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CHUNG-HUA INSTITUTION FOR ECONOMIC RESEARCH

A DEMAND SYSTEM WITH HOMOTHETIC UTILITY FUNCTION: MEASURABILITY AND EMPIRICAL RELEVANCE

MO-HUAN HSING

DISCUSSION PAPER SERIES No.9506

December 1995

Discussion papers are intended to provide prompt distribution of CIER's preliminary research work to interested scholars and to invite their discussions and critical comments.

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A Demand System with Homothetic Utility Function: Measurability and Empirical Relevance

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December 1995

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A Demand System with Homothetic Utility Function: Measurability and Empirical Relevance

Abstract

From M. H. Paston's formulation of the first order condition obtained by maximizing the constrained homothetic utility function, it is proved possible to derive a system of demand equations that could be estimated with the usual regression technique, and further to generalize and extend it to a system quite close to the linear expenditure system or the first difference forms, the latter with satisfactory testing results. The problem is rather that such a system is not a demand system as traditionally hypothesized but is a system of expenditure identities in the form of Euler's theorem, with the regression coefficients of the explanatory variables equal to the respective partial derivatives evaluated at the base-year's consumer preference.

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I. Introductory remarks

By analogy with the modern theory of technical progress, M. H. Paston (1967) assumed that taste changes are translated in each period by the consumer into new exponents in the homothetic utility function as given below:

$$u_t = A x_{1t}^{\beta 1t} x_{2t}^{\beta 2t} \tag{1}$$

where A is a constant while β_{1t} and β_{2t} are each a function of time.

It is easy to show that applying the usual procedure of utility maximization subject to the consumer's budget constraint $y_t = x_{1t} p_{1t} + x_{2t} p_{2t}$ gives the first-order condition

$$\frac{x_{1t} p_{1t}}{x_{2t} p_{2t}} = \frac{\beta_{1t}}{\beta_{2t}} \tag{2}$$

Regarding (2), Louis Phlips (1983, p.81) commented that "it would be difficult indeed to derive from [it] a system of demand equations that could be estimated with the usual regression techniques."

The present paper purports to show that it is possible to derive from (2) a demand system close to the dynamic single-equation linear expenditure system such as that developed by Houthakker and Taylor (1970, 2nd edition, ch.1) which can be estimated with the usual regression techniques. But surprisingly, such a demand system turns out to be one of linear expenditure identities in the form of Euler's theorem. We hope that this finding and the attendant observations will be of interest to the reader.

II. Derivation of a system of empirical demand equations

Let us write (2) in logarithmic form:

(3) $$\ln x_{1t}+\ln p_{1t}-\ln x_{2t}-\ln p_{2t}=\ln\beta_{1t}-\ln\beta_{2t}$$

Then, substituting $\ln(x_{2t}\, p_{2t}\, /y_t)$ for $\ln\beta_{2t}$ and rearranging terms, we have

(4) $$\ln x_{1t}-\ln\beta_{1t}=-\ln p_{1t}+\ln y_t$$

Alternatively, substituting $\ln(x_{1t}\, p_{1t}\, /y_t)$ for $\ln\beta_{1t}$ in (3), we have

(5) $$\ln x_{2t}-\ln\beta_{2t}=-\ln p_{2t}+\ln y_t$$

It is easy to demonstrate that if (1) is generalized so that the number of commodities is n (>2), then applying the same analytic procedure, we can write a demand system in the following general form:

(6) $$\ln x_{it}-\ln\beta_{it}=-\ln p_{it}+\ln y_t \qquad (i=1,\ 2,\ .\ .\ .\ ,\ n)$$

which can be transformed into

(7) $$x_{it}=\beta_{it}\left(\frac{y_t}{p_{it}}\right)$$

Differentiating (6), we obtain

(8) $$\frac{\Delta x_{it}}{x_{it}}-\frac{\Delta\beta_{it}}{\beta_{it}}=-\frac{\Delta p_{it}}{p_{it}}+\frac{\Delta y_t}{y_t}$$

Following the convention in empirical investigation, let us rewrite (8) as

(9) $$\frac{\Delta x_{it}}{x_{it-1}}-\frac{\Delta\beta_{it}}{\beta_{it-1}}=-\frac{\Delta p_{it}}{p_{it-1}}+\frac{\Delta y_t}{y_{t-1}}$$

or

(10) $$\frac{x_{it}}{x_{it-1}}-\frac{\beta_{it}}{\beta_{it-1}}=-\frac{\Delta p_{it}}{p_{it-1}}+\frac{\Delta y_t}{y_{t-1}}$$

Multiplying (10) through by x_{it-1}/β_{it} yields

$$(11) \qquad \frac{x_{it}}{\beta_{it}} - \frac{x_{it-1}}{\beta_{it-1}} = -\frac{x_{it-1}}{\beta_{it}\, p_{it-1}} \Delta p_{it} + \frac{x_{it-1}}{\beta_{it}\, y_{t-1}} \Delta y_t$$

Since it is assumed that taste changes are translated in each period by the consumer into new exponents in (1), β_{it} is in practice an indicator of consumer preference over time. Therefore x_{it}/β_{it} and x_{it-1}/β_{it-1} can be viewed as preference-corrected demand for commodity i in t and t-1, respectively——i.e., consumption in the two periods evaluated at the base-year consumer preference.

Now let us manipulate the terms on the RHS of (11) such that

$$(12) \quad \frac{x_{it}}{\beta_{it}} - \frac{x_{it-1}}{\beta_{it-1}} = \left(-\frac{x_{it-1}}{p_{it-1}} \frac{\partial p_{it}}{\partial x_{it}}\right)\left(\frac{1}{\beta_{it}} \frac{\partial x_{it}}{\partial p_{it}}\right) \Delta p_{it} + \left(\frac{x_{it-1}}{y_{t-1}} \frac{\partial y_t}{\partial x_{it}}\right)\left(\frac{1}{\beta_{it}} \frac{\partial x_{it}}{\partial y_t}\right) \Delta y_t$$

It can be readily seen that the expressions in the first brackets of the two terms are the reciprocals of the elasticities of demand for commodity i with respect to changes in p_{it} and y_t (to be denoted by $1/\eta_{pi}$ and $1/\eta_y$), respectively, while those in the second brackets are the respective preference-corrected partial derivatives also with respect to changes in p_{it} and y_t (to be denoted by $[\partial x_i / \partial p_i]_0$ and $[\partial x_i / \partial y]_0$). Thus (12) can be written as

$$(13) \qquad \frac{x_{it}}{\beta_{it}} - \frac{x_{it-1}}{\beta_{it-1}} = -\frac{1}{\eta_{p_i}} \left[\frac{\partial x_i}{\partial p_i}\right]_0 \Delta p_{it} + \frac{1}{\eta_y} \left[\frac{\partial x_i}{\partial y}\right]_0 \Delta y_t$$

From (7) it can be proved that $\eta_{pi} = -1/\beta_{it}(y_t/x_{it}p_{it}) = -1$ and $\eta_y = \beta_{it}(y_t/x_{it}p_{it}) = 1$. These results are consistent with both hypotheses of zero-degree homogeneity $\eta_{pi} + \eta_y = 0$ and unitary income elasticity $\eta_y = 1$ implied by homotheticity (for the latter, see Phlips, 1983, p.180). Hence (13) can be simplified as

$$(14) \qquad \frac{x_{it}}{\beta_{it}} - \frac{x_{it-1}}{\beta_{it-1}} = \left[\frac{\partial x_i}{\partial p_i}\right]_0 \Delta p_{it} + \left[\frac{\partial x_i}{\partial y}\right]_0 \Delta y_t$$

Now writing Δp_{it} and Δy_t as $(p_{it} - p_{it-1})$ and $(y_t - y_{t-1})$ respectively, while setting t=1, 2, ... in (14), each at a time through successive substitution, we

finally arrive at

$$\frac{x_{it}}{\beta_{it}}-\frac{x_{i0}}{\beta_{i0}}=[\frac{\partial x_i}{\partial p_i}]_0(p_{it}-p_{i0})+[\frac{\partial x_i}{\partial y}]_0(y_t-y_0) \tag{15}$$

where $\beta_{io}(=1)$ is the index number of consumer preference for the base year.

It is apparent that (15) is composed of two linear expenditure systems —one is for the base year:

$$\frac{x_{i0}}{\beta_{i0}}=[\frac{\partial x_i}{\partial p_i}]_0\ p_{i0}+[\frac{\partial x_i}{\partial y}]_0 y_0 \tag{16}$$

where the consumer preference system is assumed to be constant, and the other is for the whole period under investigation (including the base year) with the consumer preference system assumed to be changing from year to year:[1]

$$\frac{x_{it}}{\beta_{it}}=[\frac{\partial x_i}{\partial p_i}]_0\ p_{it}+[\frac{\partial x_i}{\partial y}]_0 y_t \tag{17}$$

It appears that (17) is a system of empirical demand equations we are looking for on the basis of (2). However, a cursory look makes it clear that (17) is no more than a system of expenditure identities in the form of Euler's theorem, and this characteristic becomes even clearer if (17) is transformed into the following more familiar system of demand equations:

$$\begin{aligned} x_{it}&=\beta_{it}[\frac{\partial x_i}{\partial p_i}]_0\ p_{it}+\beta_{it}[\frac{\partial x_i}{\partial y}]_0 y_t \\ &=\frac{\partial x_{it}}{\partial p_{it}}p_{it}+\frac{\partial x_{it}}{\partial y_t}y_t \end{aligned} \tag{18}$$

[1]As a matter of fact, (16) is obtainable directly from (7) if t is set to zero for the base year (and hence $\beta_{i0}=1$), then differentiating (7) totally so as to formulate Euler's theorem equivalent to (16), remembering that $\eta_{pi0}=-1$ and $\eta_{y0}=1$ as noted above. Eliminating x_{i0}/β_{i0} from (15) by substitution of (16) leads to (17).

Now we can extend (17) to include the influences of preference-corrected consumption, price and income in the preceding year *t-1*. To do this, we need only to rewrite (14) as

$$(19) \qquad \frac{x_{it}}{\beta_{it}}=[\frac{\partial x_i}{\partial p_i}]_0\ p_{it}+[\frac{\partial x_i}{\partial y}]_0\ y_t+\frac{x_{it-1}}{\beta_{it-1}}-[\frac{\partial x_i}{\partial p_i}]_0\ p_{it-1}-[\frac{\partial x_i}{\partial y}]_0\ y_{t-1}$$

Since the last three terms on the RHS of (19) sum to zero from the above analysis, (19) partakes of the same characteristics as (17) —that is : it is also a system of expenditure identities irrespective of its allowing for the past influences of preference-corrected consumption, price and income on the preference-corrected consumption in year t.

III. Empirical verification

III.a. Estimation of β_{it}

The simplest method of estimating β_{it} is to make use of the budget equation and divide $x_{it}\ p_{it}$ by the total budget y. Then $\beta_{it}=x_{it}\ p_{it}/y_{it}$ from which an index can be compiled on the basis of the available data on x_{it}, p_{it} and y_t.

An alternative method of estimating β_{it} is to make use of (9). Then we have

$$(20) \qquad \frac{\Delta\beta_{it}}{\beta_{it-1}}=\frac{\Delta x_{it}}{x_{it-1}}+\frac{\Delta p_{it}}{p_{it-1}}-\frac{\Delta y_t}{y_{t-1}}$$

and hence

$$(21) \qquad \beta_{it}=\beta_{it-1}(1+\frac{\Delta\beta_{it}}{\beta_{it-1}})$$

starting from t=1 (so $\beta_{it-1}=\beta_{i0}=1$).

We have estimated β_{it} by both methods for three broadly classified commodities, i.e., nondurables, durables and services, using U.S. data from 1960 to 1990 for illustrative purposes, and found that the two sets of β_{it}

time series exhibit about the same trends for the period under investigation as shown in the statistical appendix to this paper.

III.b. Specification and estimation of the demand system[2]

We propose to estimate both (17) and (19) which are specified as follows for regression, using the second set of β_{it} indexes in the statistical appendix:

$$\frac{x_{it}}{\beta_{it}} = a_{i1}p_{it} + a_{i2}y_t + \varepsilon_{it} \qquad (i=1, 2, 3) \tag{22}$$

$$\frac{x_{it}}{\beta_{it}} = \alpha_{i1}P_{it} + \alpha_{i2}Y_t + \alpha_{i3}\frac{x_{it-1}}{\beta_{it-1}} + \alpha_{i4}P_{it-1} + \alpha_{i5}Y_{t-1} + u_{it} \tag{23}$$

The results of estimation of these two equations by applying the Ordinary Least Squares Method as generally used and the Seemingly Unrelated Method as developed by Zeller (1962)[3] are respectively summarized in Tables 1 and 2, while the price and income elasticities derived from th results of estimation in Table 2 are presented in Table 3.

III.c. Evaluation of the results of estimation

At first glance, the results of estimation in both Tables 1 and 2 look quite satisfactory. The coefficients of all explanatory variables in the estimated equations carry the right signs and exhibit the desired properties, with a_{i2}'s (the average marginal shares of y_t spent on x_{it}/β_{it}) summing to nearly one and a_{i3}'s (the coefficients of x_{it-1}/β_{it-1}, see (19)) each exactly equal to one—all with rather small standard deviations. The most striking of all is that R^2 for each and every estimated equation is equal to unity!

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[2]In this connection, it is noteworthy that until now, p_i and y have represented, respectively, absolute price and current-price income, instead of relative price and constant-price income, as in demand studies in general. Now, we also need to follow this practice. But it can be demonstrated that these changes will not make any difference to the coefficients of the respective explanatory variables.

[3]This is used to take care of the potential problem of the error terms of the equations in each system not being independent.

While these observations seem to lend credibility to both Tables 1 and 2, a more careful scrutiny suggests that the results of estimation presented in Table 1 for (22) actually leave much to be desired. One easily observable and not too serious problem is that the sum-totals of a_{i2}'s remain somewhat less closer to one than the corresponding estimates in Table 2 for (23), by both Ordinary Least Squares estimation and Seemingly Unrelated estimation. A far more serious problem comes to the surface with the price and income elasticities derived from the results of estimation in Table 1 by either of the two methods failing to sum to about zero as do the results of estimation in Table 2. As a matter of fact, what are obtainable from Table 1 are but a set of improbably small (generally smaller than 0.1) price and income elasticities for each commodity group by the Ordinary Least Squares estimation and another set for each commodity group summing to more or less one by the Seemingly Unrelated estimation (not shown).

Apparently, these glaring divergences are primarily due to the different forms of (22) and (23), the former being of a form involving high serial correlation in the residual errors in regression as witnessed by the very low D-W statistic as compared with (23). Faced with such a situation, the common procedure would be to work with first differences. However, tracing the process of the derivation of equations back to (14) in Section II makes it plain that (19) is but a rearrangement of the terms of (14), which is precisely the first-difference form of (17). Being the estimating equations appropriate to (19) and (17), (23) is virtually the first-difference form of (22) although, when it comes to regression, a_{i1} and a_{i2} deviate slightly from a_{i4} and a_{i5} respectively as explained in footnote 4.

It is precisely for this reason that (23) performs so much better than (22) in reducing the serial correlation of the residual errors. As a result, the estimates of the price and income elasticities for the three commodity groups on the basis of the results of the estimation of (23) are nearly perfect, whereas the same elasticities based on the estimation of (22) are so improbable that they are not worth mention. Therefore, we suggest that only the results of estimation in Tables 2 and 3 be taken for consideration, while the results of estimation in Table 1 should be forgotten altogether.

As far as this part of empirical exercise is concerned, the results of estimation probably could stand any standard test; although the D-W statistic in Table 2 is still on the low side, especially for the third group of

Table 1 Results of Estimation of (22)

	a_{i1}	a_{i2}	R^2	D.W.
	Ordinary Least Squares Method			
$\frac{x_{1t}}{\beta_{1t}}$	-89.6258	0.4893	0.9991	0.167
	(32.4951)	(0.0109)		
$\frac{x_{2t}}{\beta_{2t}}$	-282.9700	0.2551	0.9966	0.090
	(20.3879)	(0.0052)		
$\frac{x_{3t}}{\beta_{3t}}$	392.8537	0.2102	0.9989	0.119
	(38.5911)	(0.0144)		
	Seemingly Unrelated Method			
$\frac{x_{1t}}{\beta_{1t}}$	-63.2629	0.4807	1.000	0.173
	(28.4334)	(0.0097)		
$\frac{x_{2t}}{\beta_{2t}}$	-277.3022	0.2538	1.000	0.089
	(18.2869)	(0.0048)		
$\frac{x_{3t}}{\beta_{3t}}$	403.6474	0.2062	1.000	0.121
	(33.4491)	(0.0126)		

Table 2 Results of Estimation of (23)[4]

	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	R^2	D.W.
Ordinary Least Squares Method							
$\frac{x_{1t}}{\beta_{1t}}$	-1380.2983	0.4769	1.0085	1378.1282	-0.4801	1.000	1.459
	(55.7234)	(0.0142)	(0.0167)	(57.0495)	(0.0144)		
$\frac{x_{2t}}{\beta_{2t}}$	-562.9031	0.1446	1.0511	553.6057	-0.1498	1.000	1.962
	(61.0605)	(0.0083)	(0.0134)	(60.8139)	(0.0092)		
$\frac{x_{3t}}{\beta_{3t}}$	-1007.6543	0.3601	1.0056	1020.4744	-0.3667	1.000	1.055
	(26.1931)	(0.0047)	(0.0061)	(26.1304)	(0.0051)		
Seemingly Unrelated Method							
$\frac{x_{1t}}{\beta_{1t}}$	-1350.5351	0.4795	1.0100	1347.9824	-0.4834	1.000	1.468
	(53.3118)	(0.0141)	(0.0162)	(54.5849)	(0.0143)		
$\frac{x_{2t}}{\beta_{2t}}$	-609.7968	0.1452	1.0533	600.4035	-0.1510	1.000	1.949
	(56.8433)	(0.0083)	(0.0133)	(56.6335)	(0.0091)		
$\frac{x_{3t}}{\beta_{3t}}$	-986.9520	0.3586	1.0053	999.9991	-0.3652	1.000	0.995
	(24.0938)	(0.0047)	(0.0060)	(24.0326)	(0.0050)		

[4]Notice that here a_{i1} and a_{i2} deviate slightly from $a_{i4}$and a_{i5} respectively. To be exact, $a_{i1}=1/\beta_{it}(\partial x_{it}/\partial p_{it})$; $a_{i4}=1/\beta_{it}(\partial x_{it}/\partial p_{it-1})=1/\beta_{it}(\partial x_{it}/\partial p_{it})(\partial p_{it}/\partial p_{it-1})=a_{i1}(\partial p_{it}/\partial p_{it-1})$. Similarly, $a_{i2}=1/\beta_{it}(\partial x_{it}/\partial y_t)$; $a_{i5}=1/\beta_{it}(\partial x_{it}/\partial y_t)(\partial y_t/\partial y_{t-1})=a_{i2}(\partial y_t/\partial y_{t-1})$. These results do not strictly conform to the postulates in (19) because, when it comes to regression, the coefficients of p_{it} and y_t would necessarily be somewhat different from those of p_{it-1} and y_{t-1} respectively.

Table 3 Price and Income Elasticities

	Ordinary Least Squares			Seemingly Unrelated Method		
	Nondurables	Durables	Services	Nondurables	Durables	Services
P_{it}	1.0394	-0.7915	-1.0686	-1.0170	-0.8575	-1.0466
Y_t	1.0397	0.7923	0.9980	1.0455	0.7956	0.9938
P_{it-1}	1.0391	0.7863	1.0773	1.0164	0.8528	1.0556
Y_{t-1}	-1.0337	-0.8107	-1.0037	-1.0406	-0.8172	-0.9996
Σ	0.0057	-0.0236	0.0030	0.0043	-0.0263	0.0032

Note: From (22) we know that $a_{i1}=1/\beta_{it}(\partial x_{it}/\partial p_{it})$, $a_{i2}=1/\beta_{it}(\partial x_{it}/\partial y_t)$, $a_{i4}=1/\beta_{it}(\partial x_{it}/\partial p_{it-1})$, $a_{i5}=1/\beta_{it}(\partial x_{it}/\partial y_{t-1})$. So the respective price and income elasticities are $a_{i1}*p_{it}/(x_{it}/\beta_{it})=(\partial x_{it}/\partial p_{it})\ (p_{it}/x_{it})$, $a_{i2}*y_t/(x_{it}/\beta_{it})=(\partial x_{it}/\partial y_t)(y_t/x_{it})$, $a_{i4}*p_{it-1}/(x_{it}/\beta_{it})=(\partial x_{it}/\partial p_{it-1})(p_{it-1}/x_{it})$ and $a_{i5}*y_{t-1}/(x_{it}/\beta_{it})=(\partial x_{it}/\partial y_{t-1})(y_{t-1}/x_{it})$.

commodities by both Ordinary Least Square estimation and Seemingly Unrelated estimation, one would suspect there is much room for further improvement,[5] given the three-commodity grouping for illustration.

The reader is reminded that both (22) and (23) are systems of linear identities in the form of Euler's theorem as stressed in Section II. We accept (23) while discarding (22) simply because the former is far closer to perfect performance in regression and thus provides an unequivocal empirical confirmation of our hypothesis—that is, what looks like a system of demand equations derived from the homothetic utility function is in fact but a system of linear identities in the form of Euler's theorem. It is believed that our job is well done.

IV. Further confirmation

To further strengthen our argument, let us recall that, apart from the regression coefficient of x_{it-1}/β_{it-1}, which has been proved to be exactly one, as pointed out above, the regression coefficients of p_{it}, p_{it-1} and y_t, y_{t-1} presumably correspond to the respective partial derivatives evaluated at the base-year's consumer preference and represented by $[\partial x_i/\partial p_i]_0$ and $[\partial x_i/\partial y]_0$ respectively.

However, there is neither an effective method for directly measuring these latter coefficients for comparison with the corresponding regression coefficients,[6] nor factual data available for cross-checking, as in the case

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[5]We have estimated (14) in the form of

$$x_{it}/\beta_{it}=a_{i1}\Delta p_{it}+a_{i2}\Delta y_t+a_{i3}(x_{it-1}/\beta_{it-1})$$

and found that the results of estimation are no better than those of (23) as shown in Table 2, yet with a totally unacceptable Durbin-Watson tatistic.

[6]To be sure, we have devised a formula for the purpose. Though this formula has been proved to be quite useful elsewhere (Hsing, 1995) it does not seem very effective in the present case, probably because the expenditure system under study consists of single-commodity equations.

of the measurement of aggregate production function.[7] Fortunately, there remains an indirect test. The idea is that, if the regression coefficients of p_{it}, p_{it-1} and y_t, y_{t-1} for the three groups of commodities equal the corresponding partial derivatives evaluated at the base year's consumer preference, then if the base year is changed from 1960 to 1990, the former set of regression coefficients obtained with 1960 as base year and the latter set of regression coefficients obtained with 1990 as base year should maintain a relation of proportionality, as $\beta_{i1990}[\partial x_i/\partial p_i]_0/\beta_{i1960}[\partial x_i/\partial p_i]_0$ and $\beta_{i1990}[\partial x_i/\partial y]_0/\beta_{i1960}[\partial x_i/\partial y]_0$ (where $\beta_{i1960}=1.000$) would do, and the proportionality factor is readily identifiable as the index number of consumer preference for 1990 (with 1960 as base year) for each group of commodities as shown in the statistical appendix.

For this indirect test, we have changed the base year of the said indexes of consumer preference from 1960 to 1990 to re-estimate (23) for the three groups of commodities, and have found the results of estimation equally satisfactory. In Table 4, we compare the average absolute values of the regression coefficients of p_{it} and p_{it-1} and of y_t and y_{t-1} from the estimation with 1960 and 1990 as the respective base year. For illustrative purposes, we choose to limit the comparison to estimation by the Ordinary Least Squares Method.

Table 4 Comparison of Regression Coefficients for a Test of Proportionality

	Nondurables	Durables	Services
Base year:1960			
(1) $\lvert a_{i1}+a_{i4}\rvert$ /2	1379.42	558.45	1014.06
(2) $\lvert a_{i2}+a_{i5}\rvert$ /2	0.4785	0.1472	0.3634
Base year:1990			
(3) $\lvert a_{i1}+a_{i4}\rvert$ /2	967.81	556.90	1358.34
(4) $\lvert a_{i2}+a_{i5}\rvert$ /2	0.3358	0.1469	0.4868
Proportionality			
(3)/(1)	0.7016	0.9972	1.3395
(4)/(2)	0.7018	0.9980	1.3396
Index no. of consumer Preferencr for 1990	0.7017	0.9976	1.3395

[7]In this case we have annual data on wage and rental which can be taken as the marginal productivities of labor and capital ($\partial X/\partial L$ and $\partial X/\partial K$) respectively. See Hsing (1992).

Judged by the closeness of the proportionality of the average absolute values of the respective regression coefficient of p_{it} and p_{it-1} and of y_t and y_{t-1} in the estimation of (23) with 1960 and 1990 as the respective base year to the appropriate index numbers of consumer preference for 1990 as shown in Table 4, one can not but be convinced that our estimated regression coefficients unquestionably can be taken as the respective partial derivatives evaluated at base year's consumer preference. Therefore, it can be safely said that the system of expenditure equation (23), which we have estimated with satisfactory results, is not only a system of expenditure identities in the form of Euler's theorem, but is a system of expenditure identities with all the regression coefficients of the explanatory variables equal to the respective partial derivatives evaluated at base-year's consumer preference. In other words, it has nothing to do with the hypothetical demand system whatsoever.

V. Summary and conclusion

The foregoing analysis may be summarized as follows: Contrary to Phlip's view, it is possible to use Paston's formulation of the first order condition obtained by maximizing the constrained homothetic utility function to derive a system of demand equations that could be estimated with the usual regression techniques, and further to generalize and extend it to a system quite close to the well—known linear expenditure system with satisfactory testing results. The problem is rather that such a system is not a demand system as traditionally hypothesized but is a system of expenditure identities.

In another paper (1995) this author has proved the same, starting from a general unspecified demand system. Even more noteworthy is that the hypothetical production function has also been proved to be immeasurable by a similar procedure and the so-called empirical production function deduced from it turns out to be but a linear output identity (Hsing, 1992). All this suggests that both the empirical relevance of orthodox economic theory and the plausibility of modelling the behaviors of both consumers and producers are suspect and deserve serious reconsideration.

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Statistical Appendix
Indexes of the Consumer's Preferences

year	Method 1: $\beta_{it}=x_{it}p_{it}/y_t$					
	Nondurables		Durables		Services	
1960	1.0000	1.4095	1.0000	1.0496	1.0000	0.7448
1961	0.9949	1.4023	0.9338	0.9801	1.0269	0.7649
1962	0.9758	1.3753	0.9880	1.0370	1.0312	0.7681
1963	0.9570	1.3489	1.0315	1.0826	1.0384	0.7735
1964	0.9457	1.3329	1.0549	1.1072	1.0437	0.7774
1965	0.9368	1.3204	1.0926	1.1468	1.0416	0.7758
1966	0.9397	1.3245	1.0889	1.1429	1.0396	0.7743
1967	0.9245	1.3031	1.0618	1.1145	1.0653	0.7935
1968	0.9124	1.2860	1.1092	1.1641	1.0639	0.7924
1969	0.9069	1.2782	1.0934	1.1476	1.0752	0.8008
1970	0.9078	1.2796	1.0103	1.0603	1.1006	0.8198
1971	0.8782	1.2378	1.0627	1.1154	1.1173	0.8322
1972	0.8628	1.2160	1.1040	1.1588	1.1215	0.8353
1973	0.8691	1.2250	1.1199	1.1754	1.1092	0.8262
1974	0.8911	1.2559	1.0148	1.0651	1.1181	0.8328
1975	0.8812	1.2421	1.0034	1.0532	1.1328	0.8438
1976	0.8580	1.2094	1.0716	1.1247	1.1372	0.8470
1977	0.8373	1.1802	1.0995	1.1540	1.1516	0.8578
1978	0.8271	1.1658	1.0896	1.1436	1.1663	0.8687
1979	0.8407	1.1850	1.0353	1.0866	1.1683	0.8702
1980	0.8481	1.1953	0.9304	0.9765	1.1935	0.8890
1981	0.8387	1.1822	0.9081	0.9531	1.2112	0.9021
1982	0.8142	1.1476	0.8791	0.9226	1.2481	0.9296
1983	0.7864	1.1084	0.9323	0.9785	1.2624	0.9403
1984	0.7703	1.0857	0.9891	1.0382	1.2624	0.9403
1985	0.7482	1.0546	1.0128	1.0630	1.2797	0.9532
1986	0.7251	1.0220	1.0462	1.0980	1.2951	0.9646
1987	0.7191	1.0136	1.0125	1.0627	1.3127	0.9777
1988	0.7072	0.9968	1.0150	1.0653	1.3253	0.9871
1989	0.7083	0.9983	0.9981	1.0476	1.3294	0.9902
1990	0.7095	1.0000	0.9528	1.0000	1.3426	1.0000

Statistical Appendix
Indexes of the Consumer's Preferences (continued)

year	Method 2:(20) and (21) in the text					
	Nondurables		Durables		Services	
1960	1.0000	1.4251	1.0000	1.0024	1.0000	0.7465
1961	0.9949	1.4179	0.9327	0.9350	1.0268	0.7666
1962	0.9750	1.3895	0.9908	0.9933	1.0310	0.7697
1963	0.9557	1.3620	1.0369	1.0394	1.0383	0.7751
1964	0.9439	1.3451	1.0624	1.0649	1.0435	0.7790
1965	0.9343	1.3315	1.1059	1.1086	1.0409	0.7770
1966	0.9370	1.3353	1.1056	1.1083	1.0385	0.7753
1967	0.9216	1.3133	1.0775	1.0802	1.0644	0.7946
1968	0.9089	1.2952	1.1296	1.1324	1.0627	0.7934
1969	0.9031	1.2870	1.1136	1.1164	1.0743	0.8020
1970	0.9040	1.2883	1.0266	1.0291	1.0998	0.8211
1971	0.8739	1.2453	1.0826	1.0853	1.1165	0.8335
1972	0.8578	1.2224	1.1301	1.1329	1.1203	0.8363
1973	0.8638	1.2310	1.1504	1.1532	1.1076	0.8268
1974	0.8860	1.2626	1.0406	1.0431	1.1165	0.8335
1975	0.8761	1.2485	1.0288	1.0313	1.1314	0.8446
1976	0.8524	1.2147	1.1031	1.1057	1.1353	0.8475
1977	0.8312	1.1845	1.1352	1.1380	1.1497	0.8583
1978	0.8209	1.1698	1.1255	1.1283	1.1644	0.8693
1979	0.8343	1.1890	1.0683	1.0709	1.1666	0.8709
1980	0.8417	1.1995	0.9581	0.9605	1.1915	0.8895
1981	0.8324	1.1863	0.9352	0.9375	1.2091	0.9026
1982	0.8079	1.1514	0.9051	0.9073	1.2457	0.9300
1983	0.7795	1.1109	0.9657	0.9680	1.2598	0.9405
1984	0.7630	1.0873	1.0299	1.0324	1.2593	0.9402
1985	0.7405	1.0552	1.0576	1.0602	1.2766	0.9531
1986	0.7175	1.0225	1.0950	1.0977	1.2919	0.9644
1987	0.7115	1.0139	1.0590	1.0616	1.3096	0.9777
1988	0.6994	0.9968	1.0630	1.0655	1.3222	0.9871
1989	0.7005	0.9983	1.0455	1.0480	1.3264	0.9902
1990	0.7017	1.0000	0.9976	1.0000	1.3395	1.0000

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