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## Modeling and Pricing Rain Risk

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## Modeling and Pricing Rain Risk


#### Abstract

In this paper we price a precipitation option based on empirical weather data from Germany using different pricing methods, among them Burn Analysis, Index Value Simulation and Daily Simulation. For that purpose we develop a daily precipitation model. Moreover, a decorrelation analysis is proposed to assess the spatial basis risk that is inherent to rainfall derivatives. The models are applied to precipitation data in Brandenburg, Germany. Based on simplifying assumptions of the production function, we quantify and compare the risk exposure of grain producers with and without rainfall insurance. It turns out that a considerable risk remains with producers who are remotely located from the weather station. Another finding is that significant differences may occur between the pricing methods. We identify the strengths and weaknesses of the pricing methods and give some recommendations for their applications. Our results are relevant for producers as well as for potential sellers of weather derivatives.


Keywords: weather risk, weather derivatives, precipitation model, basis risk JEL Classification: C8, Q14, Q54

## 1 Introduction

It is well known that weather is an important production factor in agriculture. Unfortunately, this production factor can hardly be controlled. In fact, weather risks are a major source of uncertainty in crop production. Traditionally, producers try to compensate for the negative economic consequences of bad weather events by buying insurance. However, in the midnineties a new class of instruments has emerged, namely weather derivatives. Generally spoken, weather derivatives are financial instruments that allow to trade weather related risks. These instruments include futures, options, and swaps. Common underlyings are derived from temperature, rainfall or wind. Weather derivatives have the advantage that they are not af-
fected by moral hazard or adverse selection, which may be a serious problem for insurance companies. On the other hand, a considerable risk may remain with the producer when using weather derivatives, because individual yield variations in general are not completely correlated with the relevant weather variable. Until now it has not been clear if weather derivatives would permeate in agriculture. Actually, literature increasingly deals with the question if weather derivatives can also play a role as risk management tools in the agribusiness. Most of the existing literature focuses on weather derivatives which are related to temperature (e.g. Richards et al. 2004, Turvey 2001, van Asseldonk and Oude Lansink 2003). We suspect, however, that rainfall is of greater importance in agriculture than temperature. Nevertheless, only few papers analyze the risk reducing potential of rainfall derivatives in agriculture. First steps in that direction are taken in Stoppa and Hess (2003) and Berg et al. (2005).

The aim of this article is to develop a daily precipitation model which can be used as a basis for the assessment of precipitation related weather derivatives or index-based insurances. Such a model has not been applied to agricultural questions so far, although it offers advantages compared to a direct estimation of the precipitation distribution. Another important aspect which has been mentioned in the literature but not actually dealt with is the quantification of the basis risk of weather derivatives. In this context it means the uninsurable risk resulting from the difference of the weather index at the derivative's reference point and the location of the agricultural production. While this aspect is less important concerning temperature related derivatives, it cannot be neglected when the effectiveness of precipitation derivatives is analyzed because of the high spatial variability of precipitation. The article is structured as follows: In Section 2 a statistical model for the estimation of daily precipitation is presented. Furthermore, the statistical relationship between precipitation at different places is described with a de-correlation analysis. In Section 3 an empirical application of these concepts follows. Using precipitation data of the region of Brandenburg/Germany, a put option as well as a call option on the cumulated rainfall for different accumulation periods are priced and the effect
on the risk exposure of farms is examined. Simple pricing procedures like the burn analysis and the index value simulation serve as a benchmark. The paper ends with conclusions on the proposed pricing methodology and the efficiency of weather derivatives with respect to hedging against precipitation risks (Section 4).

## 2 A precipitation model

There are two alternatives with regard to the modeling of precipitation risk. On the one hand, the distribution of the weather event (e.g. a weather index) can be estimated, either parametrically or non-parametrically. The studies available up to now in the agribusiness field follow this course (Berg et al. 2005, Turvey 2001, Stoppa and Hess 2003). On the other hand, a daily precipitation model can be developed, from which the relevant weather index is then derived. This procedure is more complex initially yet favorable for two reasons: Firstly, the ways in which the daily precipitation model can be used are very flexible within the scope of a „daily simulation", because practically all yield relevant events like the sums of precipitation for different accumulation periods, dry spells or extreme precipitation can be determined for any periods of time (Srikanthan and McMahon 2001). In contrast, a direct estimate of the distribution of the precipitation index is usually only valid for a particular index. Secondly - and this seems even more important than the higher flexibility - the accuracy of daily based models is higher due to a considerably larger number of observed values (Brix et al. 2002). For this reason, the pricing and the analysis of the effectiveness of temperature related derivatives mainly result from daily temperature models.

A precipitation model should be able to capture the following characteristics of daily rainfall:

- The probability of rainfall occurrence obeys a seasonal pattern. Rainfall in Europe for example, is more likely in winter than in summer.
- The sequence of wet and dry days follows an autoregressive process. This means, the probability of a rainy day is higher if the previous day was wet.
- The amount of precipitation on a wet day varies with the season. Rainfall in Europe is more intensive in summer than in winter.
- The volatility of the amount of rainfall also changes seasonally. In Europe it is higher in summer than in winter.

In the following, a daily precipitation model is described which can depict the characteristics mentioned. According to Moreno (2002) and Cao et al. (2004), the stochastic process of daily precipitation can be decomposed into a stochastic process for the binary event "rainfall" and "dryness" respectively, and a distribution for the amount of precipitation is given such that it describes a rainy day. Consider the random variable $X_{t}$ :

$$
X_{t}=\left\{\begin{array}{l}
0, \text { if day } t \text { is dry }  \tag{1}\\
1, \text { if day } t \text { is rainy }
\end{array}\right.
$$

It is assumed that $X_{t}$ follows a first order Markov chain. The probability $p_{t}$, that it will rain on day $t$ can be calculated as:

$$
\begin{align*}
& p_{t}=p_{t-1} \cdot q_{t}^{11}+\left(1-p_{t-1}\right) \cdot q_{t}^{01} \\
& t=1,2, \ldots, n \tag{2}
\end{align*}
$$

$q_{t}^{01}$ describes the transition probability of rain on day $t$ and dryness on the previous day $t-1$. Analogously, $q_{t}^{11}$ stands for the transition probability between two successive rainy days. Note that the transition probabilities $q_{t}^{01}$ and $q_{t}^{11}$ vary with time.

The precipitation amount $y_{t}$ is modeled as a sequence of continuous random variables with independent distributions. In the literature, various distributions with a non-negative domain are discussed, among others the exponential distribution and the gamma distribution (Woolhiser and Roldan 1982). The mixed exponential distribution has proven to be especially flexible (Wilks 1999). The density function is:

$$
\begin{equation*}
f\left(y_{t} \mid X_{t}=1\right)=\frac{\alpha_{t}}{\beta_{t}} \cdot \exp \left(\frac{-y_{t}}{\beta_{t}}\right)+\frac{1-\alpha_{t}}{\gamma_{t}} \cdot \exp \left(\frac{-y_{t}}{\gamma_{t}}\right) \tag{3}
\end{equation*}
$$

with $0 \leq \alpha_{t} \leq 1$ and $0<\beta_{t}<\gamma_{t}$
The parameters of the mixed exponential distribution $\alpha_{t}, \beta_{t}$ and $\gamma_{t}$ are also time varying. Thereby the seasonality of precipitation is taken into account. In this form however, the model is not estimable. In order to reduce the number of the parameters to be estimated, each of the time varying parameters is developed by a finite Fourier series:
$\theta_{t j}=a_{j 0}+\sum_{k=1}^{m_{j}}\left[a_{j k} \cdot \sin \left(\frac{t \cdot k}{N}+b_{j k}\right)\right]$,
where $\theta_{t 1}=q_{t}^{01}, \theta_{t 2}=q_{t}^{11}, \theta_{t 3}=\alpha_{t}, \theta_{t 4}=\beta_{t}, \theta_{t 5}=\gamma_{t}$ and $N=\frac{365}{2 \cdot \pi}$
$a_{j k}$ and $b_{j k}$ denote the Fourier coefficients, and $m_{j}$ is the maximum number of harmonics needed to specify the seasonal cycles. The Fourier coefficients for $\alpha_{t}, \beta_{t}$ and $\gamma_{t}$ as well as for $q_{t}^{01}$ and $q_{t}^{11}$ are estimated simultaneously by maximizing the following log-likelihood functions (Woolhiser and Pegram 1979):

$$
\begin{align*}
& \ln L_{1}=\sum_{t=1}^{365}\left[c_{t}^{00} \cdot \ln \left(1-q_{t}^{01}\right)+c_{t}^{01} \cdot \ln \left(q_{t}^{01}\right)+c_{t}^{10} \cdot \ln \left(1-q_{t}^{11}\right)+c_{t}^{11} \cdot \ln \left(q_{t}^{11}\right)\right]  \tag{5}\\
& \ln L_{2}=\sum_{t=1}^{T} \ln \left[f\left(y_{t} \mid X_{t}=1\right)\right] \tag{6}
\end{align*}
$$

In (5) $c_{t}^{i j}$ is the observed number of transitions from state $i$ at day $t-1$ to state $j$ at day $t$.
A weather derivative always refers to a particular weather station. The fact that the farm site is usually not located at this weather station results in a basis risk. One can expect the risk to increase with the distance to the weather station. This basis risk can be quantified by means of a de-correlation analysis. Rubel (1996) proposes the following nonlinear de-correlation function for the modeling of the spatial relationship of precipitation in Europe:

$$
\begin{equation*}
\rho(d)=e_{1} \cdot \exp \left(-e_{2} \cdot d^{e_{3}}\right) \tag{7}
\end{equation*}
$$

Herein $\rho$ denotes the correlation coefficient between the precipitation at different places and $d$ the distance between the weather station and the farmer's production site. $e_{1}, e_{2}$, and $e_{3}$ are parameters to be estimated. In spite of the de-correlation analysis being a popular instrument in meteorology, two points should be considered critically. Firstly, the de-correlation function is invariant of the direction. Thus, topographical differences potentially influencing the precipitation are neglected. Secondly, Embrechts et al. (1999) point out problems of using correlation coefficients when the underlying distributions are not elliptical. In spite of these weaknesses the concept of the de-correlation analysis is used in this study.

## 3 Valuation of rainfall options for grain producers in north-eastern Germany

### 3.1 Background, methods and data

Grain production in north-eastern Germany, Brandenburg in particular, is highly affected by rainfall risk. During the relevant months of April and May, Brandenburg has had between 4.5 mm and 136.3 mm of precipitation (with a mean of 80.2 mm ) in the last 20 years, and the grain yields have fluctuated similarly. The high correlation between rainfall and yields results from the sandy soil having little water storing capacity and the lack of irrigation. Currently there is no possibility to insure against yield losses caused by low rainfall. In view of the extreme crop failures in the years of drought 2000 and 2003, in which the government had to provide disaster relief in order to save farmers from becoming insolvent, there is a pronounced interest to introduce some kind of rainfall insurance. By purchasing a put option on a rainfall index, a grain producer is (partially) insured against revenue losses due to little precipitation in the growing season. Moreover we consider a call option on cumulated rainfall during the harvesting period which compensate grain producers for quality losses and timeliness costs. In the following, let us assume that both options are available in the OTC market.

The reference point is the weather station Berlin-Tempelhof which is central for Brandenburg. The contract specifications for the derivatives considered here are summarized in Table 1.

Table 1: $\quad$ Specification of rainfall options

|  | Option 1 | Option 2 |
| :--- | :--- | :--- |
| Type | Put | Call |
| Payoff | $V \cdot \max \left(K-I_{t}, 0\right)$ | $V \cdot \max \left(I_{t}-K, 0\right)$ |
| Index $I_{t}$ | $\sum_{t=01.04 .}^{31.05 .} y_{t}$ | $\sum_{t=01.07 .}^{15.07 .} y_{t}$ |
| Strike price $K$ | 91 mm | 30 mm |
| Tick size $V$ | $1 €$ per 1 index points | $1 €$ per 1 index points |
| Discount rate $r$ | $5 \%$ | $5 \%$ |
| Time to expiration $\tau$ | 9 months | 9 months |

The estimation of the precipitation model is based on rainfall data measured in BerlinTempelhof from 1 January 1948 to 31 August 2004 ( $T=20$ 683). Records from 23 weather stations in Brandenburg from 1 January 1983 to 31 December 2003 are available for the calculation of the de-correlation function. The weather stations are evenly distributed in the Brandenburg region within a radius of 100 km from Berlin. Pricing of the options is carried out by a burn analysis, an index value simulation and a daily simulation. These procedures are briefly described below. A more detailed description and critical discussion is found in Zeng (2000) and in Cao and Wei (2000).

## Burn Analysis (Historical Simulation)

In a non-parametric burn analysis, an empirical distribution of the rainfall index is derived from the historical rainfall data which consist of 56 observations here. Based on the empirical distribution, hypothetical payoffs of the option are determined for each of the 56 years and
discounted with the risk-free interest rate. The price of the option is the mean of the discounted payoffs.

## Index Value Simulation

An index value simulation requires determining a parametric distribution for the rainfall index. We use the EXCEL add-in BEST-FIT to test for the most appropriate distribution. Only distributions with a non-negative domain were considered. According to the Chi Square test, the Kolmogorov Smirnov test and the Anderson Darling test, the Weibull distribution in the case of the accumulation period „April/May", and the Erlang distribution in the case of the accumulation period „1st half of July" show the best fit to the 56 empirical observations. From these distributions, values for the precipitation index are randomly drawn 50000 times, and the discounted payout of the option is determined. As before the option price is the average discounted payoff.

## Daily Simulation

In this approach the daily precipitation is simulated instead of the rainfall index using the model described in previous section. The rainfall index is then derived from the simulated sample paths by summing up daily precipitation in the relevant accumulation period. The subsequent steps are identical to the index value simulation.

Note that all three procedures suffer from a theoretical shortcoming in that there is no guarantee for the calculated option prices being arbitrage free. A well-known problem with pricing weather derivatives is that weather cannot be traded and hence it is impossible to construct a risk-free hedge portfolio. However, this is a prerequisite for the application of no-arbitrage models like the Black Scholes pricing formula. Different approaches to treat this problem are proposed in the literature (cf. Alaton et al. 2002, Richards et al. 2004). All methods suggested require the market price for weather risk to be quantified. The market price for weather risk could be derived implicitly from price quotations, if a market already existed for these weather derivatives. As this is not the case in the situation considered, the only resort is to
parameterize this unknown value. We refrain from doing so here, as such an analysis has already been carried out in the papers mentioned above. Moreover, the no-arbitrage trait of derivative prices is not that relevant in the assumed situation of an OTC trade, as trading usually does not occur during the contract term. Still, when interpreting the results, one should take into account, that the option prices depicted, i.e. the costs of coverage from the farmer's point of view, constitute lower limits to the actual costs that are to be expected, as the sellers (insurance companies, banks) will charge risk premiums.

### 3.2 Results

## Estimation of the daily precipitation model

In order to estimate the parameters of the daily precipitation model (1) to (4), the likelihood functions (5) and (6) are maximized with a genetic algorithm. The parameters $m_{j}$ are determined using the Akaike Information Criterion (AIC). Figure 1 shows the actual and the estimated daily rainfall in the course of a year. Figure A1 in the appendix depicts the related transition probabilities $q_{t}^{01}$ and $q_{t}^{11}$. Obviously the model does not only fit the yearly average but also the seasonality of the rainfall amounts well. From Figure A2 it can be seen that the standard deviation of the estimated daily precipitation is actually higher in summer than in winter. Hence, the model reflects the aforementioned characteristics of daily rainfall.

Figure 1: Observed and estimated average daily precipitation (weather station Berlin-Tempelhof)


A pitfall of the daily precipitation model is the underestimation of the variance of cumulated rainfall over a period of several weeks. This underestimation of the variance has already been observed in a different context and has been termed „low frequency variability bias" (Dubrovsky et al. 2004). For example, the sample variance of the precipitation for the period of April/May is 1436, whereas the daily precipitation model only shows a value of 882 . One can expect that the daily simulation will also result in biased option prices because options prices are sensitive towards volatility. Hansen and Mavromatis (2001) discuss various methods to reduce the low frequency variability bias. In this study we take several measures: Firstly, the transition probabilities $q_{t}^{01}$ and $q_{t}^{11}$ are estimated by their empirical sample counterparts, which show a clearly higher variability than those based on the Fourier series (see Figure A1). Moreover, the parameters of the mixed exponential distribution $\alpha_{t}, \beta_{t}$ and $\gamma_{t}$ are determined in such a way that the resulting variances exactly fit the sample variance of the daily rainfall amounts shown in Figure A2b in the appendix. Secondly, following Dubrovsky et al. (2004), a second order Markov process instead of a first order one is estimated. Thus, longer sequences of consecutive rainy and dry days respectively may occur, which leads to a higher variance of the cumulated precipitation. In what follows the original and the modified daily precipitation model are called „daily simulation I" and „daily simulation II", respectively. The calculation of the spatial basis risk is exemplified for the rainfall sum in April and May (Option 1). First, the correlation coefficients of the precipitation index between pairwise weather stations are determined. Next, the distances between the stations are quantified, which serve as an explanatory variable in the non-linear regression function (7). The parameter estimates for the de-correlation function are: $e_{1}=0.9331, e_{2}=0.0009$ and $e_{3}=1.2183$. Figure 2 shows the graph of the de-correlation function which has the expected negative slope. While the correlation of the total precipitation is approximately 0.9 between BerlinTempelhof and a station at a distance of 25 km , this value decreases to about 0.5 at a distance
of 200 km . An $\mathrm{R}^{2}$ of 0.66 proves that the estimated de-correlation function can be considered a good approximation to the empirical correlations. Nevertheless, it should be noted that the topographical situation in Brandenburg matches the assumption of a direction-independent relationship, and that this assumption may apply less, for example, to mountainous regions. Moreover, the scatter plot in Figure 2 shows heteroscedasticity, i.e. the relationship between distance and correlation becomes more imprecise with increasing distance.

Figure 2: De-correlation function for cumulated precipitation in Brandenburg, Germany


## Valuation of the rainfall options

Table 2 presents the option prices obtained by applying the burn analysis, the index value simulation and both variants of the daily simulation. The burn analysis and the index value simulation lead to similar prices for both options. The similarity of the results implies that the Weibull distribution for the accumulation period „April/May" and the Erlang distribution for the accumulation period „1st half of July" approximate the respective empirical distribution of the precipitation index well. In contrast, the prices calculated with the daily simulation are lower compared to those obtained with the burn analysis. This difference is significant for the put option in the case of the daily simulation I. The reason is the aforementioned underestimation of the volatility. This effect can be reduced, yet not eliminated, by the modification of the
daily precipitation model described above. Furthermore, the differences of the pricing methods depend on the derivative to be valuated. Thus, all four methods result in similar prices for the call option referring to a shorter accumulation period.

Table 2: Option prices for different valuation models

|  | Burn | Index Value | Daily | Daily |
| :--- | :---: | :---: | :---: | :---: |
|  | Analysis | Simulation | Simulation I | Simulation II |
| Put option | 14.28 | 15.17 | 10.58 | 11.91 |
| Call option | 7.68 | 8.05 | 7.05 | 7.52 |

## Analysis of the hedging effectiveness

Finally, we investigate how the availability of a precipitation derivative affects the risk exposure of grain producers located at different distances from the weather station by using the put option as an example. As we are mainly interested in the geographic basis risk here, we will simplify matters by neglecting other production risks. In addition, it is assumed that the relation between the cumulated rainfall in April and May and the grain yield is linear and that both variables are perfectly correlated. Due to these simplifying assumptions, it is possible to interpret the rainfall distributions directly as profit distributions. The distributions for the rainfall index are generated with the Index Value Simulation using the Weibull distribution and the de-correlation function estimated before. Table 3 and Figure 3 show that a producer located in the immediate vicinity of the weather station can eliminate the downside risk by acquiring a put option. The variance of the returns can be reduced by about 65 percent. The expected value of the profit distributions does not change, since the option price is defined as the expected value of the payouts according to the actual probabilities (as opposed to the riskneutral ones). As mentioned before, this price constitutes a lower limit; due to risk premiums and transaction costs, the actual prices will be higher, which leads to a downward shift of the distributions with an option. Furthermore, it becomes clear that the risk reduction potential of the weather derivatives diminishes with increasing distance. When interpreting the results the
simplifying assumptions regarding the „production function" should be kept in mind. In reality, yields are not perfectly correlated with the rainfall index resulting in a further reduction of the hedging effect (cf. Berg et al. 2005).

Figure 3: $\quad$ Profit distributions with and without hedging


Table 3: Parameters of the profit distributions with and without hedging at different locations

| without option |  |  |  |  |  | with option |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance (Correla- <br> tion coefficient) | $\mathbf{0} \mathbf{~ k m ~ ( 1 . 0 0 )}$ | $\mathbf{0} \mathbf{~ k m ~ ( 1 . 0 0 )}$ | $\mathbf{2 5} \mathbf{~ k m ~ ( 0 . 8 9 )}$ | $\mathbf{1 0 0} \mathbf{~ k m ~ ( 0 . 7 3 )}$ | $\mathbf{2 0 0} \mathbf{~ k m ~ ( 0 . 5 3 )}$ |  |
| Mean |  | 90.5278 | 90.5346 | 90.5360 | 90.53443 | 90.5347 |
| Variance |  | 1468.9080 | 555.7463 | 689.3560 | 906.1495 | 1182.3950 |
|  | $5 \%$ | 31.5255 | 75.2600 | 55.7921 | 44.8016 | 36.9530 |
|  | $10 \%$ | 41.8978 | 75.2600 | 61.8344 | 54.3015 | 47.0091 |
|  | $15 \%$ | 49.7326 | 75.2600 | 66.0318 | 60.3195 | 54.6038 |
|  | $50 \%$ | 88.2350 | 75.2600 | 85.8361 | 87.8728 | 88.8872 |
|  | $90 \%$ | 141.8256 | 126.0856 | 126.2684 | 130.4654 | 135.8326 |
|  | $95 \%$ | 157.3603 | 141.6203 | 141.6340 | 144.0778 | 149.9206 |

## 4 Conclusions

An economic analysis of weather derivatives from the viewpoint of a potential buyer requires the solution of three interrelated problem areas: firstly, the statistical modeling of the relevant weather variables; secondly, the quantification of the relationship between the weather variables and the production; and thirdly, the development of a theoretically consistent pricing model. This paper focuses on the first aspect. A daily precipitation model is specified, from which indices can be derived that determine the payoff of the derivative. Based on this model "fair prices" can be calculated, which constitute a lower bound for the value of a derivative. A comparison with other, simpler procedures reveals clear differences in the pricing of the weather derivatives. First of all this finding underscores the importance of the model choice. However, it is difficult to draw an unequivocal conclusion regarding the superiority of one of the valuation approaches. On the one hand, application of the daily simulation has the advantage of yielding smaller confidence intervals for the resulting indices and prices compared with the non-parametric burn analysis and the index value simulation (see Brix et al. 2002, p. 139). On the other hand, the danger of a rather sophisticated daily precipitation model being wrongly specified is relatively high; such a risk is precluded when the precipitation index is estimated directly. In the present application it turned out that a daily simulation model tends to underestimate the volatility of monthly rainfall. We conclude that the preferential statistical approach to option pricing depends, among other things, on the availability of data and on the context of its application. A systematic model validation on the basis of quasi-ex ante forecasts is recommended as a subject of further research.

Regardless of the issue of the appropriate pricing method, the following practical conclusions can be drawn: The risk reducing effect of precipitation derivatives is much more regionally confined than it is the case with temperature related derivatives. In the example of Brandenburg considered here, the correlation between the precipitation index of the weather station Berlin-Tempelhof and a remote farm site decreases to a value of 0.75 at a distance of 100 km .

If one additionally takes into account the stochastic relation between precipitation on the one hand and production or returns on the other hand - which has been ignored in this study - the use of rainfall derivatives as risk management tools in agriculture appears questionable. It follows that potential suppliers of rainfall insurances should introduce a dense network of weather stations as reference points for the rainfall index in order to increase the attractiveness of this type of insurance, although this may lead to a fragmented demand. Moreover, the specification of adequate weather indices also requires further studies. The cumulated rainfall, on which we focus here, may not be specific enough from the viewpoint of many producers. For example, not only the rainfall sum is relevant for the yields but also the rainfall distribution over time. The precipitation model presented in this paper can be utilized to explore the effectiveness of any kind of rainfall based index.

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## Appendix

Figure A1: Conditional transition probabilities

$-q_{t}^{01}$ empirical —— $q_{t}^{01}$ Fourier series $\cdots \cdots q_{t}^{11}$ empirical $-\boldsymbol{-} \cdot q_{t}^{11}$ Fourier series

Figure A2: Conditional mean and standard deviation of daily precipitation
a) Mean

b) Standard deviation


