ZEF-Discussion Papers on Development Policy No. 231

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Migration when social preferences are ordinal: Steady state population distribution, and social welfare

Bonn, February 2017

Oded Stark, Migration when social preferences are ordinal: Steady state population distribution, and social welfare, ZEF – Discussion Papers on Development Policy No. 231, Center for Development Research, Bonn, February 2017, pp. 33.

ISSN: 1436-9931

Published by:
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Acknowledgements

I am indebted to two referees for thoughtful and inspiring comments, and to Gianmarco Ottaviano for advice and guidance. I benefited greatly from the suggestions and remarks of Wiktor Budzinski, Marcin Jakubek, Grzegorz Kosiorowski, Mariel Monica Sauler, Anna Solovyeva, and Ewa Zawojska.

Abstract

This paper adds three dimensions to the received literature: it models migration when the individuals' preferences regarding their relative income are ordinal, it works out the resulting spatial steady state distribution of the individuals, and it shows that the aggregate of the individuals' migration choices in the spatial steady state distribution sums up to the social optimum. This finding does not apply when the individuals' preferences regarding their relative income are cardinal. We highlight the importance of the assumption about the nature of the individuals' social preferences (whether ordinal or cardinal) to studying and predicting their migration behavior, and to elucidating the consequences of that behavior for social welfare.

Keywords: Ordinal preferences; Distaste for low relative income; An ordinal measure of income relative deprivation; Interregional migration; Steady state spatial distribution; Social Welfare

JEL Codes: C61, C62, D50, D60, D62, I31, R13, R23, Z13

1. Motivation

By now there is widespread recognition that comparisons with others impinge significantly on wellbeing and elicit substantial behavioral responses. In general, people would rather have a high income than a low income, and a high income-conferred rank than a low income-conferred rank. The incorporation of a dimension of relative income implies that income is valued in relation to the incomes of others with whom people naturally compare themselves (the "reference group" or the "comparison group"). In this paper, the preference for high rank-conferred income is expressed as distaste for low rank in the income hierarchy. Engaging in interpersonal comparisons affects the individuals' sense of wellbeing and influences their behavior, including in relation to migration. We present a first brush attempt to model the migration of an individual as a rank-seeking strategy when the individual's rank deprivation is measured as the distance from the top rank.

Evidence from econometric studies, experimental economics, social psychology, and neuroscience indicates that humans routinely engage in interpersonal comparisons, and that the outcome of that engagement affects their sense of wellbeing. People are discontented when their consumption, income or social standing falls below that of those who constitute their "comparison group". Examples of studies that recognize such discontent include Stark and Taylor (1991), Zizzo and Oswald (2001), Luttmer (2005), Fliessbach et al. (2007), Blanchflower and Oswald (2008), Takahashi et al. (2009), Stark and Fan (2011), Stark and Hyll (2011), Fan and Stark (2011), Card et al. (2012), and Stark et al. (2012). Stark (2013) presents corroborative evidence from physiology. The overwhelming weight of the evidence supports the notion of a strong asymmetry: the comparisons that affect an individual's sense of wellbeing significantly are the ones rendered by looking "up" the hierarchy, whereas looking "down" does not appear to be of much consequence or to deliver satisfaction. (For example, Andolfatto (2002) demonstrates that individuals are adversely affected by the material wellbeing of others in their comparison group when this wellbeing is far enough below their own.) Cohn et al. (2014) find that in choosing their level of work effort, workers respond to increased relative deprivation but not to increased "relative satisfaction." Frey and Stutzer (2002), Walker and Smith (2002), and Stark (2013) review a large body of evidence that lends support to the "upward comparison" view. In the analysis that follows, the interpersonal comparisons are of income-based rank, and low status is conceptualized as low rank.

The idea that rank-seeking is an important motive for human behavior is not a novelty of this paper. A study of the relationship between pay and wellbeing by Brown et al. (2008) reveals that the wage rank of workers influences significantly the satisfaction that they derive from their pay. Powdthavee (2009) finds that an individual's position in the wealth hierarchy within his local community affects his perceived economic wellbeing more than comparing his income with the mean income of the community. Boyce et al. (2010) argue that individuals gain utility (general life satisfaction) mainly from the rank (or position) of their income within a comparison group rather than from their (absolute) income. In its February 27, 2016 issue, *The Economist* magazine reports the following finding of the Eurobarometer survey, which has tracked self-reported happiness for over four decades: "According to Eurostat, the EU's statistical office, the only metric consistently correlated with European happiness is relative income. Moving one step up the income ladder increases happiness in every country in the EU."

Indeed, writings both in economics and in sociology have long maintained that individuals have a strong preference for high (social) rank, and are stressed when they have low (social) rank. Smith has remarked that "the desire of … obtaining rank among our equals, is, perhaps, the strongest of all our desires" (Smith, 1759, Part VI, Section I, Paragraph 4). Veblen (1899) has shown that higher pay to others can depress one's utility. Maslow (1943) views status as a basic human need, and Huberman et al. (2004, p. 103) infer from a study of five societies that "subjects valued status independently of any monetary consequence." There is considerable evidence from research in modern economics to the effect that the desire to escape low rank motivates workers to exert more effort (Neckermann and Frey, 2008; Kuhnen and Tymula, 2009; Kosfeld and Neckermann, 2011; Duffy and Kornienko, 2010), and students to perform better (Bandiera et al., 2009; Azmat and Iriberri, 2010).

The modern-day evidence nicely supports Smith's assessment of the power of the incentive to escape low rank as distinct from the desire for the tangible benefits associated with high rank. We provide a base for correcting the disregard of the distaste for low ordinal rank as an explanatory variable of migration behavior and for testing further the role of the distaste for falling behind others in the income hierarchy in propelling migration.

Admittedly, considerable empirical evidence finds that relative deprivation, which can be interpreted as a measure of low cardinal rank, is a statistically significant explanatory variable of migration behavior. Stark and Taylor (1991) show that relative deprivation increases the probability that household members will migrate from rural Mexico to the US to work. The significance of relative deprivation as an explanatory variable of labor migration has received additional support in several more recent studies. Quinn (2006) reports that relative deprivation is a significant motivating factor in domestic migration decisions in Mexico. Stark et al. (2009) explore the relationship between aggregate relative poverty, which is functionally related to aggregate relative deprivation, and migration. Drawing on Polish regional data, they demonstrate that migration from a region is positively correlated with the aggregate relative deprivation in the region. Czaika (2011) finds that, in India, relative deprivation is an important factor in deciding whether a household member should migrate, especially over a short distance. Basarir (2012) observes that people in Indonesia are willing to bear a loss of absolute wealth if there is a relative wealth gain from migration. Jagger et al. (2012) report that relative deprivation is a significant explanatory variable of circular migration in Uganda. Vernazza (2013) concludes that, even though interstate migration in the US confers substantial increases in absolute income, the trigger for migration is relative deprivation (low relative income), not low absolute income. Drawing on data from the 2000 US census, Flippen (2013) shows that both blacks and whites who migrate from the North to the South generally have average lower absolute incomes than their stationary northern peers, yet enjoy significantly lower relative deprivation, and that the relative deprivation gains for blacks are substantially larger than those for whites. Hyll and Schneider (2014) use a data set collected in the German Democratic Republic in 1990 to show that aversion to relative deprivation increased the propensity to migrate to western Germany.

Somewhat surprisingly, this body of work did not test for the role of distaste for low income rank as an alternative explanatory variable, nor did it study how the use of different measures of social preferences might affect the predictions of migration outcomes and the corresponding social welfare consequences. After all, and as eloquently noted by Bilancini and Boncinelli (2008), models based on ordinal rank may predict very different behavior from models based on cardinal rank. As an illustration of the distinction between incentives to migrate under the two types of rank, consider the following example. Assume that a group consists of i and j, and that i is poorer than j. Then, under distaste for low cardinal rank, the migration incentive of i will be different when j's income is 3 as opposed to when *j*'s income is 2. Under distaste for low income- based rank measured by position in the income hierarchy, however, the migration incentive of *i* will be the same because in both cases *i* is the second in the income hierarchy.

If, contrary to the cardinality assumption of the received empirical inquiries, people are concerned about experiencing low ordinal rank and tailor their migration behavior in response to this concern, then the migration outcome is unlikely to replicate the migration outcome reached under the assumption that people are concerned about experiencing low cardinal rank. Holding incomes constant which, as explained below, enables us to concentrate on the "pure" migration response to experiencing low ordinal income rank, we study next both this response and its consequences for the spatial distribution of the population and for social welfare.

An ordinal perspective has an empirically-related practical edge over a cardinal perspective. If a model assumes that people make migration decisions on the basis of income comparisons with others in their reference group, then it is presumed that they know / observe the incomes of others. This presumption can impose a heavy burden on the information structure. However, people can often rank themselves relatively easily in the income distribution of their reference group (for example, by answering a question such as: "Am I richer than other members of my reference group?"). Thus, it is more likely that migration will be found to be explained by social preferences when these preferences are conceived to be ordinal.

We characterize the steady state distribution of a population of *n* individuals who are homogeneous in preferences and heterogeneous in incomes. The individuals who, to begin with, are in region A can migrate at no cost to themselves between region A and region B. We make three main assumptions: (1) that the individuals exhibit strong social preferences; (2) that their incomes are held constant; and (3) that the two regions are sufficiently similar. The reason for making assumption (1) is given in the preceding paragraphs. It co-implies that the reason for making assumption (2) is to allow us to concentrate on essentials, namely to facilitate a study of the pure effect on migration outcomes of location-specific dissatisfaction that arises from falling behind others in the income distribution. Social preferences take the form of distaste for occupying a rank that is lower than the rank of others in the income hierarchy of the individual's reference group; in other words, social preferences represent the negative influence of unfavorable income comparisons on the individuals' sense of wellbeing. A joint rationale for making assumptions (2) and (3) is as follows. We can think of the individuals as having exogenous levels of some skill, *s*, with income, *I*, being a function of such skill and the region, *r* and, thus, $I(s,r)$. Assuming that the individuals' skill is smoothly locationally transferable so that *s* does not change when *r* changes, and that the regions are similar in the sense that the function $I(s,r)$ does not change (or changes only slightly) with respect to *r* between region A and region B *or* that the two regions are similarly developed in the sense that changes in absolute income do not have a substantial effect on utility for a fixed relative income, enables us to concentrate on distaste for low rank in the income hierarchy as a motivation for migration. Because incomes are held constant, the wellbeing of an individual is solely a function of the extent to which the individual's location aligns with his social preferences.[1]

[1] In earlier work reported in this journal (Stark et al., 2012), the dynamics of skill formation was shown to be linked with comparison group affiliation. Specifically, the acquisition of human capital was assumed to bring about a change of location in social space and revision of the comparison groups. Skill levels were viewed as occupational groups. Moving up the skill ladder by acquiring additional human capital, in itself rewarding, was shown to lead to a shift in the individual's inclination to compare himself with a different, and on average better-paid, comparison group, in itself penalizing.

2. A baseline model

If people can migrate at no cost between two regions, and if their migration decisions are induced by rank considerations, what will the distribution of the population between the two regions look like? When individuals' migration choices are driven by a desire to minimize their rank deprivation, will the aggregate of their migration moves sum up to a steady state distribution of the population between the two regions? Will it lead to a spatial distribution that is optimal from a social welfare point of view?

In order to respond to these questions, we begin with a simple representation. The population consists of $n \geq 4$ individuals (where *n* is a natural number) such that individual 1 has income x_1, individual 2 has income x_2, and so on, where $x_1 < x_2 < ... < x_n$. At the outset, all the individuals are in region A. We let (empty) region B come into being or become accessible, such that migration between the two regions is possible, and is cost free. We assume that in all relevant respects the two regions are identical, so there is no reason arising from a difference in the regions' amenities for the individuals to prefer one region to the other; improvement in rank is the sole reason for migration. When, in terms of the outcome of social comparisons, the regions are equally attractive (a tie), the individuals do not migrate. Once the individuals are in a region, the region becomes instantaneously their exclusive sphere of comparison. However, the individuals can relocate as many times as they wish in response to the actual distribution of other individuals between the two regions. Put differently, the individuals base their migration decisions on the observed current state, without simultaneously forming expectations as to how other individuals will behave. For ease of exposition, we label the steps in the migration process as periods, with the initial period (when all the individuals are in region A) being referred to as zero:

A	B
n $n-1$ $n-2$ . . . 1	

Figure 1. The initial (period zero) distribution of the individuals.

Claim 1: By period $n/2$ if *n* is even, or by period $(n-1)/2$ if *n* is odd, a steady state distribution of the population will be reached. In the steady state, the population will be divided between the two regions evenly if *n* is even, or evenly but for one individual if *n* is odd.

Proof: In period 1, individual *n* who does not have an incentive to migrate will stay in region A, whereas all the other individuals will move to region B in order to gain a higher rank.

Next, each of the individuals $n-3,\ n-4,\ \ldots,\ 1$ observes that he can obtain a higher rank (second) if he were to move back to region A. Thus, individuals $n-3,\ n-4,\ \ldots,\ 1$ move to region A, and in period 2 the distribution of the individuals will be *n* in region A, $n-1$ and $n-2$ in region B, and the remainder of the population in region A. (Individual $n-2$ will not move back to region A because of the assumption of no migration when there is a tie.)

Once again, some individuals from region A, specifically $n-5,\ n-6,\ \ldots,\ 1$, will have an incentive to move to region B. We can see that by period $n/2$ if *n* is even, or by period $(n-1)/2$ if *n* is odd, all comings and goings will come to halt, and we will have the steady state distribution of the population depicted in Figure 2.

A	B
n	
	$n-1$
	$n-2$
$n-3$	
$n-4$	
	$n-5$
	$n-6$
$n-7$	
$n-8$	
	$n-9$
$\vdots$	$\vdots$

Figure 2. The steady state spatial distribution of the population under ordinal preferences.

What remains to be characterized is the location of the two lowest income individuals, namely the location of individuals 1 and 2. It turns out that the whereabouts of these two individuals depends on whether *n* is even or odd, and on whether when *n* is even, whether *n* or $n-2$ is a multiple of 4, and when *n* is odd, whether $n-1$ or $n-3$ is a multiple of 4. Specifically, we have the following characterization. When *n* is even, then 1 and 2 are in different regions: if *n* is a multiple of 4, then 1 is in region A, and 2 is in region B; if $n-2$ is a multiple of 4, then 2 is in region A, and 1 is in region B. When *n* is odd, then 1 and 2 are in the same region: if $n-1$ is a multiple of 4, then they are in region A; if $n-3$ is a multiple of 4, then they are in region B.

In conclusion, in the steady state distribution of the population between region A and region B, the population is divided between the two regions evenly or evenly but for one individual. Q.E.D.

3. Robustness of the outcome of "an even or an even but for one individual" steady state distribution to a relaxation of assumptions

The results reported in the preceding section are not contingent on the assumption that all the individuals in region A who presume that their income rank will be higher in region B migrate to region B simultaneously. This assumption can be relaxed without jeopardizing the result of convergence to a steady state outcome, and this relaxation does not affect the characterization of the steady state distribution itself if the "migration corridor" is narrow in the sense that it allows only one individual to migrate per period, and the most "rank deprived" individual migrates first, the second most "rank deprived" migrates second, and so on. A steady state distribution will once again be reached with the population divided evenly or evenly but for one individual between the two regions, yet naturally, and differing from Section 2, reaching the steady state will take longer, with the number of periods depending on whether *n* is even or odd.

Claim 2: Assume that only one individual can migrate per period, such that the most "rank deprived" individual migrates first. Then, a steady state will be reached with the population divided evenly or evenly but for one individual between the two regions. If *n* is even, it will take $n^2/4$ periods to reach the steady state, and if *n* is odd, it will take $(n^2-1)/4$ periods.

Proof: The number of periods needed to reach the steady state is the sum of the migration moves of the individuals. Obviously, individual n never moves between region A and region B (so his contribution to the sum total of moves is zero). When they can, individual $n-1$ and individual $n-2$ move once from region A to region B and then they stay there. Individuals $n-3$ and $n-4$ move twice: as soon as they can, in the first time they move to region B and then, after the migration of individuals $n-1$ and $n-2$ to region B, as soon as it is possible for them, they move back to region A. Analogously, we can calculate that each individual $k=1, 2, ..., n-1$ moves exactly $\left\lfloor \frac{n-k+1}{2} \right\rfloor$ times where the operator $\lfloor x \rfloor$ denotes the biggest integer that is not greater than *x*. Consequently, the number of periods it takes to reach the steady state when n is even is

$$0+1+1+2+2+3+3+\ldots+\frac{n-2}{2}+\frac{n-2}{2}+\frac{n}{2}$$
$$=2+4+6+\ldots+(n-2)+\frac{n}{2}=\frac{n}{2}\cdot\frac{n-2}{2}+\frac{n}{2}=\frac{n^2}{4},$$

and when n is odd, it is

$$0+1+1+2+2+3+3+\ldots+\frac{n-1}{2}+\frac{n-1}{2}$$
$$=2+4+6+\ldots+(n-1)=\frac{n+1}{2}\cdot\frac{n-1}{2}=\frac{n^2-1}{4}.$$

Q.E.D.

Because we have assumed that $n\geq 4$, then $\frac{n^2}{4}>\frac{n}{2}$ and $\frac{n^2-1}{4}>\frac{n-1}{2}$, so the time it takes to reach the steady state is indeed longer under a narrow "migration corridor" than under no constraint on the "width" of the "migration corridor."

As another generalization, we consider an income distribution in which several individuals have the same income. In this case, all the individuals who have the same income occupy the same rank, and all the individuals who occupy a given rank behave identically. Then, the results obtained in Section 2 do not depend on the constellation of having one individual of each income. (As in the baseline model, there are no constraints on the "width" of the "migration corridor.")

Claim 3: Let there be *l* individuals of each income, where *l* is a natural number other than zero. Population size is then $n\cdot l$. Akin to Section 2, a steady state spatial distribution will be reached such that the population will be distributed between the two regions equally if *n* is even, or equally but *l* if *n* is odd.

Proof: The proof follows straightforwardly upon multiplication of the number of occurrences of each income value in the income hierarchy by *l*. Then, *l* individuals with income *n* will stay in region A, *l* individuals with income $n-1$ and *l* individuals with income $n-2$ will migrate to and stay in region B, and so on. Q.E.D.

The assumption that the individuals base their migration decisions on the observed current state without forming expectations as to how other individuals will behave simultaneously can also be relaxed. Then, in just one period, a steady state distribution of the population between region A and region B will be reached such that half or half but for one of the members of the population will be in each of the two regions. (Once again we make the assumptions of the baseline model and refer to a population consisting of *n* individuals.) We thus have the following claim.

Claim 4: If the individuals are rational and far-sighted, they will be distributed equally or equally but for one individual between the two regions, and this distribution will be reached in just one period.

Proof: Individual *n* (the richest individual) will stay in region A. Individual $n-1$ will move to region B and stay there. Knowing that, individual $n-2$ will stay in region A. By the same logic, aware of the location choices of individuals *n*, $n-1$, and $n-2$, individual $n-3$ will move to region B and stay there, and so on. Q.E.D.

Relaxation of the assumption that migration is cost-free need not interfere with the result reported in Claim 4.

As a constellation of a strictly positive cost of migration when the individuals are assumed to be rational and far-sighted, we assume that each migration move requires a cost $0 < c < x_1$, and that this cost is the same for everyone. (Additional assumptions concerning *c* follow.) An individual will migrate if the value of the rank improvement that he will obtain upon migration is higher than the cost of migration. We assume that the individual's valuation of the rank improvement is equal to the rank gain itself, namely that the value of the gain of one rank is 1, the value of the gain of two ranks is 2, and so on. Thus, and as an example, if migration confers a gain of two ranks, and if the cost of migration is less than 2, then migration will be undertaken.

Given the assumptions in the preceding paragraph, individual *n* will stay in region A. Individual $n-1$ has the second rank in region A, and will have the first rank upon moving to region B; he will gain an improvement of one rank if he decides to migrate to region B. If $c<1$, this individual will migrate, and if $c>1$, this individual will stay in A. Thus, if $c<1$ (individual *n* is in region A, individual $n-1$ is in region B), the rank of individual $n-2$ cannot be improved upon migration; in both regions he will have the second rank. Therefore, if $c<1$, individual $n-2$ will stay in region A. If $c>1$ (individuals *n* and $n-1$ are in region A), then by moving to region B individual $n-2$ can improve his rank - from the third in region A to the first in region B (a rank improvement of two). If $1<c<2$, this individual will migrate, and if $c > 2$, this individual will stay in A. We next consider the migration choice of individual $n-3$. If $c < 1$ (individuals *n* and $n-2$ are in region A, individual $n-1$ is in region B), individual $n-3$ can improve his rank by moving to region B - from the third in region A to the second in region B. Given that $c < 1$, this individual will migrate. If $1 < c < 2$ (individuals *n* and $n-1$ are in region A, individual $n-2$ is in region B), individual $n-3$ can improve his rank upon migration - from the third in region A to the second in region B. However, given that the cost of migration is higher than the value of the rank improvement ($c > 1$), this individual will not migrate. If $c > 2$ (individuals *n*, $n-1$, and $n-2$ are in region A), individual $n-3$ can improve his rank upon migration - from the fourth in region A to the first in region B. Thus, if $2 < c < 3$, this individual will migrate, whereas if $c > 3$, this individual will stay in region A. The migration choice of individual $n-4$ again depends on the magnitude of the cost of migration:

for $c < 1$ (individuals *n* and $n-2$ are in region A; individuals $n-1$ and $n-3$ are in region B), individual $n-4$ cannot improve his rank by migration and, thus, he will stay in region A;

for $1 < c < 2$ (individuals *n*, $n-1$, and $n-3$ are in region A; individual $n-2$ is in region B), individual $n-4$ can improve his rank by migration - from the fourth in region A to the second in region B. Because $c < 2$, which implies that the cost of migration is smaller than the value of the rank improvement, this individual will migrate;

for $2 < c < 3$ (individuals *n*, $n-1$, and $n-2$ are in region A; individual $n-3$ is in region B), individual $n-4$ can improve his rank by migration - from the fourth in region A to the second in region B. Because $c > 2$, which implies that the cost of migration is higher than the value of the rank improvement, this individual will stay in region A;

for $c > 3$ (individuals *n*, $n-1$, $n-2$, and $n-3$ are in region A), individual $n-4$ can improve his rank by migration - from the fifth in region A to the first in region B. Thus, if $3 < c < 4$, individual $n-4$ will move to region B, and if $c > 4$, individual $n-4$ will stay in region A.

On the basis of the preceding analysis, the following generalization can be made:

for $c < 1$, every second individual starting from individual $n-1$ downwards (namely moving in the income hierarchy towards the “poorer” individuals) will be in region B, while the other individuals will be in region A;

for $1 < c < 2$, the two "richest" individuals (namely individual *n* and individual $n-1$) will be in region A, and every second individual starting from individual $n-2$ downwards will be in region B, while the other individuals will be in region A;

for $2 < c < 3$, the three "richest" individuals (namely individual *n*, individual $n-1$, and individual $n-2$) will be in region A, and every second individual starting from individual $n-3$ downwards will be in region B, while the other individuals will be in region A;

and so on.

Consequently, for $k-1 < c < k$, k "richest" individuals will be in region A, and every second individual starting from individual $n-k$ downwards will be in region B, while the other individuals will be in region A. It follows then that unless the population is particularly small, or unless the migration cost is extremely high, the obtained steady state spatial distribution will be an approximately even division of the population between the two regions despite relaxation of the assumption of a zero cost of migration.

4. Social welfare

A particularly interesting question to pose is whether, from a social welfare point of view, the aggregate of the individuals' migration responses to experiencing unfavorable income rank yields the optimal outcome. Taking a utilitarian stance, in the current setting where incomes are held constant, we naturally equate the maximal social welfare with the minimal sum of ranks. In constructing the social welfare function, we assign equal weights to the utilities of all the individuals. In turn, we assume that the individuals derive utility from income, and disutility from low rank. Given that the individuals' incomes are held constant, social welfare is maximized when the sum of the individuals' rank positions is minimized. We consider again the baseline setting in which each income level is held by one individual.

Definition 1: Social welfare under rank preferences is the negative of the sum of the ranks of the individuals.

For example, when two individuals with different incomes are in region A, then the sum of the first rank of one of them and of the second rank of the other is $1+2=3$ rank positions; when the lower-ranked individual moves to empty region B, then the sum is $1+1=2$ rank positions. Social welfare in the latter case, at -2, is higher than social welfare in the former case, at -3.

In general, when individuals pay no heed to the consequences of their actions for the wellbeing of others, we would not expect the aggregate of their behaviors to yield the social optimum. Not so in the current case, however. To see this, we put ourselves in the shoes of a social planner who seeks to distribute the individuals between the two regions in such a manner as to obtain the smallest sum of ranks.

Claim 5: The objective of bringing social welfare under ordinal ranking to a maximum is achieved upon any allocation of the individuals in which they are distributed evenly (or evenly but for one) between the two regions.

Proof: The proof is by contradiction.

Assume that there are *n* individuals, where *n* is a natural number. Without loss of generality, we refer to a case of an even *n*. An analogous procedure to the one presented below can be conducted for an odd *n*, yielding qualitatively the same outcome.

Assume then that the individuals are distributed evenly between region A and region B. Then, social welfare, measured as per Definition 1, is

$$-2\cdot(1+2+...+n/2)=-2\cdot\frac{n}{2}\cdot\frac{1+n/2}{2}=-\frac{n(n+2)}{4}=-\frac{n^2+2n}{4}.$$

We let *k* individuals, where $k=1, 2, ..., (n/2)-1$, change their location. This means that there will be $(n/2)+k$ individuals in one region, and $(n/2)-k$ in the other region. Consequently, social welfare will be

$$\begin{aligned}&-\{[1+2+...+(n/2+k)]+[1+2+...+(n/2-k)]\}\\&=-\left[(n/2+k)\cdot\frac{1+(n/2+k)}{2}+(n/2-k)\cdot\frac{1+(n/2-k)}{2}\right]\\&=-\left(\frac{n^2}{4}+\frac{n}{2}+k^2\right)<-\left(\frac{n^2}{4}+\frac{n}{2}\right),\end{aligned}$$

where the inequality means that following this change in the distribution of the individuals between the two regions, social welfare is lowered. Q.E.D.

This result is intriguing because although the individuals act of their own accord, they achieve the *socially* preferred outcome, as they are distributed equally or equally but for one individual between two regions. This socially optimal distribution (of an even or of an even but for one divide) is reached in each of the steady state outcomes reported in the settings of Claims 1-4.

Will the result that the aggregate of the individuals' behaviors yields the socially optimal outcome be obtained if the distress arising from falling behind others in the income hierarchy is measured in a cardinal manner rather than in a rank-based manner? The answer is no. In order to present the answer in a neat manner, we consider a specific distribution of the incomes under which the socially optimal outcome is analytically tractable.

Let the constellation of incomes of a population of n individuals be given by $x_1<x_2<\ldots<x_n$. We express the distress that arises from falling behind others in the income hierarchy by means of the following cardinal measure of relative deprivation, where RD_i denotes the relative deprivation of individual i.

Definition 2: $RD_i \equiv \frac{1}{n}\sum_{k=i+1}^{n}\left(x_k - x_i\right)$ for $i = 1,\ 2,\ ...,\ n-1$

$$RD_n \equiv 0.$$

A rationale, background, and applications of this measure are provided in Stark (2013). Under the constellation of incomes in which $x_i = i$ (namely the income of individual *i* is *i*), we obtain the following simpler formulation of RD_i.

Definition 3: $RD_i \equiv \frac{1}{n}\sum_{k=i+1}^{n}\left(k - i\right)$ for $i = 1,\ 2,\ ...,\ n-1$

$$RD_n \equiv 0.$$

As before, and to begin with, in period zero the *n* individuals are in region A. In the subsequent period all the individuals who experience relative deprivation and believe that they will experience none upon migrating to region B move to region B. Namely:

A	B
n	
	$n-1$
	$n-2$
	$\vdots$
	2
	1

Figure 3. The steady state spatial distribution of the population under cardinal preferences.

Claim 6: Under cardinally-measured distaste for low relative income, the division in which *n* is in region A and the remainder of the population is in region B constitutes the spatial steady state distribution.

Proof: We consider individual k, where $k = 1,\ 2,\ ...,\ n-1$, who in period 1 weighs whether to stay in region B or whether to move back to region A. We refer to individual *k*'s relative deprivation as $RD_{k|k\in B}$ when he is in region B, and as $RD_{k|k\in A}$ when he is in region A. We have that

$$RD_{k|k\in B} = \frac{1}{n-1}\left[(k+1)-k+(k+2)-k+...+(n-1)-k\right] = \frac{n-k}{2}\cdot\frac{n-k-1}{n-1},$$

and that

$$RD_{k|k\in A} = \frac{n-k}{2}.$$

Because $\frac{n-k-1}{n-1} < 1$, individual k will prefer to stay in region B. And because this holds for any $k = 1, 2, ..., n-1$, none of the $n-1$ individuals in region B will have an incentive to migrate back to region A, nor will individual *n* who is not relatively deprived have an incentive to migrate and, thus, the observed state, as depicted in Figure 3, is the spatial steady state. Q.E.D.

Comment. The same result as the one reported in Claim 6, namely the result that the division in which *n* is in region A and the remainder of the population is in region B constitutes the spatial steady state distribution, will be obtained when cardinal relative deprivation is measured as the distance from below the mean income, namely when Definition 2 is replaced by $RD_i \equiv \max\{\bar{x} - x_i, 0\}$ where $\bar{x}$ is the average income in the region in which individual *i* is located, and Definition 3 is replaced by $RD_i \equiv \max\{\bar{x} - i, 0\}$. The proof is in the Appendix.

In the utilitarian setting, this steady state distribution with individual *n* in region A, and all the other individuals in region B is not, however, the socially optimal outcome. The distribution that confers the highest level of social welfare requires the aggregate of the individuals' levels of relative deprivation to be minimized, assuming, again, that in constructing the social welfare function an equal weight is assigned to each of the individuals.

Definition 4: Social welfare under cardinal ranking is the negative of the sum of the levels of relative deprivation of the individuals.

Claim 7: Social welfare under cardinal ranking is maximized when individuals $1, \ldots, i-2, i-1$ are in one region, and individuals $i, \ldots, n-1, n$ are in the other region, where $i=(n/2)+1$ if *n* is even, or where $i=(n+1)/2$ or $i=(n+3)/2$ if *n* is odd.

Proof: Because the proof is tedious, it is relegated to the Appendix.

As a consequence, when position in the income hierarchy is measured cardinally, there is a role for the social planner to interfere in order to achieve the socially preferred outcome.

A reason for the difference between the nature of the social welfare outcomes obtained under the two types of preference seems to be that under rank preferences, the social optimum is much easier to obtain in the sense that only the numbers of the individuals in each of the regions matter, not the distributions of their incomes and, thus, there are many social optima, so achieving any is relatively easy. Under a cardinal measure of relative deprivation, however, there is only one global optimum and, thus, it is far more difficult to achieve that optimum without synchronizing the actions of the individuals. It is worth adding that the socially optimal outcome under "cardinal ranking" is also the socially optimal outcome under "ordinal ranking," but only one out of the many socially optimal outcomes under "ordinal ranking" constitutes the socially optimal outcome under "cardinal ranking."

Clearly, the socially preferred outcome under "cardinal ranking" is not a steady state spatial distribution (for example, using the notation of Claim 7, individual *i* has an incentive to move to the other region in order to decrease his relative deprivation). Therefore, the role of the social planner is not only to help achieve the welfare maximizing spatial configuration, but also to shield it from individual actions that are likely to go against the social interest.

5. Complementary considerations

5.1 An alternative measure of income rank: a negative result

The rank measure that we have used throughout is "pure" in the sense that being placed second, say, in the income hierarchy is second, regardless of the size of the population. Consider rank as if it were a claim for a prize, and suppose that there are two prizes. Then, if an individual occupies the second rank he is awarded a prize, if he occupies a lower rank he is not awarded a prize. In terms of getting or not getting a prize, it is immaterial to the individual whether the field has, say, $n = 3$ or $n > 3$ individuals.

Formally, if rank is made dependent on the size of the population such that holding a higher rank is preferable to a lower rank *and* having a *given* rank in a larger population is preferable to having the same rank in a smaller population, then the analysis presented in this paper will collapse; a steady state will not be reached. To see that, we take again the case of individuals whose incomes are $x_1 < x_2 < ... < x_n$, and we set *n* at 4. To begin with, let 4, 3, 2, and 1 be in region A. As before, individuals 3, 2, and 1 move to region B. But under the new interpretation of the rank preferences, 4 will want to be in region B (because being the first out of four is preferable to being the first out of one), so all the individuals will then be in region B. Now 3, 2, and 1 will prefer to be in region A, 4 will prefer likewise, and so on. Thus, a steady state will not be reached. The same argument applies even if we net out the individuals who hold a top rank, but endow the other individuals with a preference to hold a given rank when there are more individuals in their region. In such a case, the sequence of moves will be:

A	B
4	
3	
2	
1	

$\rightarrow$

A	B
4	
	3
	2
	1

$\rightarrow$

A	B
4	
	3
	2
1	

$\rightarrow$

A	B
4	
	3
2	
1	

$\rightarrow$

A	B
4	
	3
2	
	1

$\rightarrow$

A	B
4	
	3
	2
	1

$\rightarrow$ …

Figure 4. The sequence of migration moves of a population of four individuals that does not result in a steady state spatial distribution under ordinal preferences.

We see that the process repeats itself ad infinitum and a steady state is not reached.

5.2 A re-interpretation of the cardinal measure of relative deprivation as a distance from a mean

The relative deprivation index presented in (the first part of) Definition 2 can be rewritten in a slightly different form. Multiplying and dividing the index by $n-i$, we obtain:

$$RD_i = \frac{n-i}{n}\left[\frac{1}{n-i}\sum_{k=i+1}^{n}(x_k - x_i)\right] = (n-i)\frac{1}{n}\left(\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_i\right) = (n-i)\left[\frac{1}{n}\left(\tilde{x}_i - x_i\right)\right], \qquad (1)$$

where $\tilde{x}_i \equiv \frac{1}{n-i}\sum_{k=i+1}^{n} x_k$ is the average income of the individuals whose incomes are higher than the income of individual *i* (these are the individuals who are positioned to the right of individual *i*, namely higher up, in the income distribution).

5.3 A re-interpretation of the cardinal measure of relative deprivation as a composite index of a rank impact term and a cardinal impact term

We can think of the most right hand side of (1) in a novel way, viewing RD_i as the product of a pure rank impact term $(n-i)$ and a cardinal impact term $\left[\frac{1}{n}\left(\tilde{x}_i - x_i\right)\right]$. For the case of Definition 3, we can write the two far ends of (1) as:

$$RD_i = (n-i)\left[\frac{1}{n}\left(\tilde{j} - i\right)\right]$$

where $\tilde{j}$ is the average income of the individuals whose incomes are $i+1, i+2, \ldots, n$.

The term $n-i$ expresses the rank distance of individual *i* from the top rank, where "distance" is measured by the number of ranks higher up. This is the measure used in the basic model and in the extensions of the model in Sections 2 and 3. Seen this way, the standard cardinal measure of relative deprivation has a pure rank preferences component imbedded in it, and a cardinal preferences component. This is revealing in the sense that the distress from trailing behind others can be decomposed into the distress from occupying a rank other than the top rank, measured by $n-i$, and the distress arising from a positive magnitude of the income differences between the higher incomes of others and one's own income. An empirical cardinal preferences model of migration will be based on a utility representation that incorporates RD_i as displayed in (1), whereas an empirical ordinal preferences model of migration will be based on a utility representation that incorporates only the $n-i$ term, as if definitionally setting the $\frac{1}{n}\left(\tilde{x}_i - x_i\right)$ part in (1) equal to one. This conceptual differentiation illustrates how the two migration models could be distinguished empirically.

6. Conclusions

There is considerable empirical evidence that, holding other considerations constant, lagging behind others in the income (or wealth) distribution prompts migration. In the received literature, the manner in which this trailing behind is measured is cardinal, and the social consequences of the migration response to this "bad" are typically not traced. Specifically, the existing research does not perform robustness checks on its conclusions by employing other measures that represent distress from falling behind others in the income hierarchy such as, for example, an income-based measure of rank. Thus, there is room, if not a need, for such an analysis. As already noted in the Introduction, models that employ ordinal rank may predict starkly different behavior from models employing cardinal rank (Bilancini and Boncinelli, 2008), and there is no certainty as to which type of measure, ordinal or cardinal, adequately represents people's preferences. An interesting possibility would be to revisit past empirical studies and re-estimate econometric models employing an ordinal measure of relative deprivation instead of the cardinal measure. The use of an ordinal measure, which brings into the regression less information about people's position in the income distribution in relation to others, can lead to results that differ from those reported when relative deprivation is measured cardinally. Suppose that the income gap at origin increases, say from between $2y$ and y to between $3y$ and y, where y is a positive real number; that there are many individuals with each of the two incomes; and that holding all else constant, this increase in the income gap is actually not observed to bring about an increase in the propensity to resort to migration by the low-income individuals. Concluding then that the individuals are not motivated to migrate by relative deprivation considerations, which could be correct if relative deprivation is sensed and measured cardinally, will be erroneous if relative deprivation is rank-based; the increase in the income gap leaves ranks intact and, thus, under rank-based references, we would not expect a change in migration behavior / the incentives to migrate. Put differently, holding the incomes of other individuals constant, a lowered rank for a given individual always implies an increase in the individual's relative deprivation measured cardinally, but the converse is not true, namely an increase in the individual's relative deprivation measured cardinally does not necessarily imply an increase in his rank-based relative deprivation.

Our analysis supplements the received empirical migration inquiries in three ways. It models migration when preferences are rank-based, it works out the resulting spatial steady state distribution, and it shows that the aggregate of the migration decisions of the individuals at the steady state sums up to a distribution that is optimal from a social welfare point of view.

Of course a setting such as the one presented here, in which distaste for low rank motivates behavior that leads to the social optimum is not enough to explain why preferences for higher rank that have conveyed evolutionary advantages millennia back in time are still with us. But it adds a reason, if we maintain that higher social welfare and evolutionary edge are positively correlated.

Appendix

Proof of the result reported in Claim 6 when cardinal relative deprivation is measured as distance from below the mean

To begin with, in period zero the *n* individuals are in region A. In the subsequent period, all the individuals who are relatively deprived - in this case, the individuals whose incomes are lower than the average income in region A - will migrate to region B, while the other individuals will remain in region A. Thus, individuals $n, n-1, ..., m$ where $m = \frac{n}{2} + 1$ if *n* is even, and individuals $n, n-1, ..., m$ where $m = \frac{n+1}{2}$ if *n* is odd, will remain in region A, whereas individuals $m-1, ..., 2, 1$ will migrate to region B. But now the average income in region A becomes higher, so in the subsequent period the individuals whose income is below the average income of those remaining in region A become relatively deprived and they will, thus, be better off migrating to region B. This process will continue until only individual *n* remains in region A.

We note that none of the individuals who have migrated to region B will find it attractive to return to region A even after the subsequent arrivals in region B of the higher income individuals. Thus, again, a spatial distribution such that individual *n* is in region A while individuals $1, 2, ..., n-1$ are in region B constitutes the steady state spatial distribution. To see this, consider individual k, $k = 1, 2, ..., n-1$. The average income in region B in the "alleged" steady state distribution is $\frac{n}{2}$, and this is lower than $\frac{n+k}{2}$, the average income that individual k will experience if he were to return to region A. Thus, if $k \geq \frac{n}{2}$, then individual k does not have an incentive to migrate back to region A because he is not relatively deprived in region B. And if $k < \frac{n}{2}$, namely if individual *k* is relatively deprived in region B, then his relative deprivation there is $\frac{n}{2} - k$, and this is lower than his relative deprivation will be in region A, which is $\frac{n+k}{2} - k$. Hence, no further migration will occur. Q.E.D.

In this case, reaching the spatial steady state will take $\lfloor \log_2(n-1) \rfloor + 1$ periods, where the symbol $\lfloor x \rfloor$ denotes the biggest integer that is not greater than *x*. For example, when $n = 9$, the number of periods it takes to reach the steady state will be $\lfloor \log_2(9-1) \rfloor + 1 = 3 + 1 = 4$. Equivalently, we have that reaching the spatial steady state will take k periods, where k is an integer such that $2^{k-1} < n \leq 2^k$, so that, for example, when $n = 9$ we have that $k = 4$ because then *n* is between $2^3 = 8$ and $2^4 = 16$. We next show the equivalence of these two formulas, namely the equivalence of k_1 and k_2 when $2^{k_1 - 1} < n \leq 2^{k_1}$ and $k_2 = \lfloor \log_2(n-1) \rfloor + 1$.

Suppose that for $n > 1$, $n \in \mathbb{Z}_+$, the numbers k_1 and k_2 are defined in the following manner:

(1.) k_1 is an integer such that $2^{k_1 - 1} < n \leq 2^{k_1}$;

(2.) $k_2 = \lfloor \log_2(n-1) \rfloor + 1$.

We proceed in five steps.

(i) We restate definition (1.) as follows: k_1 is an integer such that $2^{k_1 - 1} \leq n - 1 < 2^{k_1}$. (This definition is equivalent to definition (1.) because $2^{k_1 - 1}$, n, and 2^{k_1} are all integers, which follows from the assumption that $n > 1$.)

(ii) We look at definition (2.) and denote $x = \log_2(n-1)$, which means that $2^x = n - 1$.

(iii) Because (from (i)) $2^{k_1 - 1} \leq n - 1 = 2^x < 2^{k_1}$, it has to be that $k_1 - 1 \leq x < k_1$.

(iv) By the definition of $\lfloor . \rfloor$, $\lfloor x \rfloor$ is the biggest integer that is not larger than x. Therefore, from (iii) it follows that $\lfloor x \rfloor = k_1 - 1$.

(v) Because $\log_2(n-1) = x$, then it is also the case that $\lfloor \log_2(n-1) \rfloor = \lfloor x \rfloor$.

Thus, from (iv), (v), and the definition of k_2, we have that $k_1 = \lfloor x \rfloor + 1 = \lfloor \log_2(n-1) \rfloor + 1 = k_2$.

Proof of Claim 7

In line with Definition 4, the maximum of social welfare under cardinal ranking is reached when total relative deprivation (*TRD*) is minimized, where *TRD* is the sum of the levels of relative deprivation experienced by the members of a given group, and relative deprivation is specified as in Definition 2. To find the division of a population of *n* individuals between region A and region B that confers the highest social welfare, we proceed in two steps. First, given the size of the groups in the two regions, we show that the minimum *TRD* is reached when high income individuals are in one of the regions, and low income individuals are in the other region. (That is, the minimum is reached when the income of *any* individual who is in one region is higher than the income of *any* individual who is in the other region.) Second, given such a distribution, we show that the minimum *TRD* is reached when half of the individuals (for an even *n*) or half of the individuals but for one (for an odd *n*) are in one region, with the remainder of the individuals in the other region.

Lemma A1: Let *n* be a fixed positive integer. Consider an ordered vector $\left(a_1, a_2, ..., a_n\right)$ where $a_1 < a_2 < ... < a_n$ and the a_i's are positive integers. Let $S(a_1, a_2, ..., a_n) = \sum_{k=1}^{n}\sum_{j=1}^{n}\left|a_k - a_j\right|$. Then $S(a_1, a_2, ..., a_n)$ reaches its minimum if and only if $a_{i+1} = a_i + 1$ for $i = 1,\ 2,\ ...,\ n-1$.

Proof: Because the a_i's are positive integers, for any $k, j = 1, 2, ..., n$, we have that $\left|a_k - a_j\right| \geq \left|k - j\right|$ with the minimal possible value of $\left|a_k - a_j\right|$, namely $\left|k - j\right|$, obtained when $\left(\underset{\text{for } i=1,2,...,n-1}{a_{i+1} = a_i + 1}\right)$. Consequently, the sum $S(a_1, a_2, ..., a_n)$ reaches its minimum if and only if $a_{i+1} = a_i + 1$ for $i = 1,\ 2,\ ...,\ n-1$. Q.E.D.

Corollary A1: Consider an ordered vector of incomes $\left(1, ..., n-1, n\right)$, where *n* is a positive integer. Let the incomes be distributed between the two regions such that an ordered vector of incomes in region A is $(k_1, k_2, ..., k_{n_A})$, and an ordered vector of incomes in region B is $(j_1, j_2, ..., j_{n_B})$, where $n = n_A + n_B$. The total relative deprivation of the population is the sum of the levels of total relative deprivation experienced in each of the two regions, namely $TRD = TRD_A + TRD_B$. Then, if *n*, n_A, and n_B are fixed, *TRD* reaches its minimum if and only if $(j_1, j_2, ..., j_{n_B}) = (1, 2, ..., n_B)$ or $(k_1, k_2, ..., k_{n_A}) = (1, 2, ..., n_A)$; that is, if and only if either

A	B
n $\vdots$ n_B+1	
	n_B $\vdots$ 1

or

A	B
	n $\vdots$ n_A+1
n_A $\vdots$ 1	

Figure A1. The *TRD*-minimizing spatial distribution of the population under cardinal preferences.

Proof: We note that $TRD_A = \dfrac{S(k_1, k_2, ..., k_{n_A})}{2n_A}$, and that $TRD_B = \dfrac{S(j_1, j_2, ..., j_{n_B})}{2n_B}$, where the function $S(\cdot)$ is as defined in Lemma A1. Thus, for fixed n_A and n_B, minimizing TRD_A is equivalent to minimizing $S(k_1, k_2, ..., k_{n_A})$, and minimizing TRD_B is equivalent to minimizing $S(j_1, j_2, ..., j_{n_B})$. We denote the distribution of incomes between region A and region B that minimizes *TRD* by $(k_1^*, k_2^*, ..., k_{n_A}^*)$, $(j_1^*, j_2^*, ..., j_{n_B}^*)$.

Without loss of generality, we assume that $n \in (k_1^*, k_2^*, ..., k_{n_A}^*)$. The proof proceeds by contradiction. Assume that $(k_1^*, k_2^*, ..., k_{n_A}^*) \neq (n_B+1, ..., n)$. This means that $(j_1^*, j_2^*, ..., j_{n_B}^*) \neq (1, ..., n_B)$. Then, by Lemma A1, we have that $S(k_1^*, k_2^*, ..., k_{n_A}^*) > S(n_B+1, ..., n)$, which implies that $TRD_A(k_1^*, k_2^*, ..., k_{n_A}^*) > TRD_A(n_B+1, ..., n)$. By analogy, we obtain that $TRD_B(j_1^*, j_2^*, ..., j_{n_B}^*) > TRD_B(1, ..., n_B)$. Thus, $TRD((k_1^*, k_2^*, ..., k_{n_A}^*), (j_1^*, j_2^*, ..., j_{n_B}^*))$ $> TRD((n_B+1, ..., n), (1, ..., n_B))$, which contradicts the assumption that *TRD* reaches its minimum at $(k_1^*, k_2^*, ..., k_{n_A}^*)$, $(j_1^*, j_2^*, ..., j_{n_B}^*)$. Hence, the minimum is obtained when $(k_1^*, k_2^*, ..., k_{n_A}^*) = (n_B+1, ..., n)$ and $(j_1^*, j_2^*, ..., j_{n_B}^*) = (1, ..., n_B)$.

By a similar reasoning, it can be shown that when $(k_1^*, k_2^*, ..., k_{n_A}^*) \neq (1, ..., n_A)$ and $(j_1^*, j_2^*, ..., j_{n_B}^*) \neq (n_A+1, ..., n)$, the minimum of *TRD* is not reached.

In conclusion, *TRD* reaches its minimum at either of the two configurations exhibited in Figure A1. Q.E.D.

Building on Corollary A1, we now consider the size of the two groups that brings *TRD* to a minimum.

Let $(n,...,i)$ be in region A, and let $(i-1,...,1)$ be in region B. Then,

$$
\begin{aligned}
TRD_A &= RD_n + RD_{n-1} + ... + RD_i \\
&= 0 + \frac{1}{n-i+1} + \frac{1+2}{n-i+1} + \cdots + \frac{1+2+\cdots+n-i}{n-i+1} \\
&= \frac{1+(1+2)+...+(1+2+...+n-i)}{n-i-1} \\
&= \frac{(n-i)(n-i+2)}{6}.
\end{aligned}
$$

We obtain this result as follows. We note that

$$
\begin{aligned}
\sum_{k=1}^{n}(1+2+...+k) &= \sum_{k=1}^{n}\frac{(1+k)k}{2} = \frac{1}{2}\sum_{k=1}^{n}k + \frac{1}{2}\sum_{k=1}^{n}k^2 = \frac{1}{2}\frac{(1+n)n}{2} + \frac{1}{2}\frac{n(n+1)(2n+1)}{6} \\
&= \frac{n(n+1)(n+2)}{6}.
\end{aligned}
$$

Substituting $n-i$ for *n* in the last but one expression of TRD_A yields the last expression of TRD_A. By a similar procedure we obtain that

$$
\begin{aligned}
TRD_B &= RD_{i-1} + RD_{i-2} + ... + RD_1 \\
&= 0 + \frac{1}{i-1} + \frac{1+2}{i-1} + \cdots + \frac{1+2+\cdots+(i-1-1)}{i-1} \\
&= \frac{1+(1+2)+(1+2+...+i-2)}{i-1} \\
&= \frac{i(i-2)}{6}.
\end{aligned}
$$

Therefore, $TRD = TRD_A + TRD_B = (1/6)[(n-i)(n-i+2) + i(i-2)]$. We seek to solve $\min_{1 \le i \le n} TRD$. In order to find the minimum, we temporarily treat *i* as a continuous variable. (This treatment, as to be seen momentarily, does not affect the solution value.) Because $dTRD/di = (1/3)(-n+2i-2)$ and $d^2TRD/di^2 = (2/3) > 0$, we have that the minimum of *TRD* is reached when $dTRD/di = 0$, namely when $-n+2i-2=0$.

Thus, for an even *n*, the *i* that brings *TRD* to a minimum is $i^* = (n/2)+1$, and the corresponding value of *TRD* is $TRD^* = (1/12)(n^2 - 4)$. For an odd *n*, a direct calculation yields that when $i = (n+1)/2$, or when $i = (n+3)/2$, then, in both cases, $TRD = (1/12)(n^2 - 3)$. Therefore, if *n* is an odd number, the *i* that brings *TRD* to a minimum is $i^* = (n+1)/2$ or $i^* = (n+3)/2$. In sum, the sizes of the two groups that bring *TRD* to a minimum are the same or the same but for one. This completes the proof of Claim 7.

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