Staff Paper #40 December 1980

Measurement and Forecasting of Agricultural Productivity

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1. Introduction

If future increases in the world-wide demand for food are to be satisfied at socially acceptable price levels and in the absence of a triage system based on ability to pay, then the level of U.S. as well as world supply of food must increase. Furthermore, if these objectives are to be met consistently over a continuum of harvests rather than under the expected conditions of some long-run equilibrium planning period, then the timing of supply increases is also critical. The level of supply at any point in time is argued by economists to depend upon 1) the state of technology or technical knowledge employed to combine resources and 2) the levels at which resources are committed. It follows that changes in supply can only come from changes in the technology employed or in the levels of input resources.

The nature of the technology employed in production (applied technology) must be distinguished from the best practice technology known at a point in time because new technology is not instantaneously diffused throughout an economy. Changes in applied technology can be argued to depend upon changes in the rate at which new technical knowledge diffuses (the diffusion rate) and/or on changes in the best practice technology. The diffusion rate depends upon a complex set of social, political, economic as well as physical issues at both the producer and the market levels. The rate at which any new technology will diffuse through an economy would be extremely difficult to predict. Any prediction would carry with it a large risk of error. Changes in the best practice technology likewise are highly unpredictable due to their dependence upon the serendipity of discovery and innovation as well as its coordination with needs. It is fair to conclude that a prediction that future changes in applied technology will allow supply to keep pace with growth in demand would carry with it a high risk of error. The objective of this paper is to consider the problem of measurement and prediction of the role of changes in applied technology in changing product supply in agriculture. Although the explanation of applied technical change could be based upon a knowledge of or separate explanation of innovation effort and diffusion processes the present paper will focus only on measurement of applied technical change and measurement of supply increases resultant from changes in inputs which are under direct control by managers of production. Here after, we drop the label "applied" and use the briefer label of only technical change or productivity to mean changes in applied technology or applied productivity.

The relevance of the predictability of technical change and its impact on supply for the management of agricultural land can be seen if the role and predictability of the levels of resources committed to the production of food are considered. Broadly speaking, these resources can be classified as either 1) uncontrollable by the production manager over any duration of time, or exogeneous inputs, or 2) controllable within a short period of time, or endogeneous inputs. Climate can be placed within the former category, while in the latter might be placed inputs which depending upon economic feasibility are allocated to the production of food. Obviously, if a particular input's availability is so restricted as to lead to non-price rationing then that input could also be thought of as in the first category. This possibility implies marketed inputs would not necessarily fall into the endogeneous input category. By definition the changes in use of exogeneous inputs is unpredictable. In contrast, changes in utilization of endogeneous inputs depend upon the interaction of relative prices and the contribution of the inputs to increasing output. Forecasts of the growth in utilization of these inputs depends upon forecasts of future relative prices as well as the highly uncertain nature of technology which will be employed in the future. Furthermore, the ability to forecast utilization depends upon the input remaining endogeneous. The experience of the seventies serves as a sufficient basis to conclude that future prices and even physical availability of particular inputs cannot be predicted with certainty.

Within this setting of uncertainty, the broadly classified land input to agriculture could play an important role. Whether inputs derived from land resources are direct substitutes or complements for marketed inputs, they offer flexibility in meeting future needs. Furthermore, the physical and economic feasibility of adoption of new technologies is conditioned by the nature of land resources available. The conclusion can be drawn that in the face of substantial uncertainty concerning the availability and timing of the occurrence and adoption of new technologies as well as the economic feasibility of expanded use of marketed inputs, the management of land resources provides a form of insurance that future demands can be met. Thus, while the present paper will not address issues of land management, it will attempt to outline the nature and sources of uncertainty characterizing forecasts of changes in technology; and thereby, lend support for the importance of land management policy.

Perhaps the greatest reflection of the uncertainty which characterizes forecasts of future technical change is the disagreement among the forecasts of researchers studying the problem. As indicators of the level of technology these studies focus on a variety of measures of the productivity of resources employed in food production. The definition of the productivity measure which best indicates the level of technology is itself a crucial issue in the forecasting of technical change. Past studies have employed two measures: yield per acre and total factor productivity. The usefulness of each of these definitions will be considered in more detail in the next section. Evans [1980], and Jensen [1978] have argued that biological limits are soon to be met if the current process of change in productivity is extrapolated. Their case is based on the presumptions that 1) the contributions of past genetic innovation, improved management practices, and increased use of energy intensive inputs have been largely exhausted and 2) that the potential for future contributions from changes in these categories are unlikely. An opposing view has been taken by Ruttan [1980] and Heady [1980] who have argued that past innovations have not even been completely adopted in the U.S. much less in LDC counties. In addition, they argue that future gains in productivity can be expected to follow from research and development expenditures just as they have in the past. Lu and Quance [1979] also emphasize the importance of research and development expenditures as the fuel for future improvements in productivity. The debate appears to hinge upon the forecast of the return in increased productivity which is likely to result from continued research and development expenditures. The pessimists believe no return can be expected because of the existence of biological limits; the optimists believe those limits are illusionary.

2. Definitions and Measures of Technical Changes

2.1 Representation of the Effects of Technical Change on Production Functions

Before proceeding to consider the measurement or forecasting of technical change, it is useful to review essential definitions. The agricultural production process must be distinguished from the traditional manufacturing process on two bases. First, agricultural processes typically result in multiple outputs being produced by a single firm. In part, this results from attempts to fully utilize available resources and from efforts to reduce risk by diversification. However, there also exist physical benefits from diversification which mean that inputs employed in the production of one output also indirectly affect the production of other outputs (e.g. use of pesticides). Economists label such production processes as joint. Secondly, agricultural production relies and is affected by inputs which are not under the control of the production manager (or producer) such as climatic events, and pest or weed infestations. In order to discuss concepts and measurement of technological change, it is necessary to introduce mathematical notation concerning the relationship between inputs and outputs. Given that the agricultural firm employs a multitude of inputs to produce a variety of outputs, it is necessary to employ a generalization of the traditional single output production function used in microeconomics. To describe the outputs and inputs, the following vectors will be employed:

Y: 1 x m vector of outputs, indexed $i=1,\ldots m$

X: 1 x n vector of inputs which are variable in the short-run, indexed by $h=1,\ldots n$,

θ: 1 x p vector of inputs which are fixed in the short-run, indexed $r=1,\ldots p$, and

Z: a general vector of products, i.e.$(Y:X:\theta)$.

Using this notation, the multiple output, multiple input production function, or more generally, the product transformation function will be written:

$$2.1) \quad Y_1 = G(\hat{Y},X;\theta)$$

where $\hat{Y}$ represents the vector $(Y_2,\ldots Y_m)$.

Two approaches have been taken to specify the way in which the input-output relation is affected by technical change. Solow [1959, 1962] and Salter [1960] have argued that technical change is typically incorporated or "embodied" in new products (or new vintages of existing products) and, therefore, the input-output relation is specific to the vintage of the inputs involved. Technological change can be embodied in new capital goods or in quality improvements in other inputs such as labor, pesticides or seeds. An alternative to the embodiment hypothesis is that of disembodied technical change. That is, changes in the input-output relation result from changes in the technique of combining inputs, or know-how. In agriculture, such changes would include changes in cultural practices such as spacing of plants or timing of activities. Jorgenson [1966] notes that the possibility of embodied technical change implies that investment, research, and development expenditures represent possible indicators of technical change. Education and expenditures for the dissemination of information would play a role in fostering both types of technical change.

The effect of either form of technical change can be stated in terms of instability of the functional form of the product transformation function. By the addition of a time subscript to the transformation function to indicate that at any time t during the period of time $T_H \varepsilon T$, the function G can be thought of as conditioned by the existing state of knowledge denoted by the vector K_H. That is,

$$2.2) \quad Y_{1t} = G_t(\hat{Y}_t, X_t; \theta_t, K_H) \quad \forall\ t \,\varepsilon\, T_H .$$

As written, the functional relation between inputs and outputs in 2.2) captures the essence of the argument presented in the introduction. Changes in the levels of outputs can occur as a result of changes in the level of technology employed as represented by the function G_t or from changes in the levels of inputs (X_t, θ_t) employed in production. By definition, inputs in the vector X_t are controllable in the short-run by producers while those in θ_t are inflexible in the short-run. As noted the vector θ_t contains input flows which can be changed only in the long-run (e.g., buildings and equipment or the flows from land resources) as well as those exogeneous inputs which are never controllable such as the productive characteristics of climatic events.

Although from a theoretical viewpoint, technological change is reflected in changes in $G_t(\cdot)$, empirical measurement relies on the impact of technical change on the efficiency of inputs. That is, the effect of technical change can be equivalently stated either in terms of: 1) changes in the functional relation among inputs and outputs as represented by $G_t(\cdot)$ or 2) for a given functional relation (say $G(\cdot)$) which is constant over time, the effects can be expressed as changes in the quantity of efficiency units of inputs employed in production. The latter approach is the standard relied upon for measurement of technical change and will be employed here to discuss alternative measures of technical change and productivity.

That is, define the function G such that:

2.3) $Y_{1t} = G(\hat{Y}_t, \tilde{X}_t, \theta_t) \quad \forall \; t \in T$

where $\tilde{X}_t$ is a vector of unobservables defined by:

2.4) $\tilde{X}_t = f_x(K_t, X_t) \quad \forall \; t \in T$

where K_t is in general, a 1 x k vector of characteristics of the state of technical knowledge; however, for simplicity it will be assumed a scalar.

The functions $f_x(\cdot)$ are traditionally interpreted as translating the physical units of X_t into efficiency units of $\tilde{X}_t$. By substitution and composition of functions,

2.5) $Y_{1t} = G(\hat{Y}_t, X_t, \theta_t, K_t) \quad \forall \; t \in T$

where G is given an appropriately new definition.

2.2 Alternative Definitions of Productivity

With this representation the extent and impacts of technical change can be measured. The object of traditional concern has been productivity, a term which now deserves more precise definition. Although by definition, the functional representation of the product transformation function provides a full characterization of the productive relation between inputs and outputs, productivity measures attempt to provide efficient summaries of that relation. Clearly, the measure of interest depends upon the questions of interest. Where a particular input is of interest due to the private or social implications of its use, measures of partial productivity comparing output flows to the utilization of that input may be of interest. For example, if an input's supply is limited, measures of the average product of that input could serve as a guide for both short- and long-run non-price allocation decisions. If the input were variable in the short-run and its allocation were based on profit maximizing choice, then its partial productivity would depend on prices as well as technical change. However, the usefulness of such measures for the measurement of technical change even when the input is fixed in the short-run is limited.

For example, if the average product of a fixed input L were of interest as a partial productivity measure (PP), the change in PP over time would reflect changes in other inputs and changes in L, in addition to changes in technology. Consider the single output (q) case where:

2.6) $q = g(X,L,K)$

$PP = q/L,$

$$2.7)\quad \overset{\circ}{PP} = \frac{\partial PP}{\partial t}\,\frac{1}{PP} = \overset{\circ}{q} - \overset{\circ}{L}.$$

Using 2.6)

$$2.8)\quad \overset{\circ}{q} = \Sigma\,\frac{\partial g}{\partial X_h}\,\frac{X_h}{q}\,\overset{\circ}{X}_h + \frac{\partial g}{\partial L}\,\frac{L}{q}\,\overset{\circ}{L} + \frac{\partial g}{\partial K}\,\frac{K}{q}\,\overset{\circ}{K}$$

and by substitution:

$$2.9)\quad \overset{\circ}{PP} = \Sigma\,\frac{\partial g}{\partial X_h}\,\frac{X_h}{q}\,\overset{\circ}{X}_h + \left(\frac{\partial g}{\partial L}\,\frac{L}{q} - 1\right)\overset{\circ}{L} + \frac{\partial g}{\partial K}\,\frac{K}{q}\,\overset{\circ}{K}\,.$$

If the hypothesis that choices maximize profits is maintained, then

$$2.10)\quad \frac{\partial g}{\partial X_h}\,\frac{X_h}{q} = \frac{R_h\,X_h}{pq} \qquad \forall\ h=1,\ldots n$$

and

$$2.11)\quad \overset{\circ}{X}_h = \frac{\partial X_h}{\partial P}\,\frac{P}{X_h}\,\overset{\circ}{P} + \sum_{k=1}\,\frac{\partial X_h}{R_k}\,\frac{R_k}{X_h}\,\overset{\circ}{R}_k + \frac{\partial X_h}{\partial L}\,\frac{L}{X_h}\,\overset{\circ}{L}$$

$$+ \frac{\partial X_h}{\partial K}\,\frac{K}{X_h}\,\overset{\circ}{K}$$

where R is a 1 x n vector of input prices,

P is a 1 x m vector of output prices (in the single output case m=1).

By substitution, $\overset{\circ}{PP}$ is seen to reflect changes in prices, technology as represented by $\overset{\circ}{K}$ and any changes in L. That is,

$$2.12)\quad \overset{\circ}{PP} = \Sigma \frac{\partial g}{\partial X_h} \frac{X_h}{q} \left(\frac{\partial X_h}{\partial P} \frac{P}{X_h} \overset{\circ}{P} + \Sigma \frac{\partial X_h}{\partial R_k} \frac{R_k}{X_h} \overset{\circ}{R}_k \right)$$

$$+ \left[\left(\frac{\partial g}{\partial L} \frac{L}{q} - 1 \right) + \Sigma \frac{\partial g}{\partial X_h} \frac{X_h}{q} \frac{\partial X_h}{\partial L} \frac{L}{X_h} \right] \overset{\circ}{L}$$

$$+ \left(\frac{\partial g}{\partial K} \frac{K}{q} + \Sigma \frac{\partial g}{\partial X_h} \frac{X_h}{q} \frac{\partial X_h}{\partial K} \frac{K}{X_h} \right) \overset{\circ}{K}$$

The conclusion can be drawn that $\overset{\circ}{PP}$, the change in output yield from input L, is accounted for by effects of three types of changes: 1) changes in output and input prices (the first term on the right-hand side of 2.12)), 2) changes in use of the restricted input L (the second term) and 3) changes in technology (the third term). In the multiple output case, and where there exist exogeneous inputs, similar effects can be isolated. Given this result it should be clear that changes in technology cannot in general be identified from a series of yields. Only when prices, the levels of exogeneous inputs (e.g. of the restricted input L) remain constant will the yield series serve as a basis for measuring technical change.

If a measure of productivity is desired which solely reflects changes in output not accounted for by changes in inputs, equation 2.8) suggests an obvious alternative. Consider the single output case. If an index of total inputs X_I is defined such that

$$2.13)\quad \overset{\circ}{X}_I = \Sigma \frac{\partial g}{\partial X_h} \frac{X_h}{q} \overset{\circ}{X}_h + \frac{\partial g}{\partial L} \frac{L}{q} \overset{\circ}{L} ,$$

then the measure of total factor productivity (TFP) defined

$$2.14) \quad TFP = q/X_I$$

will satisfy the objective. That is, by substitution of 2.13) into 2.8)

$$2.15) \quad \overset{o}{TFP} = \overset{o}{q} - \overset{o}{X}_I = \frac{\partial g}{\partial K} \frac{K}{q} \overset{o}{K}.$$

Inspection of 2.9) and 2.10) indicates that $\overset{o}{TFP}$ can be written as the sum of $\overset{o}{PP}$ and an adjustment for the contributions of all inputs other than L to the change in output. At a theoretical level, the conclusion can be drawn that if it is the identification of the impact of technical change on the level of output that is of interest, then only the concept of total factor productivity is appropriate. This point is made more clearly by the relationship between $\overset{o}{TFP}$ and $\overset{o}{PP}$ which can be easily established by combination of 2.7) and 2.15):

$$2.16) \quad \overset{o}{TFP} = \overset{o}{PP} - \overset{o}{L} - \overset{o}{X}_I.$$

That is, total factor productivity change which is by definition the effect of technical change on output can be obtained from a measure of partial productivity change based on a yield series only if changes in yields are adjusted for changes in the restricted input L and all other inputs (as captured by X_I). If measures of the first and second terms on the right hand side of 2.12) could be obtained, the impact of technical change on changes in yield of q from L could be obtained from substracting those terms from the partial productivity measure $\overset{\circ}{PP}$ in 2.12) and untangling the remaining terms to identify $\overset{\circ}{TFP}$ which is by definition a measure of the impact of technical change on output. This, however, would obviously be a circuitous route to take to measure $\overset{\circ}{TPF}$ and would require the same data as required to directly measure $\overset{\circ}{TPF}$.

Extending this argument to the multiple output case requires an important change in the definition of TFP. If an index of total output (Q_I) could be defined, e.g., such that

$$2.17)\quad \overset{\circ}{Q}_I = \sum_{i=1}^{m} \frac{\partial G}{\partial Y_i} \frac{Y_i}{Y_1} \overset{\circ}{Y}_i$$

and if

$$2.18)\quad TFP = Q_I/X_I,$$

then

$$2.19)\quad \overset{\circ}{TFP} = \overset{\circ}{Q}_I - \overset{\circ}{X}_I.$$

However, in order for either Q_I or X_I to exist the product transformation function would have to be separable with respect to inputs and outputs. In such a case, the product transformation function could be written:

$$2.20)\quad K(Y) = J(X)$$

and using 2.19), the Jorgenson and Griliches [1967] definition of TFP is derived. See Section 4.3 for a detailed discussion of the implications of this restriction.

Where the product transformation function is not separable with respect to inputs and outputs an alternative approach must be taken. A possibility is suggested by the asymmetric form of the product transformation function, 2.1). Specifically, one output (e.g. Y_1) can be chosen as an index of output level and TFP defined such that:

$$2.21) \quad \overset{\circ}{TFP} = \overset{\circ}{Y}_1 - \overset{\circ}{G}$$

where

$$2.22) \quad \overset{\circ}{G} = \sum_{i=2}^{m} \frac{\partial G}{\partial Y_i} \frac{Y_i}{Y_1} \overset{\circ}{Y}_i + \sum_{h=1}^{n} \frac{\partial G}{\partial X_h} \frac{X_h}{Y_1} \overset{\circ}{X}_h$$

$$+ \sum_{r=1}^{p} \frac{\partial G}{\partial \theta_r} \frac{\theta_r}{Y_1} \overset{\circ}{\theta}_r + \frac{\partial G}{\partial K} \frac{K}{Y_1} \overset{\circ}{K}$$

or in more convenient notation,

$$= \sum_{r=1}^{m} S_i \overset{\circ}{Y}_i + \sum_{h=1}^{n} S_h \overset{\circ}{X}_h + \sum_{r=1}^{p} S_r \overset{\circ}{\theta}_r + S_k \overset{\circ}{K}$$

or in convenient notation and to make the time dependence explicititive rewrite 2.22) as:

$$2.23) \quad \overset{\circ}{G} = \sum_{i=2}^{m} \tilde{G}_{it} \overset{\circ}{Y}_{it} + \sum_{h=1}^{n} \tilde{G}_{ht} \overset{\circ}{X}_{ht} + \sum_{r=1}^{p} \tilde{G}_{rt} \overset{\circ}{\theta}_{rt} + \overset{\circ}{A}_t$$

Differentiating 2.1) with respect to time and substituting $\overset{\circ}{Y}_1$ into 2.21), the relation between $\overset{\circ}{TFP}$ and technical change is established:

$$2.24)\quad \overset{\circ}{TFP} = \frac{\partial G}{\partial K}\frac{K}{Y_1}\overset{\circ}{K} = \tilde{G}_{1K}\overset{\circ}{K}$$

While this measure provides a measure of technical change it does so by focusing on the impact on the level of Y_1. Although this renders the measure dependent upon the choice of output used to indicate the level of production, conditional upon that choice, 2.24) provides an acceptable measure of technical change. If the distinction between outputs and inputs is dropped, then even the single output index $\overset{\circ}{TFP}$ is based on the arbitrary choice of output as the net product which is to be employed to indicate the effects of technical change. We may, therefore, conclude that the product asymmetric definition of total factor productivity presented in 2.24) exactly parallels that which is traditionally used for single output production processes.

2.3. Measurement of Biases in Technical Change

In addition to the impact of technical change on the level of output, its impact on the allocation of inputs is important to understand. As technology changes, some inputs become more productive while others become less productive. These effects are revealed as producers change the mix of inputs to be used in production despite the absence of any changes in relative prices. As will become clear later in the paper, knowledge of the nature of these effects in the past is crucial for the forecasting of future trends in productivity. Following Hicks' [1932] terminology, the impacts of technical change on product transformation (i.e., the nature of $f_x(K_t)$) can be classified by the ultimate effect on the marginal rates of substitution (or transformation) between products. That is, from 2.4) K_t becomes an argument in the transformation function and as it changes, the surfaces of the production possibilities set viewed in various product spaces are stretched or shrunk and tracings of the contours of these surfaces (isoproduct curves) are shifted or skewed. Because of the matching of the slopes of these isoproduct curves with price ratios which is required by profit maximizing choice of product combinations, the impacts of technical change are revealed by changes in chosen product combinations which occur in the absence of changes in price ratios or fixed factors. Although such changes in product combinations are not readily observable, for a given specification of technical change they are identifiable via econometric methods.

By Hicks' classification, technical change has no effect on relative product choices (i.e., neutral) if the technical change does not disturb the marginal rate of substitution (transformation) measured along a ray through the product choice made within the previous technical regime. The technical change will result in changes in relative product choices only when it results in a change in the marginal rate of substitution (transformation). For example, in (X_h, X_k) input space between time t-1 and t, technical change is Hicks' X_h

$$2.25)\quad \left.\begin{array}{l}\text{saving}\\ \text{neutral}\\ \text{using}\end{array}\right\}\ \text{relative to } X_k \text{ if } \frac{\partial G(\hat{Y}^{\circ},X^{\circ},\theta^{\circ},K_t)}{\partial X_k} \Big/ \frac{\partial G(Y^{\circ},X^{\circ},\theta^{\circ},K_t)}{\partial X_h} \overset{>}{\underset{<}{=}} \frac{\partial G(\hat{Y}^{\circ},X^{\circ},\theta^{\circ},K_{t-1})}{\partial X_k} \Big/ \frac{\partial G(Y^{\circ},X^{\circ},\theta^{\circ},K_{t-1})}{\partial X_h}$$

where Y°, X°, θ° are initial positions of Y, X, θ respectively.

3. Measurement of the Impacts of Technological Change

The discussion in Section 2 established that in general changes in yield reflect changes in inputs due to changes in prices, fixed inputs and technology in addition to the direct effect of technical change on output level achievable from a fixed quantity of a particular input, say L. Only during periods when relative prices and the use of the fixed input L (upon which the partial productivity measure is based) remain constant will the yield series provide a basis for measuring the impact of technical change on output levels. Thus, although in terms of apparent operational simplicity yield series are attractive, as a basis for inference concerning the nature of technical change, relative prices do not remain constant. For this reason, it was argued that if yield series are to be relied upon to study the impacts of technical change, their use involves all complications and data requirements that are also involved in the measurement of total factor productivity change. A corrolary which will be discussed in Section 6 is that forecasts of technical change which are made in the context of forecasts of yields will be characterized by at least the same degree of possible error in forecast as would forecasts of total factor productivity change. The principal difference in the two approaches is that a forecast of total factor productivity change is a forecast of the impact of technical change on output levels, whereas a forecast of yields also involves a forecast of levels of prices and fixed inputs, making the forecast of technical change only implicit. The problem of forecasting the level of changes in either yield or total factor productivity relies upon accurate measurement and explanation of the level of these variables. This latter issue will be the subject of this and the following section.

Measurement of the levels of yields, total factor productivity or of biases in technical change on relative input use requires the use of a model which explains their levels in terms of levels of other variables. By the definition of PP, it is the yield of q per unit of L. The level of L is pre-determined or exogeneous and, thereby, requires no further explanation. In contrast, the level of q is chosen by the producer in the context of technological as well as economic feasibility. Similarly, the levels of inputs X (the changes in which are involved in the definition of total factor productivity change 2.21) are determined by producers as allocation decisions in the context of technological and economic feasibility. The conclusion can be drawn that whether the focus of interest is yield or total factor productivity, an economic model of the choice of inputs and outputs is required. As will be seen, specification of such a model of choice must include maintained hypotheses or assumptions concerning the economic goals or objectives of the producer, physical constraints which limit or restrict choices, and functional characteristics of the product transformation function. Even in the absence of a detailed statement of these assumptions, they are at least implicit in any model of choices, see Weaver [1980]. Thus, while the functional form of the product transformation function has been recognized as a dominant issue in the measurement of total productivity change, it is of equal importance in the explanation of the level of yield to land, or any other partial productivity measure. Similarly, although the applied literature has recognized the necessity and importance of assumptions concerning the objectives and constraints that determine choices of inputs and outputs, these items are of equivalent importance in the measurement of the level of yields. The remainder of this section will consider these points in more technical form.

3.1 Total Factor Productivity

Section 2 noted that an immediate implication of technical change was that the functional form of $G_t(\cdot)$ would change over time. In order to measure the impacts of technical change, an algebraic form must be assumed for the product transformation function. Even if yields are the subject of interest, specification of a functional form for a yield equation implicitly involves specification of a functional form for $G_t(\cdot)$. This follows from the fact that the yield model must recognize that yield is a choice and so its relationship with prices and fixed inputs is determined by the assumed objectives of the producer and the functional form of G, see Weaver [1980]. The instability of G_t can be thought of as instability in the parameters of its algebraic form.

The nature of that parameter instability must be specified if the function is to be estimated using a sample drawn from more than one technological regime. In the terminology of Section 2, this would require specification of the elements of K_t. However, if K_t is recognized as a vector of unobservables determined by both embodied and disembodied technical changes, then only two courses are possible, each involving errors in measurement.

As one alternative, if the determinants of technical change could be identified and functionally related to K_t, then by composition of functions those determinants could be directly introduced as arguments in the product transformation function. Although this may appear to be infeasible given current understanding of the process of innovation and its adoption, at a general level expenditures on investment, research and development and education might be identified and introduced. This is the course chosen (with variations) by Denison [1957], Griliches [1960, 1963, 1964], Fishelson [1971], Hayami [1969], Hayami and Ruttan [1971], Huffman [1978], and Lu, Cline and Quance [1979]. Denny, Fuss and Waverman [1979] have employed indicators of the extent of adoption of new technologies. However, as David and Van DeKlundert [1965] argue a useful alternative is to separate the steps of measurement and explanation of determinants of the technical change. As indicated by 2.24) an accurate estimate of $\overset{\circ}{TFP}$ would provide such a measure in the absence of any explanation. The only remaining problem is the estimation of the product transformation function without any measure of K. To the extent that the elements of K are uncorrelated with current levels of elements in the information vector (P_t, R_t, θ_t), exclusion of a measure of K will bias only constant terms in models of technology or choice.

A strategy which falls between the two alternatives noted above is based on the recognition that if innovation and adoption are processes which result in a continuous process of technical change, at least a portion of the impact of technical change on output level will be continuously related to time. If each new innovation is adopted according to a time related process (e.g., one following a logistic function as assumed by Griliches [1957]), but new innovations are rapidly and continuously introduced, then the convolution of the dynamic impacts of these processes on output could be expected to bear a constant functional relation with time. This descriptive approach will be adopted here to allow decomposition of the unexplained residual measured by $\overset{o}{TFP}$ into a component that is related to time and one that is not.

Following this approach the changes in K will be assumed to occur with an annual frequency (i.e., $T_h = 1 \;\forall\; h$) and to be monotonically related to t by a twice differentiable function, e.g., define for each element K_{gt} of the vector K_t

$$3.1) \quad K_{gt} = K_g(t) \qquad \forall \; g=1,\ldots K.$$

By substitution,

$$3.2) \quad X_h = f_x(K(t),X_h) = f_x'(t,X_h) \qquad \forall \; h=1,\ldots n$$

which allows

$$3.3) \quad Y_{1t} = G(\hat{Y}_t, X_t; \theta_t, t)$$

As written in equation 2.21) $\overset{o}{TFP}$ presents the basis for a Divisia [1926] index. Specifically, if $\overset{o}{TFP}$ is integrated between two points in time, say t and t*, then we can define the Divisia index number of total factor productivity as $\overset{\circ}{TFP}_D$.

$$3.4)\quad \overset{\circ}{TFP}_D = \frac{A(t^*)}{A(t^\circ)} \equiv \frac{Y_1(t^*)/Y_1(t^\circ)}{G_I(t^*)/G_I(t^\circ)}$$

where

$$3.5)\quad \frac{G_I(t^*)}{G_I(t^\circ)} = \exp\left[\int_{t^\circ}^{t^*} \overset{\circ}{G}\right] = \exp\left[\int_{t^\circ}^{t^*}\left(\sum_{i=2}^{m} S_i(t)\,\overset{\circ}{Y}_i(t) + \sum_{h=1}^{n} S_h(t)\,\overset{\circ}{X}_h(t) + \sum_{r=1}^{p} S_r(t)\,\overset{\circ}{\theta}_r(t)\right)\right]$$

It is important to note the caveats offered by Hulten [1973] concerning the integrability of 2.21).

Although the Divisia index number would provide a precise measure of technical change, it relies upon continuous measures of production elasticities, outputs and inputs. In essence 3.4) represents a ratio of index numbers of the numeraire output to an aggregate of all other outputs and inputs. In order to obtain measures of this index using discrete data various index number formulae have been employed as discrete approximations to the Divisia index. The problem of measurement of total factor productivity can, therefore, be thought of as one of selection of an index number formula based on discrete data which provides a close approximation to the continuous Divisia index. From a more general perspective, the Divisia index represents a transformation of the product transformation function G(•) which can be interpreted as aggregating the net products in vectors $\hat{Y}$ and X and can be labeled as an aggregator function. The problem is to select a method of measuring $G_I(\cdot)$ using discrete data. This approximate measure of $G_I(\cdot)$ will be labeled $\hat{G}_I(\cdot)$. As in the case where a measure of the product transformation G(•) is of interest, this involves specification of a functional form for $\hat{G}_I(\cdot)$ and choice of a method for measuring its parameters. It is intuitive that since $\hat{G}_I(\cdot)$ is an approximation of the aggregator function $G_I(\cdot)$, the form of the discrete index should also be related to the form of $G_I(\cdot)$.

In order to approximate the continuous form $G_I(\cdot)$ by a discrete form, the way in which the elasticities $S_i(t)$ change between discrete points in time must be specified. At the extreme, the elasticities $S_i(t)$ can be assumed to be approximately constant over time, then

$$3.6)\quad \ln G_I^*/G_I^\circ \cong \sum_{i=2}^{m} S_i \ln Y_i^*/Y_i^\circ + \sum_{h=1}^{n} S_h \ln X_h^*/X_h^\circ + \sum_{r=1}^{p} S_r \ln\theta_r^*/\theta_r^\circ$$

where the superscripts (*,°) indicate whether measurement occurs at t* or t°, respectively.

For cases where small changes occur for any product, say Z, then

$$3.7)\quad \ln Z^*/Z = Z^*/Z - 1$$

allowing 3.6) to be rewritten as:

$$3.8)\quad G_I^*/G_I^\circ \stackrel{\sim}{=} \sum_{i=1}^{m} S_i\, Y_i^*/Y_i^\circ + \sum_{h=1}^{n} S_h X_h^*/X_h^\circ + \sum_{r=1}^{p} S_r\, \theta_r^*/\theta_r^\circ$$

Index number formulae for approximating the Divisia index are based on alternative measures of the elasticities S in either 3.6) or 3.8). In each case recognition is given to the fact that the shares do not remain constant and some alternative is chosen to allow either 3.6) or 3.8) to approximate their change.

Conditional upon the integrability of 2.21) and profit maximizing allocation, the Divisia index is exact in the same sense that where

$$3.9)\qquad q = g(X;t)$$

$$\overset{\circ}{TFP} = \frac{g(X^*;0)}{g(\overset{\circ}{X};0)} = \frac{G_I^*}{G_I^\circ}$$

That is, the Divisia index is exact for all functional forms of G. Where discrete approximations to G_I^*/G_I° are employed, Diewert [1976] has examined the criteria for choice of an index number formula. A criterion that has persisted in the literature, e.g., Konyus and Byushgens [1926], has been based on the recognition that if the functional form of g(•) were known, the exact form of the index formula satisfying 3.9) would be implicitly specified. In this way, the Laspeyres and Paasche quantity indexes can be shown to be exact for Leontief fixed coefficients technology.

However, given that the functional form is typically unknown Diewert [1976] has suggested that a superlative index formula be chosen, i.e., one which is exact for a flexible functional form employed as a second-order approximation to the unknown function. Thus, two sources of errors can occur when index numbers are used: a functional approximation error due to the approximation of the unknown functional form of G and an index error following from the use of index formulae which are not exact for the desired level of approximation. By definition, use of a superlative index results in only functional approximation error. To conclude, at a theoretical level, the preferred index number formula is implied by the presumed functional form of the product transformation function G and its transform G_I.

Given that the form of the aggregator function G_I is typically unknown, it is preferable to adopt the superlative index which is exact for the flexible form employed as an approximation of the aggregator function. However, one issue remains. In the absence of a criteria which can be applied prior to estimation for selecting the flexible form to employ, are substantive errors likely to result from the use of a superlative index which is not exact for the flexible form employed for the aggregator? For example, the Tornqvist index has been shown to be exact for homogeneous translog aggregator functions. What errors would result if it were employed when the aggregator function is of the generalized Leontief form? From a theoretical perspective, Diewert [1978, 1979] has established that superlative indexes provide second-order differential approximations to each other at a point where prices and quantities are equal to those in the base year.

Parkan [1975] has compared index number series based on the Tornqvist direct and implicit price indexes with the Fisher ideal index and found they agreed up to three significant figures. Diewert [1978] extended this comparison to include the Vartia, chained Laspeyres and chained Paasche price indexes and found similar results. Although the similarity of the alternative superlative index numbers would be expected given Diewert's [1978] results, the accuracy of the chained Laspeyres and Paasche indexes is surprising. Diewert [1978] shows that as would be expected these indexes provide first-order differential approximations. Their accuracy is, therefore, interpretable as evidence that second-order effects were small at least in Diewert's sample. When a fixed base period is employed, Diewert [1978] finds this accuracy dissolved.

The remaining issue involved in measurement of total factor productivity is measurement of the elasticities (S) involved in the discrete index number formula that is selected. Two approaches have been taken here. Under a maintained hypothesis concerning the behavioral objectives of the firms or sector under study, e.g., profit maximization, first order conditions for optimization of that objective link the elasticities of production of variable products to observable profit shares. Similarly, by Hotelling's lemma the elasticities of the profit function are linked to observable profit shares as in 2.10). In the first approach, if all products are variable, observable profit shares are assumed to be exact measures of the elasticities of interest and the index number of interest is simply calculated from a series of prices, quantities and observed shares.

An alternative is hypothesized that due to the usual sources of stochastic error, the equality of shares and elasticities holds only at the population expectation. In this case, share equations such as 2.10) are econometrically estimated and fitted values are employed in the calculation of the index number of technical change. Besides the relaxed hypothesis concerning the relation between the share and the elasticity implicit in this alternative, several other advantages are apparent. First, where some products are not variable the relation between the observable profit share and the elasticity is left unspecified by the behavioral hypothesis. In this case, the econometric approach allows estimation of the elasticity which is interpretable as the shadow value of the product as a share of profit. Secondly, the econometric approach allows for statistical inference concerning the sources of change in the elasticities, e.g., relative output or input prices or changes in levels of fixed factors. Where the objective of the firm is cost minimization with respect to an exogeneously established output level, the cost share equations allow measurement of the scale elasticity (see, e.g., Berndt and Khaled [1979]).

Summarizing this section, the measurement of total factor productivity was seen to rely upon the resolution of a number issues concerning the way in which production decisions are made, and how they are related to their determinants. Even if these issues are not explicitly addressed, any index of total factor productivity, if it is to have any meaning as a measure of the impact of technical change on outputs, implicitly is consistent with a particular resolution of the issues. Specifically, the issues of critical importance were argued to include: 1) a hypothesis concerning the objectives or operating goals of the firms in the industry under study, 2) a hypothesis concerning which inputs are fixed for the firm in the short-run and which are variable, 3) the algebraic, functional relation between inputs and outputs, and 4) a hypothesis concerning the way in which that functional relation has changed over time. Needless to say, the evidence upon which resolution of these issues could be based is extremely weak. At best, a researcher can only make reasonable decisions; but, at best even these decisions lead to indeterminable errors in the measurement of total factor productivity.

3.2 Requirements of Empirical Measurement of Partial Productivity or Yields

The fact that accurate empirical measurement of the level of yields requires the resolution of each of the difficult issues which need to be resolved for the measurement of total factor productivity should be apparent from the intimate dependence of measures of total factor productivity on a model of choice. By definition, a partial productivity index or yield is the ratio of an output, the level of which can be influenced by the producer through choice of inputs, and the quantity of a fixed input. Yield is, therefore, as was established in Section 2 a choice variable. As such, accurate measurement relies upon the specification of 1) the objectives of the firms in the industry, 2) the inputs which are fixed or beyond the control of the firm, and 3) the functional relation among inputs and outputs. These are exactly the same issues which must be resolved for the measurement of total factor productivity. The conclusion can be drawn that measurement of yields or partial productivity is subject to the same set of possible errors in model specification that characterizes measurement of total factor productivity.

3.3 Requirements of Empirical Measurement of Biases in Technical Change

The definitions of bias in technical change were presented above (2.25) in terms of marginal rates of substitution. However, to facilitate the measurement of biases, they can be translated into a set of conditions on the change in input choices X^*. That is, technical change can be said to be Hicks'

$$X_h \quad \left.\begin{matrix}\text{Saving}\\ \text{Neutral}\\ \text{Using}\end{matrix}\right\} \quad \text{relative to } X_k \text{ if } d\ln X_h^* \Big|_c \gtreqless d\ln X_k^* \Big|_c$$

$$\text{or equivalently if } \left.\frac{d\ln X_h^*}{d\ln X_k^*}\right|_c \gtreqless 1.$$

where c is the set of relative prices and choices of all other products

However, the levels at which variable inputs are utilized are determined by the firm depending upon objectives, fixed inputs and the form of the product transformation function. Again, measurement of biases as in the case of measurement of total factor or partial productivity requires the specification of a model of choices made by firms under study. The accuracy of all resulting measurement is dependent upon the ability of that model of choice to accurately predict or explain actual choices made by the firms.

3.4 Conclusion

The above discussion leads to the reasonable conclusion that measurement of the level of productivity regardless of which specific definition is used will be subject to a substantial, yet unquantifiable error. As was noted this error results from errors in the specification of the model of choice upon which productivity measures must be based as well as errors in measurement and use of data. In addition to these unquantifiable errors there exist the usual sampling errors which characterize the point estimates of the parameters derived from the model of choice and used in the calculation of the productivity measure of interest.

4. Empirical Models as a Basis for Measurement of the Impacts of Technological Change

4.1 Statistical Characteristics of Productivity Measures

The discussion in Section 3 has established that measurement of each of the indicators of productivity must be based on a model of choice which provides a basis for obtaining estimates of parameters or weights involved in the calculation of the index. Whether these parameters come from econometric models or from other rules of calculation or assumptions, they represent sample statistics that are point estimates of the true, unknown parameters. Thus, the estimates represent summaries of sample information and the point estimate employed can be thought of as having been drawn from a distribution of values of the sample statistic.

Similarly, the productivity estimate itself represents a sample statistic that is a point estimate of the true, unobservable productivity. If the parameter estimates were obtained from an econometric model, then an estimate of the sampling variance of the estimate will be available and the sampling error of the productivity measure can be assessed. Unfortunately, no direct measure of specification or measurement error can be obtained. However, when econometric models are employed, estimated residuals provide an approximate measure.

4.2 Alternative Methods of Estimating Total Factor Productivity

The U.S.D.A. total productivity index provides an example of an index based on point estimates of parameters where the variance of those estimates is unknown. Although the estimates are assumed to be exact with zero variance, if uncertainty is acknowledged concerning the model of choice upon which they are implicitly based, then their variance would not be zero. While this series is exemplary in many respects (see Christensen [1975]), the absence of any indication of the uncertainty that characterizes each year's index, has led to the acceptance of the series as point estimates with zero sampling error. As noted in Section 3, the error of an estimate of a total factor productivity index follows from error in measurement of the elasticities of output Y_1 with respect to changes in other outputs and changes in input levels which were noted by S_i, S_h, S_r. For example, based on the assumption that firms maximize profits and that the product transformation function is characterized by constant returns to scale, the production elasticity S_h can be linked to observable price data. In the single output case:

$$4.1)\quad S_h = \frac{\partial Y_1}{\partial X_h}\frac{X_h}{Y_1} = \frac{R_h X_h^*}{C}$$

where C is minimum total cost given (R,θ) and an output level Y, and X_h^* is the optimal choice of X_h.

A point estimate of S_h can be obtained from input expense data and under the assumption that the distribution of that estimate has zero variance.

Alternatively, if uncertainty is acknowledged concerning the objective of the firm, or the characteristics of its product transformation function, then the equality in 4.1) is broken and a stochastic error could be introduced. For example, the elasticity could be measured by

$$4.2)\quad S_h = \frac{\partial Y_1}{\partial X_h}\frac{X_h}{Y_1} = \frac{R_h X_h^*}{C} + \varepsilon_h$$

where $\varepsilon_h \sim N(0,\sigma^2)$

In this case, the expense share provides a point estimate which is characterized by variance. Specifically,

$$4.3)\quad E(S_h) = R_h X_h^*/C$$
$$V(S_h) = V(\varepsilon_h)$$

The construction and estimation of an econometric model of choices presents the basis for an approach to the measurement of production elasticities or shares which takes a further step in the direction of measuring the production elasticites under relaxed assumptions which acknowledge the uncertainty which characterizes measurement. An early classic attempt along these lines was presented by Griliches [1963] where production elasticities were estimated using data on inputs and outputs. As econometric estimates their relation to the true, unobservable elasticities can be rigorously considered.

Recent advances in economic theory and econometrics allow the relaxation of several restrictive assumptions underlying Griliches' work. First, the levels of inputs and outputs observed and employed by Griliches do not represent arbitrary combinations along the product transformation function. Only controlled laboratory experimentation could generate such data. Instead the observed levels of inputs and outputs at any point in time are interrelated by the fact that they were chosen by producers as not only technically efficient, but also economically efficient given producer objectives, prices, fixed factors and technical efficiency. Given this result estimates of the production elasticities must be based upon a model of choice. Secondly, Griliches assumed that inputs and outputs are homothetically separable and, therefore, all outputs can be meaningfully aggregated into one aggregate output. As was noted in Section 2.1 this implies that input choices can be made independently of output choices. Finally, by the use of the Cobb-Douglas algebraic functional form, Griliches was forced to assume that the elasticity of substitution between inputs was unity.

A similar approach under relaxed assumptions has been proposed by Weaver [1977] based on application of duality theory (see e.g., Fuss and McFadden [1978]) and an explicitly stated model of choice. In general terms, given a hypothesis concerning the objectives of the firm and technology it faces, the production elasticities of variable inputs are linked to prices as in 4.1) by the necessary conditions or rules for achievement of the firm's objectives. For example, suppose the firm is hypothesized to maximize expected profits in the face of price uncertainty, and a multiple output product transformation. That is, the firm chooses a (m x 1) vector of outputs Y and an (n x 1) vector of variable inputs to solve:

$$4.4)\quad \max_{Y,\, X} \ \pi = P'Y - R'X$$

$$\text{s.t.}\quad Y_1 - G(\hat{Y}, X;\theta,K)$$

where P is a m x 1 vector of expected output prices and R is a n x 1 vector of input prices.

The necessary conditions for this problem's solution can be written:

$$4.5a)\quad P_1G_i + P_i = 0 \qquad \forall\ i=2,\ldots m$$

$$4.5b)\quad P_1G_h - R_h = 0 \qquad \forall\ h=1,\ldots n$$

The solution of these structural equations allows the derivation of traditional output supply and input demand functions:

$$4.6)\quad Y_i^* = Y_i\ (P,\ R;\ \theta,\ K) \qquad \forall\ i=1,\ldots m$$

$$4.7)\quad X_h^* = X_h(P,R;\theta,K) \qquad \forall\ h=1,\ldots n$$

From 4.6) and 4.7), the production elasticities can be written as equal to shares of expected revenue earned by output # 1:

$$4.8)\quad G_iY_i/Y_1 = -\ P_iY_i/P_1Y_1$$

$$4.9)\quad G_hX_h/Y_1 = R_hX_h/P_1Y_1$$

Given an algebraic form for G(•) the system of equations in 4.6) and 4.7) or those in 4.8) and 4.9) could be used to directly estimate the production elasticities. Given these estimates desired productivity estimates can be derived.

A convenient alternative is suggested by duality theory. By substitution of equations 4.6) and 4.7) into the definition of expected profits in 4.4), the expected profit function can be derived:

$$4.10)\quad \pi^{*} = P'Y(P,R;\theta K) - R'X(P,R;\theta K)$$
$$= \pi(P,R;\theta,K)$$

By differentiation and use of 4.5), Hotelling's lemma establishes:

$$4.11)\quad \partial\pi/\partial P_{i} = Y_{i}(P,R;\theta,K) = Y_{i}^{*} \quad \forall\ i=1,\ldots m$$

$$4.12)\quad \partial\pi/\partial R_{h} = -X_{h}(P,R;\theta,K) = -X_{h}^{*} \quad \forall\ h=1,\ldots n$$

Differentiating the profit function 4.10) with respect to K and writing the result in terms of percentage changes (indicated by a dot), the percentage change in profit can be written:

$$4.13)\quad \mathring{\pi} = \Sigma\eta_{i}\mathring{P}_{i} + \Sigma\eta_{h}\mathring{R}_{h} + \Sigma\eta_{r}\mathring{\theta}_{r} + \mathring{\beta}$$

where η represents the elasticity of profit with respect to a change in the subscripted determinant of choice (P, R, or θ), and

$\mathring{B}$ represents an index of the effect of technical change on profits, or a measure of total profit diminution.

As is demonstrated in Weaver [1977, 1981] and as has also been established by Christensen, Jorgenson and Lau [1973] in other contexts, $\overset{\circ}{B}$ represents a simple transformation of the multiple output measure of total factor productivity change $\overset{\circ}{TFP}$:

4.14) $\overset{\circ}{TFP} = \overset{\circ}{B}/M_1$

where $M_1 = P_1 Y_1^* / \pi$.

Several important conclusions follow immediately from this result, see Weaver [1977, 1981] for details. First, it is noted that by use of 4.11) and 4.12),

4.15) $M_1 = Y_i^* P_i / \pi = \eta_i = \frac{\partial \pi}{\partial P_i} \frac{P_i}{\pi} \equiv \tilde{\pi}_i(P,R;\theta,K)$

4.16) $M_h = X_h^* R_h / \pi = - \eta_h = - \frac{\partial \pi}{\partial R_h} \frac{R_h}{\pi} = - \tilde{\pi}_h(P,R;\theta,K)$

where $\tilde{\pi}_i$ and $\tilde{\pi}_h$ are logarithmic derivatives of the profit function 4.10).

If 1) algebraic functional form is specified for $\pi(\;)$, 2) $\tilde{\pi}_i$ and $\tilde{\pi}_h$ are appropriately derived from that form and 3) additive, random disturbances are added to each equation 4.15) - 4.16), then this system can be econometrically estimated and sample estimates of η_i and η_h determined. These, in turn, could be used for the calculation of $\overset{\circ}{B}$ and $\overset{\circ}{TFP}$. By their dependence upon an explicit model of choice the specification and measurement errors involved in $\overset{\circ}{B}$ and $\overset{\circ}{TFP}$ could be rigorously addressed. By their dependence upon an explicit econometric model, their sampling errors could be calculated. Finally, by use of one of many of the recently proposed flexible functional forms (e.g., translog or generalized Leontief), the resulting estimates can be freed of restrictive assumptions such as input-output separability, homotheticity, or unity elasticities of substitution, which have characterized past research.

At first consideration this approach may appear to require the researcher to specify a behavioral hypothesis while direct measurement of TFP or yield would not. However, as has been argued above the need to specify and explicitly incorporate a behavioral hypothesis is mandated not by the particular methodology employed but by the nature of the data. Specifically, observed levels of outputs and inputs are chosen by producers and their use in empirical measurement of productivity, therefore, requires specification of a behavioral hypothesis.

4.3 Alternative Models of Yield

This point can be made more sharply in the context of an estimate of partial productivity or yield. Suppose it is hypothesized that firms attempt to maximize the expectation of profits since output prices are unknown at planting, that they face a product transformation function such as 2.1), and that total land utilization L as well as its allocation among outputs, L_i is also fixed during the production period. By using 4.4), and 4.6) the partial productivity of output i relative to land L used for production of i, L_i, can be written:

$$4.17) \quad PP_i \equiv Y_i^*/L_i = Y_i(P,R;\theta,K,L)/L_i$$

The validity of the assumption that land allocation L_i to output i is fixed is, of course, not likely since land reallocation is possible when production plans are made. If, instead, L_i is chosen by the firm each production period, then 4.17) could be written:

$$\begin{aligned} PP_i &= Y_i^*/L_i^* = \\ &= Y_i(P,R;\theta,K,L)/L_i(P,R;\theta,K,L) \\ &= F_i(P,R;\theta,K,L). \end{aligned}$$

For the typical agricultural case, the vector θ would contain 1) measures of pre-season climatic events and expectations concerning the occurrance of such events during the growing season, 2) flows from marketed inputs which are fixed in the short-run and 3) flows of non-marketed inputs such as soil characteristics. The functional form of $F_i(\cdot)$ is clearly dependant upon the form of $G(\cdot)$. Despite the reassuring exactness with which 4.17) is derived once the model of choice is given, the model of choice is itself subject to substantial uncertainty. In order to empirically implement 4.17) a functional form for $F_i(\cdot)$ must be chosen or derived from $G(\cdot)$, and the elements of the vectors (P,R,θ) identified and measured. The resulting estimate of yield for any year in the sample is subject to random and sampling error. However, in addition it is characterized by the same unquantifiable specification error which characterizes measurement of $\overset{o}{\mathrm{TFP}}$.

Strong evidence concerning the uncertainty that characterizes the specification and empirical implementation of a model such as 4.17) is provided by the existence of a wide variety of yield models. Nichol and Heady [1975] base their model on fertilizer responsiveness and the impact of price changes on fertilizer use. Swanson and Nyankor [1979] focus exclusively on the time trend in yields. However, in addition to these studies there exists an extensive literature focused on the effects of prices, climate and technical change on yields. For example, Stallings [1961], Shaw and Durost [1962], Shaw [1964], Bauer [1965], Oury [1965], Williams [1969], and Thompson [1969a and b, 1970] are each papers focused on the interaction of climate and technology. A single equation approach to the study of the relation between yields of corn to output prices is found in recent work by Houck and Gallagher [1976] while Luttrell and Gilbert [1976] focus on the properties of the probability distribution from which the yield series might be drawn. However, as has been argued above, because yields are influenced by production choices, models of yields must be based upon a simultaneous consideration of climate and technology within a defensible model of choices made by the firm. The choice model developed by Weaver [1977, 1980, 1981] provides an example of an attempt to integrate these factors in a model of choice. A brief description of this model can be found in Section 4.4 below.

4.4 Alternative Measures of the Biases of Technical Change On Input Utilization

Section 2.3 presented a definition of what Hicks has defined as an indicator of bias in the impacts of technical change on input utilization. Section 3.3 noted how this definition could be written in terms of changes in relative input use. Lianos [1971] provided evidence concerning the nature of this bias on the use of labor and found that technical change had been Hicks' labor-saving during the period 1949-1968. Although Lianos' study was path breaking, its current usefulness is limited by its reliance upon an extremely simplified representation of the product transformation function. Specifically, his approach assumes 1) outputs are homogeneously separable from inputs and

2) that inputs can be represented by two aggregate categories: labor and capital. His methodology did relax the assumption of constant returns to scale and employed an algebraic form for the product transformation function which did not a priori restrict the elasticity of substitution between labor and capital to be unity. The last two specifications represented welcomed advances; however, the two restrictive assumptions limit the study's usefulness.

Although the assumption of input-output separability was certainly a standard of the time, its validity is an empirical issue. The effect of this separability can be seen as follows. If technology is input-output separable, then the product transformation function 2.1) can be equivalently written:

$$4.18) \quad Y_I = F(Y) = H(X) = X_I$$

where Y_I is an aggregate output index, see Weaver [1977, 1981]. In general, it might be expected that relative input utilization depends upon the output mix of the firm. By definition input-output separability implies input use is independent of output mix and relative output prices. An immediate implication of this is that as output mix changes no changes in input mix are induced. If, in fact, technology is input-output separable, then the residual change in input mix not accounted for by changes in relative input prices or levels of fixed input flows can be attributed to technical change (if there are no other errors in specification or measurement). Under these conditions an inference concerning biases of technical change can be made. On the other hand, if input-output separability is assumed, but does not characterize technology then the residual change in input mix would reflect the effects of changes in relative output prices and output mix in addition to changes induced in response to technical change. The conclusion can be drawn that the inappropriate imposition of the separability restriction can lead to substantial errors in measurement of total factor as well as biases in productivity if, in fact, output mixes have changed.

The implications of Lianos' representation of all inputs by two aggregate measures labeled labor and capital is not easy to determine. Implicitly, this specification assumes the inputs aggregated in the capital account are homogeneously separable from those in the labor account and that choice of which labor inputs are used can be made independently of the choice of which capital inputs are used. This specification decision may have been necessary due to the restrictive properties of algebraic functional forms available at the time. However, one important implication of its adoption is that the biases of technical change on relative use of important subcategories of inputs such as petroleum products or fertilizer could not be identified, and biases measured for the two aggregate inputs may be erroneous.

Binswanger [1974] addressed this latter issue by disaggregating the input vector into aggregates of land, labor, machinery, fertilizer and "other." This possibility largely resulted from the introduction of the "translog" algebraic functional form by Christensen, et. al. [1971]. However, Binswanger did employ the restrictive assumptions of input-output separability as well as homotheticity of technology. If technology were homothetic, changes in the scale of operation (or analogously the level of aggregate output) would have no effect on relative input utilization, see Weaver [1977, 1981]. Thus, as the output which can be produced by a given bundle of inputs increases due to technical change, the expansion of output would not induce a change in relative input use. If technology were homothetic, the residual of change in relative input use which remains after the accounting for the effect of changes in prices and fixed factors would reflect the biases of technical change (in the absence of other specification or measurement errors). However, if homotheticity is inappropriately assumed, this residual change in relative input use would also include a measure of the changes of relative input use induced by changes in output levels. Binswanger concludes that during the post-war period technical change has resulted in greater fertilizer and machinery use and a reduction in labor use. While these results are consistent with an intuitive consideration of trends in the agricultural sector, the caveats discussed above weaken the confidence that should be placed in the inference that these trends were induced by biases in technical change.

An additional assumption made by both Lianos and Binswanger was that the parameters of their model could be interpreted as representing an aggregation over individual farms. Lianos used a time series of data aggregating over all farms in the U.S. While Binswanger used data aggregating across farms in states, he assumed that technology was identical up to neutral effects across states. Because of the extensive variation of production alternatives facing farmers across the U.S. such geographical aggregation masks many important questions relating to the effects of technical change in particular homogeneous production regions.

For example, what have been the biases of technical change in the corn-soybean region of the central mid-west or in the wheat-corn-small grains region of the northern plains?

Weaver [1977, 1980, 1981] has presented a model of choice of inputs and outputs which relaxes some of the restrictive assumptions employed by Lianos and Binswanger. Specifically, multiple outputs and multiple inputs are allowed and no restrictions are placed on the regularity properties (e.g. homotheticity) of the product transformation function. Furthermore, the uncertainty that faces the farmer, the effects of climatic events and the existence of government policies are also introduced into the model.

The model was estimated for a post-war time series [1950-1970) of state level aggregate data for North and South Dakota, two states where production alternatives are dominated by wheat, feed grains, and livestock. Inputs hypothesized to be variable in the short-run were divided into the following categories: petroleum products, fertilizer, labor, building and machinery services, and operating supplies. Input flows and factors beyond the control of the farmer in the short-run were hypothesized to include: land, preseason precipitation, the wheat acreage allotment, the feed grain base, and a time trend representing changes in technology. The existence of input-output separability in this sample of data was tested and rejected. Results based on a model which was free of restrictive assumptions concerning input-output separability or homotheticity indicate that technical change had a biased effect on relative input use in the sample. Specifically, biases appeared to have been labor saving, and fertilizer using relative to all other inputs. While the effect on relative capital utilization was to reduce capital use relative to all inputs except labor in which case capital use was relatively increased. Petroleum product use was found to have been increased by technical change relative to all inputs except fertilizer. That is, although technical change led to increased use of both petroleum products and fertilizer relative to other inputs, fertilizer use was increased relative to petroleum product use.

5. Forecasting Productivity Change

Given an abililty to measure productivity at a point in time or its change over a past period of time, to what extent can the same methodology be relied upon to forecast future levels or rates of change of productivity? The issues involved in forecasting productivity based on past experience will be reviewed in this section.

5.1 Theoretical Considerations

As was noted in Section 2.1 the essence of technical change is structural change in the functional relation between outputs and inputs. In order to achieve any measure of productivity it was argued that a stable functional relation must be introduced by the redefinition of inputs in terms of efficiency units. However, empirical implementation of econometric based measures of productivity required specification of a vector of observable factors (labeled K) which translated physical input units into efficiency units. Although it might be argued that the structural change resultant from technical change has been and will be a smooth process, such a specification can at best represent a hypothesis, the validity of which is characterized by substantial uncertainty.

If structural change is not a smooth process which continues through time, then its predictability is severely limited. In the presence of systematic, persistent processes which can be modeled or explained, econometric forecasts can be accepted only on the condition that these past processes will continue. However, even conditional upon such continuation, forecast errors increase as the distance from the sample of past observations increases. The increase in this forecast error is accentuated as sampling error increases or uncertainty concerning the validity of the model specification increases. Figure 1 presents an illustration of this well-known relation between forecast error and distance of extrapolation.

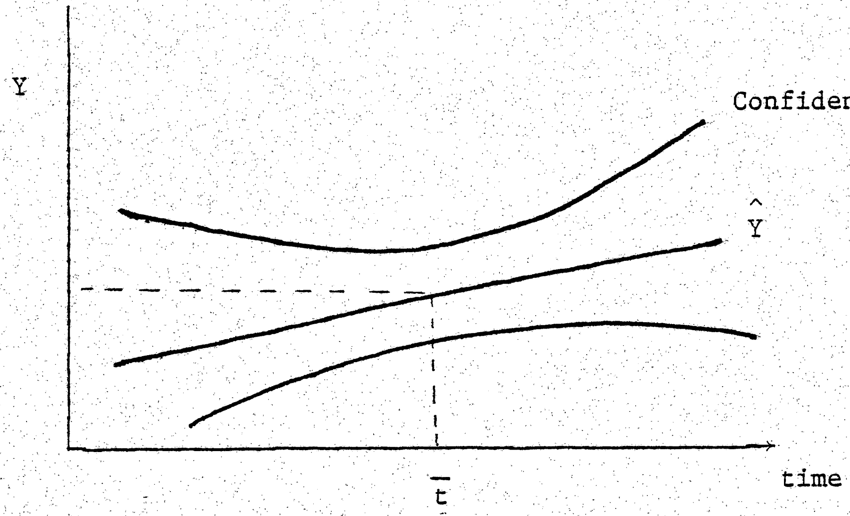

Figure 1. Forecast Error for a Variable Y Measured at $\bar{t}$.

An additional caveat should accompany forecasts of growth rates. Namely, if the growth rates are forecast to persist over a long period of time the compounding affect of growth must be recognized and its rates and the implied levels of yields at various points in the future are reported in Table 1 using wheat as an example.

Table 1

Implications of Alternative Growths Rates of Wheat Yield

Year	Yield Levels Assuming Annual Average Growth Rates of						
	.33%	.50%	.75%	1.00%	1.25%	1.5%	2.0%
1980	31	31	31	31	31	31	31
2000	33.13	34.25	35.99	37.83	39.742	41.75	46.06
2010	34.25	36.00	38.79	41.78	44.99	48.46	56.15
2020	35.41	37.84	41.80	46.15	50.95	56.24	68.45

The importance of these issues is dramatized by an example. As point estimates, the credibility or more appropriately the uncertainty which characterizes productivity forecasts can only be established by considering their forecast errors. That is, as a point estimate the forecast is a point drawn from a distribution of possible values. In the usual case, it represents the mean of these possible forecasts. Although the variance of this distribution (the variance of the forecast) can be readily estimated, typical forecasts of technical change have not considered them (see e.g. S.C.S. [1980], C.E.Q. [1980]. Suppose for illustrative purposes, that the portion of total variation in yield that is explained by a model is 95%. Despite this substantial ability of the model to explain variation in the sample, its accuracy in forecasting (or the variance about its mean forecast) depends not only on the R^2 (=.95), but also on the variation in yields that is to be explained and the differences between the forecasted values of determinants of yield and their sample means. These additional factors could lead to a substantial forecast error. For example, if the point forecast of yield for one year away from the sample were 31.387 representing a 1.25% growth in yield from the previous year's 31.000, it is conceivable that the variance of the forecast would be large enough that the true value of the next year's yield could lie anywhere between 28 and 34. By implication, the implied growth rate could lie anywhere between ± 9.67%.

The magnitude of this variance of forecast and, therefore, the range of uncertainty which would characterize a forecast of percentage change would increase as the forecast period is extended. As this occurs, the model's abililty to forecast is discounted, however a further error is introduced by errors in forecasts of the levels of the determinants of yields in future years. The conclusion must be drawn that the uncertainty that characterizes a forecast twenty years into the future is considerable. In fact, as the previous example has illustrated it is unlikely that if the forecast error were taken into consideration that existing models which present mean forecasts of 1.0% growth in productivity (by any definition) could reject the hypothesis that actual growth would be -1.0%. Given this large range of possible error, point forecasts of productivity change are of limited value. Instead the variance of the forecast, or other characteristics of its distribution (if Bayesian methods are employed) are essential information in any forecast. Carrying this lesson back to the problem of forecasting yields, if the hypothesis of negative or zero growth in total factor productivity cannot be rejected by the models upon which forecasts are based, then it follows that extensive increases in other inputs may be necessary to maintain or increase present yields in the future.

5.2 Introduction of Subjective Information into the Forecast

An important limitation in the usefulness of the past in forecasting the future is that valuable current information may be ignored. Furthermore, only one method as represented by a model is typically chosen for integrating and synthesizing past information into a forecast. These two issues are interrelated in the sense that multiple sources of information which are potentially valuable for a forecast may be available.

Selection of a forecast from a single model and based on a particular data base is an implicit vote of confidence that with probability one that model's forecast is superior to all others. An alternative is suggested by Bayesian methods, see Zellner [1971]. Specifically, expert opinion and current evidence should be explicitly introduced into the forecast through a prior distribution. By the same logic, a rationale can be constructed for introduction of forecasts based on alternative models. The uncertainty which was argued in previous to characterize models of choice grants credibility to such a strategy. Johnson and Rausser [1978] have reviewed methods along these lines. In essence, a weighted average forecast is called for where the weights represent subjectively assigned probabilities that particular forecasts will be correct.

5.3 Econometric Forecasts of Yields or Total Factor Productivity

The issues raised in the last two sections can be placed in focus by considering actual methods available for forecasting yields or total factor productivity. In the case of yields an empirical form for yield equation 4.17) presents the basis for a traditional econometric forecast. For example, if at time t a forecast is desired for yield at time t' = t + λ, and the empirical form of 4.17) is given by:

5.1) $PP_{t'} = F(P_{t'},R_{t'};\theta_{t'},K_{t'},\beta) + \varepsilon_{t'}$

where β represents the vector of population parameters of the algebraic for Y(.),

$\varepsilon_{t'}$ is the population disturbance $\varepsilon_{t'} \sim N(0,\sigma^2)$,

then $PP_{t'}$ represents a forecast given the vector of determinants ($P_{t'}$, $R_{t'}$ $\theta_{t;}K_{t}'$) and the sample disturbance $U_{t'}$. $PP_{t'}$ is a random variable the distribution of which is derived from the distribution of $U_{t'}$. Given normality of the distribution of $U_{t'}$, $PP_{t'}$ is also normally distributed and its distribution is fully characterized by its mean and variance. This mean forecast is given by

5.2) $\hat{PP}_{t'} = F(\hat{P}_{t'},\hat{R}_{t'}, \hat{\theta}_{t'}, \hat{K}_{t'} \hat{\beta})$

where "^" indicates a forecasted value or for

β an estimate,

and is traditionally chosen as the forecast of the model. The error in the forecast,

$$5.3)\quad PP_{t'} - \hat{PP}_{t'} = [E(PP_{t'}) - \hat{PP}_{t'}] + [PP_{t'} - E(PP_{t'})]$$

results from two quantifiable sources. The first of these is a sampling error represented by the difference between $\hat{PP}_{t'}$ the forecast from the sample estimate of the population regression line and the forecast $E(PP_{t'})$ given by the population regression line, i.e.

$$5.4)\quad E(PP_{t'}) = F(\hat{P}_{t'}, \hat{R}_{t'};\ \hat{\theta}_{t'}, \hat{K}_{t'}, \beta)$$

The second results from the random difference between $PP_{t'}$ and $E(PP_{t'})$, or the population disturbance.

Implicitly, this classic partitioning of the forecast error assumes that the form and arguments of PP(.) in 4.17) are known to the researcher. In the more realistic case, this information is not available and the specification of the form and arguments of PP(.) is subject to great uncertainty. Acknowledging this additional source of error (specification error), the first type of error $(E(PP_{t'})-\hat{PP}_{t'})$ would be increased, but in a nonquantifiable way. The conclusion can be drawn that in the absence of a perfect specification, the variance of the forecast discussed in the previous sections can only be interpreted as an estimate of the lower bound of uncertainty characterizing the forecast.

Finally, an additional error in the forecast is assumed zero by the partition of the error in 5.3). This additional error follows from the errors in the forecasts of the levels of the determinants ($\hat{P}_{t'}$, $\hat{R}_{t'}$, $\hat{\theta}_{t'}$, $\hat{K}_{t'}$) which are required for a forecast of PP. The error in the forecast of each of these can be partitioned into sampling error and error due to random disturbance where an implicit element in the sampling error is specification error. The compound effect of these additional errors is to increase the forecast error and its variance for $PP_{t'}$.

Ruttan [1980] presents a good practical example of the possible magnitude of these errors. He notes that during the 1950's, Ruttan [1956] had forecasted that a continuation of relatively slow historical productivity growth rates (1.0 percent per year) could permit substantial growth in output; however, the realized growth rate in productivity change was in excess of 2.2 percent per year.

Similar errors in forecasts can be expected to accompany long-range forecasts of total factor productivity. Econometric forecasting of total factor productivity requires construction of a model of the determinants of technical change. The recent study of Lu, Cline and Quance [1979] is exemplary of this approach. It was established in Section 3.1 that a measure of total factor productivity could be written:

$$5.5)\quad \overset{\circ}{TFP}_t = \overset{\circ}{Y}_{1t} - \overset{\circ}{G}_t(\hat{Y},X,\theta,K) = \frac{\partial G}{\partial K}\,\frac{K}{G}\,\overset{\circ}{K}_t = G_K\,\overset{\circ}{K}_t$$

Obviously, forecasting of TFP cannot proceed by simply forecasting the levels of determinants of Y_1 which are the arguments of $G(\cdot)$, i.e. ($\hat{Y}$, X, θ, K). In addition to these forecasts, a forecast based on 5.5) would also require a forecast of $\overset{\circ}{Y}_{1t}$ which depends upon a forecast of the level of technical change. An alternative is to rely on the right hand side of 5.5). Given a historical estimate of the elasticity of G with respect to

K, a forecast can be based on forecasts of $\overset{\circ}{K}$. The usefulness of such a forecast would depend upon the accuracy of the forecast of $\overset{\circ}{K}$, the accuracy of the measurement of G_K and most importantly the absence of any change in G_K in the future. It is reasonable to conclude that reliability of a forecast based on these required conditions would not be high.

Because the elements of K are likely to be unobservable, an attractive approach is to employ observable measures of the determinants of K. The problem, of course, is that the theory of technical change is not well-developed and may not lead to relations which are easily quantifiable. Lu, Cline and Quance [1979] provide an interesting attempt to forecast total factor productivity by explaining K_t. Specifically, they employ the U.S.D.A. measure of TFP reviewed in Section 3.1) and attempt to explain its historical variation as determined by the level of climatic inputs (W) (which are excluded from the U.S.D.A. measure of TFP), the educational level of farmers (E), production (R) and non-production (NR) oriented research and extension expenditures.

The theoretical link between this approach and the theoretical frameworks reviewed in Section 3 can be seen by recalling that

$$TFP = Q_I/X_I$$

if inputs are assumed separable from outputs and

$$5.6)\quad \overset{\circ}{TFP} = \overset{\circ}{Q}_I - \overset{\circ}{X}_I .$$

If $\overset{\circ}{Q}_I$ is affected by climatic factors and $\overset{\circ}{K}_t$ as argued in Section 3, and if $\overset{\circ}{X}_I$ excludes any measure of these climatic effects, then

$$5.7)\quad \overset{\circ}{TFP} = \tilde{G}_K\overset{\circ}{K} + \tilde{G}_W\overset{\circ}{W}$$

where $\overset{\circ}{W}$ is the growth rate is climatic factors, and $\tilde{G}_K$, $\tilde{G}_W$ are production elasticities of G with respect to K and W.

If K is determined by present and past levels of (R, NR, E), then

$$5.8)\quad K_t = K(\tilde{R}, \widetilde{NR}, \tilde{E})$$

where $\tilde{R}$, $\widetilde{NR}$, $\tilde{E}$ are vectors of present and past value of (R_t, NR_t, E_t), and

$$K_t = K_R \overset{\circ}{\tilde{R}}_t + K_{NR} \overset{\circ}{\widetilde{NR}}_t + K_E \overset{\circ}{\tilde{E}}_t$$

where K_R, K_{NR}, K_E are vectors of elasticities of K_t with respect to changes in the elements of $\tilde{R}$, $\widetilde{NR}$, $\tilde{E}$.

and by substitution,

$$5.9)\quad \overset{\circ}{\text{TFP}}_{t_\circ} = \tilde{G}_K (K_R \overset{\circ}{\tilde{R}}_t + K_{NR} \overset{\circ}{\widetilde{NR}}_t + K_E \overset{\circ}{\tilde{E}}_t) + \tilde{G}_W \overset{\circ}{W}_t .$$

If TFP is integrated between two points in time, say t° and t^*, then

$$5.10)\quad \text{TFP} = \frac{A(t^*)}{A(t^\circ)} = T(\tilde{R}_t, \widetilde{NR}_t, \tilde{E}_t, W_t)$$

Following the logic of Sections 3 and 4, the form of T(·) (as is the form chosen to calculate TFP) is directly dependent upon the functional form of the product transformation function G(·). Ideally, the forms of TFP and T(·) should be consistent. The composition of each of the vectors of determinants of K depend upon lags in the processes relating them to K. Lu, Cline and Quance assume T(·) to be of the Cobb-Douglas form in the elements of $\tilde{R}$, $\tilde{NR}$ and $\tilde{E}$ and transcendental in W_t. The resulting equation can be written as a linear logarithmic form:

$$5.11) \quad \ln TFP_t = \alpha_t \ln\tilde{R}_t + \delta_t \ln NR_t + \beta_t \ln\tilde{E}_t + \gamma_t W_t$$

If the parameter vectors $(\alpha_t, \delta_t, \beta_t, \gamma_t)$ are assumed constant over a particular sample period and if an additive normally distributed disturbance ε is added to 5.11), then it can be estimated by ordinary least squares. Lu, Cline and Quance estimate such a form of 5.11) using an Almon lag to determine the lagged relations between the hypothesized determinants of K_t and lnTFP. The resulting equation is used to forecast future levels of TFP from which growth rates in TFP are calculated.

While this approach presents a forecast which relative to other econometric forecasts leaves little room for improvement, the forecast errors are nonetheless subject to the forecast errors discussed above in the context of yield forecasting. It is unfortunate that Lu, Cline and Quance do not report the quantifiable variances of their forecasts. Although a standard deviation, maximum and minimum are reported in Tables 4-6 of their paper these represent characteristics of the variation of the mean forecast generated for different values of the climatic index and having no relation to the distribution of the forecast error. Although alternative forecasts are offered given different growth rates in R and NR no information is provided concerning the accuracy of any of these forecasts. The conclusion must be drawn that as has been argued in prior sections the possibility of substantial error in these forecasts is great.

5.4 Non-Econometric Forecasts of Technical Change

Section 5.2 noted that one weakness in econometric forecasts is their reliance on past information and their implicit assumption of persistence of the process which historically has generated the variable for which a forecast is desired. An immediate implication of this latter assumption is that current information may not be incorporated into the forecast. In many cases current information may in fact indicate that with high probability the structure of past processes will change rendering the past of less importance for a forecast. An alternative is suggested by the common practice of forecasters to adjust the constant term in econometric models to account for new, current information which may not be incorporated in the model. The Bayesian and composite forecasting approach reviewed in Section 5.2 presents still another alternative. The Delphi approach presents an operational method of constructing a forecast which relies heavily on current information.

An excellent example and application of such an approach is presented by Lu, Cline and Quance [1979]. In order to assess the possible impact of new agricultural production technologies which appear on the frontier, a group of experts were interviewed, possible new technologies were identified, subjective probability distributions for the occurrance and adoption of the new technologies were constructed and impacts on the total factor productivity for crops and livestock activities were assessed. Combination of this information allows the construction of probability distributions of possible future productivity levels. Lu, Cline and Quance present the means of these distributions as point forecasts; however, they provide no indication of their possible dispersion or possible error associated with the point forecasts. A maximum point forecast is provided which assumes early occurrance and rapid adoption; however, this measure cannot be interpreted as providing a measure of the uncertainty or variance of the forecast error.

5.5 Forecasting Yields from Forecasts of Total Factor Productivity Change

Section 2.3 noted that the growth in yields could be written in terms of the index of total factor productivity change and changes in output due to changes in inputs. Specifically, for the single output case:

$$5.12)\quad \overset{\circ}{PP} = \overset{\circ}{TFP} + \overset{\circ}{X}_I - \overset{\circ}{L}$$

where $\overset{\circ}{X}_I$ is the growth rate of the index of total inputs,

$\overset{\circ}{L}$ is the growth rate of the fixed input L with respect to which yield is measured.

The forecast of each of the elements of $\dot{PP}$ is a random variable with a forecast error. In the simplest case where the distributions of these errors for the components are independent, the variance of the forecast of yield will always exceed that of a forecast of $\overset{\circ}{TFP}$, the index of the effect of technical change on output. In such a case,

$$5.13)\quad \hat{\sigma}^2(\overset{\circ}{PP}) = \hat{\sigma}^2(\overset{\circ}{TFP}) + \hat{\sigma}^2(\overset{\circ}{X}_I) + \hat{\sigma}^2(\overset{\circ}{L})$$

where $\hat{\sigma}^2(Z)$ indicates the estimated forecast error of Z.

Although it may be true that a forecast of yield is of ultimate interest to policy formation, 5.13) illustrates that the possible error in any forecast of the growth rate of yield depends critically not only on accurate forecasts of total factor productivity, but also on the forecast errors of forecasts of changes in input uses as captured by $\overset{\circ}{X}$ and $\overset{\circ}{L}$. These changes depend upon relative prices and exogeneous factors facing the firm as indicated in Section 2. A forecast of yields, therefore, at least implicitly carries with it a forecast of these determinants of input utilization.

6. A Consideration of Alternative Technical Change Forecasts

Previous discussion has noted the uncertainty that characterizes the specification of models which can be used for forecasting as well as that which is associated with a particular forecast. Despite this uncertainty, or risk of error the assumed levels of productivity change in the S.C.S. [1980] and C.E.Q. [1980] offer only point estimates of future growth rates. S.C.S. [1980] employs a forecast a growth rate of "agricultural productivity" varying between .8% and 1.1% while C.E.Q. [1980] relies on a forecast of the growth rate of total output of food as 2.2%. If it is assumed that the S.C.S. [1980] forecast is one of total factor productivity in agriculture, then its consistency with the C.E.Q. [1980] projection of total output growth can be assessed by employing the definition of total factor productivity given in Section 2, i.e.

$$6.1)\quad \overset{\circ}{TFP} = \overset{\circ}{Q}_I - \overset{\circ}{X}_I - \overset{\circ}{L}$$

$$6.2)\quad \overset{\circ}{Q}_I = \overset{\circ}{TFP} + \overset{\circ}{X}_I + \overset{\circ}{L}$$

That is, growth in outputs results from either changes in technology as measured by TFP or changes in the levels of inputs, $\overset{\circ}{X}_I$ or $\overset{\circ}{L}$. If the two forecasts are consistent, then the growth in the level of input utilization as measured by X_I or L would have to fall in the range of 1.1% to 1.3%. Because fixed inputs θ other than land (L) were included in the definition of X_I, the growth in the level of input use could occur as a result of purchased inputs, the expansion of other fixed input use, or expansion of land use.

The accuracy of any forecast is difficult to assess as Section 5 has argued. However, in the case of these point forecasts no discussion is offered concerning the origins of suspected or actual variance of the forecasts. Following the discussion of Section 5, if these variances are of usual magnitude (e.g. 10% of the mean forecast), then the forecasts could take on a wide range of values.

7. Usefulness of Forecasts: The Costs of Increased Output

Previous sections have focused on the problems of measuring the level of productivity at any point in time and of forecasting the change in productivity in the future. This and the following section will briefly consider two important uses of this information. The present section will focus on determination of the possible costs of further increases in output while Section 8 will consider the usefulness of the distribution of the forecast error for an assessment of alternative methods of achieving specific goals for the supply of food. While it is beyond the scope of this paper to present an exhaustive enumeration or consideration of these costs, an overview of their general nature is appropriate.

The costs of achieving any level of output fall into two broad categories: those internal to the firm and those external to the firm. Within each category there exist two types of costs: pecuniary and non-pecuniary. The pecuniary costs in each case are observable costs which are borne by either the firm, as in the case of higher input expenditures, or by other firms or consumers, as in the case of higher input expenditures which at least in the short-run are resultant from increases in input prices induced by industry-wide expanded use of input. Other pecuniary costs borne by the firm might include higher short-run production costs resultant from input supply bottlenecks or fixity which prevent the firm from adjusting its resource mix to be consistent with the best technology cost function. In the long-run, higher production cost might be incurred as a result of myopic behavior which could lead to substitution of slowly renewable soil inputs for highly priced marketed inputs. External pecuniary costs would include such costs as those incurred by increased soil erosion or chemical run-offs. In addition, the opportunity cost of changing the output mix should also be considered.

Non-pecuniary costs whether internal or external to the firm result from preferences of the firm's management or society which are unrelated to monetary values. For this reason, their enumeration requires knowledge of these preferences. An example is the traditional concensus in the U.S. that family farming is an institution which deserves protection. If scale economies indicated that substantial increases in output could be achieved by reorganization of family sized farms into larger scale production units, then the loss of family farming would incur a non-pecuniary external cost. At this extreme, when the external cost may be borne by all of society it is labeled a social cost. Non-pecuniary costs which may be internal to the firm are constituted by the vast array of social, political and cultural preferences which often affect production decisions and the adoption of new technology.

While measurement of non-pecuniary costs is extremely difficult and involves methodologies which are independent of those employed to measure or forecast productivity, an estimate of pecuniary costs of expanded output can be obtained from information required to measure or forecast productivity. Estimates of the short-run cost impacts of increased use of inputs can be directly obtained from parameter estimates obtained by the estimation of a profit function as proposed in Section 4. Given a projected scenario for prices and levels of fixed factors, projections of input-use, output supply and short-run cost and profit are easily attainable. In addition, changes in costs and profits which could be expected to result from changes in scale of operation or output mix could also be forecasted.

External pecuniary costs of expanded output would be more difficult to forecast. Certainly, the research and extension expenditures necessary for attainment of target levels of productivity could be determined using a methodology such as that used by Lu, Cline and Quance. The probable costs of converting land to be suitable for crop production could also be estimated. Perhaps most difficult to project would be the external costs incurred by expanded use of particular types of inputs. For example, petroleum intensive input expansion depends upon supply and price levels which are difficult to forecast. However, demand for particular pesticides, herbicides or fertilizer constituents are highly responsive to output price levels (see e.g., Weaver [1980]), the effects of expanded demand on prices would require careful study of the competitive characteristics of the local markets for these products.

8. An Insurance Approach to Food Security Based on The Distribution of the Forecast Error

The importance of the distribution of the forecast error for the consideration of alternative means of achieving various food security objectives can be illustrated by an example. Suppose that a forecast of total factor productivity change is obtained from a composition of econometric forecasts and expert opinion and that an estimate of the distribution f(·) of the forecast error T is also available. For example, if

8.1) $\mathring{T} = \mathring{TFP} - \hat{\mathring{TFP}}$

then

8.2) $\mathring{T} \sim N(0,\sigma^2_{\mathring{T}})$

where $\hat{\mathring{TFP}}$ is the mean forecast

$\sigma^2_{\mathring{T}}$ as noted is Section 7 is not a constant but depends upon 1) σ^2, the variance of the disturbance in the relation used to forecast TFP, and 2) $\sigma^2_{\mathring{TFP}}$, the variance of the sampling error.

Social preferences concerning food security would provide a basis for the definition of a welfare function which would describe the welfare resultant from particular events or types of insecurity in the supply of food. For example, suppose the focus of social concern were the relation between the growth rate in demand ($\overset{\circ}{Q}_d$) and that of the supply of food ($\overset{\circ}{Q}_s$); however, the costs (C) of achieving various levels of balance between these growth rates is also of social importance. Suppose these preferences could be represented by a welfare function $W(\cdot)$ defined at a particular time t as:

$$8.3) \quad W = W(B, C)$$

where

$B = \overset{\circ}{Q}_s - \overset{\circ}{Q}_d$,

$C = C(\overset{\circ}{\pi}, E)$ where $C_1 < 0 \quad C_2 > 0$,

$\overset{\circ}{\pi} = M_y(\overset{\circ}{P} + \overset{\circ}{Y}) - M_x(\overset{\circ}{R} + \overset{\circ}{X})$,

M_y, M_x are vectors of revenue and expenditure shares of profit,

$\overset{\circ}{E} = E(\overset{\circ}{Y}, \overset{\circ}{X}, \overset{\circ}{L})$ represents the external cost of growth, and time dating subscripts have been omitted.

Note that cost C can be considered as including both pecuniary $\overset{\circ}{\pi}$ and non-pecuniary internal as well as external costs in the vector $\overset{\circ}{E}$. Finally, suppose the growth rates of demand and supply are determined as follows (in matrix notation):

$$8.4) \quad \overset{\circ}{Q}_d = \eta^d_p \overset{\circ}{P} + \eta^d_\phi \overset{\circ}{\phi}$$

8.5) $\overset{\circ}{Q}_s = \eta^s_P \overset{\circ}{P} + \eta^s_R \overset{\circ}{R} + \eta^s_L \overset{\circ}{K} + \eta^s_L \overset{\circ}{L}$

where η^d, η^s are vectors of demand and production elasticities.

Now, in the context of planning the problem is to select the growth rates of the elements of $\overset{\circ}{Y}$, $\overset{\circ}{X}$, and $\overset{\circ}{L}$ for given scenarios of $(\overset{\circ}{P}, \overset{\circ}{\phi}, \overset{\circ}{K}, \overset{\circ}{R})$. Since these latter growth rates are random, and endogeneous in the long-run assume the planner seeks to maximize the expected present value discounted stream of welfare $\tilde{W}$ resultant from time t to a horizon H which would result from a particular set of actions $(\overset{\circ}{Y}, \overset{\circ}{X}, \overset{\circ}{L})$. The distributions of $(\overset{\circ}{P}, \overset{\circ}{R}, \overset{\circ}{K}, \overset{\circ}{\phi})$ can be derived from the distribution of respective forecast errors. Suppose that the actions which optimize W are labeled $(\overset{\circ}{Y}^*, \overset{\circ}{X}^*, \overset{\circ}{L}^*)$. Now suppose that the worst possible case outcome for $(\overset{\circ}{P}, \overset{\circ}{R}, \overset{\circ}{K}, \overset{\circ}{\phi})$ occurs. The loss in welfare can be written:

8.6) $D = \tilde{W}(\overset{\circ}{Y}^*, \overset{\circ}{X}^*, \overset{\circ}{L}^*) - W^{\circ}(\overset{\circ}{Y}^*, \overset{\circ}{X}^*, \overset{\circ}{L}^*)$

where $\overset{\sigma}{W}$ is the present value discounted stream of welfare which actually resulted from $(\overset{\circ}{Y}^*, \overset{\circ}{X}^*, \overset{\circ}{L}^*)$ given the worse possible outcome for $(\overset{\circ}{P}, \overset{\circ}{R}, \overset{\circ}{K}, \overset{\circ}{\phi})$, while $\tilde{W}$ represents maximum expected welfare given the subjective distribution held for $(\overset{\circ}{P}, \overset{\circ}{R}, \overset{\circ}{K}, \overset{\circ}{\phi})$.

If W satisfies traditional neo-classical properties, then D will be positive and interpretable as the loss incurred as a result the occurrance of the worse possible case. Alternatively, the value D multiplied by the probability of the occurrance of the worst case can be interpreted as the insurance value of protection from the worse possible outcome of ($\mathring{P}$, $\mathring{R}$, $\mathring{K}$, $\mathring{\phi}$), i.e., the amount the planner should be willing-to-pay to avoid the worst outcome. In the case of food security it is likely that there exist a number of unexceptable or the worst possible scenarios which could occur with non-zero probability. The probabililty of each of these can be determined from the forecast error distributions.

One form of insurance could be thought of as the expense incurred as a result of increasing the availability of a particular input beyond the level which would result from optimization of $\tilde{W}$. Specifically, consider an increase in $\mathring{L}^*$. By definition, of $\tilde{W}$ and the assumption that $\partial^2 W/\partial L^2 <$ 0,

$$\frac{\partial \tilde{W}(\mathring{Y}^*, \mathring{X}^*, \mathring{L}^*)}{\partial \mathring{L}} < 0$$

whereas, for $(\mathring{P},\mathring{R},\mathring{K},\mathring{\phi})$ less than $(\mathring{P}^*,\mathring{R}^*,\mathring{K}^*,\mathring{\phi}^*)$ which leads to $\tilde{W}$,

$$\frac{\partial W^\circ}{\partial \mathring{L}} > 0$$

Therefore,

$$\frac{\partial D}{\partial \mathring{L}} = \frac{\partial \tilde{W}}{\partial \mathring{L}} - \frac{\partial W^\circ}{\partial \mathring{L}} < 0$$

Similar results hold for $\mathring{Y}$ and $\mathring{X}$. It follows then that loss incurred as a result of undesirable possible outcomes can be reduced by control of ($\mathring{Y}$, $\mathring{X}$, $\mathring{L}$). The problem with this approach is that $\mathring{Y}$ and $\mathring{X}$ (as interpreted as marketed inputs) may not be as directly controllable as might $\mathring{L}$, the growth rate of land stocks. If this were the case, a strong argument could be constructed for the management of land stocks to achieve a growth rate which reduces the possible loss to an acceptable level. Clearly, the role of controllable determinants of technical change such as research and extension expenditures must also be recognized. The work of Lu, Cline and Quance [1979], Ruttan [1980], Evenson [1968], and Griliches [1964] establishes a strong relation between these expenditures and productivity change. However, given the uncertainty which characterizes the lag in their effects and the magnitude of their effects, land management may provide the most expedient, least-cost alternative for insuring against the losses which would occur if worse possible scenarios were realized.

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