FOOD SYSTEM DEMAND ESTIMATION: COMBINING SAMPLE INFORMATION WITH SLUTSKY RESTRICTIONS

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INTRODUCTION

The determinants of consumer demand for food commodities are important to agricultural policy makers and forecasters. Model specifications typically include as explanatory variables own-price, prices of close substitutes and complements, income, and perhaps an indicator of taste and preferences. Elasticities obtained from these specifications tend to be sensitive to the choice of sample period and modifications in the specification including alternative variable combinations and functional forms (King, Wohlgenant). These _ad hoc_ specifications result in models that often produce unsatisfactory forecast and policy analyses.

A complete demand system for food commodities is an alternative to these _ad hoc_ specifications. Earlier works by Brandow, George and King, and Hassan and Johnson utilized Slutsky restrictions from individual consumer demand theory to "construct" a matrix of demand parameters. More recently, Huang and Haidacher employed constrained maximum likelihood estimation and Slutsky restrictions to simultaneously estimate a system of food demand.

When preferences are homothetic and the distribution of income is independent of prices, the market demand function has all the properties of the individual consumer demand function (Eisenberg). However, generally, and in particular non-homothetic, preferences need not satisfy the Slutsky restrictions (Sonnenschein 1973a, 1973b; Diewert). The importance of these results for applied demand analysis is clear: strong assumptions are needed to justify use of the Slutsky restrictions in estimating market demand functions.

Empirically, the question of when the Slutsky restrictions carry over to market data has been the subject of extensive investigation (Deaton and Muellbauer). For market data, the Slutsky restrictions are usually rejected (Barten, Byron, Court, Lluch). Of course, these restrictions can be imposed by the utility function for the demand system, but this latter approach and variants that permit testing of the Slutsky restrictions and, in fact, separability assumptions are not feasible for disaggregated demand systems (Berndt et al., Christensen et al.).

Problems with multicollinearity and observational errors in market data, however, preclude the relevance of an unconstrained system. The application of prior information to demand system parameters is an appealing alternative to fully constrained or unconstrained systems. Bayesian inference is one method of applying prior information to demand systems. Keifer uses the theoretical restrictions applicable to a single consumer as prior information in a Bayesian procedure to obtain parameter estimates for labor supply and household expenditures. Theil and Goldberger's mixed estimation technique is another method of applying prior information. Recently, Safyurtlu et al., have shown that Slutsky restrictions can be applied in market demand systems estimation locally and stochastically.

In this analysis, the procedures suggested by Safyurtlu et al., are applied to a system of market food demand for the United States. Comparisons of the results are discussed based on unrestricted ordinary least squares estimations, exact restrictions of the Slutsky conditions, and mixed estimation of the demand system, where the restrictions are applied locally and stochastically. Substantial differences in the predictive abilities of the estimated equations are found based upon (the degree of) restriction imposition.

MODEL

Linear Demand Systems

For the local, stochastic approximation to the demand system, linearity in the structural parameters is assumed. Let the vectors of T observations on the $i = 1, \ldots, n$ commodity groups be denoted y_i. The concomitant observations on prices and income (or total food expenditure) are denoted by the matrix X_i of dimension $T\mathrm{x}((n+1) = \ell)$. The demand equation for the ith commodity (or group) is then

$$y_i = X_i \beta_i + u_i, \tag{1}$$

where β_i is a conformable parameter vector and u_i is the vector of disturbances distributed with mean 0 and variance covariance matrix Ω_{ii}.

The full system of n demand equations can be expressed as

$$y = X\beta + u \tag{2}$$

where $y' = (y_1', y_2', \ldots, y_n')$, $\beta' = (\beta_1', \beta_2', \ldots, \beta_n')$, $u' = (u_1', u_2', \ldots, u_n')$ and X is a $nT\mathrm{x}n\ell$ block diagonal matrix with diagonal submatrices X_i (Zellner).

Mixed Estimation

Using the mixed estimation procedure, Paulus showed how stochastic prior information on a subset of the parameter vector (income elasticities) could be effectively combined with the sample data to yield own price elasticity estimates that are much more precise (small standard deviation) than those of the sample. Mixed estimation was used to combine prior and sample information because of its flexibility and ease of implementation. Furthermore, the process resulted in a significant reduction in the effective number of unconstrained parameters in a model.

For the consumer optimization problem, there are $(n^2-n)/2$ symmetry restrictions, n homogeneity restrictions, and one Engel aggregation restriction. Moreover, these restrictions are appropriate for the individual consumer only for selected prices, quantities, and income or implicitly, budget proportions unless separability assumptions are imposed.

The market data include effects of heterogeneity of preferences, proxy variables for prices and incomes, and differences in the household production functions to mention a few of the reasons for inaccuracies in the Slutsky conditions. These inaccuracies are the basis for imposing the Slutsky restrictions stochastically.

The full set of stochastic restrictions based on the Slutsky conditions can be written

$$r = R\beta + v \tag{3}$$

where R is a matrix of dimension $Jx((n) = K)$ with $J < K$ and r is a conformably defined vector of constants. The mixed estimation problem (Theil and Goldberger) is completed by assumptions on the distribution of the elements of v which are assumed distributed $E(v) = 0$, and $E(vv') = \sigma_v^2 I = V$.

The mixed estimator is

$$\hat{\beta}^* = (X'\Omega^{-1}X + R'V^{-1}R)^{-1}(X'\Omega^{-1}y + R'V^{-1}r) \tag{4}$$

with covariance matrix

$$\text{var}(\hat{\beta}^*) = (X'\Omega^{-1}X + R'V^{-1}R)^{-1}. \tag{5}$$

It is easily shown that $\text{var}(\hat{\beta}) - \text{var}(\hat{\beta}^*)$, where $\hat{\beta}$ is the least squares estimator, is positive semidefinite (Fomby, Hill, and Johnson). The stochastic prior restrictions can be weighted more or less strongly relative to the sample data by incorporating a factor $1/\omega$. As $\omega \to \infty$, the mixed estimator approaches the OLS estimator. Then the mixed estimator is

$$\hat{\beta}_\omega = (X'\Omega^{-1}X + R'PV^{-1}P'R)^{-1}(X'\Omega^{-1}y + R'PV^{-1}P'r) \tag{6}$$

where P is a diagonal matrix with elements equal to $\omega^{-1/2}$. An _ad hoc_ method of appropriately weighting the restrictions is to evaluate them using demand parameters estimated from the sample data alone. By applying these parameters and the average expenditure proportions, $\bar{r}$ and $\bar{R}$, distributions of the residuals for the Slutsky restrictions can be calculated. It is emphasized that specifying the Slutsky restrictions using $\bar{R}$ and $\bar{r}$ at their mean values implies the restrictions are "more true" for values of prices and quantities near the reference values than for other values of prices and quantities. This is the reason for defining the estimators as local approximations to the market demand system.

DATA

Data for the market demand relationships are retail price indexes, per capita food consumption and total per capita food expenditures. All price indexes and expenditures are deflated by a nonfood price index consistent with the suggestion by LaFrance regarding the estimation of incomplete demand systems (i.e., the nonfood category was not estimated). This deflator is the most general way to attain zero degree homogeneity with constant prices and income elasticities for an incomplete system of demand equations.

Food consumption was disaggregated into five specific meat groups -- beef and veal, pork, poultry, fish, and other meats -- and seven other food groups -- eggs, dairy, fruits and vegetables, cereals and bakery products, sugars and sweeteners, and non-alcoholic beverages. Price indexes, food expenditures, consumption levels, and population data were obtained from two USDA bulletins Food Consumption, Prices, and Expenditures (FCPE), and FCPE, 1963-83. Annual quantity, price, and expenditure data span years 1951-1983.

Budget share weights were derived from value aggregates for food items for the periods 1957-1959 and 1965-1967. These value aggregates were found in FCPE.

RESULTS

The twelve commodity system for food was estimated using double logarithmic functional form. As a result, all reported coefficients are constant price and expenditure elasticities with respect to the food commodity. The model was estimated using ordinary least squares, exactly restricted least squares, and mixed estimation where restrictions were imposed both locally and stochastically.

The coefficient estimates from the ordinary least squares estimates are reported in Table 1. With the exception of fish, dairy, and sugars, all of the own price elasticities have the expected sign. However, only six of these are statistically significant. Of the 122 cross price elasticities, only 24 are significant.

The exactly restricted least squares estimates are reported in Table 2. Homogeneity, Engel aggregation and symmetry conditions hold exactly using the unrounded estimates. With the exception of fish and dairy, all of the own price elasticities have the expected negative sign, and only one of these is statistically insignificant. Many more, 96 of the 122, cross price elasticities were significant when compared to the unrestricted OLS estimates.

The results obtained from the mixed estimation procedure are shown in Table 3. (In this analysis, the sample data and the Slutsky restrictions were weighted equally. Other weighting schemes have been investigated and are available from the authors.) As with the unrestricted and restricted estimates, the own price elasticities for fish and dairy are positive. Only one of the correctly signed own price elasticities is insignificant. Almost half (47) of the cross price elasticities are significant. The expenditure elasticities for all the commodities with the exception of poultry and sugar declined when compared to the exactly restricted estimates.

TABLE 1

UNCONSTRAINED OR ORDINARY LEAST SQUARES ESTIMATES OF ELASTICITIES AND THEIR STANDARD ERRORS FOR TWELVE FOOD GROUPS IN THE UNITED STATES, 1951-1983

	Beef	Pork	Poultry	Fish	Other Meats	Eggs	Dairy	Fruits and Vegetables	Cereals	Sugars and Sweeteners	Fats and Oils	Beverages	Expenditures	Constant
Beef	-.53	.36	-.02	.00	-.66	-.12	.71	.56	-.12	-.11	-.13	-.26	1.20	-2.98
	.46	.22	.13	.19	.56	.12	.40	.32	.56	.22	.21	.10	.33	2.05
Pork	-.15	-1.01	.32	.16	.39	-.10	.49	-.26	-.24	-.03	.04	-.12	.71	-.26
	.26	.12	.07	.11	.32	.07	.22	.18	.32	.12	.12	.06	.19	1.15
Poultry	-.36	-.12	-.61	.04	.86	-.07	-.21	-.07	-.02	-.03	-.04	-.03	1.01	-2.45
	.19	.09	.05	.08	.23	.05	.16	.13	.23	.09	.09	.04	.14	.85
Fish	-.26	.11	.05	.53	.25	-.05	.05	.03	-.91	.26	-.03	-.11	.05	2.09
	.24	.11	.07	.10	.29	.06	.21	.17	.29	.11	.11	.05	.17	1.07
Other Meats	.30	.08	.10	-.20	-.82	-.07	1.03	-.06	.69	-.44	-.21	-.03	-.05	2.99
	.38	.18	.11	.16	.47	.10	.33	.27	.47	.18	.17	.08	.28	1.71
Eggs	.19	-.01	.25	-.35	-.15	-.14	.52	-.21	-.30	-.06	.15	.07	-.16	4.69
	.16	.08	.05	.07	.20	.04	.14	.11	.20	.08	.07	.04	.12	.72
Dairy	.27	.12	.11	-.23	-.36	.01	.02	.01	.20	-.11	.03	.08	-.19	7.01
	.14	.07	.04	.06	.18	.04	.12	.10	.18	.07	.07	.03	.10	.64
Fruits and Vegetables	.12	-.07	.11	.06	-.01	.08	-.27	-.58	.26	.05	-.06	.12	-.02	6.23
	.22	.10	.06	.09	.27	.06	.19	.15	.27	.10	.10	.05	.16	.97
Cereals	.00	-.03	.09	.08	.14	.00	.14	-.30	-.22	.21	-.08	.06	-.16	5.93
	.22	.10	.06	.09	.26	.06	.18	.15	.26	.10	.10	.05	.15	.94
Sugars and Sweeteners	-.41	-.18	-.04	.28	.40	-.07	-.05	.24	-.47	.09	-.07	-.15	.60	1.10
	.31	.15	.09	.13	.38	.08	.27	.21	.38	.15	.14	.07	.22	1.37
Fats and Oils	-.27	-.10	.09	.05	.24	-.15	-.13	-.09	.02	.09	-.30	-.11	1.10	-2.91
	.09	.09	.06	.08	.24	.05	.17	.14	.24	.09	.09	.04	.14	.87
Beverages	-.52	-.19	.27	-.02	.74	-.15	.66	.09	-.53	.09	.06	-.34	-.51	5.89
	.49	.24	.14	.21	.61	.13	.43	.34	.61	.23	.23	.11	.36	2.21

TABLE 2

EXACTLY RESTRICTED LEAST SQUARES ESTIMATES OF ELASTICITIES AND
THEIR STANDARD ERRORS FOR TWELVE FOOD GROUPS IN THE UNITED STATES, 1951-1983

	Beef	Pork	Poultry	Fish	Other Meats	Eggs	Dairy	Fruits and Vegetables	Cereals	Sugars and Sweeteners	Fats and Oils	Beverages	Expenditures	Constant
Beef	-.98	.02	-.02	.01	-.10	-.07	-.61	-.53	-.41	-.07	-.05	-.25	3.07	-14.52
	.05	.02	.01	.01	.04	.01	.03	.05	.03	.02	.01	.02	.06	.36
Pork	.18	-.89	.03	.07	-.12	-.09	-.32	-.42	-.21	-.09	.00	-.14	1.99	-8.16
	.03	.03	.01	.01	.02	.01	.03	.04	.02	.02	.01	.01	.04	.25
Poultry	.23	.16	-.77	-.03	.06	.05	-.24	-.27	-.13	.08	-.05	-.01	.91	-1.86
	.04	.03	.03	.02	.03	.01	.06	.06	.03	.02	.02	.02	.07	.41
Fish	.27	.33	-.07	.33	-.56	-.33	-.53	-.18	-.51	-.10	-.02	-.07	1.44	-6.46
	.07	.04	.03	.05	.08	.02	.09	.08	.07	.04	.03	.02	.08	.50
Other Meats	-.31	-.34	-.01	-.38	-.18	-.17	-.39	-.65	.44	-.56	-.07	-.07	2.67	-13.81
	.13	.06	.04	.05	.16	.03	.11	.09	.09	.05	.05	.03	.09	.57
Eggs	.11	-.05	.08	-.18	-.08	-.11	.36	-.19	-.14	-.19	.00	.06	.33	1.65
	.03	.02	.01	.01	.03	.01	.03	.03	.03	.02	.01	.01	.03	.21
Dairy	-.06	.01	-.02	-.04	.02	.10	.13	-.06	.09	-.13	-.06	.07	-.03	6.03
	.02	.02	.01	.01	.02	.01	.04	.04	.02	.02	.01	.01	.04	.22
Fruits and Vegetables	.08	.00	-.02	.01	-.01	-.02	-.06	-.10	-.08	.04	-.01	.08	.08	5.64
	.03	.02	.01	.01	.02	.01	.03	.04	.02	.01	.01	.01	.04	.23
Cereals	-.12	.00	-.01	-.08	.27	-.04	.15	-.14	-.17	.16	.01	.03	-.07	5.43
	.04	.02	.01	.02	.04	.01	.04	.04	.04	.02	.02	.01	.04	.26
Sugars and Sweeteners	.15	-.04	.07	-.03	-.32	-.15	-.55	.02	.23	-.14	.12	-.11	.76	.13
	.04	.03	.02	.02	.03	.01	.05	.06	.04	.03	.02	.02	.06	.37
Fats and Oils	-.04	.03	-.10	-.01	-.04	-.05	-.61	-.37	-.10	.17	-.30	-.11	1.55	-5.74
	.05	.03	.02	.03	.06	.02	.07	.06	.05	.03	.04	.02	.06	.40
Beverages	-.55	-.25	-.06	-.05	-.03	-.02	-.16	-.12	-.16	-.19	-.08	-.49	2.16	-10.58
	.04	.02	.01	.01	.02	.01	.04	.05	.02	.02	.01	.03	.07	.40
Budget Shares	.141	.089	.042	.024	.038	.038	.170	.221	.102	.054	.030	.052		

TABLE 3

MIXED ESTIMATION ESTIMATES OF ELASTICITIES AND
THEIR STANDARD ERRORS FOR TWELVE FOOD GROUPS IN THE UNITED STATES, 1951-1983

	Beef	Pork	Poultry	Fish	Other Meats	Eggs	Dairy	Fruits and Vegetables	Cereals	Sugars and Sweeteners	Fats and Oils	Beverages	Expenditures	Constant
Beef	-.86	.18	.03	.02	-.10	.00	-.04	.27	-.17	-.02	-.04	-.25	1.39	-4.15
	.07	.05	.03	.02	.05	.03	.12	.13	.08	.04	.03	.03	.12	.73
Pork	.30	-.78	.18	.10	-.05	.00	.13	-.27	-.06	-.08	.03	-.03	.52	.84
	.05	.04	.03	.02	.04	.03	.10	.10	.08	.04	.03	.02	.09	.54
Poultry	.17	.09	-.69	-.05	.13	-.01	-.28	-.02	-.04	-.06	-.02	.02	1.10	-3.00
	.05	.04	.03	.03	.05	.03	.09	.08	.08	.04	.03	.02	.08	.50
Fish	.23	.33	.05	.46	-.27	-.12	-.23	.06	-.45	.06	-.04	-.02	-.09	2.97
	.08	.05	.04	.06	.09	.03	.11	.10	.10	.05	.04	.03	.10	.64
Other Meats	-.07	-.03	.12	-.17	-.27	-.01	.32	-.04	.41	-.33	.00	-.06	-.08	3.16
	.14	.08	.04	.05	.17	.04	.14	.13	.13	.06	.05	.04	.13	.82
Eggs	.07	.01	.19	-.19	.03	-.11	.58	-.19	-.34	-.04	.09	.06	-.48	6.65
	.04	.03	.02	.02	.04	.02	.07	.06	.06	.03	.02	.02	.06	.36
Dairy	-.02	.03	.09	-.03	.00	.05	.09	-.02	.08	-.09	-.02	.05	-.40	8.33
	.04	.03	.02	.02	.04	.02	.07	.06	.06	.03	.02	.02	.06	.34
Fruits and Vegetables	.13	.02	.06	.07	-.03	.00	.04	-.38	.02	.10	-.04	.10	-.26	7.73
	.04	.04	.02	.02	.04	.02	.08	.08	.07	.03	.02	.02	.07	.44
Cereals	.03	-.01	.04	-.03	.07	-.01	.26	-.18	-.24	.19	.01	.06	-.11	5.65
	.05	.03	.02	.02	.05	.02	.07	.07	.08	.03	.03	.02	.07	.42
Sugars and Sweeteners	.11	-.10	-.01	.00	-.27	-.98	-.40	.23	.01	-.11	.04	-.08	1.06	-1.74
	.06	.05	.03	.03	.05	.03	.11	.11	.08	.05	.03	.03	.10	.62
Fats and Oils	.02	.04	-.03	-.04	-.05	-.05	-.32	-.12	-.01	.10	-.27	-.05	1.09	-2.90
	.06	.04	.03	.03	.07	.03	.10	.08	.08	.04	.04	.02	.08	.50
Beverages	.01	.01	.13	-.01	.02	.00	.24	.27	.05	-.08	.03	-.29	-.63	6.62
	.07	.06	.04	.03	.04	.03	.13	.15	.09	.05	.03	.04	.13	.82
Budget Shares	.141	.089	.042	.024	.038	.038	.170	.221	.102	.054	.030	.052		

The results of Tables 1 and 2 have many similarities with those of Huang and Haidacher and may suggest that the estimates based on the exact restrictions (Table 2) are preferred. However, historical tracking of the actual versus predicted values of the dependent variables provides contrary evidence. Table 4 illustrates the performance of the three estimation procedures over the fit period 1951-1983. Both percentage root mean squared error (PRMSE) and mean absolute percentage error (MAPE) results based on the predictive ability of the equations indicate severe problems for the restricted least squares for several of the commodities. A casual comparison of Tables 1 and 2 reveals substantial differences in the magnitudes and, occasionally, the signs of the coefficients, particularly for fish, other meats, and beverages. These are the three worst predicting equations for the restricted least squares model. The mixed estimation results (Table 3) follow more closely those of the ordinary least squares estimates (Table 1).

While the ranking of results in Table 4 is not unexpected (because no constraints are placed on the OLS estimates, better in-sample predictive performance would be anticipated), the magnitude of the differences is quite shocking. It lends support to the conclusion reached by Barten, Byron, Court, Lluch, Safyurtlu et al., and others that for market data, the Slutsky restrictions are usually rejected, particularly when imposed exactly.

The precision of the coefficients from the mixed estimation procedure is higher than those from the unrestricted OLS equations. In every case except one, the standard errors of the own price coefficients were relatively smaller for the mixed estimation, and in one case (sugars and sweeteners) a positively signed coefficient from the unrestricted case became negative and significant in the mixed estimation. All expenditure elasticities from the mixed

TABLE 4

PERFORMANCE MEASURES OF THREE ALTERNATIVE ESTIMATION PROCEDURES OF A COMPLETE FOOD DEMAND SYSTEM[a]

Commodity	Ordinary Least Squares		Exact Restricted Least Squares		Mixed Estimation	
	PRMSE	MAPE	PRMSE	MAPE	PRMSE	MAPE
Beef	.89	.04	5.67	.20	1.16	.10
Pork	.53	.01	6.19	.42	.67	.01
Poultry	.42	.01	.75	.03	.57	.04
Fish	.83	.06	10.40	.53	1.09	.01
Other Meats	1.25	.04	14.97	.65	1.62	.01
Eggs	.37	.02	3.73	.19	.49	.04
Dairy	.21	.00	1.07	.05	.29	.01
Fruits and Vegetables	.30	.01	1.19	.04	.36	.01
Cereals and Bakery	.36	.02	.81	.01	.44	.02
Sugars and Sweeteners	.54	.04	1.25	.04	.70	.03
Fats and Oils	.43	.01	2.54	.05	.52	.01
Beverages	1.58	.00	14.35	.33	1.93	.07

[a]PRMSE is percentage root mean squared error, MAPE is mean absolute percentage error.

estimation procedure had higher t-statistics relative to their unrestricted counterparts. This is consistent with the finding by Paulus.

CONCLUSIONS

An approach imposing prior restrictions from micro theory locally and stochastically was investigated for estimating market demand function and then compared with unrestricted and exactly restricted estimation procedures. The mixed estimation approach produced plausible results for the United States. The approach is somewhat limited theoretically due to the heuristic basis for imposing the stochastic restrictions. However, this approach provides a simple yet flexible method of reducing multicollinearity among the predetermined price and income variables.

The approach offers some promise for both forecasting and policy analysis. By estimating a complete demand system, the potential problems associated with other, more _ad hoc_ approaches are avoided. However, out-of-sample forecasts were not generated using the retail demand equations as part of a larger agricultural economic model; this analysis is the next projected stop in the research plan.

Finally, the analysis falls into the class of constant elasticity demand models. LaFrance has shown that although these models are relatively restrictive, they are of practical interest in applied economic analysis for utilizing available prior information (from economic theory) and for welfare analysis. The results can be used to measure consumer's surplus for changes in the prices of commodities of interest.

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