Multiplicity, Jet, and Transverse Mass dependence of Bose-Einstein Correlations in e$^+$e$^-$- Annihilation*

Wesley J. Metzger[1],

[1]IMAPP, Radboud University, 6525 AJ Nijmegen, The Netherlands

Abstract

Bose-Einstein correlations of pairs of identical charged pions produced in hadronic Z decays are analyzed for both two- and three-jet events. A parametrization suggested by the τ-model is used to investigate the dependence of the Bose-Einstein correlation function on track multiplicity, number of jets, and transverse momentum.

1   Introduction

After a brief review of relevant previous results, new *preliminary* results are presented on the dependence of the Bose-Einstein correlation function on track and jet multiplicity and transverse momentum, using a parametrization which has been found [1] to describe well Bose-Einstein correlations (BEC) in hadronic Z decay, namely that of the τ-model [2, 3],

1.1   Review

The Bose-Einstein correlation function, R_2, is usually parametrized as

$$R_2 = \gamma \left[1 + \lambda \exp \left(- (rQ)^2 \right) \right] (1 + \epsilon Q) , \qquad (1)$$

---

*This talk was also given at *XLIV International Symposium on Multiparticle Dynamics*, Bologna, 8–12 September 2014.

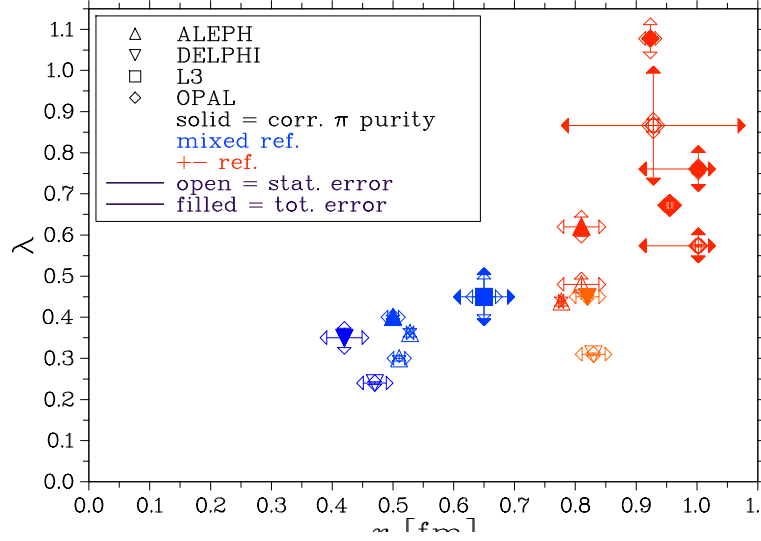Figure 1: λ and r at $\sqrt{s} = M_Z$ found in the LEP experiments [4–10].

and is measured by $R_2(Q) = \rho(Q)/\rho_0(Q)$, where $\rho(Q)$ is the density of identical boson pairs with invariant four-momentum difference $Q = \sqrt{-(p_1 - p_2)^2}$ and $\rho_0(Q)$ is the similar density in an artificially constructed reference sample, which should differ from the data only in that it does not contain BEC.

Dependence on the reference sample Two methods were frequently used at LEP to construct ρ_0: unlike-sign pion pairs from the same event, and like-sign pairs from different events. The latter method is generally referred to as mixed events. However, it must be pointed out that the observed values of the parameters r and λ depend to a great extent on which reference sample is used. This is clearly seen in Fig. 1 where the values of λ and r found for charged-pion pairs from hadronic Z decays by the LEP experiments ALEPH [4, 5], DELPHI [6], L3 [7] and OPAL [8–10] are displayed. Solid points are corrected for pion purity; open points are not. This correction increases the value of λ but has little effect on the value of r. All of the results with $r > 0.7$ fm were obtained using an unlike-sign reference sample, while those with smaller r were obtained with a mixed reference sample. The choice of reference sample clearly has a large effect on the observed values of λ and r. In comparing results we must therefore be sure that the reference samples used are comparable.

Dependence on the particle mass It has been suggested, on several grounds [17], that r should depend on the particle mass as $r \propto 1/\sqrt{m}$. Values of r found at LEP for various types of particle are shown in Fig. 2. Comparing only results using the same type of reference sample (in this case mixed), we see no evidence for a $1/\sqrt{m}$ dependence. Rather, the data suggest one value of r for mesons and a smaller value for baryons. The value for baryons, about 0.1 fm, seems very small, since the size of a proton is an order of magnitude greater. If true it is telling us something unexpected about the mechanism of baryon production.
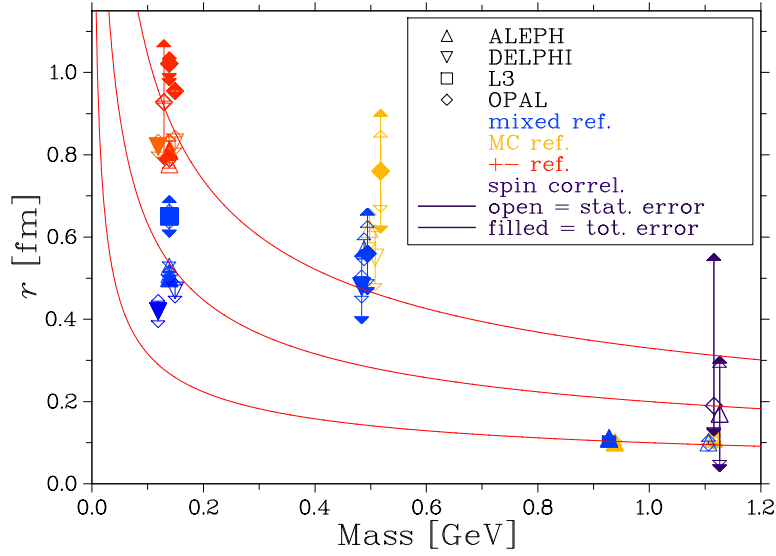
Figure 2: Dependence of r on the mass of the particle as determined at $\sqrt{s} = M_Z$ from 2-particle BEC for charged pions [4–10], charged kaons [11,12] and neutral kaons [11,13,14] and from Fermi-Dirac correlations for protons [13] and lambdas [15,16]. The curves illustrate a $1/\sqrt{m}$ dependence.

Dependence on the transverse mass However, r has been observed to depend on the transverse mass of the particle pair, [18, 19] as is shown in Fig. 3.

Dependence on particle and jet multiplicity The OPAL collaboration has studied the dependence of r and λ on the charged track multiplicity and on the number of jets [9]. They used an opposite-sign reference sample, which necessitated the exclusion of regions of Q where R_2 was too strongly affected by resonances in the reference sample. To describe the long-range correlations they introduced a quadratic term resulting in

$$R_2(Q) = \gamma \left[1 + \lambda \exp\left(-(rQ)^2\right)\right] \left(1 + \epsilon Q + \delta Q^2\right) \ . \tag{2}$$
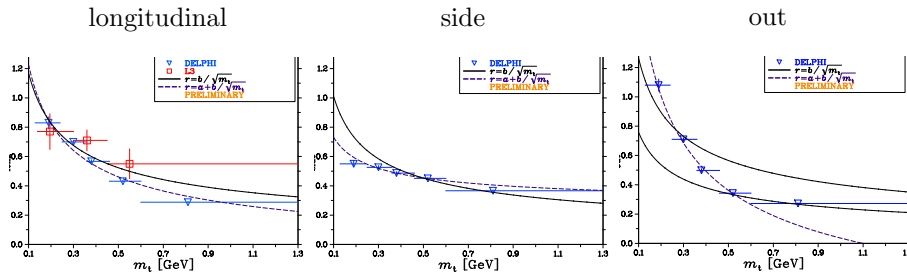

Figure 3: The transverse mass dependence of the components of r in the LCMS from Refs. 18,19.

They observed a linear rise of r with charged track multiplicity as well as an increase of r with the number of jets. The behavior of λ was the opposite. However, when only two-jet (or only three-jet) events were selected, r was approximately independent of multiplicity.

1.2 τ-model

However, the "classic" parametrization of Eq. (1) is found to be inadequate, even when it is generalized to allow for a Lévy distribution of the source:

$$R_2 = \gamma \left[1 + \lambda \exp \left(- \left(rQ \right)^\alpha \right) \right] \left(1 + \epsilon Q \right), \quad 0 < \alpha \leq 2 \tag{3}$$

This was not realized for a long time because the correlation function was only plotted up to $Q = 2\,\text{GeV}$ or less. In Ref. 1 Q was plotted to $4\,\text{GeV}$, and it became apparent that there is a region of anti-correlation $(R_2 < 1)$ extending from about $Q = 0.5$ to $1.5\,\text{GeV}$. This anti-correlation, as well as the BEC correlation are well described by the τ-model.

In the τ-model R_2 is found to depend not only on Q, but also on quantities a_1 and a_2. For two-jet events $a = 1/m_{\text{t}}$, where $m_{\text{t}} = \sqrt{m^2 + p_{\text{t}}^2}$ is the transverse mass of a particle). Parameters of the model are the parameters of the Lévy distribution which describes the proper time of particle emission: α, the index of stability of the Lévy distribution; a width parameter $\Delta\tau$; and the proper time τ_0 at which particle production begins.

We shall use a simplified parametrization [1] where τ_0 is assumed to be zero and a_1 and a_2 are combined with $\Delta\tau$ to form an effective radius R:

$$R_2(Q) = \gamma \left[1 + \lambda \cos \left((R_{\text{a}}Q)^{2\alpha} \right) \exp \left(- (RQ)^{2\alpha} \right) \right] \left(1 + \epsilon Q \right), \tag{4a}$$

$$R_{\text{a}}^{2\alpha} = \tan \left(\frac{\alpha\pi}{2} \right) R^{2\alpha}. \tag{4b}$$

Note that the difference between the parametrizations of Eqs. (3) and 4 is the presence of the $\cos$ term, which accounts for the description of the anti-correlation. The parameter R describes the BEC peak, and R_{a} describes the anti-correlation region. While one might have had the insight to add, *ad hoc*, a $\cos$ term to Eq. (3), it is the τ-model which predicts a relationship, Eq. (4b), between R and R_{a}.

A fit of Eq. (4) to L3 two-jet events is shown in Fig. 4, from which it is seen that the τ-model describes both the BEC peak and the anti-correlation region quite well. Also the three-jet data is well described [1], which is perhaps surprizing since the τ-model is inspired by a picture of fragmentation of a single string.

It must also be pointed out that the τ-model has its shortcomings: The τ-model predicts that R_2 depends on the two-particle momentum difference only through Q, not through components of Q. However, this is found not to be the case [1]. Nevertheless, regardless of the validity of the τ-model, Eq. (4) provides a good description of the data. Accordingly, we shall use it in the following.

Since the results on the dependence of the BEC parameters on particle and jet multiplicities and on transverse mass mentioned in Sect. 1.1 were obtained using the classic Gaussian parametrization, Eq. (1), and since this parametrization has been shown to be inadequate, in the rest of this paper we investigate these properties using the τ-model parametrization, Eq. (4). The results are *preliminary*.
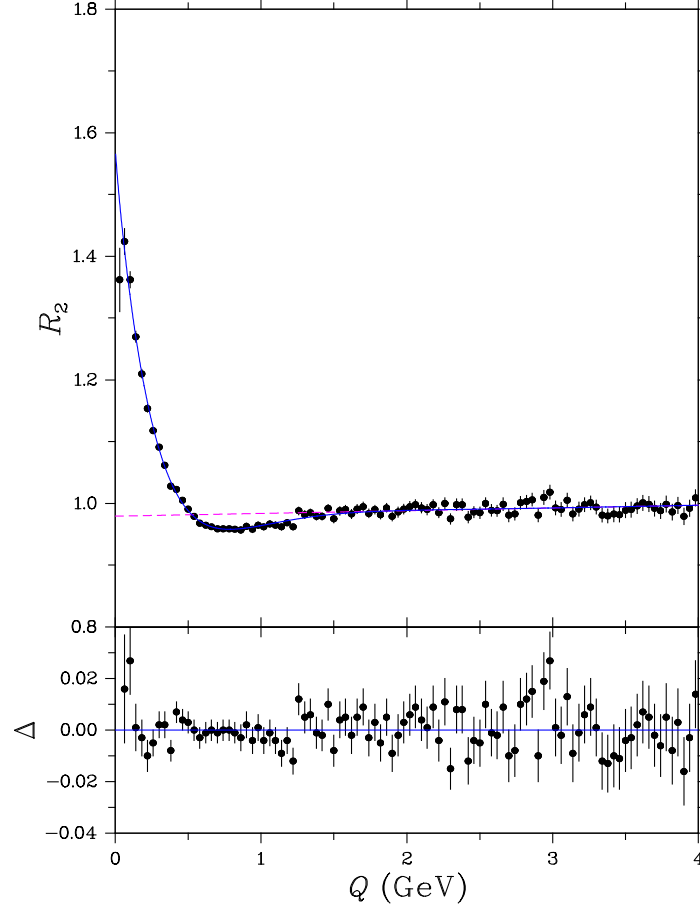
Figure 4: The Bose-Einstein correlation function R_2 for two-jet events. The curve corresponds to the fit of Eq. (4). Also plotted is Δ, the difference between the fit and the data. The dashed line represents the long-range part of the fit, *i.e.*, $\gamma(1 + \epsilon Q)$. The figure is taken from Ref. 1.

27

1.3   L3 Data

The data were collected by the L3 detector at an e^+e^- center-of-mass energy of $\sqrt{s} \simeq 91.2\,\mathrm{GeV}$. Approximately 36 million like-sign pairs of well-measured charged tracks from about 0.8 million hadronic Z decays are used. This data sample is identical to that of Ref. 1.

The same event mixing technique is used to construct ρ_0 as in Ref. 1.

Using the JADE algorithm, events can be classified according to the number of jets. The number of jets in a particular event depends on the jet resolution parameter of the algorithm, y_{cut}. We define y_{23}^{J} as that value of y_{cut} at which the number of jets in the event changes from two to three. Small y_{23} corresponds to narrow two-jet events, large y_{23} to events with three or more well-separated jets.

2   New *Preliminary* Results

The parameters of the Bose-Einstein correlation function have been found to depend on charged multiplicity, the number of jets, and the transverse mass. However these quantities are related. Both the charged particle multiplicity and the transverse mass increase rapidly with the number of jets. This is seen in Fig. 5, where the average transverse mass and the average charged multiplicity are plotted *vs.* y_{23}^{J}. In the following we investigate the dependence of R and λ on these three quantities.

An unfortunate property of the τ-model parameterization, Eq. (4), is that the estimates of α and R from fits tend to be highly correlated. Therefore, to stabilize the fits, α is fixed to the value 0.44, which corresponds to the value obtained in a fit to all events.

While we show only the results using the JADE jet algorithm, we have also performed the same analysis using the Durham algorithm. It is found to lead to the same conclusions.

2.1   Dependence of R and λ on track and jet multiplicities

The dependence of R and λ on the detected charge multiplicity,[1] is shown in Figs. 6 and 7, respectively, for two- and three-jet events as well as for all events.

For all events R is seen to increase linearly with the multiplicity, as was observed for R by OPAL. However, the same linear increase is also seen for two- and three-jet events, with R for three-jet events and for all events being approximately equal and R for two-jet events shifted lower by about a 0.5 fm. This contrasts with the OPAL observation of little dependence of r on multiplicity for two- and three-jet events.

For all events, as well as for two- and three-jet events, λ decreases with multiplicity, the rate of decrease becoming less for high multiplicity. It is higher for three-jet events than for two-jet events, with the values for all events lying in between. This contrasts with the OPAL observation that λ was higher for two-jet events, as well as the OPAL observation that the decrease of λ with multiplicity is linear.

---

[1]The charge multiplicity is approximately given by $N_{\mathrm{ch}} \approx 1.7 N_{\mathrm{ch}}^{\mathrm{det}}$.
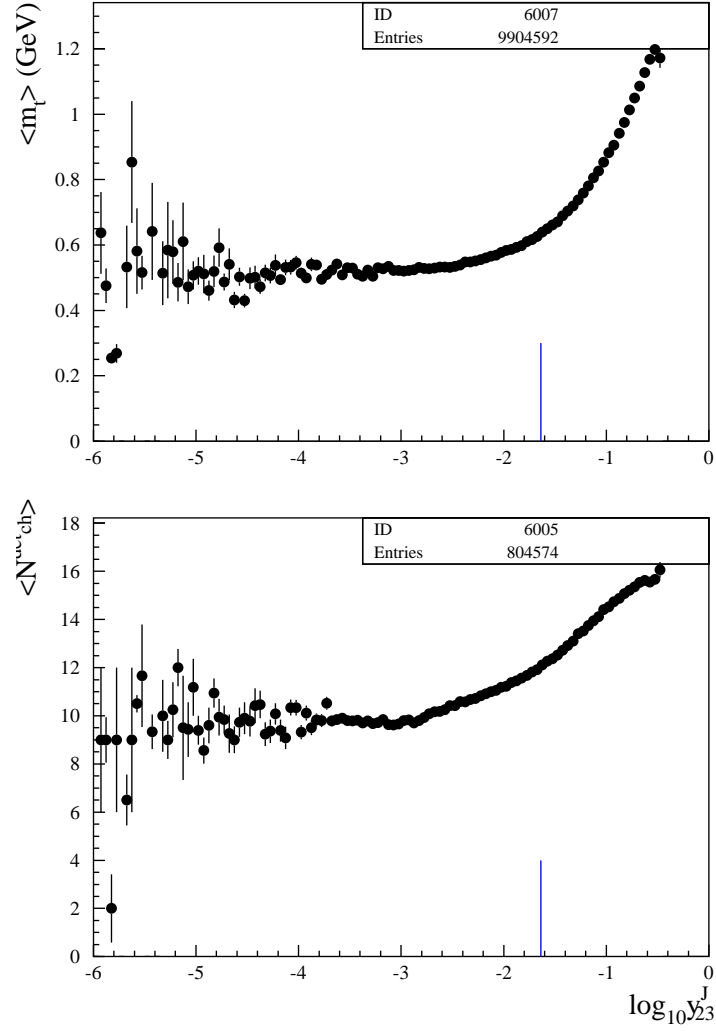
Figure 5: The average transverse mass and the average charged multiplicity as function of y_{23}^{J}. The short vertical line at $\log_{10} y_{23}^{\mathrm{J}} = -1.638$ corresponds to the cut used to define 2- and 3-jet events.
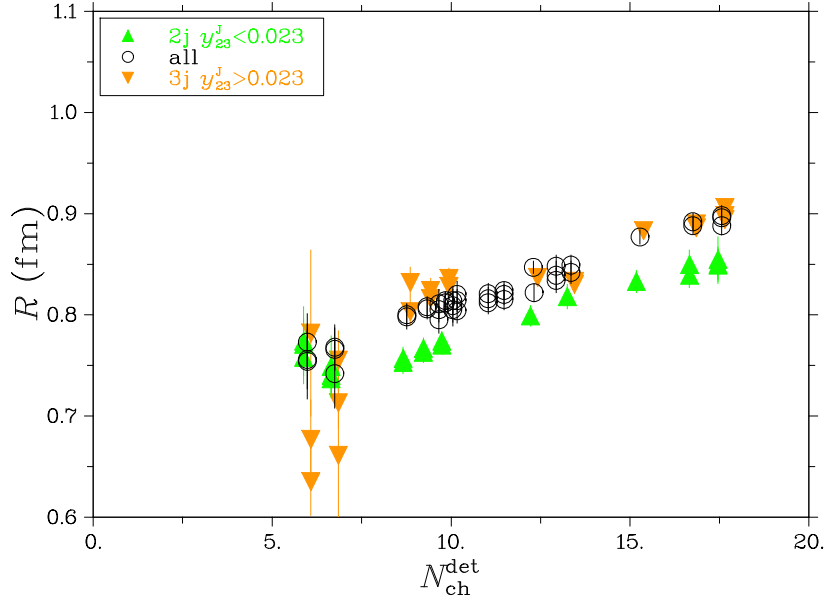
Figure 6: R obtained in fits of Eq. (4) as function of detected charged multiplicity for two-jet events ($y_{23}^J < 0.23$), for three-jet events ($y_{23}^J > 0.23$), and all events

2.2 Dependence of R and λ on trasverse mass and jet multiplicity

The dependence of R and λ on track multiplicity is shown in Figs. 8 and 9, respectively, for various selections on the transverse momentum, p_t, (or, equvilantly, m_t) of the tracks. For two-jet events both R and λ are slightly higher when both tracks have $p_t < 0.5$ GeV than when only one track is required to have so small a p_t. For three-jet events the same may be true, but the statistical significance is less; the difference decreases with multiplicity. When neither track has $p_t < 0.5$ GeV, the values of both R and λ are much lower for both two- and three-jet events. In all cases both R and λ increase with multiplicity, and the values for two-jet events are roughly equal to those for three-jet events.

3 Conclusions

The dependence of R and λ for the τ-model parametrization is different from that of r and λ found by OPAL for the usual Gaussian parametrization. However, it is unclear how much the differences depend on the use of different reference samples and how much on the parametrization used.

Multiplicity, number of jets, and transverse mass all affect the values of R and λ in the τ-model parametrization, Eq. (4).

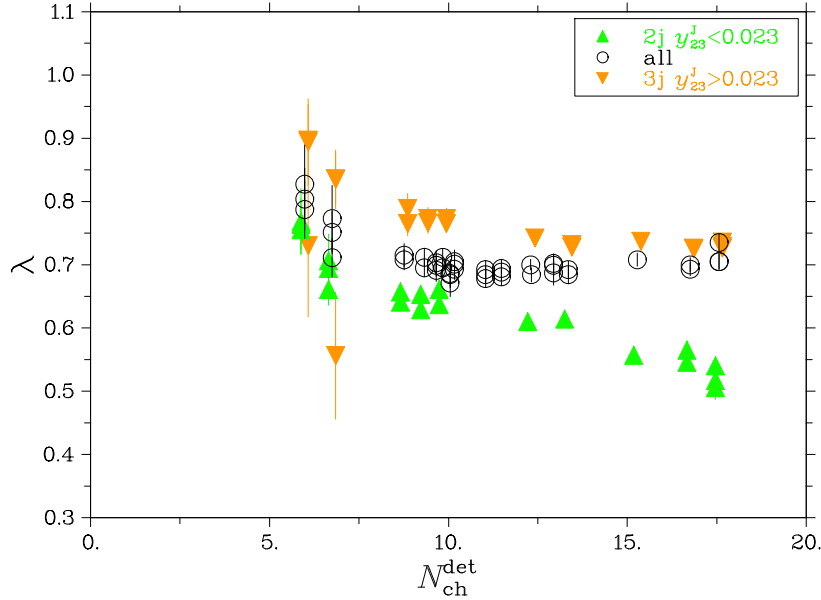Figure 7: λ obtained in fits of Eq. (4) as function of detected charged multiplicity for two-jet events ($y_{23}^{J} < 0.23$), for three-jet events ($y_{23}^{J} > 0.23$), and all events

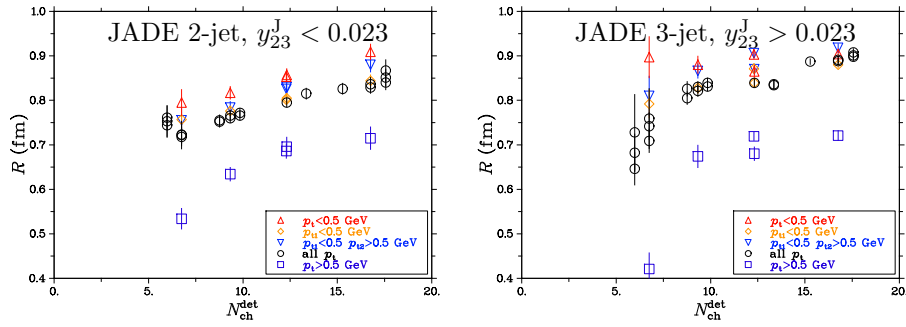

Figure 8: R obtained in fits of Eq. (4) as function of detected charged multiplicity (left) for two-jet events ($y_{23}^{J} < 0.23$) and (right) for three-jet events ($y_{23}^{J} > 0.23$) with the following selections on p_{t}: $\triangle$ both tracks having $p_{t} < 0.5$ GeV; $\diamond$ at least one track having $p_{t} < 0.5$ GeV; ∇ one track with $p_{t} < 0.5$ GeV and one with $p_{t} > 0.5$ GeV; $\circ$ all tracks; $\square$ both tracks having $p_{t} > 0.5$ GeV. Note that $p_{t} = 0.5$ GeV corresponds to $m_{t} = 0.52$ GeV.
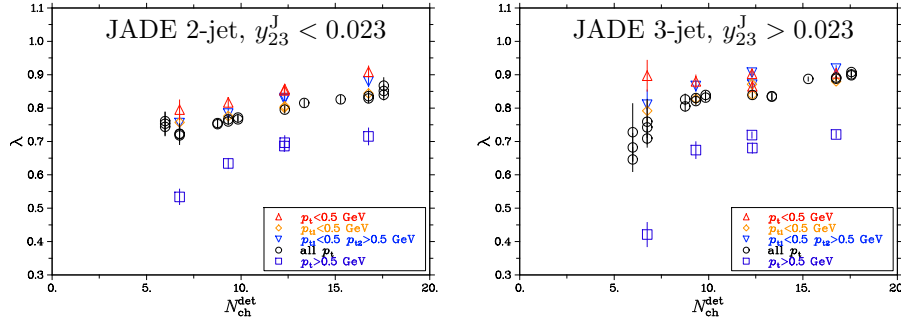
Figure 9: λ obtained in fits of Eq. (4) as function of detected charged multiplicity (left) for two-jet events ($y_{23}^{\mathrm{J}} < 0.23$) and (right) for three-jet events ($y_{23}^{\mathrm{J}} > 0.23$) with the following selections on p_{t}: $\triangle$ both tracks having $p_{\mathrm{t}} < 0.5$ GeV; $\diamond$ at least one track having $p_{\mathrm{t}} < 0.5$ GeV; ∇ one track with $p_{\mathrm{t}} < 0.5$ GeV and one with $p_{\mathrm{t}} > 0.5$ GeV; $\circ$ all tracks; $\square$ both tracks having $p_{\mathrm{t}} > 0.5$ GeV. Note that $p_{\mathrm{t}} = 0.5$ GeV corresponds to $m_{\mathrm{t}} = 0.52$ GeV.

References

[1] L3 Collab. P. Achard *et al.*, Eur. Phys. J. **C71** (2011) 1–25, 10.1140/epjc/s10052-011-1648-8.

[2] T. Csörgő and J. Zimányi, Nucl. Phys. **A517** (1990) 588–598.

[3] T. Csörgő, W. Kittel, W.J. Metzger, and T. Novák, Phys. Lett. **B663** (2008) 214–216.

[4] ALEPH Collab. D. Decamp *et al.*, Z. Phys. **C54** (1992) 75–85.

[5] ALEPH Collab. A. Heister *et al.*, Eur. Phys. J. **C36** (2004) 147–159.

[6] DELPHI Collab. P. Abreu *et al.*, Phys. Lett. **B286** (1992) 201–210.

[7] L3 Collab. P. Achard *et al.*, Phys. Lett. **B540** (2002) 185–198.

[8] OPAL Collab. P.D. Acton *et al.*, Phys. Lett. **B267** (1991) 143–153.

[9] OPAL Collab. G. Alexander *et al.*, Z. Phys. **C72** (1996) 389–398.

[10] OPAL Collab. G. Abbiendi *et al.*, Eur. Phys. J. **C16** (2000) 423–433.

[11] DELPHI Collab. P. Abreu *et al.*, Phys. Lett. **B379** (1996) 330–340.

[12] OPAL Collab. G. Abbiendi *et al.*, Eur. Phys. J. **C21** (2001) 23–32.

[13] ALEPH Collab. S. Schael *et al.*, Phys. Lett. **B611** (2005) 66–80.

[14] OPAL Collab. R. Akers *et al.*, Z. Phys. **C67** (1995) 389–401.

[15] ALEPH Collab. R. Barate *et al.*, Phys. Lett. **B475** (2000) 395–406.

[16] OPAL Collab. G. Alexander *et al.*, Phys. Lett. **B384** (1996) 377–387.

[17] Gideon Alexander, Rep. Prog. Phys. **66** (2003) 481–522.

[18] B. Lörstad and O.G. Smirnova, in Proc. 7th Int. Workshop on Multiparticle Production "Correlations and Fluctuations", ed. R.C. Hwa *et al.*, (World Scientific, Singapore, 1997), p. 42.

[19] J.A. van Dalen, in Proc. 8th Int. Workshop on Multiparticle Production "Correlations and Fluctuations '98: From QCD to Particle Interferometry", ed. T. Csörgő *et al.*, (World Scientific, Singapore, 1999), p. 37.