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VALUING UNLEVEL INCOME STREAMS

by

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ABSTRACT

The paper "Valuing Unlevel Income Streams" shows that constant income growth models based on capitalization theory can be useful in the real estate appraisal process and are theoretically appealing and defensible. We show that the O.A.R. is not always a true capitalization rate when it is extracted from properties which are experiencing growth in income due to real growth and/or inflation. A "Growth Compensated" O.A.R. may be extracted from the market which should closely correlate through time with other popular market capitalization rates. A defensible method of calculating the reversion explicitly for DCF analysis based on estimated future income and growth in income is also offered.

VALUING UNLEVEL INCOME STREAMS

Introduction

Capitalization theory is the foundation of the income approach to value. In simple form it holds that the value of an asset is determined by the income attributable to that asset. Capitalization may be defined as the conversion of an expected future income stream into a present value. The rate by which future incomes are converted or "capitalized" into a present value is the capitalization rate. Market capitalization rates are composed of risk of default and investor's time preference for money (which together can be thought of as the real capitalization rate), and the expected rate of inflation.

While analysis of financial instruments such as mortgages or bonds is a fairly straightforward application of capitalization theory, its' use in real estate appraisal is less clear-cut. For financial instruments, the valuation problem is primarily one of selecting the correct capitalization rate. Nominal income streams are known with relative certainty and are usually level ending in a known reversion. The selection of appropriate capitalization rates is just part of the real estate valuation problem however. There are also the problems of substantial uncertainty of the precise nature of an income stream which is likely changeable and the estimation of a defensible reversion amount.

This paper offers a theoretically appealing framework for dealing with changeing income streams, the extraction of appropriate capitalization rates, and the explicit justification of reversion amounts for Discounted Cash Flow (DCF) analysis. A "Growth Compensated" Overall Rate (G.C.O.A.R.) is introduced and shown to differ sometimes substantially from the traditional O.A.R. This G.C.O.A.R. should offer a better correlation through time with market capitalization rates for other popular investments. The methods offered admittedly often require computer solutions, but major clients have been applying computers to financial analysis for years and expect similar sophistication from appraisers as well.

Traditional Income Capitalization and the Reversion

A simple but instructive example of capitalization theory applied in financial analysis is the valuation of a British consol which is a bond guaranteed by the U.K. with a known perpetual income stream. Using a current market capitalization rate **i**, a level periodic income payment I_0 (denoted with the subscript 0 to indicate that it is the current payment and repeated because all future payments are equal), and the number of periods **n**, the consol is valued in time period 0

(1)

$$V_0 = I_0/(1+i) + I_0/(1+i)^2 + ... + I_0/(1+i)^n$$

which is reduced as **n** goes to infinity (see appendix) to:

(2)

$$V_0 = I_0/i$$

A point of interest is that the reversion is contained within the infinite series regardless of the holding period. No matter when the consol is resold, the next buyer will value the consol using the same I, the same i (as far as can be known in the present), and the same infinite series. That next buyer's infinite series valuation is already contained in the present series and is already discounted to the present. Of course, financial analysts do not believe that the capitalization rate **i** will remain the same if inflation is expected to be variable. Observing the past we note that even change should be expected to change. But **i** contains all that is known and expected about the future at the time of valuation. Capital gains and losses may occur due to a change in **i** but which or how much is impossible to determine in the present. The capitalization rate **i** contains an inflation premium **p** included to compensate for any erosion of currency that is presently expected.[1] Let **r** equal the real (inflation free) capitalization rate; then the nominal rate is:[2]

(3)

$$i = (1+r)(1+p) - 1.$$

The valuation of income producing real estate has some similarities to the valuation of a British consol. We can think

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1. Irving Fisher, Appreciation and Interest, (New York: Macmillan Publishing Co., Inc., 1896)

2. For a very good discussion of nominal rates see Norman G. Miller and Michael E. Solt "Using a Real Discount Rate Model Is Better than Predicting Inflation," The Appraisal Journal (April 1986): 188-197

of real estate as providing a perpetual income stream even though we realize that it will likely change sometime in the future. Unfortunately we often don't know when or how much the income stream will change. The real component of the cap rate **r** for a real estate investment is likely to be substantially higher than for the British consol due to the risk associated with the uncertain future stream of income. As in the British consol example above, where **i** is expected to eventually change and result in a capital gain or loss, a capital gain or loss can be expected eventually from a real estate investment which some investor will realize due to a change in I as well as a likely change in i.

Traditional Capitalization Rates and the O.A.R.

If the income stream is in fact expected to be level into the foreseeable future then model (2) is an appropriate simulation of expectations upon which to base a value. Herein lies the theoretical legitimacy of valuation using the O.A.R. as a true capitalization rate. Note the assumption of an expected level income stream. When this assumption is reasonable in the valuation of a subject property, the O.A.R. is the rate by which investors convert or "capitalize" the income stream into a present value; it is truely a capitalization rate. When the level returns assumption is not reasonable relative to the subject property, the O.A.R. becomes a different measure more akin to the inverse of the price/earnings ratio of a stock. This does not diminish the usefulness of the O.A.R. in income property valuation where there are suitable market data from which to extract such a ratio; indeed it is a most powerful indicator of value. It is not always, however, the rate that some clients think of as a capitalization rate. We therefore prefer to think of the O.A.R. as a ratio of current profitability when periodic income payments are not expected to be level rather than as a capitalization rate.

Suppose the expected income stream of a subject property is not expected to be level into the foreseeable future. In this case model (1) must be revised to simulate investor expectations. The I's representing periodic income payments are not all expected to be equal but can be represented algebraically using subscripts, I_0 for the current income payment, I_1 for the payment due at the end of the first period and so on through I_n

(4)

$$V_0 = I_1/(1+i) + I_2/(1+i)^2 + \ldots + I_n/(1+i)^n.$$

This model may sum to infinity as may model (1), but cannot be reduced to the same form as (2). We may extract an O.A.R. using model (2) and use it to determine a very reasonable estimate of value for an appraisal in the usual way. But in this case, unlike the case of an expected level income stream, the O.A.R. is not equal to i, the true capitalization rate of model (4). The reason of course is that I_0 in model (2) from which we derived our O.A.R. is not representative of the expected income stream. The O.A.R. is in fact a ratio of the current income payment to value but not the rate by which the expected stream of income is capitalized into a present value. This distinction is a key to understanding why the O.A.R.'s seemed low several years ago compared to popular investment market capitalization rates.

Expected Growth in Income Streams

When income is expected to change in the future, DCF analysis may be appropriate. DCF analysis is a specialized technique of capitalization in which certain of the income payments are specified explicitly and the remaining incomes are generalized. DCF requires a cap rate different than the O.A.R. and an appropriate reversion. The primary appraisal problem in this case may be the estimation of the nature of the future income pattern. Growth in income may be expected due to either expected inflation or expected real growth factors such as favorable or unfavorable supply/demand relationships in a given market. This change in expected income is positive or negative expected growth in income denoted **x** in the Institute's literature. The expected income growth rate **x** may be decomposed to reveal a real growth component **g**, and a component of inflationary expectations **p**

(5)

$$x = (1+g)(1+p) - 1.$$

The expected inflation component **p** accounts for the expected erosion of currency if inflation is thought of as a devaluation of money in general throughout the economy. Thus even if income growth **x** appears constant and zero (i.e. a level expected income stream in nominal terms) during inflationary times, the income stream is in fact being debased as inflation **p** may be positive while real growth **g** is a similar negative amount.

Explicitly committing to an income growth rate projection into perpetuity when the income only five years hence may not be certain seems presumptuous. This is what we have been doing implicitly all along however while using the traditional model (2) for valuation. The only difference is that in the traditional model, income growth is zero which we should agree is often unrealistic. This is demonstrated by examining the development of the growth model in which **x** denotes growth.

(6)

$$V_0 = I_1/(1+i) + I_1(1+x)/(1+i)^2 + \ldots + I_1(1+x)^{(n-1)}/(1+i)^n$$

is the series which leads to (see appendix) the more familiar[3]

(7)

$$V_0 = I_1\left[\frac{1-\left[\frac{1+x}{1+i}\right]^n}{i-x}\right]$$

which, when **n** goes to infinity is equal to

(8)

$$V_0 = I_1/(i-x).$$

Note that each payment after the first is assumed to equal the first payment plus an additional amount attributable to the constant growth rate **x**.

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3. A.I.R.E.A. <u>Financial Tables</u> (Chicago: American Institute of Real Estate Appraisers), page 8.

An alternative expression of the same model that does not appear in the A.I.R.E.A. literature but that is easier to interpret follows conveniently. Since growth is assumed constant, then $I_1 = I_0(1+x)$ which changes model (6) to

(9)

$$V_0 = I_0(1+x)/(1+i) + I_0(1+x)^2/(1+i)^2 +....+ I_0(1+x)^n/(1+i)^n$$

which, as n goes to infinity is

(10)

$$V_0 = I_0(1+x)/(i-x)$$

which is equivalent to (8).

Growth Compensated Overall Rate

As indicated above, these models are identical to the familiar (2) except that they contain a constant nonzero growth rate **x** in the income stream. If **x** equals 0 we have (2). We have been making income growth projections into the distant future all along, in fact to infinity, but the projection has been the usually less realistic one of zero growth. Growth models may readily be used to extract a more accurate and defensible O.A.R. from the market especially with computer assistance. Appraisers should be in the position to estimate expected growth in income **x** based on historic trends, inflationary expectations, and known supply/demand relationships. Contrary to Miller and Solt the nominal value for growth **x** should be easier to forecast than the real (inflation free) value g because nominal current incomes are observed in the market with every appraisal performed. The appraiser is well aware of historic trends in nominal form and can forecast a nominal expected growth value. Although it is sometimes helpful to think in real, inflation adjusted, terms,"we live in a nominal world" and to use the real value **g** as Miller and Solt suggest, would require making the additional forecast of expected inflation **p** and then deducting it from **x**.

After plugging in the estimate for **x** in an appropriate growth model which simulates the expected income stream, a computer may be used to extract a "Growth Compensated" O.A.R. This G.C.O.A.R. will be a true capitalization rate to the extent that the model simulates the expected income stream. The specific model used should also be realistic with respect to the expected timing of income payments as well since an incorrect timing specification will also result in an incorrect capitalization rate. The models above are generalized for exposition and reflect sometimes inappropriate annual growth rather than generally smooth stochastic growth. They also reflect annual income payments in arrears rather than the more common payment in advance. The model used to extract the G.C.O.A.R. should be constructed to account for the specific set of expectations peculiar to the subject property and the comparables.

After compensating for expected growth, G.C.O.A.R. rates should vary among properties primarily only according to the risk investors assign to different properties and should have very strong correspondence to the capitalization rates investors use in valuing financial assets with similar risk regardless of inflation.

DCF and the Reversion

The appraisal of a subject property which is expected to experience variable growth initially for a finite period of time **n** and then grow at a different constant rate **x** beyond period **n** would take the form

(11)

$$V_0 = I_1/(1+i) + I_2/(1+i)^2 + \dots$$
$$\dots + I_n/(1+i)^n + I_n(1+x)/(i-x) \times 1/(1+i)^n$$

This proposed model is just the DCF model with the reversion explicitly based on estimates of I_n and **x**.

When we forecast a lump sum reversion in DCF analysis we are implicitly forecasting either growth in income **x** or a change in the market capitalization rate **i** as they are the sources of capital gain or loss. The reversion in the model above is an estimate in as much as the income payment at the time of reversion I_n and an expected future rate of growth **x** are estimates. Note that the reversion which takes place in period **n** is essentially the constant growth model (10) discounted to the present. This explicit method of estimating the reversion is both theoretically appealing and defensible.

Example Application

Many of the generalized assumptions in the above discussion of growth models will now be altered to better suit the specific application of a growth model in the income approach for a 10-plex apartment building in Columbia, MO. Valuation will be for

March 1st. Periodic income is received on a monthly basis in advance with average annual increases of $7.50 per month typical. Landlords strive to re-negotiate leases on an annual basis to be renewed in the summer as Columbia has an overwhelming seasonal population. There is no reason to expect this pattern of nominal income growth to change in the foreseeable future. While average gross monthly income per unit is $340 per month, expenses typically run about 30% of gross for this type of property.

Comparable sales data exist to extract a G.C.O.A.R. from the market. This rate may be represented in the report as both an annual rate and as a monthly rate for computational purposes. Let the annual rate be **i** and the monthly rate be **m**. To convert from monthly to annual and vise versa the following relationships hold:

$$(1+i) = (1+m)^{12}$$
$$i = (1+m)^{12} - 1$$

$$(1+m) = (1+i)^{1/12}$$
$$m = (1+i)^{1/12} - 1$$

The growth rate **x** need only be expressed in the form of an annual rate since growth in income only occurs upon annual re-negotiation of the typical lease.

$$x = 7.50/340$$
$$= 2\%$$

Note that since

$$x = (1+g)(1+p) - 1,$$

if expected inflation p is thought by the market to be around 4% for the foreseeable future then real growth in income **g** must be approximately -2%, a negative amount implying an adverse supply/demand relationship in the market for this type of rental property. Even though real growth **g** may be negative, as long as expected inflation **p** keeps expected growth in income **x** above zero, nominal capital gains will be an inherent feature of the model as they tend to be in reality. Capital gains are normally due to nominal growth in income over time in the case of income properties.

From the gross income, 30% is typically required for operating expenses and maintenance so that current net income per month for all 10 units is

$$I_0 = (340 \text{ X } .70) \text{ X } 10 \text{ units}$$
$$= 2380$$

The appropriate model to simulate the expected stream of income which is monthly in advance is

(12)

$$\underset{\text{(March)}}{V_0 = I_0} + \underset{\text{(April)}}{I_0/(1+m)} + \underset{\text{(May)}}{I_0/(1+m)^2} + \ldots.$$

$$\ldots. + \underset{\text{(Feb., 1988)}}{I_0/(1+m)^{11}} + \underset{\text{(March, 1988)}}{I_0(1+x)/(1+m)^{12}} + \ldots.$$

$$\ldots. + \underset{\text{(Feb., 1989)}}{I_0(1+x)/(1+m)^{23}} + \underset{\text{(March, 1989)}}{I_0(1+x)^2/(1+m)^{24}} + \ldots.$$

and so on theoretically to infinity. The model is more easily written and programmed

(13)

$$V_0 = \sum_{y=o}^{\infty} \sum_{n=12y}^{(12y+11)} \frac{I_0(1+x)^y}{(1+m)^n}$$

In practice this model is easily programmed on a micro-computer and need not be computed beyond 100 years as the monthly income after the 100th year discounted to the present is not significant. If the annual cap rate was found to be 14%, the present value of the monthly income expected 100 years from March 1st would be

$$I_0(1+x)^{100}/(1+m)^{1200} = \$.04.$$

The G.C.O.A.R. could be found in a similar way that the O.A.R. is found except that the appropriate growth model would be used rather than the traditional zero growth model. With computer assistance so readily available there is no need to misspecify the frequency of income payments as annual when they are in fact monthly. To find a traditional O.A.R. we erroneously sum net monthly income payments into an annual payment in arrears and naively assume that they will not change in the foreseeable future. For comparison we will use a hypothetical comparable sale with a price of $250,000 and net monthly income payments of $2400. From model (2):

$$i = (12 \text{ X } I_0)/V_0$$

$$i = (12 \text{ X } 2400)/250,000$$

$$\text{O.A.R.} = 11.5\%$$

If income payments were received annually, it would be appropriate to find a G.C.O.A.R. rate by solving for **i** in model (10). But income is received monthly while growth in income in this case occurs on an annual basis. Model (13) must be solved for **i** by computer reiterations plugging in trial capitalization rates until the cap rate is found that results in the comparable's actual sale price using the known net monthly income and an estimate for income growth x. Only a few computer iterations are likely to be required to yield a G.C.O.A.R. rate which in the case of our hypothetical comparable is:

$$\text{G.C.O.A.R.} = 14.5\%$$

The difference of 3% between the G.C.O.A.R. and the O.A.R. is due to the incorrect specification of the frequency of income payments in the traditional annual model (accounting for about 1% error) as well as the expected growth of the income stream. In an economy characterized by accelerating inflation (such as the early 80's) we find ourselves irrationally lowering the O.A.R.

while other market rates are rising. G.C.O.A.R. rates calculated during such periods would diverge substantially from O.A.R. rates due to increasing expected inflation **p** and would correspond as we would expect to other market rates of interest with similar risk.

The value indicated by the Growth Compensated O.A.R. would be

$$V_0 = \$246,050.30$$

say,

$245,000

according to the above model programmed to run on our micro-computer. Note the computer run in Table #1.

If it is desired for the purposes of the written appraisal report, the above computation could be carried out explicitly for 5 years in a columnar format with the following reversion at the end of the 5th year.

(14)

$$\sum_{y=5}^{\infty} \sum_{n=12(y-5)}^{12(y-5)+11} \frac{I_0(1+x)^y}{(1+m)^n}$$

The value must then be discounted to the valuation date by:

$$\text{Reversion X } 1/(1+m)^{60}$$

Recall from the previous discussion of the reversion amount of a British consol that the reversion is discounted to the present in the infinite series regardless of when it actually takes place. The same principle applies in the present valuation problem. The best present estimate of the actual reversion 5 years hence should be based on present estimates of expected growth in income

x and the cap rate **i**. The estimates of these rates may very well not prevail in 5 years time but they result in a defensible reversion estimate.

Summary

We have demonstrated that growth models based on capitalization theory can be useful in the real estate appraisal process. It has been shown that the O.A.R. is not a true capitalization rate when it is extracted from properties which are experiencing growth in income, but that a true capitalization rate may be extracted from the market which we refer to as a Growth Compensated O.A.R. While the O.A.R. can certainly be used as a ratio of current profitability to achieve a value estimate, it must be recognized that the O.A.R. should not be expected to be comparable to most other market interest rates while the G.C.O.A.R. should correlate closely. Finally, a defensible method of calculating the reversion explicitly in DCF analysis based on estimated future income and expected growth in income has been offered.

TABLE #1 - THE INCOME APPROACH TO VALUE

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THE ANNUAL CAPITALIZATION RATE IS .145
MONTHLY NET INCOME IS 2380
THE EXPECTED ANNUAL GROWTH RATE OF INCOME IS .02

PERIOD	INCOME	DISC. FACTOR	PV OF INCOME	SUM
1	2380	1	2380	2379
2	2380	.9887797	2353.296	4732.296
3	2380	.9776852	2326.891	7059.187
4	2380	.9667152	2300.782	9359.969
5	2380	.9558685	2274.967	11634.94
6	2380	.9451433	2249.441	13884.38
7	2380	.9345385	2224.202	16108.58
8	2380	.9240526	2199.245	18307.82
9	2380	.9136846	2174.569	20482.39
10	2380	.9034327	2150.17	22632.56
11	2380	.8932959	2126.045	24758.61
12	2380	.8832728	2102.189	26860.8
13	2427.6	.8733623	2120.174	28980.97
14	2427.6	.8635628	2096.385	31077.36
15	2427.6	.8538734	2072.863	33150.22
16	2427.6	.8442926	2049.605	35199.83
17	2427.6	.8348195	2026.608	37226.43
18	2427.6	.8254525	2003.868	39230.3
19	2427.6	.8161908	1981.384	41211.68
20	2427.6	.8070327	1959.153	43170.84
21	2427.6	.7979776	1937.17	45108.01
22	2427.6	.789024	1915.435	47023.44
23	2427.6	.7801709	1893.943	48917.39
24	2427.6	.7714171	1872.692	50790.08
25	2476.152	.7627616	1888.714	52678.79
26	2476.152	.7542031	1867.522	54546.32
27	2476.152	.7457408	1846.568	56392.88
28	2476.152	.7373733	1825.848	58218.73
29	2476.152	.7290998	1805.362	60024.09
30	2476.152	.720919	1785.105	61809.2
31	2476.152	.7128301	1765.076	63574.27
32	2476.152	.704832	1745.271	65319.54
33	2476.152	.6969235	1725.688	67045.23
34	2476.152	.6891038	1706.326	68751.56
35	2476.152	.6813718	1687.18	70438.74
36	2476.152	.6737266	1668.249	72106.99
37	2525.675	.6661672	1682.522	73789.51
38	2525.675	.6586926	1663.643	75453.15
39	2525.675	.6513018	1644.977	77098.13
40	2525.675	.643994	1626.519	78724.65
41	2525.675	.6367683	1608.269	80332.91
42	2525.675	.6296235	1590.224	81923.13
43	2525.675	.622559	1572.381	83495.52
44	2525.675	.6155735	1554.739	85050.26
45	2525.675	.6086667	1537.294	86587.56
46	2525.675	.6018372	1520.045	88107.6
47	2525.675	.5950844	1502.99	89610.59
48	2525.675	.5884074	1486.126	91096.72

PERIOD	REVERSION	DISC. FACTOR	PV OF REV.	TOTAL
49	266332.3	.5818053	154953.6	246050.3

APPENDIX

1. $$V_o = \frac{I_o}{1+i} + \frac{I_o}{(1+i)^2} + \ldots + \frac{I_o}{(1+i)^n}$$

multiply both sides of equation by (1+i)

$$V_o(1+i) = I_o + \frac{I_o}{(1+i)} + \frac{I_o}{(1+i)^2} + \ldots + \frac{I_o}{(1+i)^{n-1}}$$

$$V_o(1+i) = I_o[1 + \frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \ldots + \frac{1}{(1+i)^{n-1}}]$$

from (1) above, $V_o = I_o[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \ldots + \frac{1}{(1+i)^n}]$

subtract V_o from $V_o(1+i)$

$$V_o(1+i) - V_o = I_o[1 - \frac{1}{(1+i)^n}]$$

$$V_o(1+i-1) = I_o[1 - \frac{1}{(1+i)^n}]$$

$$V_o(i) = I_o[1 - \frac{1}{(1+i)^n}]$$

$$V_o = \frac{I_o}{i}[1 - \frac{1}{(1+i)^n}]$$

as n goes to infinity, $\frac{1}{(1+i)^n}$ goes to zero so that

2. $$V_o = \frac{I_o}{i}$$

6. $$V_o = \frac{I_1}{1+i} + \frac{I_1(1+x)}{(1+i)^2} + \ldots + \frac{I_1(1+x)^{n-1}}{(1+i)^n}$$

$$V_o = I_1[\frac{1}{1+i} + \frac{1+x}{(1+i)^2} + \ldots + \frac{(1+x)^{n-1}}{(1+i)^n}]$$

multiply both sides of the equation by (1+i)

$$V_o(1+i) = I_1[1 + \frac{1+x}{1+i} + \ldots + \frac{(1+x)^{n-1}}{(1+i)^{n-1}}]$$

multiply both sides of V_o above by (1+x)

$$V_o(1+x) = I_1[\frac{1+x}{1+i} + \ldots + \frac{(1+x)^n}{(1+i)^n}]$$

subtract $V_o(1+x)$ from $V_o(1+i)$

$$V_o[(1+i) - (1+x)] = I_1[1 - \frac{(1+g)^n}{(1+i)^n}]$$

$$V_o(i=x) = I_1[1 - \frac{(1+x)^n}{(1+i)^n}]$$

7. $$V_o = I_1 \frac{1 - \frac{(1+x)^n}{(1+i)^n}}{(i-x)}$$

since i must be greater than x, as n goes to infinity, $\frac{(1+x)^n}{(1+i)^n}$ goes to zero so that

8. $$V_o = \frac{I_1}{i-x}$$

9. $$V_o = \frac{I_o(1+x)}{1+i} + \frac{I_o(1+x)^2}{(1+i)^2} + \ldots + \frac{I_o(1+x)^n}{(1+i)^n}$$

$$V_o = I_o[\frac{1+x}{1+i} + \ldots + \frac{(1+x)^n}{(1+i)^n}]$$

multiply both sixes by $\frac{1+i}{1+x}$

$$V_o(\frac{1+i}{1+x}) = I_o[1 + \frac{1+x}{1+i} + \ldots + \frac{(1+x)^{n-1}}{(1+i)^{n-1}}]$$

subtract V_o from $V_o(\frac{1+i}{1+x})$

$$V_o[\frac{1+i}{1+x}-1] = I_o[1-\frac{(1+x)^n}{(1+i)^n}]$$

$$V_o[\frac{(1+i)-(1+x)}{1+x}] = I_o[1 - \frac{(1+x)^n}{(1+i)^n}]$$

$$V_o[\frac{i-x}{1+x}] = I_o[1 - \frac{(1+x)^n}{(1+i)^n}]$$

since i must be greater than x, as n goes to infinity, $\frac{(1+x)^n}{(1+i)^n}$ becomes zero so that

$$V_o[\frac{i-x}{1+x}] = I_o$$

10. $V_o = I_o\frac{1+x}{i-x}$

since I_1 equals $I_o(1+x)$

8. $V_o = \frac{I_1}{i-x}$