except that the restriction that each supply equation is homogeneous of degree zero in expected prices is added. Its parameters were estimated using restricted least squares (RLS model). The final model is the constant elasticity transform (CET Model) of the OLS model. Its parameters were estimated using restricted least squares also. This paper will emphasize the theory underlying the latter two models.

The remainder of the paper is presented in the following order. First, the basic OLS model and data are briefly discussed. A section on each of the other two models follows. A summary of empirical results from this study is presented next. The paper concludes with a discussion of the relative merits of the three models and implications for future research.

The OLS Model and Data

Parameters of acreage response functions were estimated for the major crops produced in each of six geographical production regions of the United States. Between two and six crop acreage response equations were included in each region, for a total of twenty-two regional equations. The acreage equations were fitted to annual time-series observations spanning the years 1945 to 1972.[2/]

The system of acreage response equations for any given geographical region may be written in the form

$$AC_i = f(P_i, P_j, FS, G_i) \quad (1)$$

where

AC_i = acreage of the ith product,

P_i = expected price of the ith product,

P_j = vector of expected prices of alternative products,

FS = supply of all factors of production,

G_i = relevant government program variables.

The expected prices were calculated using a distributed lag model with geometrically declining weights on the lagged prices. Factor supply was included as an independent variable instead of the traditional variable, input price, because one underlying assumption of the CET model is that production in a given year is undertaken with a fixed resource bundle. This assumption can be maintained by the inclusion of a measure of the quantity of inputs used in production. Input supply was used rather than input prices in the OLS and RLS models to maintain comparability among the three models. The relevant government program variables used in this study are the prices paid for diversion in excess of that necessary for program compliance at the minimum level of participation.[3/]

The RLS Model

Intriligator has shown (pp. 191-192) that theoretical supply functions are homogeneous of degree zero in prices (*i.e.*, producers react to changes in real prices, not changes in nominal prices). If a supply function is homogeneous of degree zero in prices, the sum of all its supply elasticities (own price, cross price, diversion price, and input price) is zero. The restriction of homogeneity can be enforced using exact linear restrictions, and the parameters of the model estimated using restricted joint generalized least squares [See Theil (pp. 312-17) for a discussion of the estimation techniques].

Consider the following acreage response function:

$$AC = X_1B_1 + X_2B_2 + \varepsilon \tag{2}$$

where

AC = a T x 1 vector of acreages (T = number of observations),

X_1 = a T x K_1 matrix of "nonprice" independent variables [FS in equation (1)],

X_2 = a T x K_2 matrix of "price" independent variables [P_i, P_j, G_i in equation (1)],

B_1, B_2 = K_1 x 1 and K_2 x 1 vectors of parameters to be estimated,

ε = a T x 1 vector of random disturbances.

Homogeneity of degree zero in prices implies that the restriction

$$RB = 0 \tag{3}$$

is true, where

$$R' = \begin{bmatrix} 0 \\ \hline \overline{P}_i/\overline{AC}_i \end{bmatrix}$$ (0 is a K_1 x 1 vector and $\overline{P}_i/\overline{AC}_i$ is a K_2 x 1 vector),

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

The K_2 x 1 vector in R', $\overline{P}_i/\overline{AC}_i$, is a vector of mean prices divided by mean acreages. Equation (3) multiplies these price-acreage ratios by the partial derivatives of acreage with respect to prices (the slope coefficients, B_2), sums them, and sets the sum equal to zero. Thus the sum of the price elasticities is forced to be zero.

If the restriction is enforced and is also true, restricted joint generalized least squares (RJGLS) coefficient estimates are unbiased and have smaller variances than OLS estimates. If the restriction is false, the RJGLS estimates are biased, but they do have smaller variances than OLS model estimates.

The CET Model

If it is assumed that at the beginning of each production period all producers are faced with a fixed bundle of resources for production and that several alternative crops can be produced from this resource bundle, the relative output of any two commodities may be expressed using a production possibilities function. A measure of the shape of this production possibilities function is the elasticity of product transformation, defined in equation (4).

$$E_{ij} = \frac{d(AC_i/AC_j)}{(AC_i/AC_j)} \div \frac{d(\partial AC_i/\partial AC_j)}{d(\partial AC_i/\partial AC_j)} \qquad i \neq j \tag{4}$$

where

E_{ij} = the elasticity of product transformation,

AC_i = the acreage of crop i.

The elasticity of transformation is the percent change in product mix divided by the percent change in the marginal rate of transformation.[4/] The OLS model in equation (1) can be expressed in the form of equation (5).

$$AC_{it} = X_{1t}\beta_1 + \sum_{j=1}^{n} A_{ij}P_{jt} + \varepsilon_{it} \text{ for } i = 1, 2, 3 \qquad (5)$$

where

AC_{it} = acreage of the $_i$th crop in year t for i = 1, ..., n,

X_{1t} = a vector of non-price independent variables in year t,

P_{jt} = the expected price of the $_j$th crop (or diversion) at time t,

ε_{it} = a random error term,

β_1, A_{ij} = parameters to be estimated.

This system of equations contains n^2 price parameters (A_{ij}'s) to be estimated (n for each of n crops). The advantage of the CET model is that it greatly reduces the number of parameters that must be estimated directly.

The CET model employs the following three assumptions:

1. The elasticity of product transformation between any two crops is constant over time. This is consistent with a wide range of production possibilities function shapes as well as with both product neutral and and product biased shifts in the production possibilities functions.[5/]

2. A competitive equilibrium in the product market exists, i.e., the marginal rate of product transformation equals the inverse product price ratio.

3. The acreage equations are homogeneous of degree zero in expected prices.

The use of these three assumptions makes it possible to express equation (5) as its CET transform, equation (6).

$$AC_{it} = X_{1t}\beta_1 + \sum_{j \neq i} E_{ij}S_{ijt} + \eta_{it} \text{ for } i = 1, 2, 3 \tag{6}$$

where

AC_{it}, X_{it}, β_1, and E_{ij} are as previously defined,

S_{ijt} = the mean acreage of crop i (over time) times the share of product j in the gross revenue of products i and j times the difference of the expected prices of crops j and i, each taken relative to its mean,

η_{it} = a random error term.6/

The reduction in the number of price parameters to be estimated directly stems from the following two relationships among the elasticities of transformation:

1. $E_{ij} = E_{ji}$, (7)

2. $\sum_{j=1}^{n} E_{ij} = 0$, (8)

where n equals the number of price variables in crop equation i. The first of these relationships, symmetry, is true by definition of the elasticity of transformation. The second, homogeneity, comes from the assumption that each equation is homogeneous of degree zero in prices. The incorporation of the above two relationships among the elasticities of transformation reduces the number of price parameters that need to be estimated directly from n^2 to $\frac{1}{2}(n^2-n)$ [see table 1].

The parameters of the CET transform of equation (1), the elasticities of product transformation, were estimated using restricted joint generalized least squares. The restrictions that were imposed consisted of the symmetry relationships of the elasticity of transformation. The CET model, equation (6), has

Table 1. Delineation of the price parameters that must be estimated directly in the CET model.

Equation	Price 1	Price 2	Price 3	...	Price n
1	h	e	e	...	e
2	S	h	e	...	e
3	S	S	h	...	e
⋮	⋮	⋮	⋮		⋮
n	S	S	S	...	h

e = coefficient must be estimated directly.

S = coefficient may be calculated indirectly using the symmetry relationship.

h = coefficient may be calculated indirectly using the homogeneity relationship.

homogeneity built into the model, and therefore it does not need to be retricted as in the RLS model.

As is the case in the RLS model, if the restrictions (symmetry) are true, the coefficients will be unbiased and have smaller variances than unrestricted estimates. If false, the estimated coefficients are biased, but still have smaller variances than the unrestricted coefficients. One major difference exists between the restrictions in the CET model and those in the RLS model. The homogeneity restrictions in the RLS model are based on economic theory and there is a possibility (although remote) that the theory might not be applicable to this problem and therefore the restrictions are incorrect. The restrictions on the CET model are true by definition, and they cannot be incorrect. Therefore, if the OLS model is correctly specified (no relevant explanatory variables have been omitted), and the symmetry restrictions of the CET model are rejected, one can only conclude that the CET transform is not applicable. The rejection of the symmetry constraints indicates that at least one of the assumptions of the CET model (E_{ij} constant over time, marginal rate of transformation equals inverse product price ratio, acreage equations homogeneous of degree zero) is violated.

Empirical Results

This section of the paper will compare the OLS, RLS, and CET models with respect to the number of correct signs on estimated coefficients and with respect to the reliability of their forecasts. A summary of the percentages of estimated coefficients with anticipated and unanticipated signs is presented in table 2. None of the three models is very satisfactory in terms of the percentage of the estimated coefficients that have the anticipated signs. The results of the OLS and RLS models are nearly identical. The RLS model has a slightly higher percentage of

Table 2. Comparison of OLS, RLS, and CET Models with Respect to Signs on Estimated Coefficients[a]

Model	Anticipated Sign		Unanticipated Sign	
	Significant	Nonsignificant	Significant	Nonsignificant
OLS	19.9	33.8	11.9	34.4
RLS	22.5	31.1	11.9	34.4
CET	37.9	23.2	24.2	14.7

a/ Numbers represent percentages of the coefficients falling into each category.

Table 3. Comparison of OLS, RLS, and CET models with respect to forecasting ability[a]

Model	Smallest Forecasting Error	
	1973	1974
OLS	6	4
RLS	6	7
CET	10	11

a/ Numbers represent the number of regional crops falling into each category.

significant coefficients than the OLS model, but these two models are identical with respect to their percentages of the coefficients with anticipated signs. The CET model results are somewhat better than those of either the OLS or RLS models. A higher percentage of the CET coefficients have the anticipated sign and a higher percentage of these are significant. Also, the CET model has more coefficients with "wrong" signs that are significant.[7/]

Table 3 contains a brief summary of the accuracy of forecasts produced by the three models. The CET model forecasted the most accurately for more regional crops than either the OLS or RLS models in both 1973 and 1974. In 1973, the OLS and RLS models performed equally well in this respect, but the RLS model gave more accurate regional forecasts than the OLS model in 1974. In addition, both of the models with restrictions (RLS and CET) greatly reduced the maximum forecasting error in both 1973 and 1974.

Summary and Conclusions

In this paper, a brief description and briefer summary of empirical results of two acreage response models that explicitly incorporate economic theory were presented. The empirical results of these two models (RLS and CET) were compared to the traditional ordinary least squares approach that ignores economic theory (OLS model). All three models suffered from specification problems and therefore the empirical results were somewhat questionable at best. The OLS and RLS models produced similar results with respect to both correct signs on the estimated coefficients and forecasting ability, but the RLS model was slightly better on all accounts. The CET model, on the other hand, exceeded the RLS and OLS models by a considerable margin, with respect to both producing anticipated signs on the estimated coefficients and forecasting. For twenty of the twenty-two regional acreage response equations, the null hypothesis that the equation is homogeneous of degree zero in prices was not rejected. The symmetry constraint in the CET model was, however, rejected for four of the six geographical production regions. Since a very large percentage of the signs on the estimated coefficients are not those anticipated, it is likely that the acreage response model is misspecified. Therefore, it is unclear whether the failure of symmetry in the CET model is due to model misspecification or inapplicability of the CET model. However, even though the results of this study are certainly not conclusive and model problems do exist, it appears that the use of economic theory directly incorporated into supply estimation will likely result in "better" aggregate acreage response estimates.

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Footnotes

1/ One noteworthy exception is the research by Powell and Gruen.

2/ For a complete description of the regions, crops, and data, see Whittaker.

3/ All variables in equation (1) are discussed in considerable detail in Whittaker.

4/ The elasticity of transformation (in the product-product world) is completely analagous to the elasticity of factor substitution (in the factor-factor world) which is a measure of the shape of an isoquant.

5/ See Powell and Gruen pp. 317-18 for a detailed discussion of the flexibility of the CET model.

6/ For a mathematical derivation of (7) and a more detailed discussion of the variables involved, see Whittaker (pp. 51-55).

7/ This fact is likely caused by a reduction in the standard errors of the estimated coefficients over either the OLS or RLS models due to the addition of the symmetry restrictions.

TEXAS FIELD CROP SUPPLY RESPONSE: APPLICATION OF A LINEAR CET SUPPLY MODEL

C. Richard Shumway and Anne A. Chang*

Introduction

In his investigation of crop supply in six U. S. regions, Whittaker applied three alternative supply models, unconstrained OLS, restricted least squares (constraining the supply function to be homogeneous of degree zero), and a linear CET commodity supply model. All three models provided empirical results with a large proportion of unexpected parameter signs. Of the three, the linear CET supply model provided results most consistent with theoretical expectations and also gave the most accurate predictions.

With this background concerning the relative attractiveness of the linear CET commodity supply model for multi-product supply response estimation, two objectives are sought in this study:

a. Using a linear CET supply model, estimate Texas short-run supply response, including relevant cross relations, for six major field crops. The crops considered (corn, cotton, hay, rice, sorghum, and wheat) represent 93% of 1971-75 Texas harvested acreage and 94% of value of production.

b. Examine the sensitivity of estimated parameters to alternative specifications in model scope and in non-price variables. The sensitivity analysis will be conducted to determine whether the proportion of unexpected parameter signs estimated with the linear CET supply model is highly dependent on the number of commodities in the model or on the number and definition of non-price independent variables included.

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Model Structure

One of the underlying assumptions of the linear CET commodity supply model is that producers act like profit maximizing perfect competitors. However, with much evidence that risk averse behavior is common among agricultural producers, risk is expected to also be an important behavioral variable. A utility function with arguments in price and risk can be specified, and the equilibrium conditions that would maximize utility can be derived in a straightforward manner. The problem faced here, though, is that even with the simple quadratic utility function and assuming zero covariances, a nonlinear estimation procedure would be required to estimate the risk aversion parameter of each linear CET supply equation.

Consequently, risk is included in this supply model in an ad hoc fashion. Risk, defined as subjective variance in own-product total returns, is added to the set of independent linear variables. Estimation of its parameter then is a simple test of the previous hypothesis that producers do operate as perfectly competitive profit maximizers. If the risk parameters are significantly different from zero, that hypothesis is suspect.

Also expanding the set of relevant production possibilities surface shift variables, the following linear CET commodity supply model results:

$$(1)\quad y_{i,t} = \alpha_{i,o} + \sum_{j\neq i}^{k} \tau_{i,j} z_{i,j,t} + \alpha_{i,1} v^{*}_{i,t} + \sum_{m=1}^{4} \alpha_{i,m+1} s_{i,m,t} + \mu_{i,t}$$

$$(2)\quad \tau_{i,j} \equiv \{d(y_{i}/y_{j})\}\,(y_{i}/y_{j})/\{d(\partial y_{i}/\partial y_{j})\}\,(\partial y_{i}/\partial y_{j})$$

$$(3)\quad z_{i,j,t} \equiv (\bar{y}_{i}\bar{w}_{i,j})\,\{(p^{*}_{j,t}/\bar{p}^{*}_{j}) - (p^{*}_{i,t}/\bar{p}^{*}_{i})\}$$

(4) $\bar{w}_{i,j} \equiv \bar{p}^*_j\bar{y}_j/(\bar{p}^*_j\bar{y}_j + \bar{p}^*_i\bar{y}_i)$

where $y_{i,t}$ is supply of commodity i in year t, $\tau_{i,j}$ is the estimated elasticity of transformation parameter between commodities i and j, z is the transformed expected price variable, v^* is the expected risk variable, s is a set of four production possibilities surface shift variables, μ is the error term, p^* is expected price, $\bar{p}^*$ is the mean of expected prices, $\bar{y}$ is the mean of quantities supplied, and other terms in equation (1) are estimated parameters.

The vector of parameters τ in equation (1) is designed to measure output response along the production possibilities surface to changes in relevant price ratios. Expected prices affect the slope of the iso-revenue line. The defined shift variables directly affect the position of the production possibilities surface. Because there is a yield response as well as an acreage response to product price changes (Houck and Gallagher), this study is concerned with output response. The shift variables selected, therefore, include variables designed to reflect input levels, technology, agricultural commodity policies, and weather.

Expected Prices

Expected prices are defined following Powell and Gruen as geometric lag functions of past prices, truncated on pragmatic grounds at seven years.

(5) $p^*_{i,t} = a_i\beta_i \sum_{\ell=1}^{7} (1-\beta_i)^{\ell-1} p_{i,t-\ell} + \varepsilon_{i,t}$

where β is the coefficient of price expectation, a is the weighting factor to adjust the weights on the seven lagged price observations to sum to 1.0, and ε is the error term.

Also folowing Powell and Gruen, the coefficient of price expectation (β_i) is estimated for each commodity independently of all other model parameters. However, the estimation procedure used in this study differs. The β_i are parametrized from 0.1 to 1.0 in 0.1 increments. β_i is selected on the basis of minimizing the sum of squares between expected and observed prices over the data period.

Risk

The risk measure is subjective variance in own-commodity total returns per acre defined following Just as a geometric lag function of past variance:

$$(6) \quad v^*_{i,t} = \phi_i \sum_{\ell=1}^{\infty} (1-\phi_i)^{\ell-1} (r_{i,t-\ell} - r^*_{i,t-\ell})^2$$

where ϕ is the coefficient of risk expectation, r is returns per acre, and r^* is expected returns per acre which in turn is a geometric lag function of past returns:

$$(7) \quad r^*_{i,t} = \Theta_i \sum_{\ell=1}^{\infty} (1 - \Theta_i)^{\ell-1} r_{i,t-\ell}$$

where Θ is the coefficient of return expectation.

Like expected price, subjective risk as defined by Just is an unobserved variable and must be estimated. Following his procedure, subjective risk is partitioned into two parts which he labels unobserved and observed risk. The first is really initial risk which in this study is defined as a function of variance in returns during the seven years prior to the first observation of other model variables (i.e., 1946). The second is a subsequent risk variable and is a function of return variance for each year from 1946 to t-1.

The annual returns per acre series and the parameters ϕ and Θ fully define the subjective risk variables. The parameters are obtained as maximum likelihood estimates using OLS. To secure the greatest efficiency in estimation, the risk variables are defined while estimating other parameters of the model. This is accomplished in a three-pass procedure:

a. Three stage least squares are used to estimate the parameters of equation (1) excluding risk.

b. Because the τ's are symmetric, they are treated as known structural constants, and parameters defining risk are obtained as maximum likelihood estimates using OLS.

c. With the risk variables defined, all parameters of equation (1), including $\alpha_{i,1}$, are re-estimated.

Input Level

Perhaps the most obvious variable determining the location of the production possibilities surface is the quantity of inputs available. A proxy for the aggregate level of inputs, i.e., acreage used in the production of the six field crops, is defined as the first shift variable.

The quantity of all inputs available for production is obviously not predetermined. Input supply can change in response to anticipated input price changes which in turn are affected by expected product price changes. Based on the partial adjustment premise, an input supply equation is estimated as a function of expected product prices (formulated as a single index in order to avoid collinearity problems) and lagged input level:

$$(8) \quad x_t = b_o + b_1 p_t^* + b_2 x_{t-1} + e_t$$

where x is acreage used in production of the six field crops, p^* is average expected price of the six field crops weighted by lagged output, and e is the error term.

Like product supply, input supply is also dependent on expected prices. However, due to the complexity of the product supply model, it is estimated separately from that model[1] using generalized least squares.[2] The short-run product supply response caused both by movements along the production possibilities surface and changes in quantity of inputs used may be derived by the chain rule from equations (1) and (8).

Technology

The second major variable that shifts the production possibilities surface over time is technology. Following Powell and Gruen, we use lagged output as a measure of capacity. The coefficient of adjustment reflects technological stickiness in the adjustment of supply.

Government Policies

Agricultural commodity policies directly affect the supply of several crops included in this analysis: corn, cotton, rice, sorghum, and wheat. The policy variables used follow Houck and Ryan, Ryan and Abel, and Penn. Two variables are defined: weighted diversion payment and weighted support price. Weighted diversion payment is added as the third variable shifting the production possibilities surface. Because of interpretation problems as well as high collinearity when both support price and market price are included as independent variables in supply models, the higher of weighted support price and expected price is included in the model as the price assumed to drive supply.

Weather

Because this study is concerned with output supply response rather than acreage response, weather is a relevant shift variable and is included as the final independent variable in the model. The weather proxy variable used is an adaptation of Stallings' index.

Hypotheses

If the market is efficient and all commodities are strictly competitive for a given set of resources, all $\tau_{ij}<0$. Powell and Gruen anticipated these conditions so strongly that they not only hypothesized negative τ's but also constrained all positive τ's = 0 in deriving direct elasticities. Negative τ's are hypothesized in this study also. However, positive τ's (which imply convex production possibilities curves) may actually occur in the real world. Increasing returns to scale in two technically independent commodities is the most likely condition for such occurrence, but other possible conditions will also produce positive τ's. Whether τ is negative or positive depends on the signs and relative magnitudes of the first and second partial derivatives of the multi-product production function.

Not accounting for input supply response, the hypothesis that $\tau_{ij}<0$ implies hypotheses that cross-product supply elasticities are less than zero and derived direct supply elasticities are greater than zero.

Hypotheses for non-price variable parameters are:

$\alpha_{i,1}=0$. If producers are risk neutral, product supply will be unrelated to own-product risk. However, if they are risk averse (preferers), product supply will decrease (increase) as risk increases. Actually, two risk parameters are estimated, on observed and on unobserved risk. Both parameters are expected to be zero if producers are risk neutral.

$\alpha_{i,2}>0$. As the input level increases, the production possibilities surface shifts outward and product supply should increase.

$\alpha_{i,3}>0$. As technology develops as implied by lagged product supply, the production possibilities surface shifts outward and current product supply should increase.

$\alpha_{i,4}<0$. As the diversion payment for crop i increases, the incentive to decrease the acreage used in its production also increases.

$\alpha_{i,5}>0$. Since the weather index is positively related to the ratio of observed and expected yield, the higher the index, the further outward is the production possibilities surface and the higher is product supply.

Empirical Results

Input Supply

The estimated parameter magnitude on the input supply price variable is only half as great as the standard error, thus leading to the conclusion that input supply is insignificatly related to the weighted expected product price. Estimation of three alternative input supply formulations leads to the same conclusion. These formulations include (a) using Texas and Oklahoma index of total inputs used for agricultural production as the proxy for input level and/or (b) deleting the lagged dependent variable. Consequently, it is inferred that short-run product supply response along the production possibilities surface approximates total short-run supply response of these commodities.

Initial CET Results

Parameter estimates of the inital CET model are reported in table 1. Unexpected signs are obtained on 24% of the estimated price and shift variable parameters, including 53% of the τ's and 5% of the shift variable coefficients. Of the parameters with expected signs, 61% have t-values $\geq$ 2.0 including 57% of the τ's. Of parameters with unexpected signs, only 22% have t-values $\geq$ 2.0, including 13% of the τ's.

Table 1. Initial CET Model Estimates

			CET Price Variables						Risk[a]				Shift Variables			
									Variables		Parameters		Land	Lagged Output	Weighted Diversion	Weather
Commodity	Unit	Intercept	Corn	Cotton	Hay	Rice	Sorghum	Wheat	Unobservable	Observable	ϕ	Θ	Input	(Technology)	Payment	Index
	(1,000)															
Corn	bu.	-32,968		-.957	.632	.081	.321	.080	-95,536	-5.416	.1	.3	1.177	1.130	-23,027	700
		(14,514)[b]		(.229)	(.289)	(.309)	(.500)	(.433)	(13,379)	(1.047)			(.773)	(.095)	(18,052)	(99)
Cotton	lb.	-388,936			.034	-.003	-.579	-1.252	-927,802	17.561	.5	.1	62.370	-.165	-4,409,032	13,640
		(275,898)			(.115)	(.096)	(.154)	(.151)	(130,540)	(7.746)			(11.107)	(.047)	(726,336)	(1,182)
Hay	ton	-1,248				-.486	.256	-.458	-2,445	-.038	.1	.3	.014	.529		35
		(654)				(.166)	(.225)	(.244)	(341)	(.015)			(.030)	(.117)		(5)
Rice	cwt.	-15,495					.020	-.313	-2,285	.005	.5	.3	.223	.587		185
		(4,632)					(.185)	(.257)	(1,146)	(.003)			(.138)	(.071)		(23)
Sorghum	bu.	-35,448						.312	-214,374	2.454	.1	.3	1.661	.052	-47,349	2,851
		(48,659)						(.321)	(33,656)	(1.209)			(2.721)	(.075)	(59,877)	(366)
Wheat	bu.	-106,702							8,502	5.869	.9	.1	2.956	.115	-9,188	1,064
		(12,723)							(7,422)	(2.889)			(.607)	(.054)	(5,609)	(64)

[a]The risk variables are defined by ϕ and Θ, estimated in the second pass, and by returns per acre. See Green for variable definition and estimation procedure.

[b]Estimated standard errors are in parentheses.

Nine of the 12 estimated risk parameters have t-values $\geq$ 2.0 and one more is nearly 2.0. Although this constitutes an imperfect test, the maintained hypothesis of the linear CET supply model that producers operate as though they were profit maximizing perfect competitors appears highly suspect. Five of the estimated parameters are positive (implying risk preference), but only three have t-values $\geq$ 2.0. All seven t-values of the negative parameters exceed 1.99. Consequently, the evidence supporting an alternative hypothesis that producers are risk averse appears to be greater than evidence implying that they are either risk preferers or risk neutral.

To test the hypothesis that the τ's are symmetric, the linear CET commodity supply model is re-estimated without the symmetry constraint on the price coefficients. The symmetry hypothesis is not rejected at the 95% level of significance by the test statistic, $F_{15,116}=1.33$. Thus, opposite from Whittaker's findings, the data used in this study do not cause rejection of the symmetry condition of this expanded CET model.

The data period used in estimating the parameters of the model is 1946-76. Commodity supply for 1977 is predicted using the estimated coefficients. The predictive accuracy of this model is not particularly good. Percent error in prediction ranges from 4% to 45%, and Theil's inequality coefficient ranges from .30 to .59.

The symmetry test does not compel rejection of the linear CET commodity supply model as a valid descriptor of Texas field crop supply response. Unexpected signs cannot totally be ruled out on theoretical grounds. Further, a large proportion of unexpected parameter signs and values is consistent with the findings of prior multi-product supply studies. However, the risk neutral hypothesis of the CET model is rejected. Further, the basis for the remaining stated parameter hypothesis is strong and gives substantial reason to suspect spurious estimation of some parameters. High collinearity among independent variables is a particular problem limiting efficient multi-product supply response estimation. Therefore, the remainder of this paper is devoted to an evaluation of the sensitivity of unexpected parameter signs to alternative specifications and model scope.

Sensitivity to Reduced Model Scope

The two crops with smallest harvested Texas acreage, rice and corn, are deleted from the model and the remaining parameters re-estimated. Reducing the number of commodities in the model by 33% reduces the number of price coefficients (τ's) requiring estimation by 60%, i.e., from 15 to 6. A much smaller proportion of parameters estimated with this reduced model have unexpected signs (14%) than parameters estimated with the initial model (see table 2, model 2). The proportion of unexpected τ signs is substantially reduced (33%) while the proportion of unexpected shift variable coefficients is approximately the same (7%).

Sensitivity to Alternative Risk and Shift Variable Definitions

Although the generic variables included in the model can be strongly defended on conceptual grounds, the specific working variables selected to represent each concept are less defensible. In this portion of the sensitivity analysis, three variables are re-defined one at a time and all model parameters are re-estimated.

With risk re-defined as a three-year moving standard deviation of total returns per acre (model 3), the percent of unexpected parameter signs is the same as with the initial model (24%). This includes 33% of the τ's and 18% of the shift variable coefficients. A substantial

Table 2. Unexpected Parameter Signs, 12 Alternative Model Specifications

Variable	Model[a]											
	1. Initial	2. No Rice or Corn	3. Redefined Risk Variable	4. Redefined Input Variable	5. Redefined Technology Variable	6. No Risk, WDP, WI	7. No Risk, WDP	8. No Risk, WI	9. No WDP, WI	10. No Risk	11. No WDP	12. No WI
A. CET Price Variables:[b]												
Corn - Cotton		-			X							
Corn - Hay	X[c]	-	X	X	X	X	X	X	X	X	X	X
Corn - Rice	X	-	X			X	X	X			X	X
Corn - Sorghum	X	-		X	X						X	
Corn - Wheat	X	-	X			X	X	X		X	X	X
Cotton - Hay	X			X			X			X		
Cotton - Rice		-										
Cotton - Sorghum												
Cotton - Wheat												
Hay - Rice		-										
Hay - Sorghum	X	X		X	X						X	
Hay - Wheat												
Rice - Sorghum	X	-	X	X	X	X	X	X	X	X		X
Rice - Wheat		-			X							
Sorghum - Wheat	X	X	X		X	X	X	X	X	X	X	X
Percent Unexpected Price Parameters	53	33	33	33	47	33	40	33	20	33	40	33
B. Shift Variables:												
Input:												
Corn		-	X	X								
Cotton												
Hay			X	X		X	X	X		X	X	
Rice		-				X		X	X			X
Sorghum			X	X		X	X	X		X		
Wheat				X								
Technology:												
Corn		-			X							
Cotton	X	X	X	X	X				X	X	X	X
Hay												
Rice		-										
Sorghum												
Wheat												
Weighted Diversion Payment:												
Corn		-				-	-	X	-	X	-	X
Cotton						-	-		-		-	
Sorghum						-	-	X	-		-	
Wheat		-				-	-	X	-		-	X
Weather Index:												
Corn		-				-		-	-			-
Cotton						-		-	-			-
Hay						-		-	-			-
Rice		-				-		-	-			-
Sorghum						-		-	-			-
Wheat						-		-	-			-
Percent Unexpected Shift Parameters	5	7	18	23	9	25	11	38	17	18	11	25
C. Percent Unexpected Parameters, All	24	14	24	27	24	30	24	35	19	24	24	29

[a] WDP is weighted diversion payment; WI is weather index.

[b] The first CET price variable parameter measures the response in corn supply to a change in cotton price.

[c] Notation: X is unexpected parameter; - is a parameter not estimated.

improvement in expected signs is obtained among the τ's.

Because acreage represents only one input used in agricultural production, an alternative variable, index of total inputs used for agricultural production in Texas and Oklahoma, is substituted in model 4. Estimation of this model results in more unexpected signs (27%), but with the same improvement in estimated τ's (33% with unexpected signs).

The justification for lagged output as a proxy variable for technology is perhaps the most tenuous variable in the specified model. Substituting time as the technology proxy (model 5) results in the same proportion of unexpected parameter signs as in the initial model (24%) but with minor reduction in unexpected τ's (47%).

Re-defining any one of these three variables results in improvement in the proportion of expected τ signs (with the alternative risk and input variables providing much improvement). There is no improvement in the shift variables. Overall, the proportion of unexpected parameter signs changes little.

Sensitivity to Deletion of Risk and Shift Variables

The final sensitivity analysis focuses on reducing alternative combinations of the risk and two shift variables (policy and weather). With each of these seven models (models 6-12), the percent of unexpected τ estimates is lower than in the initial model, ranging from 20 to 40%. However, unexpected shift variable parameters are greater in all cases (11 to 38%), and the percent of all parameters with unexpected signs varies in both directions (19 to 35%).

Evaluation of Sensitivity Analysis

The initial model gives the largest percent of unexpected τ signs.

Reducing model scope, re-defining variables, and deleting combinations of selected variables improves those results, but does not always increase the proportion of expected signs on other model parameters.

Reference to the CET price variable rows of table 2 identifies 5 estimated τ's (cotton-rice, cotton-sorghum, cotton-wheat, hay-rice, and hay-wheat) that are negative in all 12 model specifications. Two more are almost always negative (positive in only one model); three are generally negative (positive in four or five models); two are generally positive (negative in three or four models); two are almost always positive (negative in only one model); and one is always positive. It is, therefore, concluded that estimated τ signs are generally quite stable to a wide variety of alternative model specifications. It is further concluded that the elasticity of transformation between corn and hay, between rice and sorghum, and between sorghum and wheat may in fact be positive. The first is positive in all model estimates.

Evaluation of Profit Maximization Hypothesis

Estimated risk parameters with t-values $\geq$ 2.0 are identified in table 3 for each of the eight models containing risk variables. In six of the models, at least 2/3 of the parameters are in this category. In none do fewer than 3/8 of the parameters have τ-values $\geq$ 2.0, thus challenging the hypothesis of profit maximizing perfectly competitive behavior. From all but one model, the evidence supporting risk averse behavior is substantially greater than that supporting the notion of risk preference.

Elasticities of Supply

Two sets of supply elasticities are reported in table 4. The range is based on the initial model and model 9 estimates. The latter

Table 3. Risk Parameters with t-values ≥ 2.0, 8 Alternative Model Specifications

Risk Variable	Model[a]							
	1. Initial	2. No Rice or Corn	3. Redefined Risk Variable	4. Redefined Input Variable	5. Redefined Technology Variable	9. No WDP, WI	11. No WDP	12. No WI
Unobservable:								
Corn	–[b]	/	/	–	–		–	
Cotton	–	–	/	–	–	–	–	–
Hay	–	–	/	–		–	–	–
Rice		/	/	–	–	–	–	–
Sorghum	–	–	/	–		–	–	–
Wheat			/	+	+	–		–
Observable:								
Corn	–	/		–	+	+	–	+
Cotton	+		+		+		+	
Hay	–		+	+	–	+	+	+
Rice		/						
Sorghum	+					–		–
Wheat	+		+	+	–			
Percent summary:								
$t < 2.0$	25	62	50	25	33	33	33	33
$t \geq 2.0$								
Negative	50	38	0	50	42	50	50	50
Positive	25	0	50	25	25	17	17	17

[a] WDP is weighted diversion payment; WI is weather index. Models 6, 7, 8, and 10 are not reported because risk variables were not included.

[b] Notation: – is a negative parameter and + is a positive parameter, each with a t-value ≥ 2.0; / is a parameter not estimated.

Table 4. Range in Supply Elasticities Estimated by the Initial Model and the Model with the Smallest Percent of Unexpected Parameters

	Elasticity with Respect to the Price of					
Commodity	Corn	Cotton	Hay	Rice	Sorghum	Wheat
Corn	-.10 to .15	-.85 to -.30	.34 to .95	0 to .05	-.45 to .26	-.10 to .05
Cotton	-.10 to -.04	.52 to .66	-.03 to 0	-.03 to 0	-.38 to -.20	-.22 to -.19
Hay	.29 to .81	-.19 to .03	.02 to .03	-.27 to -.16	-.25 to .20	-.28 to -.23
Rice	0 to .03	-.16 to 0	-.21 to -.12	.18 to .34	.02 to .37	-.27 to -.17
Sorghum	-.10 to .06	-.72 to -.38	-.07 to .05	.01 to .13	.17 to .58	.09 to .19
Wheat	-.05 to .03	-1.03 to -.86	-.18 to -.15	-.22 to -.14	.22 to .45	.84 to 1.10

model is the one with the lowest percent of unexpected price parameter estimates. Among the six-commodity models, it also has the fewest unexpected parameters on all variables combined. It has two shift variables deleted from its structure, weighted diversion payments and weather.

Corn is the only crop with a negative derived direct elasticity estimate. Although a fifth of the elasticities reverse signs between the two models, 4/10 of the elasticities vary by less than a magnitude of 0.1 Another 3/10 differ by no more than 0.2 from each other. None vary by a magnitude greater than 0.8.

Conclusions

With a large number of alternative model specifications examined in the sensitivity analysis, the percent of unexpected parameter signs ranges from 14 to 35%. The percent of unexpected τ's differs more, from 20% to 53%, (the former with model 9 and the latter with the initial model). Unexpected shift variable parameters range from 5 to 38%.

Although a considerable number of unexpected parameter signs are estimated, the symmetry test does not compel rejection of the symmetry condition of the linear CET supply model as a valid specification of Texas field crop supply response. The estimated risk parameters do challenge the hypothesis that producers act like profit maximizing perfect competitors.

Based on consistency in sign estimation, it is tentatively concluded that 1/5 of the elasticities of transformation are in fact positive. This leads to the conclusion that their cross-product supply elasticities are also positive since estimated input response to product prices is negligible. The direct supply elasticities derived from the initial model are all positive. All but one derived from the model with the fewest unexpected parameter signs are also positive. Although generally smaller in absolute magnitude than the direct elasticities, the magnitudes of estimated cross-elasticities are often substantial, suggesting that alternative product prices play an important role in determining supply response.

Footnotes

1. It is obvious that separate estimation of these two models may introduce some estimation inefficiency because of the high possibility of contemporaneously correlated error terms.

2. Because the lagged dependent variable is included as an independent variable in the input supply model, the Cochrane-Orcutt and Cooper transformations are imposed in the generalized least squares procedure to obtain efficient parameter estimates in the presence of autocorrelated errors and to obtain asymptotically unbiased estimates of the variance-covariance matrix.

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Consistent Output and Input Choice Functions
for Multiproduct Technologies

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In my introductory remarks a consistent choice model was defined as one which incorporated or could be tested for consistency with all implications of the maintained hypothesis concerning the way in which choices are made. These included linear homogeneity in prices and symmetry conditions as well as specification of functional forms which are consistent with the hypothesized technical relation among outputs and inputs. Whittaker and Shumway have each presented applications which employ supply functions which are consistent with the existence of multiproducts and may be restricted to satisfy symmetry of elasticities of transformation as well as linear homogeneity in output prices. However, their methodologies fail to accommodate the existence of input choice which is simultaneous with output supply decisions. Further, the C.E.T. model imposes a particular functional form on the supply functions which is only consistent with a particular characterization of production. Do we know *a priori* how the production function is shaped?

As an example application, I shall deal with agricultural production decisions. Suppose we hypothesize that the firms which we observe are attempting to maximize their expected profits where output prices are uncertain and they are constrained by a fixed total acreage of land available, binding government acreage controls on certain crops, and a multiple product technology. If we solve their maximization problem

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we derive a set of output supply functions and a set of variable input demand functions. By substituting these choice functions into the objective we derive an expected profit function which relates expected returns to fixed factors to the determinants of choice: expected prices and flows from fixed factors. That is, if we define expected profit as:

$$1a)\quad E(\pi) \equiv E(P'Y - r'X)$$

and technology by:

$$b)\quad Y_1 = G(\hat{Y}, X, \theta)$$

where P is a m x 1 vector of random prices for net outputs Y,

r is a n x 1 vector of prices for net inputs X, and

θ is a r x 1 vector of flows from fixed factors.

The necessary conditions for determining a maximum for 1a) subject to 1b) are:

$$2)\quad E(P_i)\ \frac{\partial G}{\partial Y_i} + E(P_i) = 0 \qquad i = 2, \ldots m$$

$$E(P_i)\ \frac{\partial G}{\partial X_h} - r_h = 0 \qquad h = 1, \ldots n$$

When the transformation function G satisfies the appropriate convexity conditions, these necessary conditions are also sufficient and may be solved for the optimal levels of inputs and outputs. We shall call these <u>provisional choices</u> since they represent planned outputs and input utilization based upon expectations concerning the future market. Thus, we shall write the provisional choices of outputs (Y_i^*) and inputs (X_h^*) which represent solutions to 1b) and 4) in vector notation as:

$$3)\quad Y_i^* = Y_i[E(P),\ r,\ \theta,\ T] \qquad i = 1, \ldots m$$

$$4)\quad X_h^* = X_h[E(P),\ r,\ \theta,\ T] \qquad h = 1, \ldots n$$

By substitution of these provisional choice functions into the definition of expected profit we obtain the provisional profit function which relates maximum values of 1) to information variables hypothesized to determine decisions:

$$5)\quad E(\pi)^* \equiv \sum_i^m E(P_i)\ Y_i^* - \sum_h^n r_h X_h^*$$

$$= \pi[E(P),\ r,\ \theta,\ T]$$

The expected profit function is the basis for a convenient derivation of the set of choice functions. By differentiating it once by the expected price of each net output we obtain the supply function for each net output. By differentiating it by each net input price we obtain the negatives of the net input demand functions. That is, by Hotelling's lemma:

$$6)\quad \frac{\partial \pi}{\partial P_i} = Y_i^*(P,\ r,\ \theta,\ T)$$

$$7)\quad \frac{\partial \pi}{\partial r_h} = -X_h^*(P,\ r,\ \theta,\ T)$$

Finally, by differentiating the profit function twice by any pair of prices we obtain the comparative-statics of choice, the basis of calculation of elasticities of choice. Furthermore, like the choice functions, the expected profit function also reflects the structural characteristics of technology.

For multiple product technologies we may have jointness in inputs, homotheticity, homogeneity or separability as structural characteristics. If the technology is smooth and highly regular we might have elasticities of substitution and transformation which are constant everywhere along the production surface. However, whether we do is an empirical question. Thus, we shall employ a functional form for the expected profit function which may be tailored to be consistent with alternative structures of production through the imposition of testable restrictions on its parameters. The form I have chosen is the translog form introduced by Christensen, et al. in 1971. By taking the logarithmic derivative of the translog profit function consistent with 5) we have:

$$8)\quad \frac{E(P_i)Y_i^*}{\pi} = \alpha_i + \sum_{j=1}^{m} \beta_{ij} \ln E(P)_j + \sum_{h=1}^{n} \gamma_{hi} \ln r_h + \sum_{r=1}^{s} \phi_{ri} \ln \theta_r + \varepsilon_i \qquad i = 1, \ldots m$$

$$\frac{r_h X_h^*}{\pi} = - \alpha_h - \sum_{i=1}^{m} \gamma_{hi} \ln E(P)_j - \sum_{k=1}^{n} \lambda_{kh} \ln r_k - \sum_{r=1}^{s} \phi_{rh} \ln \theta_r - \varepsilon_h \qquad h = 1, \ldots n$$

These represent m + n equations which may be employed to estimate choice elasticities. Looking at them another way, they are simply supply and input demand functions scaled by the appropriate price and the reciprocal of expected profit. For data, we require only revenues and expenses for each variable product employed or produced. We do not require expenses by enterprise. In addition, we require a measure of expected output prices, variable input prices and flows from fixed factors.

To proceed, a post-war [1948-1970] sample of state level aggregate data from North and South Dakota was employed. Variable inputs and outputs were aggregated in the following groups: labor (LAB), capital services (building and machinery) (CAP), fertilizer (FERT), petroleum products (PET), materials (MAT), food grains (FG), feed grains (AO), and livestock (LTK). Fixed factors were defined as including technology (measured by a time trend) (T), total farmland available for cultivation (LLDI), pre-season precipitation (R), wheat allotment (A), and feed grain base (B).

In order that these equations be consistent with the maintained hypothesis of expected profit maximization the provisional profit function must be homogeneous of degree one, monotonic in prices and Young's theorem must hold for its parameters. With these conditions imposed the share equations were estimated using an iterative Zellner estimator to ensure that any cross-equation correlation in the error terms be taken in account.

As written this system is consistent with any functional form of production. To proceed, I determined sets of parameter restrictions which are consistent with various characteristics of production. These were then imposed on a model restricted to be consistent with symmetry and linear homogeneity in prices and their statistical significance tested using joint F-tests. Table 1 reports the results of such tests.

In summary, I found that strong evidence that the present sample rejects the hypotheses of non-jointness, homotheticity and, therefore, homogeneity as well as separability of all outputs, of crops, fertilizer and materials and materials and petroleum.

To proceed, I employed a model restricted only for symmetry and linear homogeneity in prices to estimate the behavioral elasticities. The means of these are reported in Table 2. Although the limited time available prohibits a thorough discussion of these results several general points should be made. First, because a functional form was employed which allows estimation of elasticities of choice at each point on the production or profit function elasticities are expected to vary over time and cross-section. This follows from the fact that a smooth and regular production function was not assumed. Secondly, if we recall that the comparative-statics of choice are indeterminant in sign except for own price effects (see e.g., Hicks [1947]), then the signs of parameters presented in Table 2 can not be compared against a theoretically grounded hypothesis. For example, Powell and Gruen [1968] argued that the effect of change in the price of output j on the supply of output i should be negative. Although at first thought this would agree with our intuition concerning substitution, we must remember that our comparative-statics are not ceterus paribus for other choices, but only for variables which are exogeneous determinants of them. Thus, when a price changes all product (input and output) choices may be expected to change. If we hope to base our intuition concerning the effect of a price change on a two-dimentional view of a production possibilities frontier drawn in (Y_i, Y_j) space, we must not forget that adjustment in choices other than Y_i and Y_j leads to a shifts and twists of the frontier. The observed response of Y_i to the change in P_j includes, then, not only a substitution effect, but the effect of any positional change in the frontier. As Table 2 illustrates, own price elasticities are consistent with theoretical hypotheses. In general, short-run input and output choices appear to respond to price changes although some of these choices are quite inelastic.

Table 1. Test Statistics for Alternative Restrictions on the Structure of Production

Hypothesis	Non-Jointness	Homotheticity in Output Prices	Homogeneity with respect to variable inputs	Separability in Prices				
				All Outputs	Crops	Fertilizer and Materials	Capital and Petroleum	Materials and Petroleum
Number of Restrictions (q)	3	3	9	24	22	22	22	22
$F_{q,217}$	9.52	6.23	4.44	36.8	35.59	1.30	5.71	4.40
$F^{.01}_{q,217}$	3.91	3.91	2.53	1.92	1.96	1.96	1.96	

Table 2. Means of Price Elasticities of Choice, 1950-1970

Product \ Price	Wheat FG	Feed Grain AO	Livestock LTK	Labor LAB	Fertilizer FRT	Capital Services CAP	Materials MAT	Petroleum Products PET
Wheat FG	.3997*	.9239	.9976	-.5449	-.0438	-1.0178	-.5201	-.1947
	.7895**	-.6999	1.6151	.1241	.0612	-.6228	-1.2742	.0068
AO	.7858	.7354	.8282	-.7165	-.0667	-1.0567	-.2608	-.2487
	-.1200	.6379	1.4752	-.6143	-.03166	-.7090	-.4697	-.1713
LTK	1.0779	1.0463	.5551	-.6896	-.0859	-1.0978	-.5203	-.2857
	.2334	.9645	1.0110	-.5786	-.0387	-.7163	-.6994	-.1785
LAB	.8269	1.349	1.0251	-1.0161	-.0540	-1.6298	-.1963	-.3107
	-.0714	1.1902	1.7068	-.7908	-.0163	-1.3828	-.3940	-.2442
FRT	.7204	1.4586	1.5601	-.6115	-1.3774	-.0287	-.7775	-1.1254
	-1.100	1.7309	3.660	-.2950	-2.1562	5.5094	-2.0117	-3.3585
CAP	1.0835	1.3145	1.0809	-1.0146	-.0107	-1.6558	-.4707	-.3326
	.2262	1.1555	2.7970	-1.0845	.06338	-1.1034	-.7451	-.3090
MAT	1.4607	.8487	1.3472	-.3953	-.1077	-1.2382	-1.6988	-.2167
	.4683	.8367	1.8981	-.3978	-.0517	-.8046	-1.8127	-.1378
PET	.9419	1.4262	1.3041	-.9141	-.2625	-1.5534	-.3785	-.5638
	.0015	1.3339	2.1536	-.9408	-.3415	-1.5069	-.6027	-.0970

* North Dakota ** South Dakota

Because elasticities are based upon non-linear combinations of normally distributed parameters, they are not in general normally distributed. Therefore, t-values are not presented.

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